Wealth and Inheritance in the Long Run

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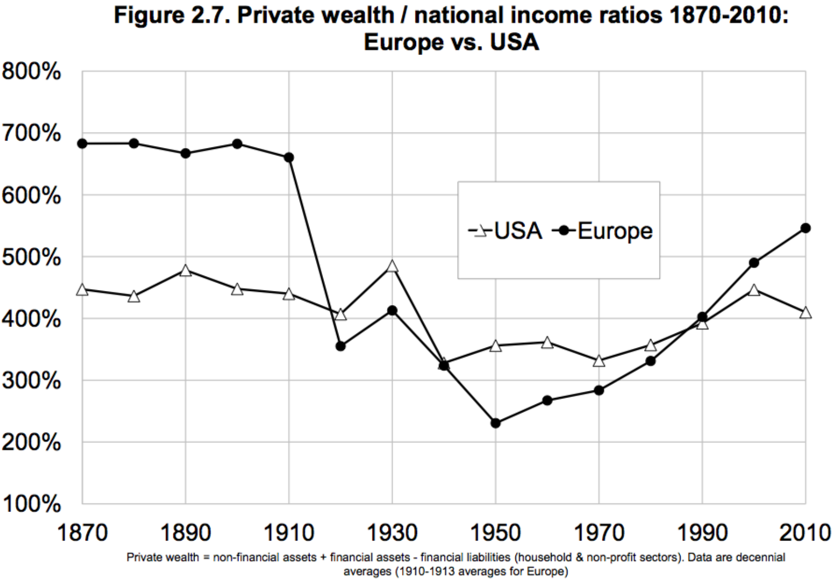
Motivation

- Their focus is on how three interrelated ratios have changed over time and across countries

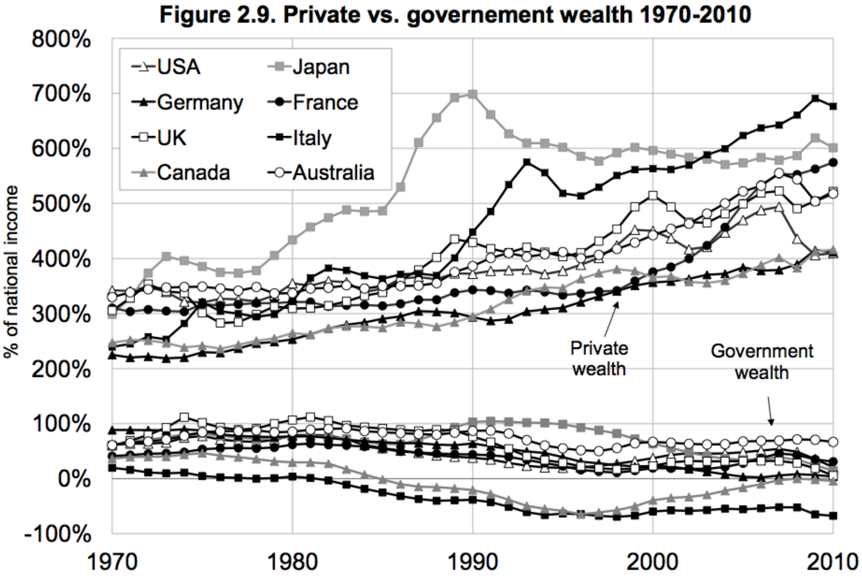
    1. Private wealth to income ratio (B_t)
    2. Ratio of wealth held by top 1 or top 10 percent
    3. Share of wealth that is inherited

- Trends in each of these can be explained by analysis of $\bar{r} - g$, the difference between the after tax rate of return on capital and the growth rate of the economy

