A Theory of Credit Scoring and the Competitive Pricing of Default Risk

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SED Annual Conference
Session: Credit and Macroeconomic Stabilization
ITAM, Mexico City, Mexico
June 30, 2018

Why do people pay back rather than file for bankruptcy?

Benefits: Ch 7 offers credit protection

Costs: Filing costs (low), bankruptcy can hurt your reputation

- **direct:** lower credit scores, higher interest rates, restricted access
- **indirect:** difficulties renting, getting hired, in relationships

Quantitative models of default get too much unsecured credit $\implies$ need additional punishments

- literature assumes **exclusion,** stigma, etc.
- we tackle the reputation problem in a quantitative framework
- delivers a theory of **credit scoring**

▸ Evidence: credit scores and prices ▸ Evidence: bankruptcy and credit scores

What we do

Can **reputation effects** simultaneously explain small amounts of unsecured credit, low interest rates, and low default rates?

Construct an **incomplete markets model with adverse selection**

- people differ in persistent wealth, income, type, and "reputation"
  - type – propensity to default, borrow too much (β), unobservable
  - reputation – lender's "best guess" of β (+ other traits $\rightarrow$ credit score)
- also transitory traits that impact default today only
- lenders have to infer types via reputation to price loans

Map the model to the data

- target wealth distribution, key credit moments
- compare model implied credit score dynamics to data

Vary the notion of punishment

- compare 3 economies: full info, benchmark, "extra" reputation

What we find

Yes — reputation matters.

Key mechanism

- low β types more like to borrow too much, default
- $\implies$ these actions signal bad type, reflected in lenders' assessment
- reflected in pricing function, reigning in credit

Quantitative results

1. compared to **full information** case, benchmark model features
   - lower levels of default (by 42%), interest rates (by 83%)
   - wider dispersion of interest rates (factor of 25)
   - why? better able to separate types
2. individuals would need to be compensated for a bad reputation
3. **non-price effects** play a role: 1% reduction in earnings for bad credit score reduces default by 28%

▸ Literature

Model environment: households

HH have **preferences** ordered by $u(c)$, s.t. to 2 unobservable shocks

- persistent: discount rate $\beta \in \{\beta_L, \beta_H\}$, drawn from $\Gamma^{\beta}(\beta, \beta')$
- transitory: additive, action-specific shocks ϵ drawn from $G^{\epsilon}(\epsilon)$

Earnings are comprised of 2 observable components

- persistent: e, follows $\Gamma^{e}(e, e')$
- transitory: z, drawn from $G^{z}(z)$

Each period, HH take **action** (d, a')

- $a' \in \mathcal{A} = \{a_1, ..., 0, ..., a_{N(a)}\}$: asset (or debt) position for next period
- $d \in \{0, 1\}$: default decision. If HH defaults ($d = 1$), then
  - HH cannot save that period ($a' = 0$)
  - and loses κ of income ($c = e + z - \kappa$) $\rightarrow$ "static" punishment

Model environment: intermediaries

- risk neutral, perfectly competitive (free entry)
- borrow at exogenous interest rate r, intermediation cost ι on debt
- observe earnings (e and z) and choices (d, a')

Inference problem: cannot observe β or $\epsilon^{(d,a')}$ when pricing loans

- β persistent $\implies$ actions can signal type
- ϵ transitory $\implies$ adds confusion
  - GEV / logit assumption $\implies$ all actions chosen with prob > 0

Solution: assign reputation, subjective prior $s = \Pr(\beta = \beta_H)$

- update via Bayes rule using observables (d, a') and $\omega = (e, z, a, s)$ to revise type score $\psi^{(d,a')}(\omega)$

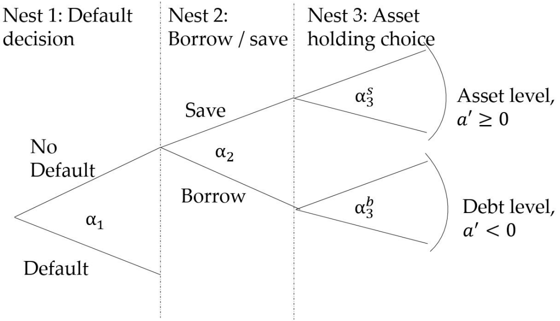
Pricing: offer discount loans at prices $q^{(0,a')}(\omega)$, where

$$q^{(0,a')}(\omega) = \begin{cases} \frac{p^{(0,a')}(\omega)}{1+r+\iota} & \text{if } a' < 0 \\ \frac{1}{1+r} & \text{if } a' \geq 0, \end{cases}$$

Timing   Type scoring and pricing

HH problem: overview

$$V(\beta,\omega,\epsilon) = \max_{(d,a')\in\mathcal{F}(\omega)} \underbrace{v^{(d,a')}(\beta,\omega)}_{\text{fundamental value}} + \underbrace{\epsilon^{(d,a')}}_{\sim\text{GEV}},$$



- α's determine correlation b/w ϵ_i shocks for i w/in each nest
- high $\alpha \implies$ low variance of ϵ shocks for options in nest
- highest $v^{(d,a')}(\beta,\omega) \implies$ modal action

