# Borrowing Constraints, Search, and Life-Cycle Inequality

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# Background

Underlying idea:

- Does wealth have a direct effect on earnings?
- Expanding directed search theory to borrowing constrained risk averse workers suggests this could be the case
  $\rightarrow$ low-wealth workers may choose to apply for lower-paying jobs with higher job-finding probability

Typically, these effects appear small but here they are larger because

- Agents face a tradeoff between precautionary savings and human capital investment
- Lower-wealth individuals are more desperate to increase savings
  $\implies$ Low-wealth individuals both apply for lower-paying jobs, and while in these jobs they invest less in human capital

# This Paper

Question:

- How do differences in **wealth**, **human capital**, and **learning ability** at **labor market entry** impact life-cycle
  - job search behavior?
  - human capital accumulation?
  - consumption?
- What channels are quantitatively important?

Method:

- Estimate a quantitative life-cycle model with directed search
  - Heterogeneity: wealth, human capital and learning ability and
  - Frictions: Credit, insurance and search
- Decompose the effect of each of the channels on life-cycle inequality

# Contribution / Literature Review

Adds insights from search theory to literature on the effects of initial conditions on life-cycle outcomes

- Huggett, Ventura, Yaron (2011)

Contributes to literature linking initial wealth to earnings through directed search with risk-averse agents

- Herkenhoff, Phillips, Cohen-Cole (2016), Eeckhout, Sephalasari (2019)

Contributes to literature on income risk and earnings processes

- E.g. Guvenen (2009), Heathcote, Storesletten, Violante (2014)

# Outcomes

Estimated model yields the following outcomes:

- Low initial wealth (first quintile) causes individuals to have
  $\rightarrow$ 6.94% lower average life-time wages
  $\rightarrow$ 4.29% lower average human capital
- Effect is stronger in the beginning of the working life

# Model

# Environment

- Life-cycle model with deterministic retirement (after $T$ period)
- Directed and on-the-job search
- Ben-Porath human capital investment: $H(h, I, \tau) = I(h\tau)^{\alpha}$.
  Stochastic human capital depreciation.
- Age-specific borrowing constraint ($\underline{a}'_t$)
- Agents are born unemployed without unemployment insurance and draws an initial state $(a_0, h_0, l_0)$
- Wages are set at a piece-rate $\mu$, which is constant over the job duration s.t.
  $Income = \mu(1-\tau)h$, Firm's profit $= (1-\mu)(1-\tau)h$
- Agents receive unemployment benefits after job loss, which are stochastically lost with probability $\gamma$ each period
  UI income: $b_{UI} = \min\{b\mu(1-\tau)h, \bar{b}\}$
  no UI income: $b_L \leq b_{UI}$

# Households' problem

States:

- Assets, human capital, learning ability, age: $(a, h, l, t)$
- Employment states: employed, unemployed with/without UI

Choices:

- Fraction of time $(\tau)$ spent on human capital investment
- Savings in each period
- Which submarket $\theta(\mu; a, h, l, t)$ to search for job in (when unemployed)

# HHS Value functions – consumption/savings

Unemployed (with UI):

$$U_t\left(b_{UI}, a, h, \ell\right) = \max_{c, a' \geq \underline{a}'_t} u\left(c\right) + \nu + \beta E\left[\left(1-\gamma\right) R^U_{t+1}\left(b_{UI}, a', h', \ell\right) + \gamma R^U_{t+1}\left(b_L, a', h', \ell\right)\right]$$

$$\text{s.t. } c + a' \leq \left(1 + r_F\right) a + b_{UI}$$

$$h' = e^{\epsilon'} h, \ \epsilon' \sim N\left(\mu_\epsilon, \sigma_\epsilon\right)$$

Employed:

$$W_t\left(\mu, a, h, \ell\right) = \max_{c, a' \geq \underline{a}'_t, \tau \in [0,1]} u\left(c\right) + \beta E\left[\left(1-\delta\right) R^E_{t+1}\left(\mu, a', h', \ell\right) + \delta R^U_{t+1}\left(b_{UI}, a', h', \ell\right)\right]$$

$$\text{s.t. } c + a' \leq \left(1 + r_F\right) a + \mu\left(1-\tau\right) h$$

$$b_{UI} = min\{max\{b\left(1-\tau\right)\mu f\left(h\right), b_L\}, \bar{b}\}$$

$$h' = e^{\epsilon'}\left(h + \ell\left(h\tau\right)^\alpha\right), \ \epsilon' \sim N\left(\mu_\epsilon, \sigma_\epsilon\right)$$