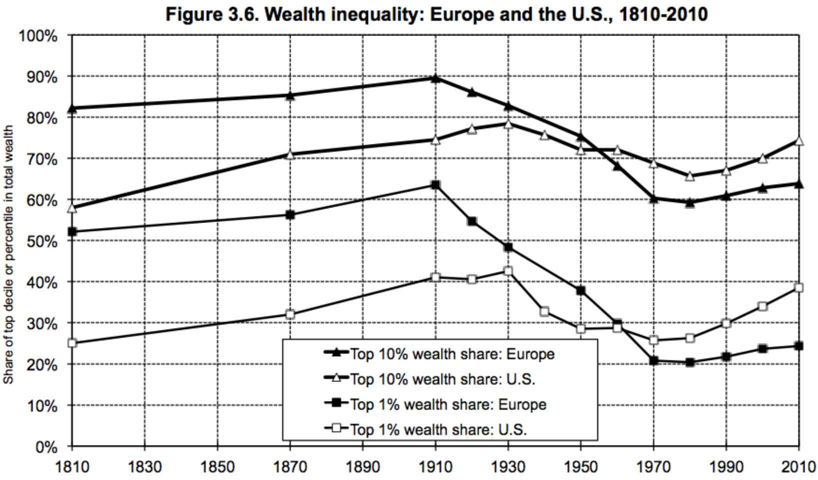
Private wealth to income ratio (B_t)



B_t and government wealth to income ratio N_t more recently



Concentration of wealth



Until the mid 20th century, wealth inequality was higher in Europe than in the United States.

Inherited wealth

- A complete definition requires extensive micro data
- Simplified definition using savings and inheritance flows from macro data
- Let I_t equal inheritance flow, and $b_t = I_t / Y_t$ be inheritance flow/income ratio
- Let S_t equal savings flow, and $s_t = S_t / Y_t$ be savings flow/income ratio
- Want to estimate $\varphi_t = W_b / W$, ratio of wealth that is inherited

Inherited wealth

- Assuming b_t and s_t are constant, and the propensity to save is the same no matter the income source:

$$\varphi = \frac{b + \varphi \alpha s}{b + s}$$

where α is the capital share.

- This simplifies to

$$\varphi = \frac{b}{b + (1 - \alpha)s}$$

- Without assuming constant flows, the formula for estimating the ratio of wealth that is inherited:

$$\varphi = \frac{\int_{t-H \leq s \leq t} e^{(r-g)(t-s)} b_s ds}{\int_{t-H \leq s \leq t} e^{(r-g)(t-s)} (b_s + (1 - \alpha_s) s_s) ds}$$

- Estimates are relatively close lower bounds to the more complicated definition using French micro data

Data

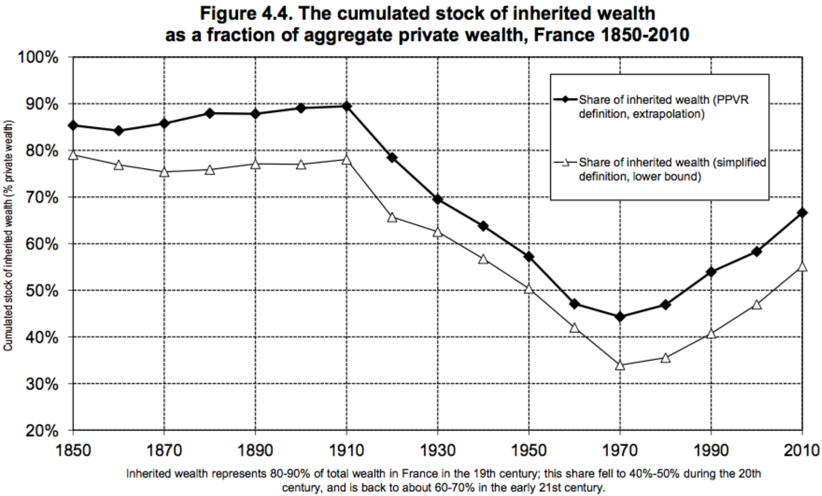


Figure 4.4. The cumulated stock of inherited wealth
as a fraction of aggregate private wealth, France 1850-2010
100%
90%
80%
70%
60%
50%
40%
30%
20%
Cumulated stock of inherited wealth (% private wealth)
1850
1870
1890
1910
1930
1950
1970
1990
2010
Share of inherited wealth (PPVR definition, extrapolation)
Share of inherited wealth (simplified definition, lower bound)
Inherited wealth represents 80-90% of total wealth in France in the 19th century; this share fell to 40%-50% during the 20th century, and is back to about 60-70% in the early 21st century.