Fundamental value | Feasibility | GEV | Nests

HH problem: decision rules

Results in 4 decision rules / probabilities

- default, $\sigma^D(\beta, \omega)$
- borrowing, cond. on not defaulting: $\sigma^B(\beta, \omega | \neg D)$
- debt [asset] level, cond. on not defaulting and borrowing [saving]: $\sigma^{a'}(\beta, \omega | \neg D, B)$ $[\sigma^{a'}(\beta, \omega | \neg D, \neg B)]$

Can combine nest-level decisions into a single function $\sigma^{(d,a')}(\beta, \omega)$, e.g.

$$\sigma^{(0,a')}(\beta, \omega) = \sigma^N(\beta, \omega)\sigma^B(\beta, \omega | N)\sigma^{a'}(\beta, \omega | N, B) \text{ if } a' < 0$$

- substitute for $\sigma \implies$ functions of only $v^{(d,a')}(\beta, \omega)$ and α's
- used by intermediary to price / assess reputation
  - show up in denominator of Bayesian posterior $\implies$ always positive (if feasible) is desirable

▸ Nests ▸ D / ND ▸ B / S ▸ a' ▸ IIA

Equilibrium definition

Definition

A **stationary recursive competitive equilibrium (SRCE)** comprises:

- pricing function $q^*(\omega)$ (vector-valued)
- type scoring function, $\psi^*(\omega)$ (vector-valued) ▸ Details
- quantal response function, $\sigma^*(\beta, \omega)$ (vector-valued)
- steady state distribution, $\mu^*(\beta, \omega)$ ▸ Details

such that

- $\sigma^*(\beta, \omega)$ is consistent with HH optimization
- $q^*(\omega)$ implies lenders break even, with repayment probabilities implied by σ^*
- $\psi^*(\omega)$ satisfies Bayes' Rule
- $\mu^*(\beta, \omega)$ is stationary

Theorem

There exists a SRCE.

Targeted model moments: wealth and credit

- select basic preference and filing cost parameters
- externally calibrate earnings process
- SMM on remaining 8 preference parameters
  - match wealth and credit market moments (25 total)

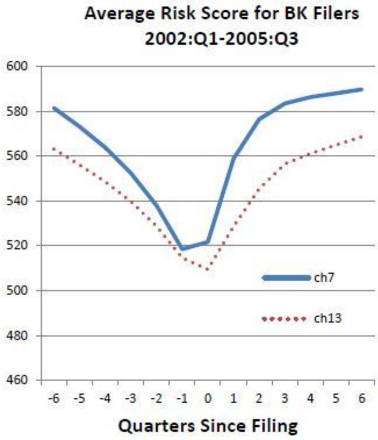
	Moment	Data	Model
Credit	Default rate (%)	0.33	0.50
	Average interest rate (%)	12.89	11.49
	Fraction of HH in debt (%)	6.49	7.13
	Debt to income ratio (%)	0.26	0.20
	Interest rate dispersion (%)	6.58	5.59
Wealth	Mean wealth to mean earnings	3.22	1.91
	Correlation b/w wealth and earnings	0.52	0.65

▸ Distributional and transition moments ▸ Data: debt ▸ Data: default

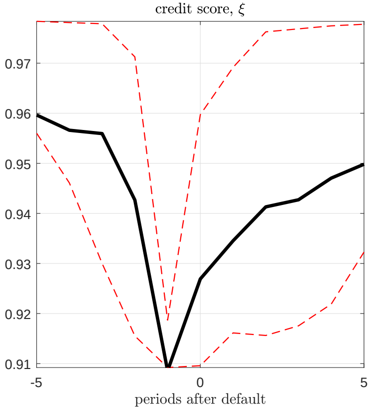
Parameterization

	Parameters	Notation	Value
Selected	CRRA	ν	3.0
	risk-free rate (%)	r	1.0
	filing costs to mean income (%)	κ	2.0
External	var. of $\log(z)$ [transitory]	σ_z	0.0421
	persistence of $\log(e)$	ρ_e	0.914
	variance of $\log(e)$	σ_e	0.206
Internal	high type discount factor	β_H	0.954
	low type discount factor	β_L	0.920
	$\beta_L \to \beta_H$ transition prob	$\Gamma^{\beta}(\beta'_H \mid \beta_L)$	0.090
	$\beta_L \to \beta_H$ transition prob	$\Gamma^{\beta}(\beta'_L \mid \beta_H)$	0.121
	EV scale parameter, default	α_1	349
	EV scale parameter, borrow / save	α_2	158
	EV scale parameter, $a \geq 0$ level	α_3^s	164
	EV scale parameter, $a < 0$ level	α_3^b	306

Bankruptcy and credit scores



(a) Data [Jagtiani and Li (2014)]



(b) Model

Other objects

3 economies

1. **Full information (FI)**
   - type observable $\implies$ no inference problem
   - obviates credit score, but actual type can directly affect prices

