Chapter 8

Consumption and labor choice

We begin the more applied part of the text to discuss how consumption and labor supply are chosen. These are, arguably, two of the most important choices households make, so they deserve special focus for this reason. They are also integral parts of most analyses in macroeconomics. We will focus on the dynastic household, but much of the insights here carry over to households with finite lifetimes. Some of the discussion will involve some details of the market structure, but the general purpose is to focus on individual choice both in the very long run and in response to short-term fluctuations. Thus, this chapter prepares for, and offers intuition behind, the analyses in the growth and business-cycles chapters that come next.

We begin with a static model. The idea here is that many aspects of labor supply can be understood in such a setting. Moreover, the static model can be closely tied to a dynamic model under balanced growth. We then introduce dynamics and illustrate with simple examples without uncertainty.

8.1 A static model

In a frictionless market with one consumption good and where leisure is valued, consumption and hours worked, along with the wage, can be viewed to be determined by labor demand and labor supply. Thus consider a static economy where the resource constraint is given by $c = F(k, n) - \delta k$, where k is capital and n is labor. In the static economy, we regard k as exogenous, and notice that we specify production by a function F before and explicitly include a depreciation cost for capital the way it would appear in a steady state where capital is held constant. A representative consumer enjoys consumption and leisure according to a function $u(c, l)$, where l is leisure; l and n sum up to a total time endowment, assumed to be 1 for simplicity. The social planner would thus simply maximize $u(F(k, 1-l), l)$ by choice of l. This choice results in the first-order condition

$$\frac{u_l(F(k,1-l)-\delta k, l)}{u_c(F(k,1-l)-\delta k, l)} = F_n(k, 1-l).$$

This equation solves for l given k and it has the interpretation that the marginal rate of substitution between labor and leisure has to equal the marginal rate of transformation between labor and consumption.

In a decentralized economy we can talk about labor demand and labor supply. The obvious decentralized market structure here is one with competitive markets for inputs and output. Firms maximize $F(k,n)-rk-wn$ by choice of k and n; this allows us to talk about each firm's demand for capital and labor. We will as usual assume that F has constant returns to scale (and is concave, so that first-order conditions can be used). An individual firm will set $F_k(k,n)=r$ and $F_n(k,n)=w$, in principle determining k and n as a function of the prices r and w. For general values for r and w, the firm decision can lead to either zero production, be ill-defined (because if r and w are such that the firm can make profits, there is no bound to how high the profits can become since the firm can scale production arbitrarily), or lead to an indeterminate solution (in the knife-edge case). However, since capital, unlike labor, is exogenous here, we can think of equilibrium r as determined residually from the firm's first-order condition for capital—it becomes whatever it has to become as a function of the outcome for the labor input. Therefore, we can think of aggregate labor demand as being determined by the firm's first-order condition for labor:

$$F_n(k,n)=w.$$

This expression gives the inverse labor demand function; solving instead for n as a function of w (and k) we obtain the labor demand function.

What does the labor demand function look like? The answer is entirely dependent on the shape of the production function. Macroeconomists almost always use the Cobb-Douglas formulation: $F(k,n)=Ak^{\alpha}n^{1-\alpha}$ where A and α are given parameters. The reason, as discussed in previous chapters, is that the labor share—total wages as a fraction of output—has remained remarkable constant in the U.S. for a long time so that one might want to require an F whose value for $F_n(k,n)n/F(k,n)=F_n(k/n,1)/F(k/n,1)$ is indeed equal to a constant (here $1-\alpha$) for all values of k/n, since k/n has grown a lot over the same period. For the Cobb-Douglas function, thus, we obtain labor demand as $n=[A(1-\alpha)]^{\frac{1}{\alpha}}kw^{-\alpha}$, an isoelastic function shifted one-for-one by the level of capital. Two remarks are worth making here, however. First, the labor share has moved significantly over time in some economies and seems to have displayed a downward trend recently, since about 2000, in many countries (including the U.S.). Second, it is not impossible to formulate a production function outside the Cobb-Douglas class that can deliver constant shares. Consider a CES: $y=(\alpha(A_kk)^{\rho}+(1-\alpha)(A_nn)^{\rho})^{1/\rho}$, where ρ regulates the substitutability between the inputs ($\rho=-\infty$ is Leontief, $\rho=0$ is Cobb-Douglas, and $\rho=1$ is perfect substitutes) and there are now two separate technology variables, one capital-augmenting and one labor-augmenting. As was pointed out in the growth chapter, we can have a balanced growth path for this formulation, regardless of the value of ρ, with y and k growing at the same rate and n staying constant if A_k is constant but A grows at the same rate as output. Thus, consider a CES function with $\rho\neq 1$ and such that A_n is growing at a roughly constant rate; then it would not be surprising to observe some movements in the labor share around a stationary value as k and n fluctuate, for whatever reason, around their their balanced paths. Supposing that $\rho < 0$ ($\rho > 0$), a declining labor share could then be interpreted as a falling (rising) value of $(A_k/A_n)(k/n)$.[1] Regardless of the degree of substitutability, for a given labor input the wage will depend greatly on the development of technology. The nature of technological change—which factor(s) is augmented—will be important for the wage too if there is more or less substitutability than Cobb-Douglas.

