Life-Cycle Models

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ITAM

March 25, 2015

Evidence

Wages across USA and Europe have been increasing since 1970

- Differences in TFP between USA and Europe affect the return of high productive workers
- Difference in education (higher in Europe because it is free)
- Stock of capital is lower in Europe than in the USA (differences in taxation)
- **Europe taxes are higher than in the USA. Europe has a bigger welfare government**

Progressive Wedge

$$PW[0.5, K0.5] = 1 - \frac{1 - \tau(K0.5)}{1 - \tau(0.5)}$$

$$(\tau(y_s))\text{Marginal Tax} \quad \geq \quad \text{Average Tax}(\bar{\tau}(y_s))$$

Assumptions

Individuals have one unit of time that can be allocated:

- Leisure
- Working
- Investing in Human-Capital

Assumptions

- Assume $\beta(1+r)=1$
- Human-Capital is produced using the following technology:

$$Q_s = A^j (h_s n_s i_s)^\alpha$$

  - A^j: Individual Ability
  - h_s: Individual stock of Human-Capital
  - $n_s i_s$: Fraction of individual working time that is spent training

Assumptions

- The cost of training is borne by workers:

$$w_s = P_h h_s (1 - i_s)$$

  - P_h: Price of Human-Capital
- Law of Motion of Human-Capital:

$$h_{s+1} = h_s + Q_s$$

Assumptions

- Labor Income:

$$y_s = w_s n_s = P_h h_s (1 - i_s) n_s = P_h h_s n_s - P_h C(Q_s)$$

  - $P_h h_s n_s$: Potential Wage

Maximization Problem

- The problem of individual type j is:

$$\max_{\{Q_s, a_{s+1}, h_{s+1}, c_s, n_s\}_{s=1}^{S}} \sum_{s=1}^{S} \beta^{s-1} u(c_s, 1 - n_s)$$

$$\text{s.t.} \quad c_s + a_{s+1} = (1 - \bar{\tau}(y_s)) y_s + (1 + r) a_s$$

$$h_{s+1} = h_s + Q_s$$

$$y_s = P_h h_s n_s - P_h C(Q_s)$$

$$\text{where} \quad C(Q_s) = \left(\frac{Q_s}{A_s^j} \right)^{\frac{1}{2}}$$

State Variables

There are three state variables:

- a_s: Assets
- h_s: Human-Capital
- s: Age (it matters because individuals retire)

Value Function

$$V(h_s, a_s, s) =$$
$$= \max_{\{Q_s, a_{s+1}, h_{s+1}, n_s\}_{s=1}^{S}} \{u((1+r)a_s + (1-\bar{\tau}(y_s))y_s - a_{s+1}, 1-n_s) +$$
$$+ \beta V(h_{s+1}, a_{s+1}, s+1)\}$$

s.t.
$$y_s = P_h h_s n_s - P_h C(Q_s)$$
$$h_{s+1} = h_s + Q_s$$

Note

- Tax Liability:

$$\bar{\tau}(y)y$$

- Marginal Income Tax:

$$\bar{\tau}(y) + \bar{\tau}'(y)y = \tau(y)$$

First Order Conditions

$$a_{s+1} : -u_c(c_s, 1-n_s) - \beta V_a(h_{s+1}, a_{s+1}, s+1) = 0$$

$$\begin{aligned} n_s : & -u_n(c_s, 1-n_s) + u_c(c_s, 1-n_s)(P_h h_s(1-\bar{\tau}(y_s)) - \bar{\tau}'(y_s)P_h h_s y_s) = \\ & = -u_n(c_s, 1-n_s) + u_c(c_s, 1-n_s)P_h h_s(1-\bar{\tau}(y_s) - \bar{\tau}'(y_s)y_s) = \\ & = -u_n(c_s, 1-n_s) + u_c(c_s, 1-n_s)P_h h_s(1-\tau(y_s)) = 0 \end{aligned}$$

$$\begin{aligned} Q_s : & \; u_c(c_s, 1-n_s)[(1-\bar{\tau}(y_s))(-P_h C'(Q_s)) - \\ & - \bar{\tau}'(y_s)y_s(-P_h C'(Q_s)) + \beta V_h(h_{s+1}, a_{s+1}, s+1)] = 0 \end{aligned}$$

First Order Conditions

Therefore...

$$u_c(c_s, 1 - n_s) = \beta V_a(h_{s+1}, a_{s+1}, s + 1) \tag{1}$$

$$u_n(c_s, 1 - n_s) = u_c(c_s, 1 - n_s) P_h h_s (1 - \tau(y_s)) \tag{2}$$

$$u_c(c_s, 1 - n_s) P_h C'(Q_s)(1 - \tau(y_s)) = \beta V_h(h_{s+1}, a_{s+1}, s + 1) \tag{3}$$