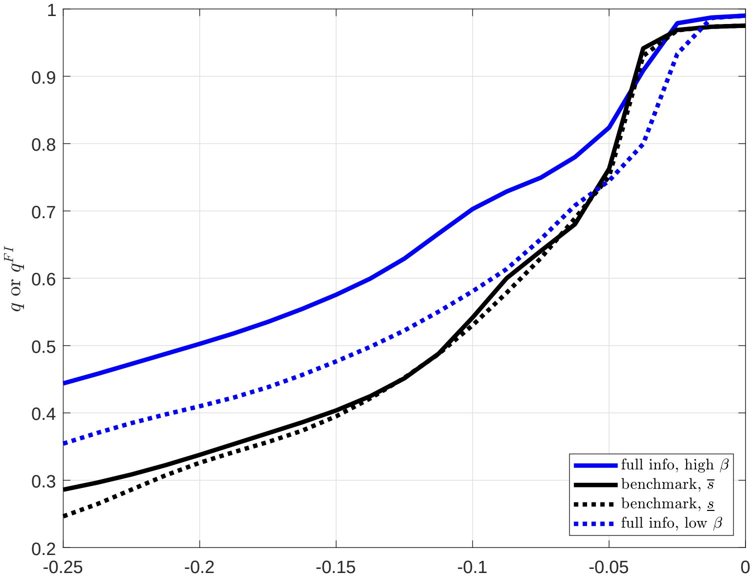
2. **Dynamic Punishment (DP)** [benchmark]
   - credit score is tracked and updated through time
   - affects loan pricing function only

3. **Dynamic Punishment with Earnings Effects (DP+)**
   - same pricing, credit scoring tracking as DP model
   - extra: good (bad) credit score raises (lowers) earnings

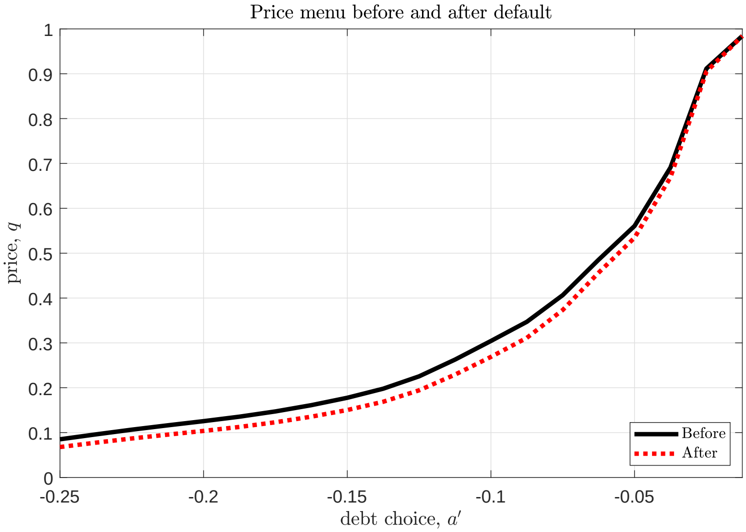
$$e + z \to e + z + (1 + \lambda)s + (1 - \lambda)(1 - s),$$

with $\lambda = 1\%$ (similar results for utility cost).

Impact of information on price schedules



Impact of default on price schedules



Dynamics around default

Reputation and the credit market

		Model		
Moment	**Data**	**FI**	**DP**	**DP+**
Default rate (%)	0.33	0.87	0.50	0.36
Average interest rate (%)	12.89	69.07	11.49	6.08
Fraction of HH in debt (%)	6.49	3.70	7.13	6.29
Debt to income ratio (%)	0.26	0.26	0.20	0.20
Interest rate dispersion (%)	6.58	76.5	5.59	3.34
Mean wealth to mean earnings	3.22	2.23	1.91	2.02
Corr b/w wealth and earnings	0.52	0.63	0.65	0.64

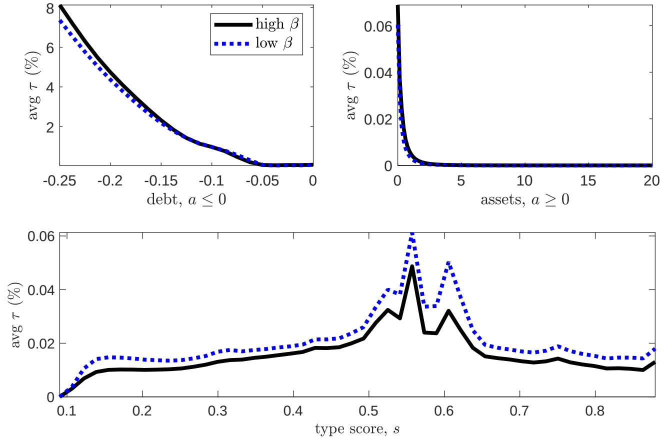
- full information economy severely punishes bad types
- DP+: even slight non-price punishment does a lot

Separation

Individuals' value of reputation

How much compensation is required to accept worst reputation?

$$W(\beta, \omega) = W(\beta, e, z, a + \tau(\beta, \omega), \underline{s})$$



Key takeaways

Model mechanism

- credit scores allow lenders to track reputation and infer default probability, price loans better
- default signals bad type, shifts interest rates up, reigns in borrowing

Quantitative results

- reputation model outperforms full information case
- non-price impact of credit score matters
- reputation is worth something, especially if have a good reputation, debt