Retired:

$$U_{T+1} = W_{T+1} = U_R\left(a\right) = \max_{a'} u\left(c\right) + \beta\left(1-\delta_D\right) U_R\left(a'\right)$$

$$\text{s.t. } c + a' = \left(1 + r\right) a + b_{Ret}$$

# HHS Value functions – job search

Search from unemployment:

$$R_t^U\left(b_{UI}, a, h, \ell\right) = \max_{\mu'} P\left(\theta_t\left(\mu', a, h, \ell\right)\right) W_t\left(\mu', a, h, \ell\right) + \left(1 - P\left(\theta_t\left(\mu', a, h, \ell\right)\right)\right) U_t\left(b_{UI}, a, h, \ell\right)$$

Search from employment:

$\lambda_E :=$ probability of being allowed to search on the job

$$R_t^E\left(\mu, a, h, \ell\right) = \max_{\mu'} \lambda_E P\left(\theta_t\left(\mu', a, h, \ell\right)\right) W_t\left(\mu', a, h, \ell\right) + \left(1 - \lambda_E P\left(\theta_t\left(\mu', a, h, \ell\right)\right)\right) W_t\left(\mu, a, h, \ell\right)$$

# Firms' problem

- Firms choose which submarket $(\mu, a, h, l)$ to post a vacancy in (at cost $\kappa$)
- By free entry:
  $\kappa = q(\theta(\mu, a, h, l)) \times J_t(\mu, a, h, l)$

Value of a filled vacancy to a firm:

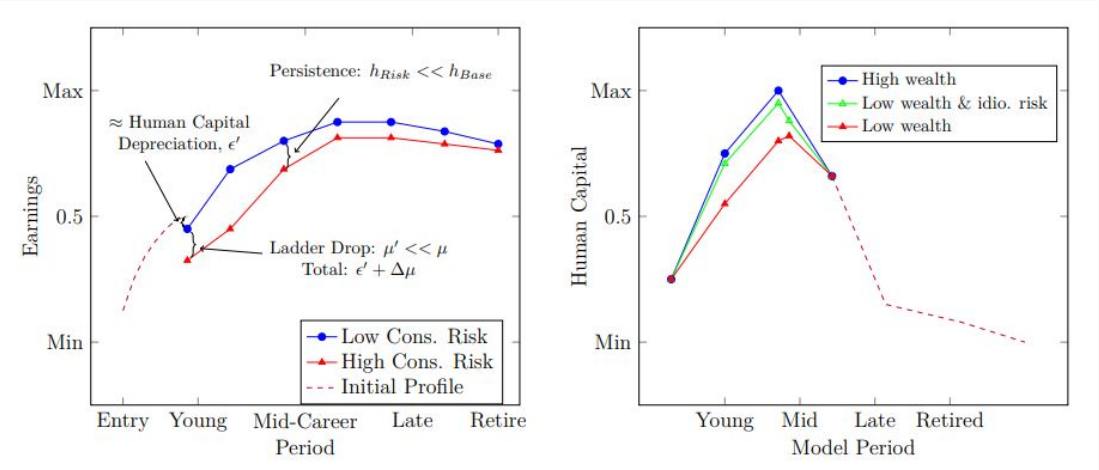
$$J_t(\mu, a, h, l) = (1-\mu)(1-\tau)h$$
$$+ \beta E[(1-\delta)(1-\lambda_E P(\theta_{t+1}(\mu', a', h', l)))J_{t+1}(\mu, a', h', l)]$$

where $h' = e^{\epsilon'}(h + l(h\tau)^{\alpha}), \epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$

# The Earnings Process

The model speaks to the large empirical literature on transitory vs permanent earnings shocks, but here the nature of shocks is *endogenous*

