A Quantitative Theory of Unsecured Consumer Credit with Risk of Default

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Objective of the paper

- To develop a workable model of unsecured consumer credit with default in the context of a competitive loan industry that reflects the U.S. legal system.

- To map the model to U.S. bankruptcy facts and to use it to ask quantitative regulatory questions.

Key Features

- Households can borrow and can and DO default. If they default, they temporarily lose access to credit markets and incur transaction costs.

- Competitive loan industry with free entry and zero costs that offer a menu of loan sizes and borrowing rates with an endogenously determined borrowing limit.

Literature

- Kehoe and Levine (1993,2001), Kocherlakota (1998), Alvarez and Jermann (2000). Lucky borrowers want to default (individual rationality constraint binds in high income states). No household actually goes bankrupt in equilibrium.

- Athreya (1999). First to study default in an environment similar to ours. Unexploited profits remain.

- Lehnert and Maki (2000). Assumes commitment on the part of lenders. There are periods where firms make negative profits.

The Plan

1. A few bankruptcy facts.

2. Start with a simple version of the model that cuts on notation and ingredients.

3. Characterize default behavior and interest rate structure.

4. Establish existence of non-trivial (with borrowing and with default) equilibria.

5. Calibrate the model to U.S. default facts. It requires expanding the model in several directions to make it empirically plausible.

6. Ask some policy questions regarding bankruptcy law.

7. Some final comments on future research. Especially with regards to fluctuations.

A Few Bankruptcy Facts

- Households default mainly through Chapter 7 of the Bankruptcy Code.

- Under Chapter 7, a household files for bankruptcy; upon successful completion of the process (a very easy thing), the household's assets are liquidated above a certain amount (varies by state), the household's debts disappear and creditors lose any rights to recover future assets; the household gets to keep its future labor earnings, and it cannot file again for seven years; after ten years, the bad credit history disappears.

- In 1998, 1,007,922 households filed for bankruptcy under Chapter 7 while 1,379,249 was total filings. Moreover, approximately 90% of Chapter 13 debt was not repaid in 1997 (WEFA data) which makes this legal figure less relevant or, more likely, a prelude to a Chapter 7 filing.

We Interpret Bankruptcy as

• If it has a good credit history, an indebted household can file for bankruptcy. This means that

- Its debts disappear, and its creditors lose any future claims to those debts.

- In the filing period, the household cannot save and must consume its current earnings.

- Its credit history turns bad.

• If it has a bad credit history, the household cannot borrow but can save. It suffers some inconveniences (bonded credit cards) that we model as a proportional γ loss of income. Upon termination of the punishment period, the hhold's credit history turns good.

• The law only prevents another bankruptcy filing within a 7 year period as well as fixing a 10 year limit to a bad credit history. We *interpret* this to mean that households cannot borrow during that amount of time. We abstract from renegotiation issues.

The Model

• There is a continuum of ex-ante identical agents with idiosyncratic shocks to earnings.

• There is free entry in the credit market. Firms operate at zero costs. We only look at one period loans.

• Interest rates on loans can only depend on the size of the loan.

• The legal system is that of the U.S.

• The risk free rate is determined exogenously (by a storage technology or a world interest rate).

Households

- Earnings: $e \in E = [\underline{e}, \overline{e}] \subset \mathbb{R}_{++}$, i.i.d. across individuals and time (relax later) with continuous cdf $F(e)$. Associated probability space $(E, \mathcal{B}(E), \mu)$.

- Borrowing/Saving opportunities: Households can hold assets $\ell \in L = \{\ell_{\min}, \cdots, 0, \cdots, \ell_{\max}\}$ finite.

- Preferences: $E_0 \left\{\sum_{t=0}^{\infty} \beta^t \, u\left(c_t\right)\right\}, \ \beta < 1.$

  $u : [0, \ell_{\max} - \ell_{\min} + \overline{e}] \to \mathbb{R}$ concave, increasing.

$$A1 : u(\underline{e}) - u(0) \geq \frac{\beta}{1-\beta}\left[u\left(\overline{e} + \ell_{\max} - \ell_{\min}\right) - u\left(\underline{e}\left(1-\gamma\right)\right)\right].$$

- Storage technology on $L^+ = (L \cap \mathbb{R}_+)$ yields gross return $\frac{1}{\hat{q}}$, where $1 > \hat{q} > \beta$.