Credit scores and prices

	Having an offer	Credit limit	Spread
VantageScore bins			
550-600	0.100***	-208.749***	1.229***
	(0.009)	(34.383)	(0.181)
600-650	0.155***	-251.378***	2.249***
	(0.010)	(34.182)	(0.186)
650-700	0.223***	-208.157***	1.693***
	(0.011)	(32.610)	(0.203)
700-750	0.252***	-100.237***	0.436**
	(0.011)	(33.416)	(0.188)
750-800	0.285***	102.084***	-0.367*
	(0.011)	(31.987)	(0.191)
800-850	0.292***	326.984***	-0.871***
	(0.012)	(35.524)	(0.190)
850-900	0.264***	577.903***	-0.969***
	(0.012)	(31.360)	(0.185)
900-950	0.254***	714.265***	-0.865***
	(0.011)	(33.499)	(0.192)
> 950	0.267***	809.787***	-0.770***
	(0.010)	(40.664)	(0.178)

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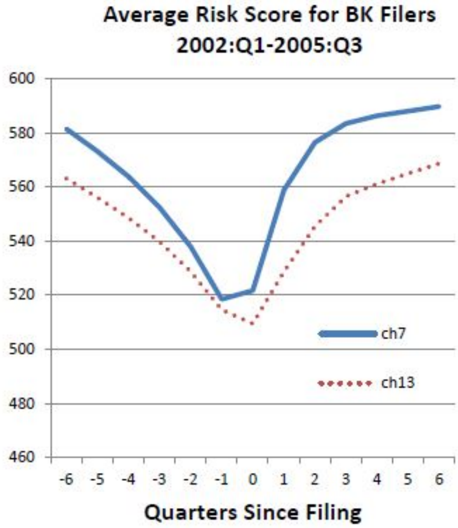
Bankruptcy and credit scores



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Some related literature

Equilibrium models of bankruptcy

- full information, exogenous punishment
  - Chatterjee et al. (2007), Livshits et al. (2007), sovereign debt
- asymmetric info, static signaling, exogenous punishment
  - Athreya et al. (2009, 2012), Livshits et al. (2015)
- asymmetric info, dynamic signaling, endogenous punishment
  - Chatterjee et al. (2008), Mateos-Planas et al. (2017)
- important issue with asymmetric info: off equilibrium path beliefs

Discrete choice models

- estimation of logit / nested logit models: McFadden (1973), Train (2009)
- dynamic models: Rust (1987)
- make sense of behavior in experimental data (quantal response equilibrium): McKelvey and Palfrey (1995, 1996)

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Timing

1. HH begin period with state (β, e, a, s)
2. HH receive transitory earnings and preference shocks
   - $z \sim G^z(z)$
   - $\epsilon = \{\epsilon^{(d,a')}\}_{(d,a')\in\mathcal{Y}} \sim G^\epsilon(\epsilon)$, which is GEV with scale $1/\alpha_j$ in nest j (details next slide)
3. Given price schedule $q = \{q^{(0,a')}(\omega)\}$, agents choose (d, a')
4. Intermediaries revise type scores from $s \to s'$ via Bayes rule and the type scoring function $\psi^{(d,a')}(\omega)$
5. Next period states are drawn:
   - $\beta' \sim \Gamma^\beta(\beta'|\beta)$
   - $e' \sim \Gamma^e(e'|e)$
   - $s' \sim \Gamma^s(s'|\psi)$

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HH problem: fundamental value

The individual's decision problem is to solve

$$V(\beta, \omega, \epsilon) = \max_{(d,a')\in\mathcal{F}(\omega)} v^{(d,a')}(\beta, \omega) + \epsilon^{(d,a')},$$

where $\epsilon = \{\epsilon^{(d,a')}\}_{(d,a')\in\mathcal{Y}}$ is drawn from a GEV distribution. GEV

- $\mathcal{F}(\omega)$ is the set of feasible actions given state ω Details

The conditional value function for a given feasible action is

$$\begin{aligned} v^{(d,a')}(\beta, \omega) &= u\left(c^{(d,a')}\right) \\ &+\beta \sum_{z',\beta',s',e'} \left(\Gamma^{\beta}(\beta, \beta')\Gamma^{e}(e, e')\Gamma^{s}(\psi, s')G^{z}(z') \right. \\ &\left. \times \int V(\beta', \omega', \epsilon') dG^{\epsilon}(\epsilon')\right) \end{aligned}$$

Back to HH overview

Budget feasibility and actions

Set of all possible default and asset choices:

$$\mathcal{Y} = \{(d, a') : (d, a') \in \{0\} \times \mathcal{A} \text{ or } (d, a') = (1, 0)\}$$

Given observable state ω and a set of equilibrium functions $f = (q, \psi)$, the set of feasible actions is

$$\mathcal{F}(\omega | f) \subseteq \mathcal{Y}$$

which contains all actions $(d, a') \in \mathcal{Y}$ such that $c^{(d,a')} > 0$, where consumption is pinned down by the budget constraint

$$c^{(d,a')} = \begin{cases} e + z + a - q^{(0,a')}(\omega)a' & \text{if } d = 0, a' < 0 \\ e + z + a - \frac{a'}{1+r} & \text{if } d = 0, a' \geq 0 \\ e + z - \kappa & \text{if } d = 1, a' = 0 \end{cases}$$

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Generalized extreme value distribution

[From Train (2009)]

Let the set of alternatives $j \in \{1, ..., J\}$ be grouped into K non-overlapping nests