- Wealthy people hit by a separation shock stay in unemployment until they find another good job
  $\rightarrow$ shock is transitory
- Poor people hit by a separation shock will search for a low-paying job that is easy to find
  $\rightarrow$ shock is permanent
- In Calibrated model: 53.43%, 42.77%, 34.93% for 1st, 3rd, 5th wealth quintile workers



# Estimation

# Estimation

- Indirect Inference (conditional MoM):
  - Select reduced-form analogs to structural model.
  - Objective: match coefficients for regressions with data & simulated data.
  - Minimize by changing structural parameters.
- Basic approach:
  - Estimate effect of wealth on job search behavior.
  - Match age-earnings regressions by initial heterogeneity.
  - Match observable marginal distributions.

# Empirical Preliminaries

- Quarterly period length, ages 23-54
- Model parameters: $\sigma = 2, r_F = 0.012$
- Power utility + unemp leisure: $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma} + \nu$
- HC Evolution: $h' = e^{\epsilon}(h + H(h, \ell, \tau)) = e^{\epsilon}(h + \ell \times (h\tau)^{\alpha})$
- Natural borrowing constraint: $\underline{a}_t = \sum_{j=t}^{T} \frac{b_L}{(1+r_F)^j}$
- Initial conditions:
  - $(a_0, h_0, \ell) \sim LN(\psi, \Sigma)$
  - Means and variances: $\mu_A, \mu_H, \mu_L, \sigma_A^2, \sigma_H^2, \sigma_L^2,$
  - Correlations: $\sigma_{AH}, \sigma_{AL}, \sigma_{HL}$

# Estimation Strategy

- Target labor market behavior of low-wealth individuals for inference on $b_L$ and $a_t$.
- Target earnings growth within job over life-cycle for inference on human capital accumulation $(\alpha_H, \mu_\epsilon, \sigma_\epsilon)$.
- Target earnings regression (Mincer) by initial heterogeneity:
  - Intercepts & slopes by initial wealth quintile $(\sigma_{AH}, \sigma_{AL})$.
  - Intercepts & slopes by standardized test (AFQT) quintile $(\sigma_{HL})$.
- Marginal Distributions:
  - Wealth: Distribution of pre-labor market wealth.
  - Human Capital: Distribution of initial earnings.
  - Learning Ability: Slope and variance of earnings growth.

# Estimation of some key parameters

Human capital depreciation

- Recall the human capital law of motion:
  $h' = e^{\epsilon'}(h + I(h\tau)^{\alpha}), \epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$
  $\rightarrow$ need to estimate the parameters $\mu_\epsilon, \sigma_\epsilon$
- Changes in earnings in the model are due to (i) human capital investment (ii) job-switches or (iii) shocks to human capital
- Workers near retirement age should not make any new investments in HC, so the earnings process of job-stayers near retirement can be used to estimate $\mu_\epsilon, \sigma_\epsilon$ (as in Huggett, 2011)

# Estimation of some key parameters

The borrowing constraint:

- Recall that the model assumes a natural borrowing constraint: $\underline{a}_t = \sum_{j=t}^{T} \frac{b_L}{(1+r)^j}$
- But still need to estimate $b_L$

Strategy:

- Estimate by regressing re-employment wage on UI for different wealth quantiles
  $\log(W_i, j+1) = \sum_{q=1}^{5} \beta_1^q \times 1_{i,q} \log(UI_i) + X_{1,i}\beta_2 + X_{2,i}\beta_3 + \epsilon_{i,j+1}$
- If higher UI leads to higher earnings in subsequent job it means that the worker was borrowing constrained
  $\rightarrow$ The response of earnings following unemployment by wealth levels identifies the borrowing constraint by wealth level
  Strategy: match $\beta_1^1$ in simulated and real data regressions

# Estimation of some key parameters

The utility from leisure

- Recall the value of leisure parameter $\nu$
  $U_t = u(c) + \nu + [...]$
- $\nu$ is identified using the assumption that workers at all wealth levels weakly prefer working to not working
- Can then show that wealthy individuals will apply to jobs with wage rate near the value of leisure

# Identifying $b_L$ and $\nu$ – intuition

First panel:

