Econ 702, Spring 2005

Problem Set 6 - Suggested Answers

Problem 1

An industry equilibrium is a set $\{y^*,\ p^*,\ x^*(s),\ n^*(s)\}$, such that

1) $p^* = p^*(y)$, output is consistent with the market demand.

2) Total output is given by $y^* = \int_S sf(n^*(s))\ dx^*(s)$

3) $n^*(s) = n(s, p^*)$, such that $f'(n^*(s)) = \frac{w}{p\ s}$, and

4) $c_e = \int_S \Pi(s)\ dx^*(s)$, where $\Pi(s)$ is given by

$\Pi(s) = \sum_{t=1}^{\infty} \left(\frac{1-\delta}{1+\ r}\right)^t [p\ s\ f(n^*(s)) - w\ n^*(s)]$.

Note that in this setting there is no non-trivial choice of the firm. The only possibility that a firm leaves the market is given by the exogenous death rate, δ. This means that the distribution $x^*(s)$ is automatically deduced from the initial distribution $\gamma(s)$. In other words the distribution $x^*(s)$ in the above formulas can be replaced by $\gamma(s)$.

Problem 2

In this problem we will need the notion of a transition function. The probability measure of a firm in current state $s \in S = \{s^1, s^2, ..., s^M\}$, ending up in the subset $B \in \mathcal{S}$, is given by

$$Q(s, B) = \sum_{s'} \Gamma_{s\ s'}\{s' \in B\}.$$

So

$$x_1(B) = \sum_s \gamma(s) Q(s,B) = \sum_s \gamma(s) \sum_{s'} \Gamma_{s\ s'}\{s' \in B\}.$$

For a 2-year old firm

$$x_2(B) = \sum_s x_1(s) Q(s,B) = \sum_{i=1}^{M} x_1\left(s^i\right) \sum_{j=1'}^{M} \Gamma_{i\ j}\{s^{\ j} \in B\} =$$

$$= \sum_{k=1}^{M} \gamma(s^k) \sum_{i=1}^{M} \Gamma_{k\ i} \sum_{j=1'}^{M} \Gamma_{i\ j}\{s^{\ j} \in B\}$$

For a 3-year old firm

$$x_3(B) = \sum_s x_2(s) Q(s,B) = \sum_{i=1}^{M} x_2\left(s^i\right) \sum_{j=1'}^{M} \Gamma_{i\ j}\{s^{\ j} \in B\} =$$

$$= \sum_{m=1}^{M} \gamma(s^m) \sum_{k=1}^{M} \Gamma_{m\ k} \sum_{i=1}^{M} \Gamma_{k\ i} \sum_{j=1'}^{M} \Gamma_{i\ j}\{s^{\ j} \in B\}$$

and so on.

Problem 3

We know that in this industry (and under the assumption of First Order Stochastic Dominance for the Markovian process), the choice of the firm will be of the form :$\{stay\ if\ s \geq s^*,\ quit\ otherwise\}$.

We need to define a stationary distribution for this industry. In equilibrium the number of firms quiting will be equal to the number of new entrants, and so

$$x^*(B) = \int_S \left[\int_{s^*}^{\bar{s}} \Gamma_{s\ s'} dx^*(s) \right] \{s' \in B\}\, ds' + \left(\int_{\underline{s}}^{s^*} dx^*(s) \right) \int_S \{s' \in B\}\, d\gamma(s'),$$

where $\int_{\underline{s}}^{\bar{s}^*} dx^*(s)$ is just the number of firms that go out of business.

Moreover, note that there is a 1-1 relation between output and productivity shock:

By the FOC of the firm,

$$f'(n(s)) = \frac{w}{p\ s} \Rightarrow\ n(s) = (f')^{-1}\left(\frac{w}{p\ s}\right)$$

and if f is strictly concave (which we assume here) $f\ '$is strictly decreasing and so is $(f\ ')^{-1}$.

So $\frac{d\ f}{d\ s} = \frac{d\ f}{d\ n}\ \frac{d\ n}{d\ s} > 0$.

This means that the firm that draws the 99 % highest shock is also the one that produces the 99 % highest output.