- Let $q_\ell \in [0, \widehat{q}]$ be the discounted price of asset position $\ell \in L$ next period (the interest rate r is $1/q - 1$.). The price schedule is $q : L \to [0, \widehat{q}]$. Let Q be the set of possible prices.

Default Options

- Household credit history, $h \in \{0, 1\}$.
- Default decision, $d \in \{0, 1\}$.
- If $h = 0$ (good credit history), choice of $d = 1$, implies
  - $\ell = 0$ (debt is wiped clean)
  - $\ell' = 0$ (cannot save in same period you default).
- If $h = 1$, (the household has a bad credit history).
  - $\ell' \geq 0$ (cannot borrow)
  - $h' = 0$ with probability $1 - \lambda$. It gets determined at calibration stage.

Budget Correspondences

Defined on (ℓ, h, d, e, q) in 3 parts.

1. Good Credit History, Don't Default: (can be empty for $e + \ell < 0$ and small q'_ℓ's).

$$B_{\ell,0,0}(e,q) = \{c \in \mathbb{R}_+, \ell' \in L : c + q_{\ell'}\, \ell' \leq e + \ell\}.$$

2. Good Credit History, Default:

$$B_{\ell,0,1}(e,q) = \{c \in \mathbb{R}_+, \ell' = 0 : c \leq e\}.$$

3. Bad Credit History:

$$B_{\ell,1,.}(e,q) = \{c \in \mathbb{R}_+, \ell' \in L^+ : c + \hat{q}\, \ell' \leq e + \ell\}.$$

Household Problem

- Denote the value function by $v_{\ell,h}(e,q)$ and let $w_{\ell,h}(q) = \int_E v_{\ell,h}(e,q)\, d\mu$. Let (if B non empty)

$$\chi_{\ell,0,0}(e,q) = \max_{c,a' \in B_{\ell,0,0}(e,q)} u(c) + \beta\, w_{\ell',0}(q).$$

$$\chi_{\ell,0,1}(e,q) = u(e) + \beta\, w_{0,1}(q).$$

- Depending on Credit History (Good or bad)

$$v_{\ell,0}(e,q) = \max \begin{cases} \{\chi_{\ell,0,0}(e,q), \chi_{\ell,0,1}(e,q)\} & \text{if } \ell < 0, \\ \chi_{\ell,0,0}(e,q), \text{ otherwise} \end{cases}$$

$$v_{\ell,1}(e,q) = \max_{c,a' \in B_{\ell,1,;}(e,a)} u(c) + \beta \left[\lambda\, w_{\ell',1}(q) + (1-\lambda)\, w_{\ell',0}(q)\right].$$

Given w^0, we obtain $w^1(w^0)$ via functions χ and v and taking expectations. This defines an operator and

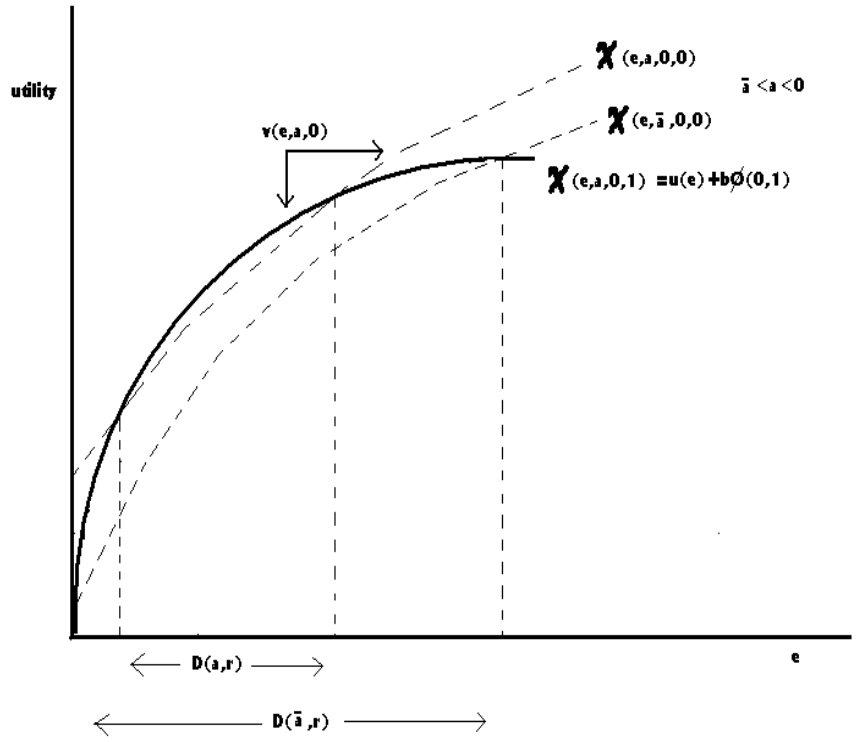
Proposition 1. *The household problem is a contraction.*

