Frisch Elasticity of Labor Supply

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Introduction

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- Frisch labor Supply. Hold the marginal utility of wealth constant.

Frisch Elasticity

The Frisch labor supply Elasticity (constant marginal utility of wealth) is defined as

$$\eta^{\lambda} = \frac{\partial n}{\partial w}\frac{w}{n}\|_{\lambda} \tag{1}$$

Simple Standard Consumer Problem

$$\max_{c_t, a_{t+1}, n} \sum_{t=0}^{\infty} \beta^t U(c_t, n_t) \tag{2}$$

subject to:

$$c_t + a_{t+1} = (1+r)a_t + w_t n_t \qquad \text{multiplier } \lambda_t \tag{3}$$

- β is the discount factor, a_t is the household asset in the beginning of period t, and n_t, units of labor.

FOC's

The FOC's are:

$$U_{c_t} = \lambda_t \tag{4}$$

$$-U_{n_t} = \lambda_t w_t \tag{5}$$

$$\lambda_t = \beta(1+r)\lambda_{t+1} \tag{6}$$

FOC's

Notice (4) and (5) are in function of λ and w:

$$\frac{\partial U(c(\lambda, w), n(\lambda, w))}{\partial c} = \lambda \tag{7}$$

$$\frac{\partial U(c(\lambda, w), n(\lambda, w))}{\partial n} = -\lambda w \tag{8}$$

Computing Frisch Elasticity

Taking derivative of (4) and (5) with respect to w, we have

$$U_{cc}\frac{\partial c}{\partial w} + U_{cn}\frac{\partial n}{\partial w} = 0 \tag{9}$$

$$U_{nc}\frac{\partial c}{\partial w} + U_{nn}\frac{\partial n}{\partial w} = -\lambda \tag{10}$$

Notice this is a system of 2 equations and two unknowns.

Computing Frisch Elasticity

Solving for $\frac{\partial n}{\partial w}$ we have:

$$\frac{\partial n}{\partial w} = \frac{\lambda U_{cc}}{[U_{cn}^2 - U_{nn}U_{cc}]} \tag{11}$$

Replacing λ from (5) we have

$$\frac{\partial n}{\partial w} = \frac{-\frac{U_n}{w}U_{cc}}{[U_{cn}^2 - U_{nn}U_{cc}]} \tag{12}$$

Computing Frisch Elasticity

Using the last expression we have that the Frisch labor supply elasticity is

$$\eta^{\lambda} = \frac{U_n}{n[U_{nn} - \frac{U_{cn}^2}{U_{cc}}]} \tag{13}$$

Example: Separable Utility

Consider the following utility function

$$U(c,n) = ln(c) - \alpha \frac{n^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \tag{14}$$

FOC's:

$$\frac{1}{c} = \lambda \tag{15}$$

$$\alpha n^{\frac{1}{\nu}} = \lambda w \tag{16}$$

Example: Separable Utility

The labor supply is given by

$$n = (\frac{w}{\alpha c})^{\nu} \tag{17}$$

The Fischer elasticity is given by

$$\eta^{\lambda} = \nu \tag{18}$$

Example: Non Separable Utility

Consider the another utility function

$$U(c, n) = \frac{(c^{\gamma}(1-n)^{1-\gamma})^{(1-\sigma)}}{1-\sigma} \tag{19}$$

The Frisch Elasticity is given by

$$\eta^{\lambda} = \frac{1-n}{n}[\frac{1-\gamma(1-\sigma)}{\sigma}] \tag{20}$$