- each alternative j belongs to a single nest B_k, $k \in \{1, ..., K\}$

Then, the transitory preference shocks $\epsilon = \{\epsilon_j\}_{j=1}^{J}$ follow a distribution with CDF

$$\exp\left(-\sum_{k=1}^{K}\left[\sum_{j\in B_k}\exp\left(-\epsilon_j/\lambda_k\right)\right]^{\lambda_k}\right)$$

Consider two alternative actions, $i \in B_k$ and $j \in B_\ell$

- if $k \neq \ell$, then ϵ_i and ϵ_j are uncorrelated
- if $k = \ell$, then ϵ_i and ϵ_j are correlated

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HH problem: overview of 3 phases

1. default (D) vs no default (N) ▸ D / ND

$$D = \{1, 0\} \text{ and } N = \{B, S\}$$

2. conditional on no default, borrow (B) vs save (S) ▸ B / S

$$B = \{(0, a')|a' < 0\} \text{ and } S = \{(0, a')|a' \geq 0\}$$

3. conditional on borrow (save), choose specific debt (asset) level ▸ a'

Disciplines the correlations b/w choices at each decision node

- with extreme value preference shocks, implies a **3-tier nested logit** structure

Analyze this problem working backwards through these three decisions.

▸ Back to tree ▸ Back to decision rules

HH problem: debt / asset choice

Using discrete choice results, **conditional on not defaulting and on borrowing**, the probability of choosing a debt level $a' < 0$ is

$$\sigma^{(0,a')}(\beta,\omega|N,B) = \frac{\chi^{(0,a')}(\omega)\exp\{\alpha_3^B v^{(0,a')}(\beta,\omega)\}}{\sum_{\tilde{a}'\in B}\chi^{(0,\tilde{a}')}(\omega)\exp\{\alpha_3^B v^{(0,\tilde{a}')}(\beta,\omega)\}}$$

- $\chi^{(0,a')}(\omega)$ is an indicator equal to 1 if action $(0,a')$ is feasible for an agent in state ω
  - formally, $\chi^{(0,a')}(\omega) = 1 \iff (0,a') \in \mathcal{F}(\omega)$

We can define the expected value of borrowing, then, via the inclusive value or logsum formula

$$W^B(\beta,\omega) = \frac{1}{\alpha_3^B}\ln\left[\sum_{a'\in B}\chi^{(0,a')}(\omega)\exp\{\alpha_3^B v^{(0,a')}(\beta,\omega)\}\right].$$

The procedure is similar for savings levels, replacing $a' < 0$ with $a' \geq 0$ and B with S in the above formulas.

▸ Back to decision rules ▸ Back to 3 phases

HH problem: borrow / save choice

Similarly, **conditional on not defaulting,** the probability of borrowing is

$$\sigma^B(\beta, \omega | N) = \frac{\chi^B(\omega) \exp\{\alpha_2 W^B(\beta, \omega)\}}{\chi^B(\omega) \exp\{\alpha_2 W^B(\beta, \omega)\} + \chi^S(\omega) \exp\{\alpha_2 W^S(\beta, \omega)\}}$$

- $\chi^j(\omega)$ is an indicator equal to one if there is any feasible action in set $j \in \{B, S\}$ for an agent with observable state ω
  - formally, $\chi^j(\omega) = 1 \iff j \cup \mathcal{F}(\omega) \neq \emptyset$ for $j \in \{B, S\}$
- similar for saving, replacing B with S above
  - 2 choices $\implies \sigma^S(\beta, \omega | N) = 1 - \sigma^B(\beta, \omega | N)$

We can define the expected value of not defaulting, then, via the inclusive value or logsum formula

$$W^N(\beta, \omega) = \frac{1}{\alpha_2} \ln \left[\chi^B(\omega) \exp\{\alpha_2 W^B(\beta, \omega)\} + \chi^S(\omega) \exp\{\alpha_2 W^S(\beta, \omega)\} \right].$$

▸ Back to decision rules ▸ Back to 3 phases

HH problem: default / no default choice

Similarly, the probability of defaulting is

$$\sigma^D(\beta,\omega) = \frac{\chi^D(\omega)\exp\{\alpha_1 W^D(\beta,\omega)\}}{\chi^D(\omega)\exp\{\alpha_1 W^D(\beta,\omega)\} + \chi^D(\omega)\exp\{\alpha_1 W^N(\beta,\omega)\}}$$

- $\chi^i(\omega)$ is an indicator equal to one if there is any feasible action in set $i \in \{D, N\}$ for an agent with observable state ω
  - formally, $\chi^i(\omega) = 1 \iff i \cup \mathcal{F}(\omega) \neq \emptyset$ for $i \in \{D, N\}$
  - $\chi^D(\omega) = 1$ if and only if $a < 0$
- similar for no default, replacing D with N above
  - 2 choices $\implies \sigma^N(\beta,\omega) = 1 - \sigma^D(\beta,\omega)$
- $W^D(\beta,\omega) = v^{(1,0)}(\beta,\omega)$

We can define an agent's total expected value, then, via the inclusive value or logsum formula

$$W(\beta,\omega) = \frac{1}{\alpha_1}\ln\left[\chi^D(\omega)\exp\{\alpha_1 W^D(\beta,\omega)\} + \chi^N(\omega)\exp\{\alpha_1 W^N(\beta,\omega)\}\right].$$