Turning to labor supply, the consumer would maximize $u(c, l)$ subject to $c = w(1-l) + (r-\delta)k$ by choice of c and l. The outcome is a first-order condition that reads $u_c(w(1-l) + (r-\delta)k, l)w = u_l(w(1-l) + (r-\delta)k, l)$. This equation implicitly defines labor supply: $n = 1 - l$ as a function of w. Not surprisingly, the shape of the utility function is thus a key determinant of labor supply. Like in the case for labor demand, one can restrict the functional form based on some long-run facts. The key long-run fact on the labor-supply side is that working hours per household have remained roughly constant over a long period of time, despite massive increases in the wage. Intuitively, it must therefore be the case that we need a utility function such that the income and substitution effects go in opposite directions and, in fact, cancel. The increasing wage will induce people to work more, since it pays off more and more to work, but for a given amount of hours worked the higher wage causes a higher demand for leisure (if leisure is a normal good) going in the opposite direction. A utility function that has the desired property is (any monotone transformation of) $cv(l)$; assume also that v is not only strictly increasing but that $cv(l)$ is strictly quasiconcave. That these preferences will be necessary to match the long-run facts will be proved in Section 8.2.1 below.

To illustrate how the household's problem is solved using this functional form, note that the first-order condition becomes $v(l)w = cv'(l)$. Let us first simplify the discussion and suppose there is no capital income. Then, with the budget substituted in, one sees that w cancels and that one obtains $v(l) = (1-l)v'(l)$, an equation that determines l based on the shape of v only: income and substitution effects cancel. If there is capital income, the first-order condition becomes $v(l) = (1 - l + (r-\delta)k/w)v'(l)$. Now an increased wage level (it is straightforward to verify it) will increase the amount of working time supplied to the market if the added wealth is positive—if $(r-\delta)k > 0$). The intuitive reason is that the income effect of an increased amount of wealth is smaller when there is other income, so the substitution effect will dominate.[2] In the long run, of course, we know from the Kaldor facts discussed in the growth section that k will grow at the same rate that w will grow, and r will be constant, so the fact that labor supply is constant is consistent with the presence of capital income: rk/w will matter for the determination of l but this variable will have no trend and therefore the outcome for l will be stationary.

Putting together labor demand and labor supply and eliminating prices it is straightforward to see that we reproduce the planner's first-order condition for l. However, the purpose of looking separately at demand and supply was to gain some intuitive insights on the firm

[1] The labor share equals $(1-\alpha)/(\alpha(A_k k/(A_n n))^\rho + 1 - \alpha)$.

[2] If the added wealth is negative, then the income effect is stronger than before and an increased wage will lead to less work.

and consumer levels into the determinants not only of hours worked but also of wages. Given the specialized preferences we use, in particular, we see that the constancy of labor supply will imply, under Cobb-Douglas production, that the wage will be proportional to Ak^{α} and hence follow its trend.