- Agents near borrowing constraint apply for jobs with higher piece-rates

Second panel:

- Wealthy individuals apply for jobs with piece-rates near the value of leisure

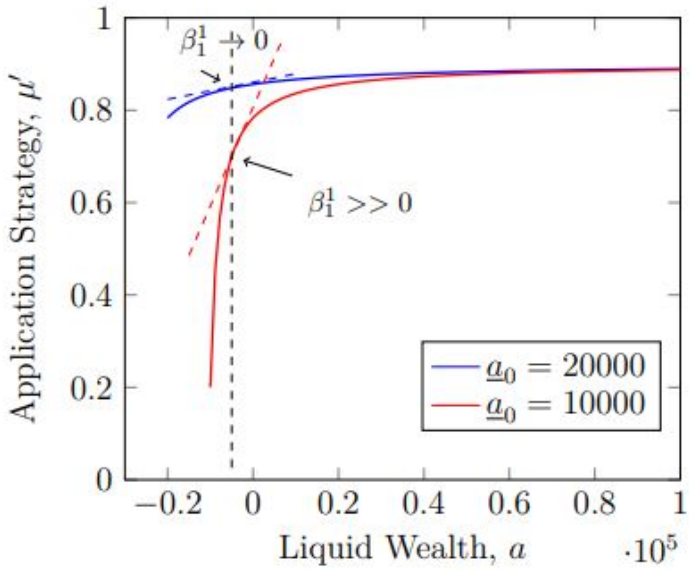



Figure 3.1: Identification of $\underline{a}_t$ and $b_L$

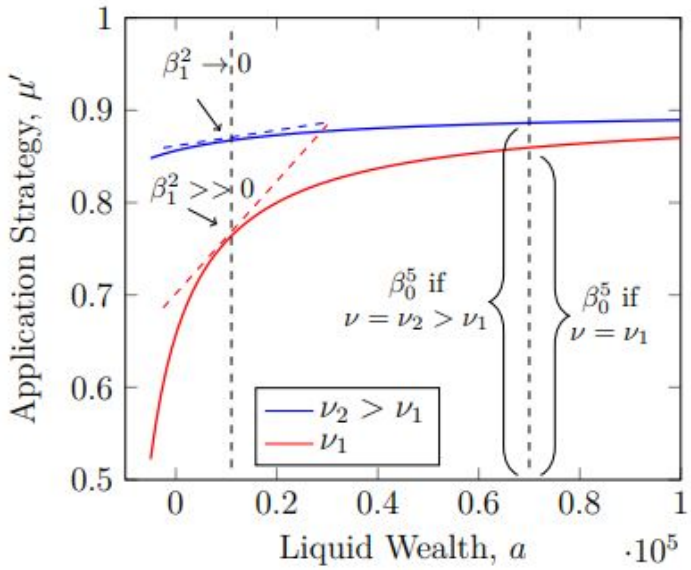



Figure 3.2: Identification of $\nu$

# Further Identification

Initial wealth distribution: $(\mu_A, \sigma_A)$

- Liquid wealth prior to entering labor market (PSID)

Initial distribution of HC and learning ability: $(\mu_H, \sigma_H), (\mu_L, \sigma_L)$

- Intuition: for a given wealth level, long-run differences in earnings are driven almost purely by learning ability.

