# Course in Heterogeneity: Econ 081 

VIII: Wealth, Wages, and Employment

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Based on joint work with Per Krusell and Jinfeng Luo

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- Price of those skills (evolution of the skill premia that very recently seems to have switched back)


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- Relate to Business Cycles Version of Aiyagari (1994)


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- We use the volatility of gross flows to estimate the extent of wage rigidity.


## Literature

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- Especially Eeckhout and Sepahsalari (2018), Chaumont and Shi (2022), Griffy (2021).


## What are the uses?

- The study of Business cycles including gross flows in and out of employment, unemployment and outside the labor force
- Policy analysis where now risk, employment, wealth (including its distribution) and wages are all responsive to policy.
- Get some insights into the extent of wage rigidity
- Life-Cycle versions of these ideas (under construction) will allow us to assess how age dependent policies fare.

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- Outside Steady State Employers commit to a wage schedule $w(z)$ that depends on the aggregate state.


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- With partial wage rigidity the model fares reasonably well with the data. A few things still to improve. (Excessive Job-to-JOB transitions)
- Similar behavior to that in the Shimer/Hagedorn-Manowski debate. Here we can try to move towards an accommodation of both points of view.

A Brief Look At Data: Relevant Volatility Properties in U.S.

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| Investment | 25 | 4.88 | 0.90 | NIPA |

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- General equilibrium: Workers own firms.


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Total job destruction is $\delta$.

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(5) Job Matching : $M(V, U)$ : Some vacancies meet some unemployed job searchers. A match becomes operational the following period. Job finding and job filling rates $\psi^{h}(\theta)=\frac{M(V, U)}{U}, \psi^{f}(\theta)=\frac{M(V, U)}{V}$.

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- Problem of the employed: (Standard)

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\begin{aligned}
V^{e}(a, w)= & \max _{c, a^{\prime}} u(c)+\beta\left[(1-\delta) V^{e}\left(a^{\prime}, w\right)+\delta V^{u}\left(a^{\prime}\right)\right] \\
\text { s.t. } & c+a^{\prime}=a(1+r)+w, \quad a \geq 0
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\text { s.t. } & c+a^{\prime}=a(1+r)+w, \quad a \geq 0
\end{aligned}
$$

- Problem of the unemployed: Choose which wage to look for

$$
\begin{aligned}
V^{u}(a)= & \max _{c, a^{\prime}, w} u(c)+\beta\left\{\psi^{h}[\theta(w)] V^{e}\left(a^{\prime}, w\right)+\left[1-\psi^{h}[\theta(w)]\right] V^{u}\left(a^{\prime}\right)\right\} \\
\text { s.t. } & c+a^{\prime}=a(1+r)+b, \quad a \geq 0
\end{aligned}
$$

$\theta(w)$ is an equilibrium object

## Characterization of a worker's decisions

- Standard Euler equation for savings

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u_{c}=\beta(1+r) E\left\{u_{c}^{\prime}\right\}
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- Up to a certain level of wealth, richer households apply to higher wages. After that, it seems not. Consistent with theory

Worker's wage application decision


## Worker's saving decision



## Firms Post vacancies: Choose wages \& filling probabilities

- Value of wage-w job: uses constant $\bar{k}$ capital that depreciates at rate $\delta^{k}(\Omega=\bar{k})$

$$
\Omega(w)=z-\bar{k} \delta^{k}-w+\frac{1-\delta^{f}}{1+r}\left[\left(1-\delta^{h}\right) \Omega(w)+\delta^{h} \Omega\right]
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- Affine in $w: \quad \Omega(w)=\left[z+\bar{k}\left(\frac{1-\delta^{f}}{1+r} \delta^{h}-\delta^{k}\right)-w\right] \frac{1+r}{r+\delta^{f}+\delta^{h}-\delta^{f} \delta^{h}}$

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$$

## Standard Stationary Equilibrium

- Functions $\left\{V^{e}, V^{u}, \Omega, g^{\prime e}, g^{\prime \mu}, w^{u}, \theta\right\}$, an interest rate $r$, and a stationary distribution $x$ over $(a, w)$, s.t.


