Negotiation

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Modeling a Match: Converntional Ways

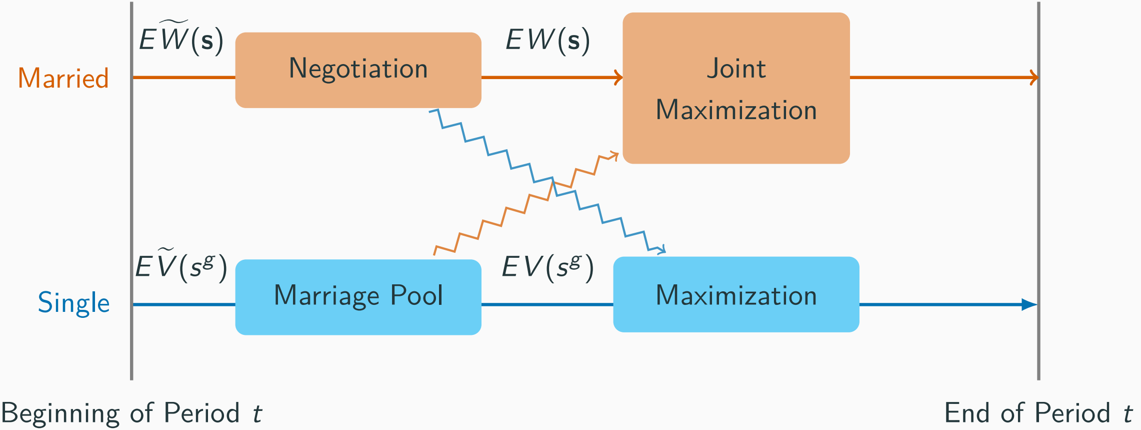
- There are several ways of modeling a match of two individuals
  - represent them by a single utility function (unitary model)
  - each has her own utility function but Pareto weight is fixed over time
  - each has her own utility function and Pareto weight changes according to the outside values (Limited commitment)
- In the first and second formulation,
  - No need to keep track of Pareto weight as a state variable
  - the resource allocation within the match is fixed over time by fixed Pareto weight or equivalence scale
  - the match dissolution happens whenever at least one of them finds her outside values exceeds inside value

Modeling a Match: Converntional Ways

- There are several ways of modeling a match of two individuals
  - represent them by a single utility function (unitary model)
  - each has her own utility function but Pareto weight is fixed over time
  - each has her own utility function and Pareto weight changes according to the outside values (Limited commitment)
- In the third formulation,
  - allocation within a match and dissolution is a result of negotiation
  - need to keep track of Pareto weight as a state variable
  - they may find a new Pareto weight that can sustain a match through negotiation even when one's outside value exceeds her inside value

Modeling a Match: Our Approach

- Limited commitment can endogenize both allocation within a match and dissolution
- But keeping track of Pareto weights is computationally burden
- Our negotiation protocol maintains both endogenous allocation choice and dissolution outcome through negotiation, while no need to keep track of Pareto weight
- Specifically, they negotiate every period with additive utility shocks to the potential outcomes (remains in a match or dissolved)
- To describe out approach, consider a situation in which a married couple decides their allocation or getting divorce.


Married
Single
$E\widetilde{W}(\mathbf{s})$
Negotiation
$EW(\mathbf{s})$
Joint Maximization
$E\widetilde{V}(s^g)$
Marriage Pool
$EV(s^g)$
Maximization
Beginning of Period t
End of Period t

Negotiation in a married couple

- Potentially two-stage game
  1. Choose *Satisfied (S)* or *Challenge (C)*
     - If both choose S, set $\lambda = \lambda^{SS}$ and stay married
     - If both choose C, get divorce.
     - If one of them chooses C, go to the next stage.
  2. The one who chooses C offer new λ, and the other decides whether accept or reject (=divorce) it
- *Challenge* and high λ offer may result in better allocations for the Challenger, but it also increases the risk of being rejected and divorce.

Negotiation in a married couple

		Husband	
		Satisfied	Challenge
Wife	Satisfied	λ^{SS}	λ^{m} or Div.
	Challenge	λ^{f} or Div.	Divorce

- First, they choose *Satisfied* or *Challenge*

Negotiation in a married couple

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Wife	Satisfied	λ^{SS}	λ^{m} or Div.
	Challenge	λ^{f} or Div.	Divorce

Satisfied

- First, they choose *Satisfied* or *Challenge*
  - if both Accept, set PW $\lambda = 1/2$

Negotiation in a married couple

		Husband	
		Satisfied	Challenge
Wife	Satisfied	λ^{SS}	λ^{m} or Div.
	Challenge	λ^{f} or Div.	Divorce

Challenge

- First, they choose *Satisfied* or *Challenge*
  - If both Challenge, they divorce

Negotiation in a married couple

		Husband	
		Satisfied	Challenge
Wife	Satisfied	λ^{SS}	λ^m or Div.
	Challenge	λ^f or Div.	Divorce

Satisfied Challenge

- First, they choose *Satisfied* or *Challenge*
  - Now suppose wife chooses *Challenge* but husband selects *Satisfied*,
- Second, wife offers λ and husband decides *Accept* or *Reject* it.
  - husband receives new PW (λ^f) offer from wife, and decides accept or reject the offer
  - λ^f is chosen so that it maximizes the expected value of the wife

Negotiation in a married couple

- We summarize the exact schedule of the negotiation process:
  - 1. Before private additive util shocks realize, decide λ to be offered
  - 2. Learn shocks of their own, but cannot observe spouse's shocks, and decide which to choose; Satisfied or Challenge
  - 3. If go to the second step, Accept or Reject proposed PW with the shock values
- In what follows,
  - EW and EV are end-of-period value functions of being married and single (after negotiation, before solving allocation problem)
  - $E\widetilde{W}$ and $E\widetilde{V}$ are start-of-period values (before negotiation)
  - $\mathbf{s}$ summarizes the state variables relevant for a married household, while s^g is the state variables of an individual with gender g