Correlations $(\sigma_{AL}, \sigma_{AH}, \sigma_{HL})$

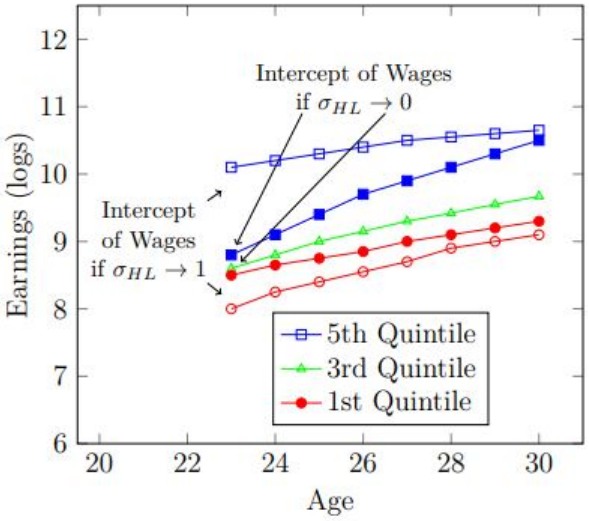



Figure 3.3: Identification of $\sigma_{HL}$

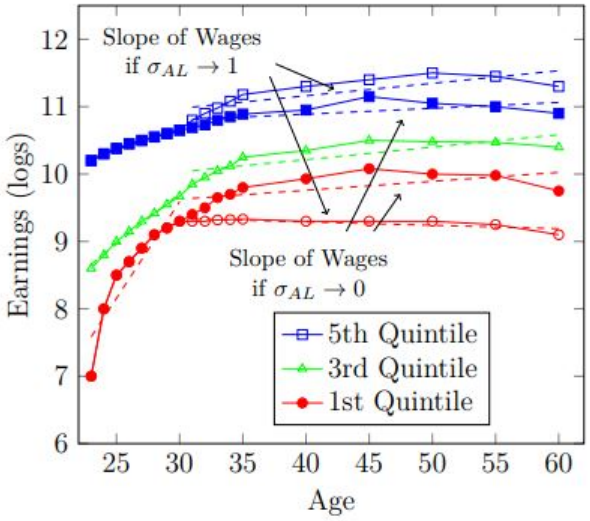



Figure 3.4: Identification of $\sigma_{AL}$

# Findings

# Questions asked of the quantitative model

The estimated model is used to address three questions:

- How do job mobility and human capital accumulation affect life-cycle earnings dynamics?
- How much inequality is caused by differences in initial conditions, and which initial conditions are the primary causes?
- How do employment and consumption risk affect job search and human capital?

# Findings

- Initial states explain 53.15% of life-cycle variation in income
- Initial wealth generally explains little of variation in earnings – on average 1.65% of income, 5.09% of consumption
  $\rightarrow$ Learning ability and initial human capital are more important

| Source | Income (%) | Consumption (%) | $h$ (%) | $\mu$ (%) |
|---|---|---|---|---|
| *Initial Conditions* | | | | |
| Wealth | 1.65 | 5.09 | 1.97 | 1.54 |
| Human Capital | 8.36 | 10.59 | 12.75 | 0.49 |
| Learning Ability | 46.02 | 49.51 | 47.22 | −0.01 |
| Combined | 53.15 | 59.90 | 62.76 | 3.03 |
| *Realized over Life-Cycle* | | | | |
| Residual of Combined | 46.85 | 40.10 | 37.24 | 96.97 |

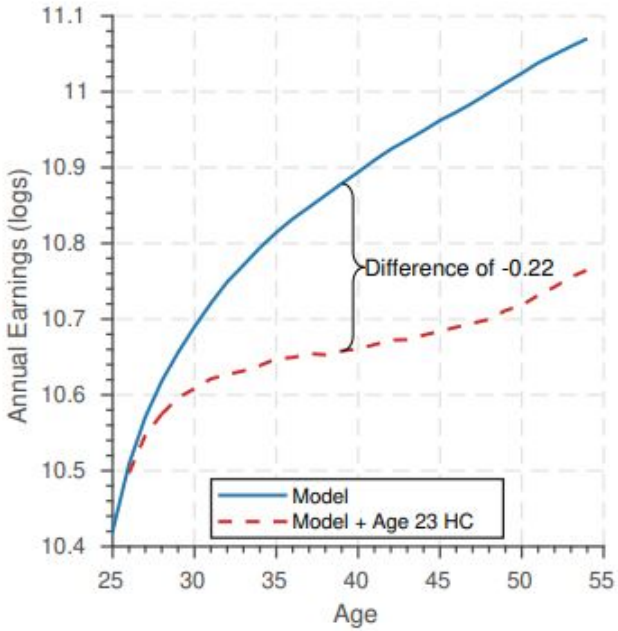
- Argues that wealth is more important for inequality amongst those in the first wealth quantile

