Problem Set 10
Suggested Solutions

Solution 1

Remembering the original problem,

$$c(V) = \min_{c,a,V^u} c + [1 - p(a)]\beta c(V^u)$$

subject to

$$\begin{aligned} V &= u(c) - a + \beta\left[p(a)V^E + (1 - p(a))V^u\right] && (1) \\ 1 &= [p'(a)\beta]\left[V^E - V^u\right] && (2) \end{aligned}$$

1. Let V^E be the value of being employed. As suggested in class, this is the solution to usual saving-consumption problem without uncertainty, since the only source of uncertainty is employment realization. The problem has the following budget constraint,

$$c + qz' = z + 1_e w + (1 - 1_e)b$$

where z is the savings, q is the discount rate, b is the unemployment benefit and 1_e is the indicator function for employment.

2. Now the household pays for the insurance once employment with a flat tax rate determined as a function of unemployment history. Now the value of unemployment is,

$$V^E = \frac{U(w - \tau(V))}{1 - \beta}$$

and the objective function of the SP is,

$$c(V) = \min_{c,a,V^u} c + \beta\{[1 - p(a)]c(V^u) - p(a)(\frac{\tau(V)}{1 - \beta})\}$$

and the constraints are same as before.

3. Let the probability that government forgets with probability α. Now the objective function and participation constraint becomes,

$$\begin{aligned} c(V) &= \min_{c,a,V^u} c + [1 - p(a)]\beta\{\alpha c(V^u) + (1 - \alpha)c(V)\} \\ V &= U(c) - a + \beta\{(1 - p(a))[\alpha V^u + (1 - \alpha)V] + p(a)V^e\} \end{aligned}$$

4. The probability of loosing the job is γ. Then the value of employment is,

$$V^e = U(c) + \beta[\gamma V^e + (1 - \gamma)V^u]$$

and basic structure of the problem does not change.

5. Now $\gamma(e)$. Probability of loosing the job depends on the job effort. This is different than the effort a thus does not effect the IC constraint

(Part II) When a is not observable, planner can only choose c and V^u. And households choose a optimally. If given c and V^u, the agent will solve

$$\max_a u(c) - a + \beta \left[p(a) V^E + (1 - p(a)) V^u\right] \tag{3}$$

FOC is

$$\left[p'(a)\beta\right]^{-1} = V^E - V^u \tag{4}$$

This FOC gives an implicit function of a as a function of V^u: $a = g(V^u)$.

Then, the planner solves her cost minimization problem,

$$c(V) = \min_{c,a,V^u} c + [1 - p(a)]\beta c(V^u)$$

subject to

$$V = u(c) - a + \beta \left[p(a) V^E + (1 - p(a)) V^u\right] \tag{5}$$

$$1 = [p'(a)\beta]\left[V^E - V^u\right] \tag{6}$$

Lagragian is

$$c + [1 - p(a)]\beta c(V^u) + \theta \left[V - u(c) + a - \beta \left[p(a) V^E + (1 - p(a)) V^u\right]\right]$$
$$+\eta \left[1 - [p'(a)\beta]\left[V^E - V^u\right]\right]$$

FOC: (c)

$$\theta^{-1} = u_c \tag{7}$$

(a)

$$c(V^u) = \theta \left[\frac{1}{\beta p'(a)} - \left(V^E - V^u\right)\right] - \eta \frac{p''(a)}{p'(a)}\left(V^E - V^u\right) \tag{8}$$

(V^u)

$$c'(V^u) = \theta - \eta \frac{p'(a)}{1 - p(a)} \tag{9}$$

Envelope condition

$$c'(V) = \theta \tag{10}$$

We see the Lagrangian multiplier associated η is positive as long as the insurance is costly otw if $\eta = 0$ (8) would imply $c(V^u) = 0$ ($a > 0 \Rightarrow \frac{1}{\beta p'(a)} = \left(V^E - V^u\right)$), which means that the constraint is binding. So,

$$\eta \frac{p'(a)}{1 - p(a)} > 0$$

Therefore, we have

$$c'(V^u) < c'(V) \Rightarrow V^u < V$$

from the strict convexity of $c(.)$. The delayed promised utility decreases over time.

Let $\theta^u = \theta - \eta \frac{p'(a)}{1-p(a)}$, then $\theta^u < \theta$, which also implies θ is decreasing over time and thus so is consumption since

$$\theta^{-1} = u_c$$

And from

$$[p'(a)\beta]^{-1} = V^E - V^u$$

we know the effort level is decreasing over time. The reason is that V^u decreases over time, thus RHS increase over time and $p(.)$ is strictly concave, therefore, a_t is increasing over time.

Solution 2

Note that the maximimum possible utility ever to be promised is promised when the lack of commitment agent gets the highest realization of the endowment shock, and after that she is always promised this level of utility and given the associated level of consumption since her participation constraint never binds again. We also know the efficient way to deliver this level of utility by the commited agent is to provide it through a constant stream of consumption. Then the we know the highest possible promised utility $\overline{w}_s$ and associated consumption level $\overline{c}_s$ must satisfy,

$$U(\overline{c}_s) + \beta \overline{w}_s = U(\overline{y}_s) + \beta V_{aut} \quad (11)$$

$$\overline{w}_s = \frac{U(\overline{c}_s)}{1-\beta} \quad (12)$$

where $\overline{y}_s$ is the highest realization of endowment shock. We also know,

$$V_{aut} = \sum_s \Pi_s U(y_s) + \beta V_{aut} \quad \Rightarrow V_{aut} = \frac{\sum_s \Pi_s U(y_s)}{1-\beta} \quad (13)$$

and these three equations solve for $\overline{w}_s, \overline{c}_s, V_{aut}$.

Solution 3

This is a result due to strict concavity of the utility function. One can think of it as a cost minimization or its dual utility maximization. We know given total amount of resources and equal prices, the utility maximazing consumption stream is a constant one and the dual argument is given a fixed amount of promised utility, the cheapest way to provide it is through constant consumption.