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$$

(3) An interest rate $r$ clears the asset market

$$
\int_{A \times(W \cup 0)} a d x=\int_{A \times(W \cup 0)} \Omega(w) d x+\mu^{0} \bar{k} .
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## Summary: Properties of Exogenous Quits Model

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- It is almost like a two-agent model (employed, unemployed) of Pissarides despite curved utility and savings


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- Adds a (smoothed) quitting motive so that conditional on wealth, high wage workers quit less often.
- Firms may want to pay high wages to retain workers.


## Endogenous Quits Model: Time-line

(1) Workers enter period with or without a job: $\{e, u\}$.
(2) Production occurs and consumption/saving choice ensues:
(3) Exogenous job/firm destruction happens.
(4) Quitting:

- The employed, e, draw shocks $\left\{\epsilon^{e}, \epsilon^{u}\right\}$ and make quitting decision. Job losers cannot search this period.
© Search: New or Idle firms post vacancies. Choose $\{w, \theta\}$.
(6) Matches occur


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$V^{e}$ and $V^{u}$ are values after quitting decision as described before.

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- The higher the wage the higher the difference bw $V^{e}$ and $V^{u}$, so longer job durations.
- Firms could pay more to keep workers longer.


## Quitting Model: Workers Problem

- Problem of the employed: just change $\widehat{V}^{e}$ for $V^{e}$

$$
\begin{aligned}
V^{e}(a, w) & =\max _{c, a^{\prime}} u(c)+\beta\left[(1-\delta) \widehat{V}^{e}\left(a^{\prime}, w\right)+\delta V^{u}(a)\right] \\
\text { s.t. } & c+a^{\prime}=a(1+r)+w, \quad a \geq 0
\end{aligned}
$$

- We let $\mu=-\alpha \gamma-\ln (2)$ so that $E\left\{\max \left[\epsilon_{1}^{\mu}, \epsilon_{2}^{\mu}\right]\right\}=0$. To avoid the option value of working we have also add $E\left\{\max \left[\epsilon_{\mathbf{1}}^{u}, \epsilon_{\mathbf{2}}^{u}\right]\right\} \quad$ to the utility of the unemployed
- Alternatively we could accept the fact that a job is an option to get utility.


## Quitting Model: Value of the firm

- Free entry condition requires that for all offered wages

$$
\bar{c}+\bar{k}=\frac{1}{1+r}\left\{\psi^{f}[\theta(w)] \Omega^{0}(w)+\left[1-\psi^{f}[\theta(w)]\right] \Omega\right\}
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$\Omega^{j}(w)$ : Value with with $j$-tenured worker.

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- Probability of retaining a worker with tenure $j$ at wage $w$ is $\ell^{j}(w)$.
(One to one mapping between wealth and tenure)

$$
\ell^{j}(w)=1-q\left[g^{e, j}(a, w), w\right]
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$g^{e, j}(a, w)$ savings rule of a $j$-tenured worker that was hired with wealth a

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- Firm's value

$$
\Omega^{j}(w)=z-\bar{k} \delta^{k}-w+\frac{1-\delta^{f}}{1+r}\left\{\ell^{j}(w) \Omega^{j+1}(w)+\left[1-\ell^{j}(w)\right] \Omega\right\}
$$

## Quitting Model: Solving forward for the Value of the firm

$$
\Omega^{0}(w)=\left(z-w-\delta^{k} k\right) Q^{1}(w)+\left(1-\delta^{f}-\delta_{k}\right) k Q^{0}(w),
$$

$$
\begin{aligned}
& Q^{1}(w)=1+\sum_{\tau=0}^{\infty}\left[\left(\frac{1-\delta^{f}}{1+r}\right)^{1+\tau} \prod_{i=0}^{\tau} \ell^{i}(w)\right], \\
& Q^{0}(w)=\sum_{\tau=0}^{\infty}\left[\left(\frac{1-\delta^{f}}{1+r}\right)^{1+\tau}\left[1-\ell^{\tau}(w)\right]\left(\prod_{i=0}^{\tau-1} \ell^{i}(w)\right)\right] .
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- New equilibrium objects $\left\{Q^{0}(w), Q^{1}(w)\right\}$. Rest is unchanged.
- Except for when there are agents in a decreasing part of the wage applying function, it is Block Recursive because wealth can be inferred from $w$ and $j$. (No need to index contracts by wealth (as in Chaumont and Shi (2022)) ).