Back to decision rules
Back to 3 phases

Independence of irrelevant alternatives (1)

For any two options (d, a') and $(\tilde{d}, \tilde{a}')$ within a given nest k, we have

$$\frac{\sigma^{(d,a')}(\beta,\omega)}{\sigma^{(\tilde{d},\tilde{a}')}(\beta,\omega)} = \frac{\frac{\chi^{(d,a')}(\omega)\exp\{\alpha_k v^{(d,a')}(\beta,\omega)\}}{\sum_{(\hat{d},\hat{a}')\in k}\chi^{(\hat{d},\hat{a}')}(\omega)\exp\{\alpha_k v^{(\hat{d},\hat{a}')}(\beta,\omega)\}}}{\frac{\chi^{(\tilde{d},\tilde{a}')}(\omega)\exp\{\alpha_k v^{(\tilde{d},\tilde{a}')}(\beta,\omega)\}}{\sum_{(\hat{d},\hat{a}')\in k}\chi^{(\hat{d},\hat{a}')}(\omega)\exp\{\alpha_k v^{(\hat{d},\hat{a}')}(\beta,\omega)\}}} = \exp\left\{\alpha_k\left(v^{(d,a')}(\beta,\omega) - v^{(\tilde{d},\tilde{a}')}(\beta,\omega)\right)\right\},$$

assuming both actions are feasible.

- this is the **IIA property**
- ratio of choice probs depend only on relative action values

▸ Back to HH decision rules ▸ More

Independence of irrelevant alternatives (2)

Is IIA sensible in our environment? Consider 2 variants

Changing the value of alternatives

- IIA is typically about changing "attributes" of an alternative
- our choices have no (differential) attributes

Changing the alternatives (i.e. changing asset grid points)

- our choices are scalar quantities, and for aggregates we can compute

$$\sum_{a'} a' \times \left[\sum_{\beta,\omega} \mu(\beta,\omega) \cdot \sigma^{(0,a')}(\beta,\omega) \right]$$

- how sensitive are such means to grid density / bounds?
  - **bounds:** only matter if "new region" is close to / includes modal choice
  - **density:** only matters ifα is low

Back to HH decision rules | Back to IIA 1 | Details | Numerical Example

IIA and grid density: details

Suppose there are N choices, $a_1, \ldots, a_N$.

- let σ_i be the decision prob on i (must have $\sum_{i=1}^{N} \sigma_i = 1$)
- define $x_{i,j} = \sigma_i / \sigma_j$ for all $i, j = 1, \ldots, N$ ($x_{i,i} = 1$)
- define the mean choice to be

$$\mu = \sum_{i=1}^{N} \sigma_i a_i = \sigma_1 \sum_{i=1}^{N} x_{i,1} a_i$$

Now, add an additional L choices, $a_{N+1}, \ldots, a_{N+L}$

- let $\tilde{\sigma}_i$ be the decision prob on i
  - now $\sum_{i=1}^{N+L} \tilde{\sigma}_i = 1$ and $\tilde{\mu} = \sum_{i=1}^{N+L} \tilde{\sigma}_i a_i$
- IIA $\implies$ $\tilde{\sigma}_i / \tilde{\sigma}_j = x_{ij}$ for all $i, j = 1, \ldots, N$ as before. Therefore,

$$\frac{\tilde{\mu}}{\mu} = \underbrace{\frac{\tilde{\sigma}_1}{\sigma_1}}_{\text{shift in choice probs}} \times \underbrace{\left[1 + \frac{\sum_{i=N+1}^{N+L} x_{i,1} a_i}{\sum_{i=1}^{N} x_{i,1} a_i}\right]}_{\text{shift in actions}}$$

Calculation | Back to IIA main | Back to HH decision rules

IIA and grid density: numerical example

Consider the following application:

- $N = 21$
- $\{a_1, ..., a_N\}$ is an evenly spaced grid on $[-0.2, 0.0]$ (every 0.01)
- $\max_i \sigma_i = 11$, so the modal action is $a_{11} = -0.1$

Back to HH decision rules
Back to IIA main

IIA and grid density: calculation

$$
\begin{aligned}
\frac{\tilde{\mu}-\mu}{\mu} &= \frac{1}{\mu}\left(\sum_{i=1}^{N+L}\tilde{\sigma}_i a_i - \sum_{i=1}^{N}\sigma_i a_i\right) \\
&= \frac{\tilde{\sigma}_{N+1}\sum_{i=N+1}^{N+L} x_{i,N+1}a_i}{\mu} + \frac{(\tilde{\sigma}_1-\sigma_1)\sum_{i=1}^{N} x_{i,1}a_i}{\mu} \\
&= \frac{\tilde{\sigma}_{N+1}\sum_{i=N+1}^{N+L} x_{i,N+1}a_i}{\sigma_1\sum_{i=1}^{N} x_{i,1}a_i} + \frac{\tilde{\sigma}_1-\sigma_1}{\sigma_1} \\
&= \frac{\tilde{\sigma}_1}{\sigma_1}\left[1+\frac{x_{N+1,1}\sum_{i=N+1}^{N+L} x_{i,N+1}a_i}{\sum_{i=1}^{N} x_{i,1}a_i}\right]-1 \\
&= \frac{\tilde{\sigma}_1}{\sigma_1}\left[1+\frac{\sum_{i=N+1}^{N+L} x_{i,1}a_i}{\sum_{i=1}^{N} x_{i,1}a_i}\right]-1
\end{aligned}
$$