8.2 A dynamic model

We restrict attention to an economy with dynastic households (a restriction that will not be important for the main points here). Thus we describe preferences by $\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$. We also use a neoclassical setting so the resource constraint simply reads $c_t + k_{t+1} = F(k_t, A_t(1-l_t)) + (1-\delta)k_t$. We will establish that a certain utility function is necessary and sufficient for exact balanced growth where the Kaldor facts are satisfied.[3] We will then look at several specific cases in order to gain intuition for how consumption and saving will be determined in dynamic economies.

8.2.1 Preference requirements for exact balanced growth

Let us now look more carefully at the preference requirements necessary for balanced growth, i.e., a formulation such that if labor-augmenting productivity grows at a constant rate it is not only feasible but also desirable to have consumption growth at that same rate but also a constant labor supply. These preferences will exhibit a constant rate of intertemporal substitution regarding consumption (or a consumption-leisure bundle) and, regarding leisure, will require that income and substitution effects cancel. This class of utility functions is given by a $\sigma \geq 0$ and a v such that

$$u(c, l) = \lim_{s \to \sigma} \frac{(cv(l))^{1-s} - 1}{1-s}.$$

There are many commonly used special cases. One is $\theta \log c + (1-\theta)\log l$. Another (more general) formulation, using labor input instead as leisure, is $\log c - B\frac{n^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}$. As we shall see, the former places rather strong restrictions on curvature, whereas the second one has curvature that is parameterized by γ. We will mainly use the latter in our illustrations.

Constant elasticity of intertemporal substitution

We will proceed by abstracting first from valued leisure and thus have $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$. Before proceeding to the argument why this function is needed, let us interpret it. The *elasticity of intertemporal substitution* between c_t and c_{t+k} is defined in terms of consumption choice as

$$\left.\frac{d \log \frac{c_{t+k}}{c_t}}{d \log R_{t,t+k}}\right|_{\bar{U}},$$

[3]That technology has to be labor-augmenting is something we will discuss and verify in detail in Chapter 9 below. This discussion involves only restrictions on technology and thus does not overlap with the discussion here.

where the notation $R_{t,t+k}$ is used to represent the relative price between the two consumption goods, i.e., the accumulated gross interest rate across the k periods and $\bar{U}$ indicates a substitution along an indifference curve, i.e., taking the utility level $\bar{U}$ as given (a "compensated" elasticity). That is, if you change the gross interest rate by 1 percent, how will the ratio of consumption levels at the two dates change (also in percent)? For the particular functional form here we will see that this elasticity is constant, independent of consumption levels. The marginal rate of substitution—which will equal the relative price—is $\frac{u'(c_t)}{\beta u'(c_{t+k})} = \left(\frac{c_{t+k}}{c_t}\right)^{\sigma} \beta^{-1}$ so for this function it is possible to write the ratio as a function of the marginal rate of substitution (or the price): $\frac{c_{t+k}}{c_t} = MRS_{t,t+k}^{\frac{1}{\sigma}} \beta^{\frac{1}{\sigma}}$ as we move along an indifference curve. Given the simple relation we have that the sought-after elasticity is constant and equal to $1/\sigma$.

It is perhaps useful to pay extra attention to the logarithmic special case because it is commonly used and has a special feature: under logarithmic preferences, income and substitution effects of an interest-rate change cancel. With logarithmic preference, the marginal rate of substitution between the two goods involved will be proportional to the ratio consumed of the two goods, and thus the relative income shares will remain constant (e.g., the ratio of c_t and c_{t+1}/R_{t+1} will be constant and equal to $1/\beta$).