## Do we get More Wage Dispersion?

- This Model has the potential to get more wage dispersion


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BUT we will see a problem

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- For the poorest, employment duration increases when wage goes up.



## Value of the firm as wage varies: The Poor

- For the poorest, employment duration increases when wage goes up.
- Firms value is increasing in the wage



## Value of the firm as wage varies: The Rich

- For the richest, employment duration increases but not fast enough.
- Firm value is slowly decreasing in wages (less than static profits).

Firm Value: Omega


## Value of the firm: Accounting for Worker Selection

- Large drop from below to above equilibrium wages.
- In Equilibrium wage dispersion COLLAPSES due to selection.

- Related to the Diamond dispersion paradox but for very different


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- However, by paying higher wages, firms attract workers with more wealth.
- Wealthy people quit more often, shrink employment duration.
- In equilibrium, the wage gap is narrow (disappears?) and the effect of wealth dominates. sraph


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(1) On the Job Search
(2) Aiming Shocks: (EV) Shocks that distort the wage applying decision.
- Direct search with noise.


## Model 3: Aiming and Quitting Shocks Time-line

(1) Workers enter period with or without a job: $\{e, u\} . V^{e}, V^{u}$ defined here.
(2) Production \& Consumption:
(3) Exogenous Separation.
(4) Quitting $\widehat{V}^{e}\left(a^{\prime}, w\right)$, determined here.
(5) Search: Firms choose $\{w, \theta\}$. The unemployed asses the value of all wage applying options, receive match specific aiming shocks $\left\{\epsilon^{w^{\prime}}\right\}$ and choose the wage level $w^{\prime}$ to apply. Those who successfully find jobs become $e^{\prime}$, otherwise become $u^{\prime}$.
(6) $\widehat{V}^{u}\left(a^{\prime}\right),\left\{\Omega^{j}(w)\right\}$ are determined with respect to this stage.
(7) Matching

## Aiming and Quitting Shocks: Household Probl

- After saving, the unemployed problem is

$$
\widehat{V}^{u}\left(a^{\prime}\right)=\int \max _{w^{\prime}}\left[\psi^{h}\left(w^{\prime}\right) V^{e}\left(a^{\prime}, w^{\prime}\right)+\left[1-\psi^{h}\left(w^{\prime}\right)\right] V^{u}\left(a^{\prime}\right)+\epsilon^{w^{\prime}}\right] d F^{\epsilon}
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no longer FOC for which wage to apply.

- After saving, the employed choose whether to quit as before

$$
\begin{aligned}
& \widehat{V}^{e}\left(a^{\prime}, w\right)=\int \max \left\{V^{e}\left(a^{\prime}, w\right)+\epsilon^{e}, V^{u}\left(a^{\prime}\right)+\epsilon^{u}\right\} d F^{\epsilon} \\
& V^{e}(a, w) \text { and } V^{u}(a) \text { are as before beginning of period values. }
\end{aligned}
$$

## Aiming and Quitting Shocks: Household Probl

- The employed solve

$$
\begin{aligned}
V^{e}(a, w)= & \max _{c, a^{\prime} \geq 0} u(c)+\beta\left[(1-\delta) \widehat{V}^{e}\left(a^{\prime}, w\right)+\delta V^{u}\left(a^{\prime}\right)\right] \\
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- The unemployed face the problem

$$
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## Aiming and Quitting Shocks Model: Value of the Firm

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- Except that now the probability of keeping a worker after $j$ periods is

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$$

- Explicitly Not Block Recursive unless contracts were indexed by wealth which is illegal.