Table 4.2: Asymmetric Effects of Wealth on Average Outcomes

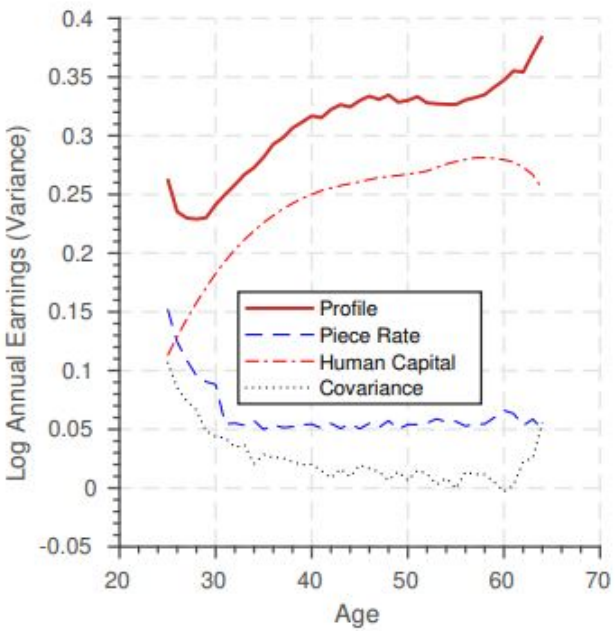
| | Income | | | | $h$ | | | | $\mu$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Counterfactual | 1st | 3rd | 5th | Ave | 1st | 3rd | 5th | Ave | 1st | 3rd | 5th | Ave |
| $\sigma_A = 0$ | 6.94 | 1.70 | −0.82 | 1.68 | 4.29 | 1.87 | −1.94 | 1.01 | 4.42 | 0.57 | 0.61 | 1.44 |
| $\sigma_H = 0$ | 6.84 | 1.70 | −4.84 | 0.55 | 8.71 | 2.48 | −3.92 | 1.63 | 1.08 | 0.45 | −0.98 | 0.23 |
| $\sigma_L = 0$ | 18.35 | −3.15 | −21.95 | −5.74 | 25.90 | 1.65 | −14.68 | 0.46 | 0.76 | −0.17 | −1.32 | −0.27 |

# Findings

- Differences in human capital explain most of variation in earnings over the life-cycle
- Early in life the differential piece-rates are also important