Back Back to HH decision rules

Lender problem: type scoring and debt pricing

Given actions and observables, type updating function is

$$\begin{aligned}\psi^{(d,a')}(\omega) &= \Pr\left(\beta' = \beta_H \mid (d,a'), \omega\right) \\ &= \sum_{\beta} \Gamma^{\beta}(\beta, \beta_H) \frac{\sigma^{(d,a')}(\beta,\omega)s(\beta)}{\sum_{\tilde{\beta}} \sigma^{(d,a')}(\tilde{\beta},\omega)s(\tilde{\beta})}\end{aligned}$$

Perfect competition, deep pockets $\implies$ breakeven pricing

$$q^{(0,a')}(\omega) = \begin{cases} \frac{p^{(0,a')}(\omega)}{1+r+\iota} & \text{if } a' < 0 \\ \frac{1}{1+r} & \text{if } a' \geq 0, \end{cases}$$

where $p(\cdot)$ is the assessed repayment probability using both the type score ψ and the decision rules σ

$$\begin{aligned}p^{(0,a')}(\omega) = \sum_{s',e',z'} \Bigg\{ & \Gamma^{s}(\psi^{(d,a')}(\omega), s')\Gamma^{e}(e,e')G(z') \\ & \times \left[s'\left(1 - \sigma^{(1,0)}(\beta_H,\omega')\right) + (1-s')\left(1 - \sigma^{(1,0)}(\beta_L,\omega')\right)\right] \Bigg\}\end{aligned}$$

Back to environment Back to Eqm.

Type scores and the likelihood ratio

Define the likelihood ratio for decisions to be:

$$x^{(d,a')}(\omega) = \frac{\sigma^{(d,a')}(\beta_H, \omega)}{\sigma^{(d,a')}(\beta_L, \omega)}$$

Then the type score updating function can be rewritten as

$$\begin{aligned}
\psi^{(d,a')}(\omega) &= \frac{\Gamma_\beta(\beta_H'|\beta_H)\sigma^{(d,a')}(\beta_H,\omega)s + \Gamma_\beta(\beta_H'|\beta_L)\sigma^{(d,a')}(\beta_L,\omega)(1-s)}{\sigma^{(d,a')}(\beta_H,\omega)s + \sigma^{(d,a')}(\beta_L,\omega)(1-s)} \\
&= \frac{\Gamma_\beta(\beta_H'|\beta_H)x^{(d,a')}(\omega)s + \Gamma_\beta(\beta_H'|\beta_L)(1-s)}{x^{(d,a')}(\omega)s + (1-s)}
\end{aligned}$$

And a simple calculation shows that as long as $\Gamma_\beta(\beta_H'|\beta_H) > \Gamma_\beta(\beta_H'|\beta_L)$

$$\frac{\partial \psi^{(d,a')}(\omega)}{\partial x^{(d,a')}(\omega)} > 0,$$

so the type score is increasing in the likelihood ratio.

Stationary distribution

Let T^* be the operator mapping a distribution of agents today into a distribution tomorrow. Then,

$$T^*(\beta,\omega,\beta',\omega') = \sigma^{(d,a')}(\beta,\omega)\Gamma^s(\psi^{(d,a')}(\omega),s')\Gamma^\beta(\beta,\beta')\Gamma^e(e,e')H(z')$$

and the distribution $\mu(\beta,\omega)$ evolves via

$$\mu'(\beta',\omega') = \sum_{\beta,\omega} T^*(\beta,\omega,\beta',\omega')\mu(\beta,\omega)$$

A stationary distribution is a fixed point of this expression.

Back to Eqm.

Targeted model moments: distributions and transitions

Earnings → Wealth ↓	Data			Dynamic Model			Static Model		
	T1	T2	T3	T1	T2	T3	T1	T2	T3
T1	0.20	0.10	0.03	0.15	0.13	0.05	0.14	0.11	0.04
T2	0.09	0.14	0.10	0.07	0.12	0.14	0.08	0.13	0.15
T3	0.03	0.09	0.22	0.03	0.09	0.21	0.04	0.09	0.22

Table: Joint distribution of earnings and wealth tertiles

Wealth, $t+2$ → Wealth, t ↓	Data			Dynamic Model			Static Model		
	T1	T2	T3	T1	T2	T3	T1	T2	T3
T1	0.76	0.22	0.02	0.83	0.12	0.05	0.50	0.18	0.31
T2	0.20	0.62	0.18	0.10	0.66	0.24	0.09	0.75	0.16
T3	0.04	0.14	0.82	0.03	0.08	0.90	0.03	0.08	0.89

Table: Wealth tertile transitions

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Data details: debt

Source: 2007 SCF

Computation details

- for debt stats, debtor $\iff$ negative net worth
  - exclude debts of greater than 120% of annual income

