Life Cycle Problem with Learning by Doing and by not Doing

Tayyar Buyukbasaran

University of Minnesota

April 2010

Model

- We add human capital and a learning process to the life cycle problem

$$\max_{c_i, n_i, h_{i+1}, a_{i+1}} \sum_{i=1}^{I} \beta^{i-1} U(c_i)$$

$$s.t.$$

$$c_i + a_{i+1} = Ra_i + wE_i h_i n_i \qquad [\lambda_i]$$

$$h_{i+1} = \Psi(h_i, n_i) \qquad [\mu_i]$$

$$0 \leq n_i \leq 1 \qquad [\eta_i]$$

$$a_1 = a_{I+1} = 0, \quad h_1 = 1$$

Learning by Not Doing

- Learning by schooling or leisure reading,
  $\Psi(h_i, n_i) = (1-\delta)h_i + (1-n_i)^\alpha$
- For convex constraint set $0 < \alpha \leq 1$, I pick $\alpha = 0.2$
- Solution methodology:
  - Learning has benefit in ALL future periods, iterating FOC of human capital: $\mu_i = \sum_{j=i+1}^{I} \beta^j (1-\delta)^{j-i} U_c E_j w_j n_j$
  - So backward induction method is better: estimate c_I, h_{I+1}, iterate using FOCs $\Rightarrow a_1(c_I, h_{I+1})$ and $h_1(c_I, h_{I+1})$
  - Initial and terminal conditions: $a_1 = a_{I+1} = 0, h_1 = 1, \mu_I = 0$

Learning by Not Doing

- Aim: c_I^* and h_{I+1}^* s.t. $a_1(c_I^*, h_{I+1}^*) = 0$ and $h_1(c_I^*, h_{I+1}^*) = 1$.
  No closed form solution.

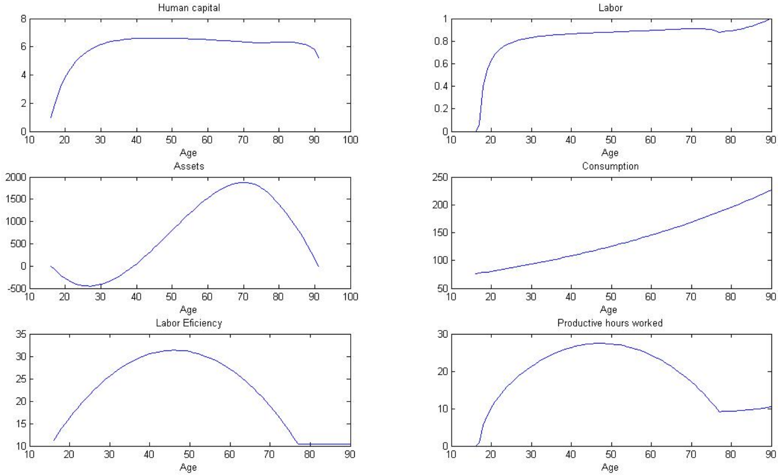
Solution Algorithm

- Define f: Given c_I, h_{I+1}, $(\mu_I = a_{I+1} = 0)$, for i from I to 1:
  - Determine n_i solving following nonlinear equation from FOCS

$$\beta^{i-1} c_i^{-\sigma} w E_i \frac{h_{i+1} - (1 - n_i)^\alpha}{1 - \delta} = \mu_i \alpha (1 - n_i)^{\alpha - 1}$$

  - Check whether corner solution for labor exists.
  - Find h_i, a_i, c_{i-1} and μ_{i-1} using FOCs. Find $a_1, h_1 - 1$
  - Therefore f: $(c_I, h_{I+1}) \longmapsto (a_1, h_1 - 1)$
- Find roots of f.

Learning by not Doing



Learning by Doing

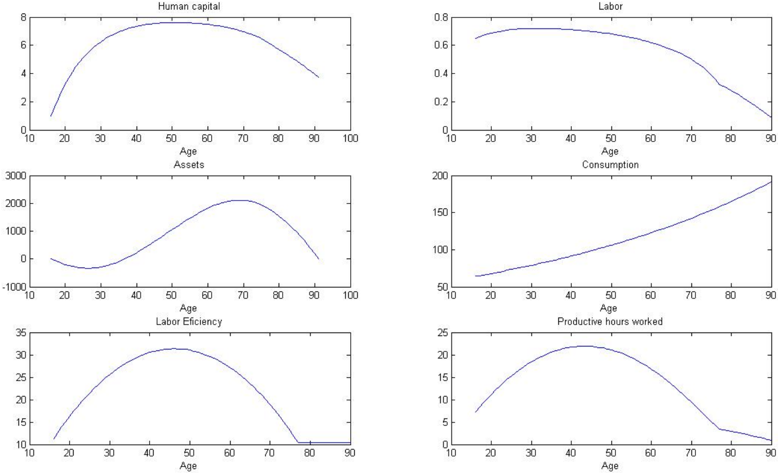
- $\Psi(h_t, n_t)$ increasing in n_t. Used following specification: $\Psi(h_t, n_t) = (1-\delta)h_i + n_i^{\theta}$. I pick $\theta = 0.2$
- We have to add leisure to utility:

$$U(c,n) = \frac{c^{1-\sigma}-1}{1-\sigma} + \Pi\frac{(1-n)^{1-\gamma}-1}{1-\gamma}$$

- Solution methodology is same (backward induction). FOCs changed. Find n_i by solving nonlinear equation

$$\beta^{i-1}c_i^{-\sigma}wE_ih_i + \mu_i\theta n_i^{\theta-1} = \beta^{i-1}\Pi(1-n_i)^{-\gamma}$$

Learning by Doing



Both

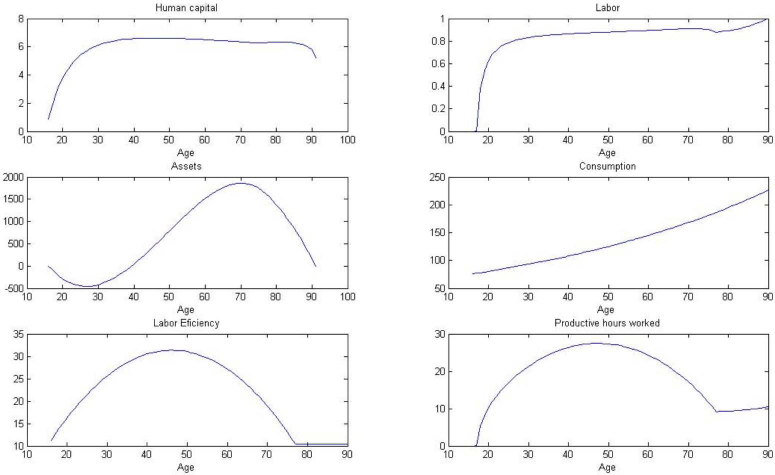
Utility is same $U(c,n) = \frac{c^{1-\sigma}-1}{1-\sigma} + \Pi\frac{(1-n)^{1-\gamma}-1}{1-\gamma}$

For this part I have used two specification:

- $\Psi(h_i, n_i) = (1-\delta)h_i + n_i^{\alpha} + (1-n_i)^{\theta}$
- $\Psi(h_i, n_i) = (1-\delta)h_i + n_i^{\theta_i}(1-n_i)^{1-\theta_i}$

Solution methodology is same for both. For first specification I take $\alpha = \theta = 0.2$

Both: specification 1



Both: specification 2

- Considering only static substitution between leisure and labor:

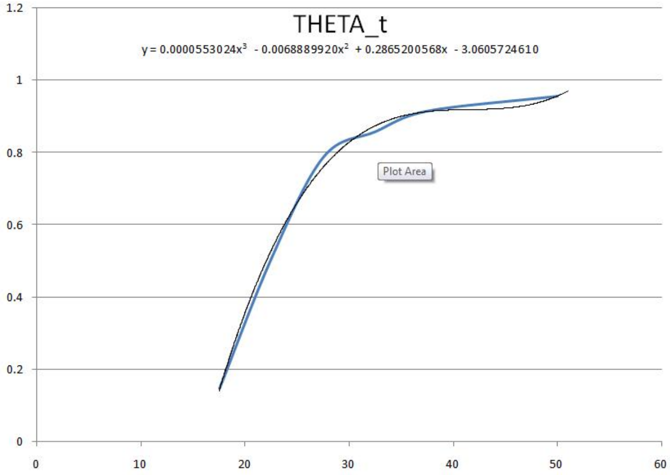
$$\max_x x^{\theta}(1-x)^{(1-\theta)}$$

would give $\frac{x}{1-x} = \frac{\theta}{1-\theta}$.

- Using schooling data and employment data I calibrate the θ_t as

$$\frac{\theta_t}{1-\theta_t} = \frac{population \quad employed}{population \quad schooling}$$

Both: specification 2



Both: specification 2