Modeling the trends

Movements in all three ratios of interest can be explained by changes in $\bar{r} - g$, the difference between the after tax return on capital and the economy's growth rate. This is shown in the following three models:

- Dynastic model
- Wealth-in-the-utility-function model
- Endogenous growth model

Dynastic model

- $Y_t = F(K_t, L_t)$, closed economy so the wealth income ratio is the same as the capital output ratio
- Dynasties maximize $V = \int_{t \geq s} e^{-\theta t} \frac{c_t^{1-\gamma}}{1-\gamma}$
- FOC in a steady state implies $r = \theta + \gamma g$
- Savings rate in the steady state: $s = \alpha \frac{g}{r} = \alpha \frac{g}{\theta + \gamma g}$
- In the long run, a fraction g/r of capital income is saved, so that dynastic wealth grows at the same rate as national income
- The saving rate $s = s(g)$ rises slower than g, so the steady state wealth income ratio $B = s/g$ is a decreasing function of g.

Wealth in the utility function

- Discrete set of generations that live for one period, with no population growth, exogenous labor productivity growth g
- Budget constraint: $c_t + b_{t+1} \leq y_t = y_{Lt} + (1 + r_t)b_t$, where b_t indicates a bequest from a previous generation and y_{Lt} is inelastic labor income
- Utility is Cobb-Douglas over consumption and $\triangle w_t = w_{t+1} - w_t$ (people like to leave their children better off than they were)
- Maximization implies fixed savings rate, $w_{t+1} = w_t + sy_t$, and in the long run again $B = s/g$

Endogenous growth

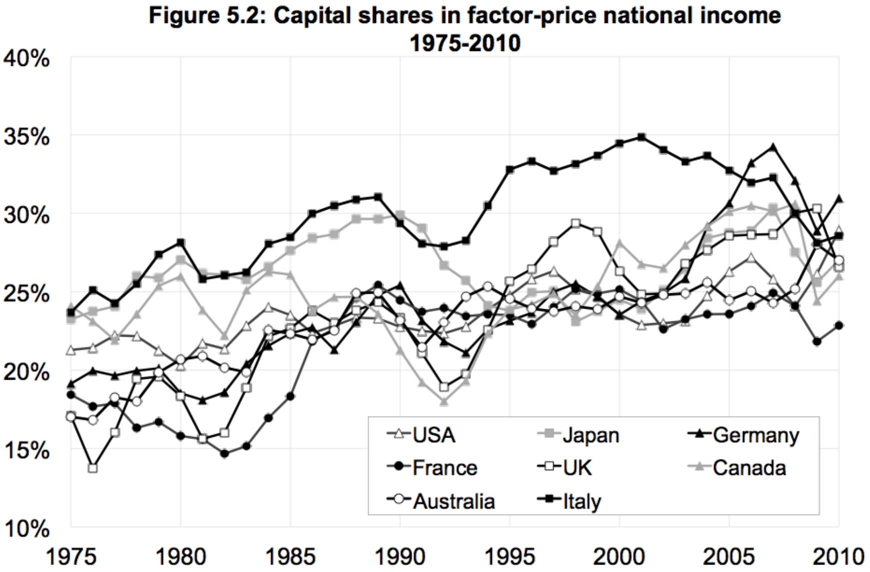
- In the AK model, the growth rate rises proportionally with the savings rate, and so $B = 1/A$, a constant

- But if the model is made more complicated (imperfect international capital flows, innovation, institutions) then the resulting growth rate rises less than proportionally with the savings rate.