The case without valued leisure

We now proceed to argue why we need a constant elasticity of intertemporal substitution in order for consumers to choose balanced consumption growth. The Euler equation has to hold for all t, or equivalently for all starting levels c_t. Thus we need to require $u'(c)/u'(gc)$ to be constant for all c. Denoting this constant $\#$, and recognizing that g would have to depend on $\#$, we have $u'(c) = u'(g(\#)c)\#$ for all c. This allows us to differentiate with respect to c to obtain $u''(c) = u''(g(\#)c)\#g(\#)$. Dividing the first equation by the second, we obtain $\frac{u''(c)}{u'(c)} = \frac{u''(g(\#)c)g(\#)}{u'(g(\#)c)}$. Multiply by c on both sides to obtain $\frac{u''(c)c}{u'(c)} = \frac{u''(g(\#)c)g(\#)c}{u'(g(\#)c)}$. Since this holds for all c and $g(\#)$ is an arbitrary positive number, this means that $\frac{u''(c)c}{u'(c)}$ is independent of c, i.e., the expression must equal a constant. Denote the constant a. Thus we have $u''(c)/u'(c) = a/c$. The "trick" here is to see that this can be written as $d \log u'(c)/dc = a \cdot d(\log c + b)/dc$ for any arbitrary constant b. Developing the expression slightly, we have $d \log u'(c)/dc = d(\log c^a + ab)/dc$. This gives that $\log u'(c) = \log c^a + B$, where B is a constant. Thus, $u'(c) = Ac^a$ for some constant A. This means that $u(c)$ is of the functional form stated. (Notice that the case $\sigma = 1$ is subsumed here—it corresponds to $a = -1$.) Restrictions of course need to be placed jointly on A and a so that u is strictly increasing and strictly concave, which leads to the formulation adopted.

Valued leisure

In the case without leisure, we derived above that $u'(c)$ has to be of the form Ac^a. What this means in a context with leisure is that $u'(c)$ must be of the form $A(l)c^{a(l)}$: l is constant on a balanced path, and thus can be an argument of any constant appearing in $u'(c)$. This means that $u(c,l)$ must be of the form $B(l)c^{b(l)} + D(l)$ or, if $a(l) = -1$, $B(l)\log c + D(l)$.

However, we need to make sure that the first-order condition for labor is satisfied along a balanced path. Taking our functional form, and replacing w by a constant, e, times c—since they need to grow at the same rate—we obtain, for $b(l) \neq 0$,

$$u_c(c,l)w = B(l)b'(l)c^{b(l)-1}ec = u_l(c,l) = B'(l)c^{b(l)} + B(l)(\log c)b'(l)c^{b(l)} + D'(l),$$

an expression that needs to be met for all c. Because it needs to hold for all c, it is clear that unless $b(l) = 0$ (the log case), $b'(l) = D'(l) = 0$ has to hold, allowing us to conclude that for a balanced growth path with l constant and equal to an arbitrary value within some given bound, $b(l)$ and $D(l)$ have to be constants. In the log case, we obtain a similar equation where $b'(l) = 0$ is still needed for the equation to hold for all c but where $D(l)$ can be a function that depends on l; however, now $B'(l)$ must be zero so $B(l)$ must be a constant. Thus, we are left with a utility function $B(l)c^b$, for $b \neq 0$, or $B \log c + D(l)$. This completes the argument.

It is perhaps instructive to point out that some commonly used utility functions do not admit balanced growth with constant labor supply. One is the case $\frac{c^{1-\sigma}-1}{1-\sigma} + B\frac{l^\psi}{\psi}$ for $\sigma \neq 1$; additivity only works if σ is equal to one (the logarithmic case). A second case is the so-called GHH utility function (Greenwood, Hercowitz, and Huffman, xyz): $\frac{(c+v(l))^{1-\sigma}-1}{1-\sigma}$. This formulation amounts to there being no wealth/income effect of a wage change. I.e., in the first-order condition for labor vs. leisure, consumption drops out and labor supply becomes a(increasing) function of the wage only. Clearly, an ever-increasing wage would then lead to ever-increasing labor supply. It should be noted that for any utility function that does not match the long-run facts it would be possible to restore the facts by introducing exogenous trend factors in the utility function. For example, in the GHH case, if one assumes $\frac{(c+B_t v(l))^{1-\sigma}-1}{1-\sigma}$, where B_t is shifting up at exactly the rate of consumption growth, the consumer would choose constant labor supply. However, such a formulation would call for a deeper explanation of the increased value of leisure, and the results would not be robust if consumption growth were to change for technological reasons.