(a) Earnings with and without human capital growth

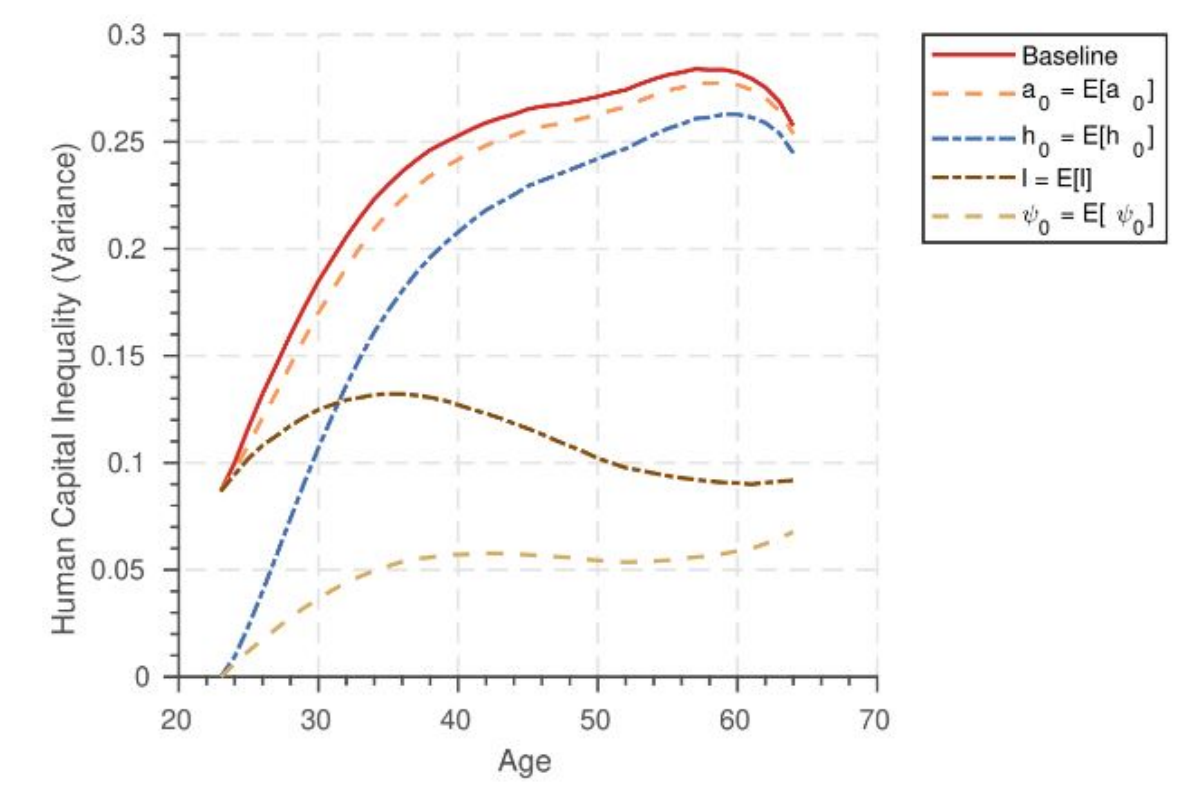



(b) Variance decomposition

# The contributors of inequality over the life-cycle

Initial wealth explains a significant part of inequality early in life

→ Conclusion(?): wealth levels are converging quite quickly over the life-cycle



# Persistence of unemployment shocks by wealth

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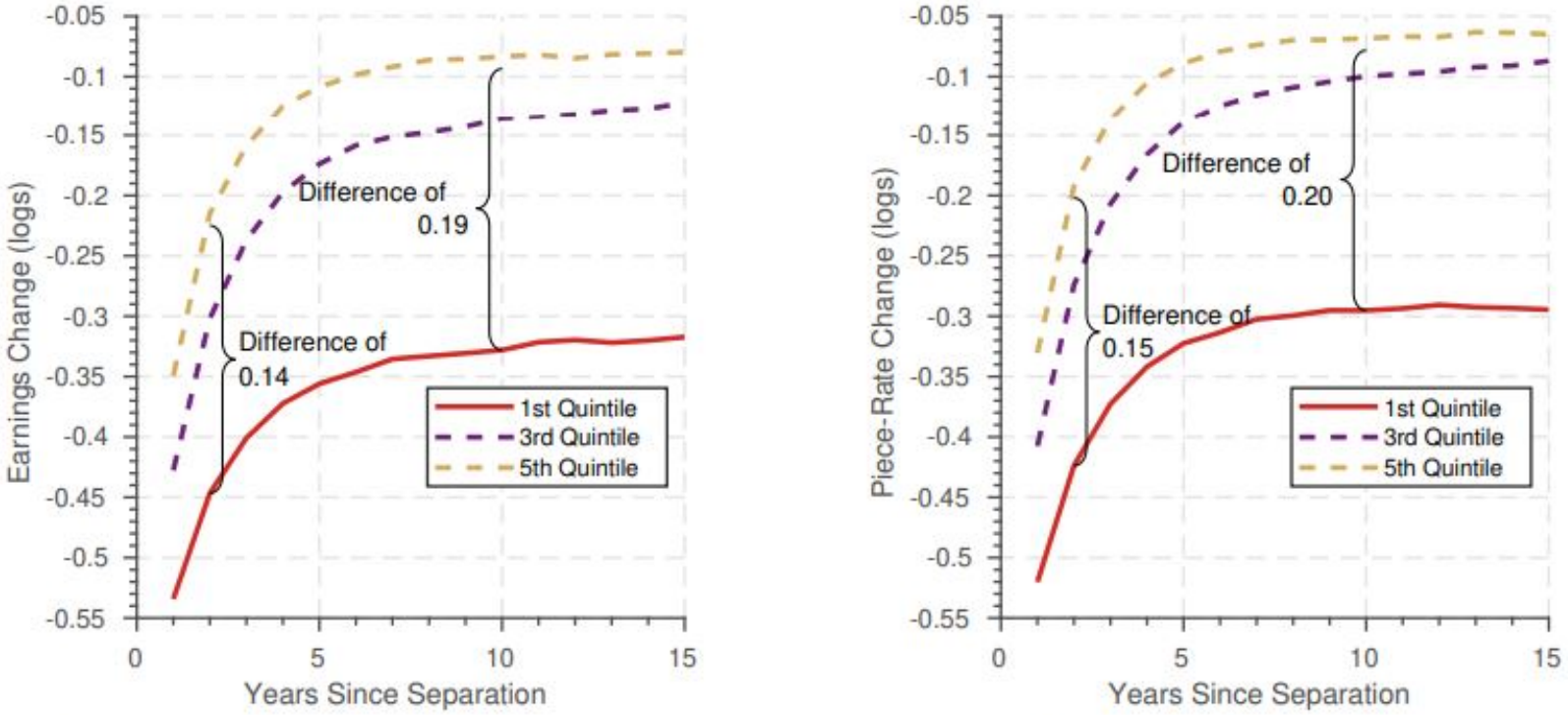