## Aiming and Quitting Shocks: Equilibrium Properties

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- Rich unemployed apply for higher wages (on average)
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## Model 4: On the Job Search: Time-line

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(7) Match

## On the Job Search: Household Probl

- After saving, the unemployed problem is

$$
\widehat{V}^{u}\left(a^{\prime}\right)=\int \max _{w^{\prime}}\left[\psi^{h}\left(w^{\prime}\right) V^{e}\left(a^{\prime}, w^{\prime}\right)+\left(1-\psi^{h}\left(w^{\prime}\right)\right) V^{u}\left(a^{\prime}\right)+\epsilon^{w^{\prime}}\right] d F^{\epsilon}
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$$

- The value of searching is

$$
V^{s}\left(a^{\prime}, w\right)=\int \max _{w^{\prime}}\left[\psi^{h}\left(w^{\prime}\right) V^{e}\left(a^{\prime}, w^{\prime}\right)+\left[1-\psi^{h}\left(w^{\prime}\right)\right] V^{e}\left(a^{\prime}, w\right)+\epsilon^{w^{\prime}}\right] d F^{\epsilon}
$$

## On the Job Search: Household choices

- The probabilities of quitting and of searching

$$
\begin{aligned}
& q\left(a^{\prime}, w\right)=\frac{1}{1+\exp \left(\alpha\left[V^{e}\left(a^{\prime}, w\right)-V^{u}\left(a^{\prime}\right)\right]\right)+\exp \left(\alpha\left[V^{s}\left(a^{\prime}, w\right)-V^{u}\left(a^{\prime}\right)+\mu^{s}\right]\right)} \\
& s\left(a^{\prime}, w\right)=\frac{1}{1+\exp \left(\alpha\left[V^{u}\left(a^{\prime}\right)-V^{s}\left(a^{\prime}, w\right)\right]\right)+\exp \left(\alpha\left[V^{e}\left(a^{\prime}, w\right)-V^{s}\left(a^{\prime}, w\right)-\mu^{s}\right]\right)} .
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- Households solve

$$
\begin{aligned}
V^{e}(a, w) & =\max _{a^{\prime} \geq 0} u\left[a(1+r)+w-a^{\prime}\right]+\beta\left[\delta V^{u}\left(a^{\prime}\right)+(1-\delta) \widehat{V}^{e}\left(a^{\prime}, w\right)\right] \\
V^{u}(a) & =\max _{c, a^{\prime} \geq 0} u\left[a(1+r)+b-a^{\prime}\right]+\beta \widehat{V}^{u}\left(a^{\prime}\right)
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$$

## OJS Quitting Probabilities, Various wealths \& Wage Density



- The rich pursue often other activities (leisure?)
- The value of the firm is again given like in the Quitting Model

$$
\begin{aligned}
\Omega^{0}(w)= & \left(z-w-\delta^{k} k\right) Q^{1}(w)+\left(1-\delta-\delta_{k}\right) k Q^{0}(w), \\
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$$
\begin{aligned}
& \ell^{j}(w)=1-\int h(w ; a) q\left[g^{e, j}(a, w), w\right] d x^{u}(a)- \\
& \quad \int h(w ; a) s\left[w ; g^{e, j}(a, w)\right]\left[\int \hat{h}\left[\widetilde{w} ; g^{e, j}(a, w), w\right] \xi \phi^{h}(\widetilde{w}) d(\widetilde{w})\right] d x^{u}(a)
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- Not block recursive but $Q^{1}$ and $Q^{2}$ are sufficient.


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8 Match

Mapping the Model to Data

Mapping the Model to Data: Adding Some Bells and Whis-

## TLES

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- Searching while on the job is slightly more inefficient than while unemployed.
- Workers hired from the ranks of unemployment require some training

$$
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$$

# Parameter Values: Period is half a quarter 

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| $\sigma$ | risk aversion | 2 |

## Parameter Values: Period is half a quarter

|  | Definition | Value in Yearly Units |
| :--- | :--- | :---: |
| $r$ | interest rate | $3 \%$ |
| $K$ | fixed capital required | 3 |
| $\delta^{f}$ | firm destruction rate | $2.88 \%$ |
| $\delta^{k}$ | capital maintenance rate | $6.38 \%$ |
| $c^{v}$ | job posting cost | 0.03 |
| $y$ | productivity on the job | 1 |
| $b / w$ | productivity at home | 0.4 |
| $\sigma$ | risk aversion | 2 |
| Matching function | $m=\chi u^{\eta} v^{1-\eta}$, OJS | $\chi=0.3$ |
|  |  | $\eta=0.5$ |