8.2.2 Labor-leisure and consumption choice over time without saving

For the rest of the chapter, we abstract from long-run growth (for notational convenience only) but allow time-varying labor-augmenting technology. The idea is now to look at a number or interesting cases in order to build intuition for how consumption and labor are determined in dynamic models. We will look at models where markets work, so that the planning problem can be analyzed.

It is straightforward to formulate the planning problem and derive first-order conditions: an (intertemporal) Euler equation and a labor-leisure tradeoff (which is intratemporal). Such a model would, in general, have nontrivial transition dynamics for saving and hours worked. The purpose is not to emphasize these, let alone characterize them in full generality, but rather to emphasize some mechanisms. For this, let us consider some extreme special cases that serve an illustrative purpose. The first case is trivial: if capital plays no role in production, i.e., $y_t = A_t n_t$ for all t, so that there is really no possibility to move resources across time, the model reduces to a static one without capital. Thus, no matter how much labor-augmenting technology moves over time, the labor supply will be constant. The reason is that income and substitution effects cancel: movements in A amount to movements in the wage, and because there is no other income than labor income, labor supply will be chosen to be constant and satisfying $v(l) = (1-l)v'(l)$ (where $n = 1-l$).

8.2.3 A linear savings technology

In a second example, suppose instead that $F(k_t, A_t n_t) = bk_t + A_t n_t$, so that capital is productive as well and there are no decreasing returns to either capital or labor. Now capital accumulation is possible at the gross rate $b + 1 - \delta$ between any two periods. Here, the planning problem will look identical to a consumer problem where the prices on capital and labor, r_t and w_t, from the firm's first-order conditions, are simply b and A_t, respectively. To simplify the problem further, assume that $\beta(b+1-\delta) = 1$. The consumer's problem then reads

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\log c_t - B \frac{n_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) \quad \text{s.t.}$$

$$c_t + k_{t+1} = w_t n_t + k_t/\beta \quad \text{and} \ .$$

Here, to simplify even more, we will suppose that $k_0 = 0$ so that all resources available for consumption derive from working during the consumer's lifetime. We will also allow capital to be negative so as to illustrate how unrestricted movements of resources over time interact in an important way with the labor-supply decision.[4] The maintained formulation can also be viewed as an open-economy interpretation where the gross international interest rate is constant and equal to $1/\beta$.

The Euler equation for this problem implies that $c_t = c_{t+1} \equiv c$ for all t, i.e., that there will be complete consumption smoothing. The labor-leisure first-order condition then reads

$$\frac{w_t}{c} = B n_t^{\frac{1}{\gamma}}.$$

The resource constraint can be expressed as a discounted sum and the resulting expression can be written $c/(1-\beta) = \sum_{t=0}^{\infty} \beta^t w_t n_t$.

We shall proceed toward solving this problem momentarily but before doing this we can already note that a very different result than in the static model is expected here. Suppose time periods are quite short (i.e., β is very close to 1), so that a wage change in a given period does not influence present-value income, and hence c, much at all. Then it follows from the first-order condition above that a wage change will change hours worked rather directly. Thus, the income effect of a higher wage in a given period is thus very limited. Moreover, the effect of wages on labor supply can be strong, namely if γ is high. Intuitively,

[4] A no-Ponzi-game restriction is assumed as well.

in this model there is intertemporal substitution of hours worked: if the wage is high at one point in time and low at another point in time the consumer works more in the former and less in the latter, while moving income across period using borrowing and lending.

From inserting the first-order condition into the resource constraint and simplifying we obtain a solution for consumption:

$$c = B^{-\gamma}(1-\beta)\sum_{t=0}^{\infty}\beta^t w_t^{1+\gamma}$$

Thus, suppose compare different wage paths such that $\sum_{t=0}^{\infty}\beta^t w_t^{1+\gamma}$ is constant. Then consumption will not change in any period. For example, suppose the wage in period t_1 goes up marginally and the wage in some other period t_2 falls so as to keep $\beta^{t_1}w_{t_1}+\beta^{t_2}w_{t_2}$ constant. Then, since c will not change, the effect of the wage changes on labor supply, expressed in an elasticity form and for each of the two time periods, is $d\log n/d\log w = \gamma$.