(a) Unemployment scarring by wealth

(b) Job ladder effect by wealth

Figure 4.2: Unemployment Scarring Decomposition

# How Risk affects Inequality

Want to assess how much of the effect is due to risk (and hence benefits from the search framework)

[TBC]

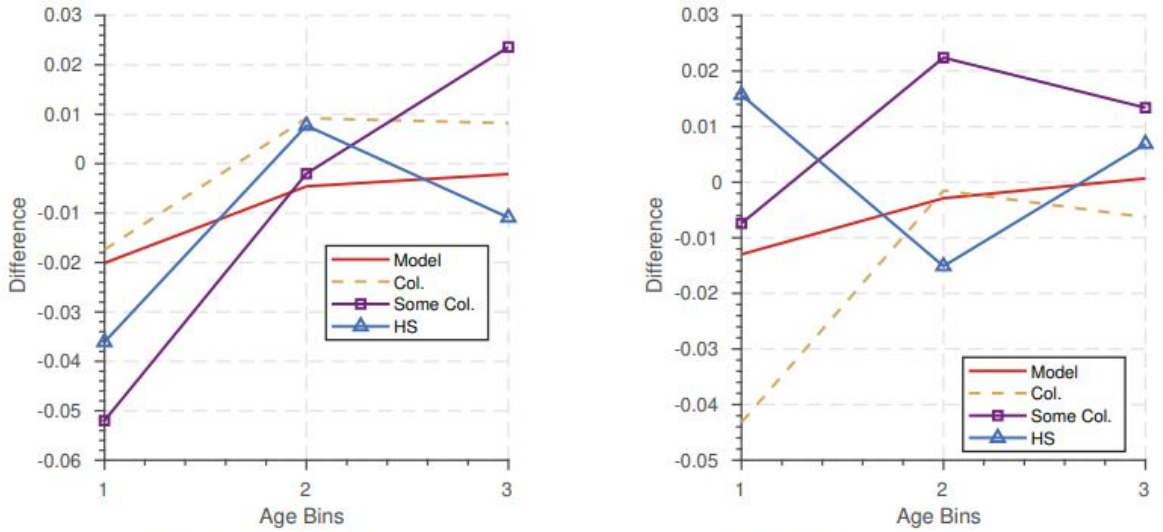
# Evidence

# Dynamics of earnings

Use PSID and NLSY to show that early-career earnings profiles by wealth level is consistent with the model

Assumption:

- Workers who have the same education level in the data have the same learning ability and initial human capital in expectation




(a) Diff. in Growth, Q1-Q3 (b) Diff. in Growth, Q1-Q2

Figure 5.1: Difference in Initial Earnings Growth by Wealth

# Conclusion and comments

- Model shows that wealth has a significant effect on early-life inequality, but diminishingly so. Would be good to know if this is due to
  - Asset holdings converging quickly over the life-cycle

  And if so
  - Is this consistent with the data?
- A drawback of the paper is that firms observe assets s.t. firms' make "discriminate" by wealth
  - Would be nice to see whether this feedback makes effect of wealth on earnings stronger or weaker.
- A key parameter of the model is the borrowing constraint, which is modelled in a very round-about way
  - Could perhaps try to match this in a more realistic way