Macroeconomics 702, Spring 2004, Qualifying Exam:

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In the following there are 19 questions for 120 points. Please answer questions that are worth 100 points. Only the first 100 points worth of possible answers count. Be as BRIEF as you can and good luck. You have 120 minutes.

Growth Models

There is an economy with many identical consumers and infinite time. Consumers have preferences

$$E\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{\left[\phi\left(c_{A,t},c_{B,t}\right)\right]^{1-\sigma}}{1-\sigma}+\alpha(1-n_t)\right]\right\}$$

where $\phi(.,.)$ is some function increasing in its two arguments $c_{A,t}$ is the consumption of apples at time t, $C_{B,t}$ is the consumption of bananas at time t and n_t is the fraction of time worked at time t.

Apples can become capital the next period, while apples do not. The technology to produce both types of fruit is given respectively by

$$z_t^i\ F^i(K_{it},N_{it}),\qquad\qquad i\in\{A,B\}$$

and where K_{it},N_{it}, are capital and labor. Shocks to productivity z^i have finite support, are independent of each other and follow a Markov chain with transition matrices Γ^i. Capital can be costlessly allocated between technologies and depreciates at rate δ.

1. (10 points) Define an Arrow-Debreu competitive equilibrium. Carefully define the commodity space, and the consumption and production possibility sets.

2. (5 points) State the two welfare theorems.

3. (10 points) State sufficient conditions (on the state of the economy, and/or on function ϕ, and or the technologies) so that in some state of the economy no apples are produced.

Suppose now that the household owns capital and rents it to firms.

4. (10 points) Define a recursive competitive equilibrium.

5. (9 points) Give a formula for the relative price of apples and bananas if both are produced. Give another formula when apples are not produced.

Now assume that the government taxes bananas at rate $\tau(z,K)$ and returns the proceeds as per unit leisure subsidies $\theta(z,K)$.

6. (6 points) Write a formula that links the equilibrium values of functions $\tau(z, K)$ and $\theta(z, K)$.

Lucas Trees

Assume there is a representative agent economy. Each agent has endowments given by z_t which follows a Markov chain with transition matrix Γ. In addition, there is a plot of land that can be used to produce. If so, there is a per period cost of α. When used the plot of land produces d_t with transition matrix Λ. The shock is observed before the decision to produce takes place. The agent has standard preferences. Assume that the endowments cannot be sold but the land can.

7. (5 points) Define equilibria recursively assuming that the land prices only depend on the current yield.

8. (5 points) Write a formula for an option to buy land tomorrow at price p_1 and then reselling it at price p_2 the period after.

9. (5 points) Imagine now that if the land is not used, it has to be idle two periods. State a formula for the prices of land in the first period of idleness. Make sure that you write when this happens.

Contracting Frictions

Consider now an economy with many identical consumers each of them with one efficient unit of labor each period that if used for fishing produces s_t units of the good (s_t is Markov with transition Γ). Preferences are given by $E\left\{\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\sigma}}{1-\sigma}\right\}$ where $\beta < 1$.

Each agent has access, in every period, to a storage technology that allows to store the good and make it available in the next period at zero cost. Assume there are no credit markets.

10. (5 points) Write the problem of the agent.

11. (5 points) Is there a stationary distribution? Briefly explain.

12. (5 points) Imagine a policy that taxes 10% of the produce and is distributed to those with the lowest s. Describe the problem that you would solve to see if you recommend such a policy (notice that you are not asked what is the best policy).

    Imagine now that there is a new agent, the Manzicster, in this economy that can commit to its word. Imagine that this agent proposes "take-it-or-leave-it offers" to all other agents. These other agents, though, can abandon the arrangement at will."

13. (8 points) Explain how does the Manzicster calculate what offers to make (which problems does he solve).

14. (7 points) Characterize as much as you can the properties of the offer made to the agent in the worst possible shape.

Industry Equilibria

Imagine an industry with demand given by $p = p(Y)$, where Y is the total level of output. Imagine that there are many firms in that industry, each indexed by its technology level s, that, how else, follows a Markov chain with transition matrix Γ and for new firms is drawn from cdf F. Imagine that capital can be rented at rate r, there is no depreciation, and that labor can be hired at rate w. Output of a firm is given by $s\ f(k, n)$, where k is capital, n is labor and f is a decreasing returns to scale production function. Imagine that the government charges 10 units of the numeraire to each firm each period for renewal of their license (a requisite for operation), 20 units for a first time license that is given before the first s is drawn. In addition, the government levies a 16% excise tax.

15. (5 points) Define a stationary equilibrium with free entry.

16. (5 points) Give a formula for the value of a firm with shock $\bar{s}$.

17. (5 points) Imagine now an aggregate multiplicative iid shock $z \in Z$ to the demand of the industry. Define as briefly as you can a recursive competitive equilibria.

Monopolistic Competition

Imagine that preferences of a representative consumer in a static world are given by

$$u(\{c(i)\}_{i\in[0,A]}) = \left(\int_0^A c(i)^\gamma\ di\right)^{1/\gamma}$$

Moreover, imagine that there each good $i \in [0, a]$ is produced by a firm that has a monopoly over its production. Output of these firms are produced with zero costs.

18. (5 points) Give an expression for the price that each firm charges, when the income of the consumer is 1.

19. (5 points) What would be the price when representative consumers own the firm? This requires the computation of the consumer's wealth.