Notice also that if we were to impose a "borrowing constraint", so that capital holdings could not fall below a certain value, then for a period in which this constraint binds—when the consumer would like to increase consumption by borrowing more—a change in the wage will have a very different effect. An increased wage would have a much stronger, positive effect on consumption and hence working hours would barely change, at least for a small enough wage increase. Similarly, a fall in the wage would make consumption fall further and have a very small effect on labor supply.

Motivated by the example above, let us finally consider a formal definition of labor-supply elasticity: the percentage change in labor supply from a one-percent increase in the wage keeping the marginal utility of wealth constant. This concept was proposed by Ragnar Frisch and is usually referred to as the Frisch elasticity. The marginal utility of wealth can be thought of in terms of goods available for consumption in any period here, and the marginal utility of such resources will equal the marginal utility of consumption. In a model where consumption and leisure are separable in utility, holding the marginal utility of wealth constant is thus equivalent to holding consumption constant. It is straightforward to see that with the preferences assumed here, the Frisch elasticity becomes precisely γ. Consider, however, the commonly used $u(c,l) = \theta\log c + (1-\theta)\log l$: what is the Frisch elasticity in this model? The first-order condition in any period reads

$$\frac{\theta w_t}{c_t} = -\frac{1-\theta}{1-n_t}.$$

To obtain the Frisch elasticity, take log on both sides and use the fact that consumption is constant so that $d\log(1-n_t) = -d\log w_t$. Because $d\log(1-n) = -(d\log n)n/(1-n)$ we thus obtain a Frisch elasticity equal to $(1-n)/n$. That is, the elasticity depends on the level of labor supply (relative to the amount of leisure). In typical calibrations, leisure is usually thought of 2/3 of the total available time and hours worked as 1/3. This implies a Frisch elasticity of 2. This number is very high compared to most microeconomic estimates, which tend to range between 0 and 1/2. For a recent survey and view of the literature, see Chetty (2009).

8.2.4 Decreasing marginal returns to savings

Suppose instead we use a more empirically reasonable production function such as the Cobb-Douglas function. Then labor income cannot be generated and transformed linearly across periods on the level of the whole economy so intertemporal substitution of leisure is not as straightforward. First, there are decreasing returns to working in a given period, and second, there are decreasing returns to saving the working income. So if one asks about the effects of, say, an increase in the level of technology in period t on hours worked, just how does the answer differ from that obtained above? Maintaining the utility function $\log c - B\frac{n^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}$, we obtain first-order conditions as follows:

$$\frac{(1-\alpha)k_t^{\alpha}A_t^{1-\alpha}n_t^{-\alpha}}{c_t} = Bn_t^{\frac{1}{\gamma}}.$$

$$\frac{1}{c_t} = \beta\frac{\alpha k_{t+1}^{\alpha-1}(A_t n_t)^{1-\alpha} + 1 - \delta}{c_{t+1}}.$$

To make the most extreme assumption of decreasing returns to capital within this framework, suppose $\delta = 1$, so that capital depreciates fully after use; this assumption is not reasonable for short time horizons but serves as illustration. Then it is straightforward to show that these equations, together with the resource constraint, imply that $k_{t+1} = \alpha\beta k_t^{\alpha}(A_t n_t)^{1-\alpha}$, i.e., that the rate of saving is constant and equal to $\alpha\beta$ (this guess was verified to solve the Euler equation in earlier chapters and still works here) and that the first-order condition for leisure simplifies to

$$\frac{1-\alpha}{n_t} = Bn_t^{\frac{1}{\gamma}}.$$

Hence, n_t becomes constant and independent of both A_t and k_t! Decreasing returns are, apparently, strong enough in this case to totally offset any desire to intertemporally substitute labor efforts. Increased productivity thus leads to increased production and consumption at all dates (though less so further into the future) but no changes in hours worked at any date. Here (and in general), the amount of hours worked depend not just on the utility function but on the production technology, though only on its elasticity with respect to labor: the higher this elasticity, the higher is working effort.

With less than full depreciation, capital accumulation becomes "more linear" and there will be intertemporal substitution of labor.