Choice of λ to offer

- Before they receive additive utility shocks ϵ, they decide what λ to offer if challenges
- Let the husband's Acceptance policy function when wife offers λ^f as $\mathbb{1}^{A,m}(\mathbf{s}, \lambda^f, \boldsymbol{\epsilon}^m)$.
- Then, a wife's optimal choice λ^f is a solution of the following problem:

$$\lambda^{f*}(\mathbf{s}) = \arg\max_{\lambda^f} \Big\{ \mathbb{E}\big[\mathbb{1}^{A,m}(\mathbf{s}, \lambda^f, \boldsymbol{\epsilon}^m) \big(EW^f(\mathbf{s}, \lambda^f) + \epsilon^f_M \big) \\ + \big(1 - \mathbb{1}^{A,m}(\mathbf{s}, \lambda^f, \boldsymbol{\epsilon}^m) \big) \big(EV^f(s^f) + \epsilon^f_S \big) \big] \Big\},$$

- where ϵ^f_{ms} is the additive util shock to wife's values when her marital status is *ms*.

Choice in the First Stage

- Each chooses Satisfied or Challenge in the first stage
- They received their private additive utility shocks, but cannot observe spouse's shocks
- Let the wife's expected values conditional on choosing Satisfied and Challenge as $\widehat{W}^{S,f}(\mathbf{s}, \boldsymbol{\lambda}, \boldsymbol{\epsilon})$ and $\widehat{W}^{C,f}(\mathbf{s}, \boldsymbol{\lambda}, \boldsymbol{\epsilon})$.
- Wife's expected value of choosing Satisfied is

$$\widehat{W}^{S,f}(\mathbf{s}, \boldsymbol{\lambda}, \boldsymbol{\epsilon}) = \underbrace{\mathbb{1}^{S,m}(\mathbf{s}, \boldsymbol{\lambda}, \boldsymbol{\epsilon}^m)\Big(EW^f(\mathbf{s}, 1/2) + \epsilon_M^f\Big)}_{\text{husband Satisfied}}$$
$$+ \underbrace{\Big\{1 - \mathbb{1}^{S,m}(\mathbf{s}, \boldsymbol{\lambda}, \boldsymbol{\epsilon}^m)\Big\}\Big[\max\Big\{EW^f(\mathbf{s}, \lambda^m) + \epsilon_M^f, EV^f(s^f) + \epsilon_S^f\Big\} - \kappa\Big]}_{\text{husband Challenge}}$$

Choice in the First Stage

- In case if wife chooses challenge, her expected value is

$$\widehat{W}^{C,f}(\mathbf{s},\boldsymbol{\lambda},\boldsymbol{\epsilon}) = \underbrace{\mathbb{1}^{S,m}(\mathbf{s},\boldsymbol{\lambda},\boldsymbol{\epsilon}^m)\mathbb{1}^{A,m}(\mathbf{s},\lambda^f,\boldsymbol{\epsilon}^m)\Big(EW^f(\mathbf{s},\lambda^f)+\epsilon^f_M\Big)}_{\text{husband Satisfied and Accept}}$$
$$+\underbrace{\Big\{1-\mathbb{1}^{S,m}(\mathbf{s},\boldsymbol{\lambda},\boldsymbol{\epsilon}^m)\mathbb{1}^{A,m}(\mathbf{s},\lambda^f,\boldsymbol{\epsilon}^m)\Big\}\Big(EV^f(s^f)+\epsilon^f_S\Big)}_{\text{otherwise}}-\kappa$$

- where κ denotes the utility cost of Challenge.

Choice in the First/Second Stage

- The policy function of choices at the first stage, Satisfied/Challenge is

$$\mathbb{1}^{S,f}(\mathbf{s}, \boldsymbol{\lambda}, \boldsymbol{\epsilon}^f) = \begin{cases} \mathit{1} \text{ if } \widehat{W}^{S,f}(\mathbf{s}, \boldsymbol{\lambda}, \boldsymbol{\epsilon}^f) \geq \widehat{W}^{C,f}(\mathbf{s}, \boldsymbol{\lambda}, \boldsymbol{\epsilon}^f) \\ \mathit{0} \text{ otherwise} \end{cases}$$

- and the policy function of choices at the second stage if husband challenges, Accept/Reject is

$$\mathbb{1}^{A,f}(\mathbf{s}, \boldsymbol{\lambda}, \boldsymbol{\epsilon}^f) = \begin{cases} \mathit{1} \text{ if } EW^f(\mathbf{s}, \lambda^{m*}) + \epsilon_M^f \geq EV^f(s^f) + \epsilon_S^f \\ \mathit{0} \text{ otherwise} \end{cases}$$

Choice in the First/Second Stage

- Thus, the start-of-period expected value of a wife is

$$E\widetilde{W}^f(\mathbf{s}) = \mathbb{E}\Big[\mathbb{1}^{S,f}(\mathbf{s},\boldsymbol{\lambda})\widehat{W}^{S,f}(\mathbf{s},\boldsymbol{\lambda},\boldsymbol{\epsilon}^f) + \{1 - \mathbb{1}^{S,f}(\mathbf{s},\boldsymbol{\lambda})\}\widehat{W}^{C,f}(\mathbf{s},\boldsymbol{\lambda},\boldsymbol{\epsilon}^f)\Big]$$

- where the expectation is taken over ϵ's.

- The husband's expected value functions and policy functions are defined symmetrically.

- Note that start-of-period expected value functions/policy functions do not depend λ as it is determined during the negotiation process (**s** does not contain λ)