Statistic	**Value**	
Total debt in group ($B)	$71.3	A
Total HH in group	7,541,007	B
Total HH in US	116,107,641	C
2007 Nominal GDP ($B)	$14,478	D
Debt / HH (A / C)	$614.50	A / C
Fraction of HH in debt (B / C)	6.493%	B / C
GDP per HH (D / C)	$124,692	D / C
Debt to income ratio ((A / C) / (D / C))	0.493%	A / D

▸ Back to targets ▸ Distributional and transition moments

Data details: default rates

Sources: all for data year 2007 for consistency with SCF (used for debt statistics)

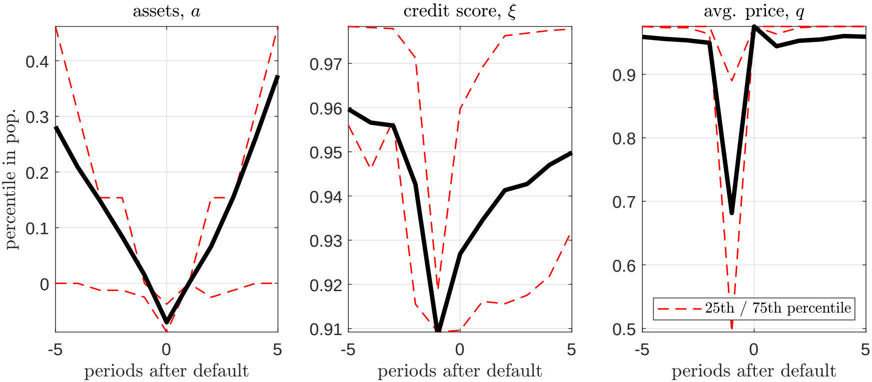
- Federal reserve chargeoff data for chargeoffs
- US courts for bankruptcy filings (nonbusiness, chapter 7)
- US census for total number of HH

Computation details

- use as denominator # of households (not individuals)
- sum up all filings across all quarters, divide by #HH from census
- then, there is a further adjustment: married couples filing together

▸ Back to targets ▸ Distributional and transition moments

Trends in agents' state around default



▸ Back to credit scores ▸ Back to prices

4 economies

1. **Full information (FI)**
   - type observable $\implies$ no inference problem
   - obviates credit score, but actual type can directly affect prices
2. **Dynamic Punishment (DP)** [benchmark]
   - credit score is tracked and updated through time
   - affects loan pricing function only
3. **Static Punishment (SP)**
   - no tracking of credit scores or assets
   - only punishment is the cost of filing
4. **Dynamic Punishment with Earnings Effects (DP+)**
   - same pricing, credit scoring tracking as DP model
   - extra: good (bad) credit score raises (lowers) earnings

Reputation and the credit market

		Model			
Moment	**Data**	**FI**	**SP**	**DP**	**DP+**
Default rate (%)	0.33	0.87	0.49	0.50	0.36
Average interest rate (%)	12.89	69.07	11.01	11.49	6.08
Fraction of HH in debt (%)	6.49	3.70	8.25	7.13	6.29
Debt to income ratio (%)	0.26	0.26	0.24	0.20	0.20
Interest rate dispersion (%)	6.58	76.5	5.17	5.59	3.34
Mean wealth to mean earnings	3.22	2.23	1.90	1.91	2.02
Corr b/w wealth and earnings	0.52	0.63	0.65	0.65	0.64

- full information economy severely punishes bad types
- DP+: even slight non-price punishment does a lot
- current work: why SP and DP performance so close?

Measuring separation: absolute distance

AD Measure	Dynamic #	Static #	Static ΔDyn	Full info #	Full info ΔDyn
Total	0.519	0.517	0.0025	0.560	-0.0786
Action	1.125	1.114	0.0106	1.492	-0.3676
saving	1.210	1.213	-0.0031	1.549	-0.3384
borrowing	0.012	0.011	0.0006	0.027	-0.0147
Default	0.034	0.029	0.0051	0.097	-0.0634

$$\text{AD} = \sum_{\omega,(d,a')} \left| \sigma^{(d,a')}(\beta_H,\omega) - \sigma^{(d,a')}(\beta_L,\omega) \right| \cdot \tilde{\mu}(\omega)$$

Alternatives:

$$\text{AD(action)} = \sum_{\omega} \left| \sum_{a'} a' \left[\sigma^{(0,a')}(\beta_H,\omega) - \sigma^{(0,a')}(\beta_L,\omega) \right] \right| \cdot \tilde{\mu}(\omega)$$

$$\text{AD(default)} = \sum_{\omega} \left| \sigma^{(1,0)}(\beta_H,\omega) - \sigma^{(1,0)}(\beta_L,\omega) \right| \cdot \tilde{\mu}(\omega)$$

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Measuring separation: alternatives

1. **Utility cost (UC) of imitation.** Let $(d_i(\omega), a_i'(\omega))$ be the modal action of type i in state ω. Compute

$$UC = \sum_{\omega} \left[v^{(d_i(\omega), a_i'(\omega))}(\beta_i, \omega) - v^{(d_j(\omega), a_j'(\omega))}(\beta_i, \omega) \right],$$

for $j \neq i$ (i.e. can do relative to either high or low type).

2. **An equilibrium approach.** Solve model twice,changing only β_L from (1) to (2) so that types are "farther apart." Compare:
    - decisions under parameterization (1)
    - decisions under parameterization (2)
    - decisions under parameterization (1) given prices from (2)

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