Envelope Conditions

$$V_a(h_s, a_s, s) = u_c(c_s, 1 - n_s)(1 + r)$$

$$V_h(h_s, a_s, s) = u_c(c_s, 1 - n_s)[(1 - \bar{\tau}(y_s))P_h n_s - \bar{\tau}(y_s)P_h n_s y_s] - \\ - \beta V_h(h_{s+1}, a_{s+1}, s + 1)$$

Then...

$$V_a(h_s, a_s, s) = u_c(c_s, 1 - n_s)(1 + r) \tag{4}$$

$$V_h(h_s, a_s, s) = u_c(c_s, 1 - n_s)P_h n_s(1 - \bar{\tau}(y_s)) + V_h(h_{s+1}, a_{s+1}, s + 1) \tag{5}$$

Equations

If we iterate in (5)...

$$\begin{aligned} V_h(h_s, a_s, s) = & \ u_c(c_s, 1 - n_s) P_h n_s (1 - \tau(y_s)) + \\ & + \beta u_c(c_{s+1}, 1 - n_{s+1}) P_h n_{s+1} (1 - \tau(y_{s+1})) + \\ & + \beta^2 u_c(c_{s+2}, 1 - n_{s+2}) P_h n_{s+2} (1 - \tau(y_{s+2})) + \\ & + ... + \\ & + \beta^{S-s} u_c(c_S, 1 - n_S) P_h n_S (1 - \tau(y_S)) \end{aligned}$$

Equations

From (3)...

$$\begin{aligned} u_c(c_{s-1}, 1-n_{s-1}) P_h C'(Q_{s-1})(1-\tau(y_{s-1})) = \\ &= \beta u_c(c_s, 1-n_s) P_h n_s (1-\tau(y_s)) + \\ &+ \beta^2 u_c(c_{s+1}, 1-n_{s+1}) P_h n_{s+1}(1-\tau(y_{s+1})) + \\ &+ \beta^3 u_c(c_{s+2}, 1-n_{s+2}) P_h n_{s+2}(1-\tau(y_{s+2})) + \\ &+ \ldots + \\ &+ \beta^{S-s+1} u_c(c_S, 1-n_S) P_h n_S (1-\tau(y_S)) \end{aligned}$$

Equations

From (1) and (4)...

$$u_c(c_{s-1}, 1 - n_{s-1}) = \beta(1 + r)u_c(c_s, 1 - n_s)$$

Hence...

$$\frac{u_c(c_{s-1}, 1 - n_{s-1})}{u_c(c_s, 1 - n_s)} = 1$$

Solution

Finally...

$$P_h C'(Q_{s-1}) \quad \rightarrow \quad \text{Marginal Cost of Investing in Human-Capital}$$

$$=$$

$$P_h \left[\beta \frac{1-\tau(y_s)}{1-\tau(y_{s-1})} n_s + \beta^2 \frac{1-\tau(y_{s+1})}{1-\tau(y_{s-1})} n_{s+1} + \dots \right.$$

$$\left. \dots + \beta^{S-s+1} \frac{1-\tau(y_S)}{1-\tau(y_{s-1})} n_S \right]$$

$$\rightarrow \quad \text{Marginal Benefit of Investing in Human-Capital}$$

Special Cases

- When the labor supply is inelastic and the taxes are constant throughout time ($n_s = 1$ and $\tau(y_s) = \tau$):

$$C'(Q_{s-1}) = \beta + \beta^2 + ... + \beta^{S-s+1}$$

  The marginal benefit **DOES NOT** depend on the tax rate.

- More progressive tax systems reduce the benefit of investing in Human-Capital:

$$\frac{1-\tau(y_s)}{1-\tau(y_{s-1})} < 1$$

Elastic Labor Supply

$\uparrow$ Marginal Income $\Rightarrow$
$\Rightarrow \downarrow$ Labor Supply $\Rightarrow$
$\Rightarrow \downarrow$ Marginal Benefit of Investing in Human-Capital $\Rightarrow$
$\Rightarrow \downarrow$ Optimal Level of Human-Capital $\Rightarrow$
$\Rightarrow$ Endogenous Labor Supply Amplify the Effect of Tax on Human-Capital

$$\text{Progressive Wedge:} \quad PW(y_s, y_{s+k}) = 1 - \frac{1 - \tau(y_{s+k})}{1 - \tau(y_s)} \cdot \frac{n_i}{n_{AVG}}$$