Characterizing Default Sets

$$D_\ell(q) = \{e \in E \ : v_{\ell,0}(e,q) \leq u(e) + \beta\, w_{0,1}(q)\}.$$

Proposition 2. *The default set is a closed interval, and $D_\ell(q)$ is non–increasing in ℓ and q. Moreover, the default probability $\mu[D_\ell(q)] = F[e^U_\ell(q)] - F[e^L_\ell(q)]$ is a continuous function of q.*

Sometimes too poor to default.



Unsecured Credit Industry

- Competitive firms with zero costs and free entry.
- Firms make loan of size ℓ at price q_ℓ.
- Firms only observe the total asset position and the credit history of the household (ℓ, h).
- Firms are one period lived.
- Households with a bad credit history are prevented from borrowing (neglect renegotiation).
- Profits (Law of large numbers)

$$\pi_\ell[a, q_\ell] = \; a \; [\; (1 - p_\ell) \; \ell \; \hat{q} \; - \; \ell \; \; q_\ell]$$

  where a is the measure of agents receiving a loan and $p(\ell)$ is the fraction of borrowers on loans of size ℓ who default.

- Zero Profit Condition:

$$q_\ell(p) = \hat{q} \; \; [1 - p_\ell]$$

A Word About Other Contracting Possibilities if firms lived longer

- Within a period state contingent contracts.

  There are no mechanisms to introduce incentive compatible contracts where agents report different endowments. This would be possible if people differed in risk aversion and lotteries were used as in Cole 1988.

- If loans were to last for more than one period.

  The introduction of, say, two period loans, is likely to change the default options of a household and hence the discounted price of these loans cannot be obtained by compounding the price of one period loans (note the shocks after the first period).

  We plan to expand the class of loans that are allowed in future work.

Equilibrium

Definition 1. *A vector of prices for assets positions $q^* \in Q$ is a competitive equilibrium for the unsecured credit economy if given q^*, household optimization induces default sets $D_\ell(q^*)$, and hence default probabilities $\mu[D_\ell(q^*)]$, so that profit maximizing firms that use prices q^* obtain zero profits.*

• It is obvious from the sequential nature of this definition that the key element in establishing the existence of equilibrium is finding a fixed point.

Proposition 3. *A competitive equilibrium exists (Brower).*

• A key element of the proof is the continuity of the $F[D_\ell(q)]$. It requires continuity of F so small changes in q do not induce large changes in default behavior.

Remarks I: Characteristics of Equilibria

• If for some $\widetilde{\ell}$, all households default, then $q_{\widetilde{\ell}} = 0$.

• There is an absolute level of debt that poses a natural lower bound on assets: that implied by having the maximum level of debt that could be paid by the luckiest household with the lowest possible interest rate. This is $\frac{\overline{e}}{1-\hat{q}}$. (polar opposite of the one in Aiyagari-94 and Athreya-99).

• Equilibria is non trivial. $q = 0$ is not an equilibrium (for fine enough grid).

• In any equilibrium there is always default (for fine enough grid). This follows from the fact that for the maximum debt level everybody defaults, and from Assumption 1.

Remarks II: Stationarity and Computation

• Note that we have not referred to a stationary measure of agents, we do not need to. Storage technology guarantees that prices are not affected by the distribution of agents: any equilibrium q is consistent with any initial condition.