Hence, the critical shock, $\hat{s}$, that we are interested in, is given by

$$\int_{s^*}^{\hat{s}} dx^*(s) = \ 0.99.$$

The output produced by the 1% largest firms is given by

$$\int_{\hat{s}}^{\bar{s}} s\ f(n(s))\, dx^*(s)$$

The ratio of the output of the 1% largest firm over total output is

$$\frac{\int_{\hat{s}}^{\bar{s}} s\ f(n(s))\, dx^*(s)}{\int_{s^*}^{\bar{s}} s\ f(n(s))\, dx^*(s)}.$$

Problem 4

In this problem the only difference is that the firm first observes the value of the initial shock and then decides if it's going to operate or not. Again, with First Order Stochastiv Dominacne the decision rule is of the form $\{stay\ if\ s \geq s^*,\ quit\ otherwise\}$.

But in the previous model the firm has to pay the fixed cost for at least one period (the first period), which is not true here. (If the firm draws a really bad shock it has the right to walk away).

The profit function is given by

$$\Pi(s) = \max\left\{0,\ \max_n \left[p\ s\ f(n) - w\ n - \ c_0\ + \frac{1}{1+\ r}\ \sum_{s'} \Gamma_{s\ s'} \Pi(s')\right]\right\}.$$

An industry equilibrium is a set $\{y^*,\ p^*,\ x^*(s),\ \ n^*(s)\}$, such that

1) $p^* =\ p^*(y)$, output is consistent with the market demand.

2) Total output is given by $y^* = \int_{s^*}^{\bar{s}} sf(n^*(s))\ dx^*(s)$

3) $n^*(s) =\ n(s, p^*)$, such that $f'(n^*(s)) = \frac{w}{p\ s}$, and

4) $c_0 = \int \pi(s)\gamma(s)$.

Problem 5

Note that in this problem the labor force of the last period is a state variable for the firm. This means that the state space will also be different. The new state space is given by $X = S \times N$. N is the set of the possible values of labor force. For convenience assume that it is bounded, i.e, $N = \left[0, \bar{N}\right]$, where $\bar{N} < \infty$.

Moreover, maintaining the assumption of FOSD of the Markovian process, the choice of the firm will be characterized by a threshold, s^*. However, here this critical point will not be the same for all firms. It will depend on the other state variable n^{-1}. That is, each firm has its own $s^*\left(n^{-1}\right)$, which depends on the last periods number of workers for that particular firm.

Assuming that there is a cost of firing equal to a per worker, the profit function is given by

$$\Pi\left(s,n^{-1}\right)=$$

$$\max\left\{-a\ n^{-1},\ \max_{n}\left[p\ s\ f\left(n\right)-w\ n-\ a\left(n^{-1}-n\right)\ \{n^{-1}>n\}\ +\tfrac{1}{1+\ r}\ \sum_{s'}\Gamma_{s\ s'}\Pi\left(s',n\right)\right]\right\}.$$

In equilibrium the number of firms that quit will be equal to the number of firms entering the industry. So the formula for the stationary distribution is

$$x^{*}\left(B\right)=\int_{S}\int_{N}\left[\int_{s^{*}\left(n^{-1}\right)}^{\bar{s}}\int_{N}\Gamma_{s\ s'}dx^{*}\left(s,n^{-1}\right)\right]\ \{\left(s',n\left(s,n^{-1}\right)\right)\in B\}\ ds'dn+$$

$$+\left(\int_{\underline{s}}^{s^{*}\left(n^{-1}\right)}\int_{N}dx^{*}\left(s,n^{-1}\right)\right)\int_{S}\{\left(s',n\left(s,0\right)\right)\in B\}\ d\gamma\left(s'\right),$$

where

$B\in\mathcal{X}$, the set of subsets of the (new) state space,

$\int_{\underline{s}}^{s^{*}\left(n^{-1}\right)}\int_{N}dx^{*}\left(s,n^{-1}\right)$ is the number of firms that quit (or- in equilibrium- enter the market),

and $\int_{S}\{\left(s',n\left(s,0\right)\right)\in B\}\ d\gamma\left(s'\right)$ is the probability measure that a new firm will end up in B (that's why $n^{-1}=0$)