Problem Set 11
Suggested Solutions
Econ 702, Spring 2005

Solution 1

$P(.)$ is positive for some V. Let the promised value of always giving the agent y_1 (lowest possible endowment). This is calculated by the following:

$$V_1 = \sum_{t=0}^{\infty} \beta^t u(y_1)$$

Then the value of the moneylender who promised V_1 is:

$$P(V_1) = \sum_t \beta^t \sum_s \Pi_s (y_s - y_1)$$

This is trivially strictly positive (as long as there is a positive probability of realization of $y_s > y_1$).

On the other hand, $P(.)$ is negative for some V. Let the promised value of always giving the agent y_S (highest possible endowment). This is calculated by the following:

$$V_S = \sum_{t=0}^{\infty} \beta^t u(y_S)$$

Then the value of the moneylender who promised V_S is:

$$P(V_S) = \sum_t \beta^t \sum_s \Pi_s (y_s - y_S)$$

This is trivially strictly positive (as long as there is a positive probability of realization of $y_s < y_S$).

Solution 2

We know

$$V^A = \frac{\sum_s \Pi_s u(y_s)}{1-\beta}$$

Thus,

$$V_{AM} = \frac{u\left(\overline{c_S}\right)}{1-\beta} = u\left(y_S\right) + \beta \frac{\sum_s \Pi_s u(y_s)}{1-\beta}$$

Because y_S is the best possible endowment shock,

$$\begin{aligned} \frac{u\left(\overline{c_S}\right)}{1-\beta} &= u\left(y_S\right) + \beta \frac{\sum_s \Pi_s u(y_s)}{1-\beta} \\ &< u\left(y_S\right) + \beta \frac{\sum_s \Pi_s u(y_S)}{1-\beta} \\ &= \frac{u(y_S)}{1-\beta} \end{aligned}$$

Therefore, we know

$$\overline{c_S} < y_S$$

Solution 3

Note that a complete proof of this requires more than what we cover below but to have the basic idea following should suffice. The recursive SPP problem is,

$$P(V) = \max_{\{c_s,\omega_s\}_{s=1}^S} \sum_s \Pi_s[(y_s - c_s) + \beta P(\omega_s)] \tag{1}$$

subject to

$$u(c_s) + \beta\omega_s \geq u(y_s) + \beta V^A \quad \forall s \tag{2}$$

$$u(Y_s - c_s) + \beta P(\omega_s) \geq u(Y_s - y_s^1) + \beta V^A \quad \forall s \tag{3}$$

$$\sum_s \Pi_s[u(c_s) + \beta\omega_s] \geq V \tag{4}$$

where $Y_s = y_s^1 + y_s^2$. Note that $P(.)$ here traces out the constrained Pareto frontier in this problem, i.e. the set of sustainable allocations. Sustainability here refers to the allocations being Subgame Perfect. So given ω_s the value promised to the other agent is $P(\omega_s)$. Note that the autarky is the worst possible SPE (requires proof but it is true), i.e. worst punishment for anyone violating a contract and thus participation constraints have the value of autarky as the promised value of walking out of a contract. We know then, $\omega_s \in [V^A, V_{\max}]$ for some $V_{\max}$. Now suppose first PC binds, than we know $c_s \leq y_s^1$ since $\omega_s \geq V^A$, if second PC binds as well we have $c_s \geq y_s^1$ since we know $P(\omega_s) \geq V^A$. Then if both PCs bind we have $c_s = y_s^1$ and $\omega_s = V^A = P(\omega_s) \Rightarrow V^A = P(V^A)$ which cannot be true as long as there exist some SP allocation that is not autarky, which can be shown to be the case.