• Sometimes we can use Tarski's FPT to prove existence for which we need an operator to be nondecreasing in q, that is a logical candidate to use to find equilibria by successive approximations.

1. Guess an initial discount price $q = \hat{q}$.

2. Given q, solve the h'hold problem. Find the value function and default intervals for every $\ell \in L$. We apprx v functions with splines, good for default intervals.

3. Compute the new q that yields zero profits. If equal go to 4. If different, update q and go to 2.

4. Compute the stationary distribution by successive approximations. Compute its relevant statistics.

Mapping the Model to Data

We use 1998 data and take all U.S. households except for

- Older than 65.
- In the top wealth quintile.
- With total debts lower than average yearly household earnings.

	Value
Average Earnings	100.
Total assets	153.
Negative assets	2.7
Percentage of Defaulters	1.2

Only some Default/Debt within our Theory

• [1] of those defaulters, about 2/3 file under Chapter 7.

• [2] of those defaulters there are various reasons adduced:

Reasons adduced for defaulting	
Loss of job	12.2%
Marital Distress	14.3%
Credit Missmanagement	41.3%
Health Care	16.4%
Lawsuits and Harassment	15.9%

• So we target .5% of defaulters as being within the realm of our theory.

• Also, we target our model to have half of the debt held by households with negative asset position. So we target 1.5% of earnings as the level of debt. (We are possibly underrepresenting debt (Gross and Souleles-01).

Our Theory is Limited .....

It is hard to get a lot of debt and a lot of default.

- If default is not painful, then many households will default and as a consequence borrowing limits are close to zero and there there is low debt.

- If default is painful then households take measures to avoid having to default (they save a lot) and even though borrowing limits are far from zero, there is little debt and very little default.

...., so we expand it

But we manage to achieve it with only one ex–ante type of consumers by means of

- Demographic turnover. Despite both defaulting and poor being unpleasent there are many young peple like that. This works by killing people and replacing them with others with zero assets.

- Concentration of bad luck. There are some events that ocurr once and no more, at least for a while. We are thinking of divorce for example.

  - Shocks to Preferences, $\theta \in \{\theta_1, \theta_2\}$, $\Gamma_{\theta,\theta'}$, shock to marginal utility: need to consume now.

  - Persistency of Shocks. $z = \{z_1, z_2\}$, $\Gamma_{z,z'}$, $F(e|z)$.

From the point of view of the theory, these extensions are cosmetic, and all the results go through.

The Baseline Model Economy

In the left column we have our baseline model economy. In the right the long run implications of the decrease of punishment period from 7 to six years. Note that all is as it should be.

Statistic	Data	Model
Avg. Duration of Bad Credit History	10 years	10 years
Earnings	100.	100.
Earnings Gini	.44	.39
Total assets	153.	154.
Wealth Gini	.63	.55
Negative assets	-1.5	-1.5
Defaulters	.50%	.56%
Delinquent	?	3.6%

Change Policy: Forget Bad Credit Faster

Statistic	10 Year	5 Year
Prob$(h' = 0 \vert h = 1)$	10.%	20.%
Earnings	100.	100.
Earnings Gini	.39	.39
Total assets	152.52	154.54
Wealth Gini	.549	.542
Negative assets	-1.52	-1.29
Defaulters	.562%	.697%
Delinquent	3.64%	2.81%
Amount Defaulted	.253	.272

- Less commitment so
  - Less credit
  - More Defaulters
  - More Savings
  - More Default
  - Less households with Bad Credit

But not by much. Population turnover is key.

Conclusions

• We have a computable model of default where it is the poor people that default, that resembles the legal system, where the loan industry is in equilibrium, capable of replicating some of the main U.S. bankruptcy facts, and useful for answering questions about policy.

Next?

- Explore the implications of the coexistence of one period loans with multiperiod loans.
- Use this structure to better assess the implications of changing the legal definitions of bankruptcy.
- Evaluate a variety of policy changes. Need to compute transitions.
- Expand the model into questions of monetary policy. How changes in real interest rate propagate throughout the economy. Should we see changes in real activity? Increases in bankruptcies?
- Apply these techniques to housing issues.