Steady state capital share

- Assume the following CES production function:
$Y = F(K, L) = (aK^{\frac{\sigma-1}{\sigma}} + (1-a)L^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$
- The rate of return on capital is $r = F_K = aB^{-1/\sigma}$, and the capital share is $\alpha = rB = aB^{\frac{\sigma-1}{\sigma}}$. So we see that if $\sigma > 1$, then the capital share is an increasing function of the wealth income ratio, but if $\sigma < 1$, then it is a decreasing function of the wealth income ratio.

Capital share



Piketty-Zucman model

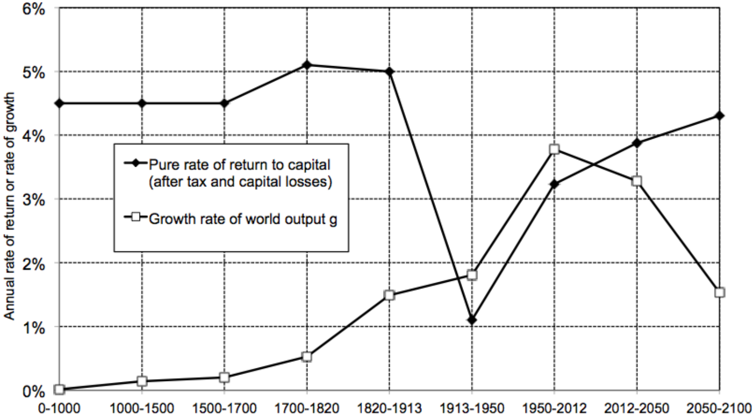
- Discrete model (each period can be interpreted as a year or a generation)
- Population $N_t = [0, 1]$, so $W_t = w_t$ and $Y_t = y_t$ (aggregate and average wealth and income are the same)
- Effective labor input grows exogenously: $L_t = N_t h_t = h_0(1+g)^t$, and domestic output is $Y_{dt} = F(K_t, L_t)$
- Each individual i receives labor income $y_{Lti} = y_{Lt}$ and has the same annual rate of return $r_{ti} = r_t$
- Agents choose c_{ti} and w_{ti} to maximize utility $V(c_{ti}, w_{ti}) = c_{ti}^{1-s_{it}} w_{ti}^{s_{ti}}$, where s_{ti} is a wealth or bequest taste parameter, which are drawn from an i.i.d. process with mean less than 1.
- Budget constraint is $c_{ti} + w_{t+1i} \leq y_{Lt} + (1+r_t)w_{ti}$

Implications of the model

- Maximization implies that individual wealth transition follows $w_{t+1i} = s_{ti}[y_{Lt} + (1 + r_t)w_{ti}]$
- Wealth-income ratio $B_{t+1} = s\frac{1-\alpha_t}{1+g} + s\frac{1+r_t}{1+g}B_t = \frac{s}{1+g}(1 + B_t)$. Note there is convergence only if $\omega = s\frac{1+r_t}{1+g} < 1$.
- In steady state, $B_t = \frac{s}{g+1-s} = \frac{\tilde{s}}{g}$, where $\tilde{s} = s(1 + B) - B$ is the steady state savings rate in terms of national income
- Wealth will converge to a Pareto distribution (Hopenhayn Prescott 1982) that has a fatter tail when ω is larger
- If $r - g$ goes from 2% to 3%, then if $s = 20\%$ and generations last 30 years, the resulting Pareto distribution goes from having a top 1% wealth share of 20-30% to one with 50-60%

$\bar{r} - g$ over time

Figure 5.5. After tax rate of return vs. growth rate at the world level, from Antiquity until 2100



The rate of return to capital (after tax and capital losses) fell below the growth rate during the 20th century, and may again surpass it in the 21st century.

Predictions

- Piketty foresees $\bar{r} - g$ continuing to increase in the future
  - International competition for capital, decreasing capital tax rates push up $\bar{r}$
  - Slower population and productivity growth, so g will be low
- According to the above models, an increase in $\bar{r} - g$ will increase the wealth-income ratio, the share of wealth going to the top of the distribution, and the portion of wealth that is inherited

Questionable assumptions

- Capital and wealth are treated as the same thing
- Diminishing rates of return on capital are assumed away
- Housing and land are treated as equivalent to other types of capital