Steady State Allocations in Yearly Units: Endog Quits \& OJS
interest rate ..... 0.030
avg consumption ..... 0.652
avg wage ..... 0.683
avg wealth ..... 2.938
stock market value ..... 3.015
avg labor income ..... 0.653
consumption to wealth ratio ..... 0.222
labor income to wealth ratio ..... 0.222
quit ratio ..... 0.061
unemployment rate ..... 0.087
job losers ..... 0.089
wage of newly hired unemp ..... 0.619
std consumption ..... 0.013
std wage ..... 0.004
std wealth ..... 3.875
mean-min consumption ..... 1.956
mean-min wage ..... 1.153
UE transition ..... 1.152
total vacancy ..... 0.826
avg unemp duration ..... 0.531
avg emp duration ..... 9.108
avg job duration ..... 0.317
OJS move rate ..... 2.368

## Job Finding Probability Curves



Wage Distributions: Baseline


## Wage Distributions: Comparing with lower OJS




## Wage Applications of the Unemployed by Wealth



Wage Applications of U and $\bar{w}$ and densities of all


## Summary of Steady States

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- Obviously, not a good theory of wealth inequality. Should complement it with other mechanisms.
- But it can deliver gross flows (3\% per month OJS and a bit less for quits).

Aggregate Fluctuations

## Introduce Aggregate Shocks (in a small open economy)

- We now pose a standard aggregate shock


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- We use the Boppart et al. (2018) way of solving aggregates


## Baseline: IRF to z shock: Typical Response when wages suf-

## FICIENTLY FLEXIBLE



Figure 1: Wages


Figure 2: Unemployment Rate

- Obviously New wages move more than average wages
- Some response of unemployment


## Baseline: IRF to z shock



Figure 3: J2J transitions


Figure 4: J2J search \& JFP

- Too much responsive j2j transitions
- Due to improved job finding probabilities, not more searchers


## Assessing Performance in terms of standard hp-Filtered 2ND MOMENTS

- 1st order data moments are from standard database: CPS, JOLTS, LEHD and NIPA.
- 2nd order data moments are from Haefke et al. (2013), Campolmi and Gnocchi (2016), Brown et al. (2017) and Fujita and Nakajima (2016).


## Productivity Shock $\rho=0.95$ : Relative Volatility

- Wage adjustment estimate $\varphi^{w}=.8$ :



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| Model | Data |
| :---: | :---: |
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- Unemployment moves way way too little


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Table 2: Correlation with Contemprary Output: Only Productivity Shock

- Correlations are too large but appropriate


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- The amount of wage rigidity implied is small


## Conclusions I

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## Conclusions II

- Exciting set of continuation projects:
(1) Incorporate movements in and out of the labor force.
(2) Endogenous Search intensity on the part of firms and in general abandon the constant zero profit entry condition (Qiu (2022))
(3) Aiming Shocks to soften correlation between wages and wealth
(4) Efficiency Wages: Endogenous Productivity (firms use different technologies with different costs of idleness)
(5) Move towards more sophisticated household structures (more life cycle movements, multiperson households).


## Extensions

## Outside the Labor Force

## Outside the Labor Force Model: Time-line

(1) Workers enter period with or without a job: $V^{e}, V^{u}$.
(2) Production \& Consumption:
(3) Exogenous Separation
(4) In the beginning of the period non Workers get a shock to the utility of either searching or not searching. They then choose whether to sit out and not search or to search. It is an extreme value shock.

Workers get a utility injection equal to the expected utility of the maximum of those two shocks to get no bias in the value of working versus not.
(5) Quitting? Searching? Neither?:
(6) Search
(7) $\widehat{V}^{u}\left(a^{\prime}\right),\left\{\Omega^{j}(w)\right\}$ are determined with respect to this stage.

8 Match

## Firms choose Search Intensity

- The number of vacancies posted is chosen by firms
- Easy to implement
- Slightly Different steady state


## Free entry with variable recruiting intensity

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0=\max _{\bar{c}}\left\{v(\bar{c}) \psi^{f}[\theta(w)] \frac{\Omega(w)}{1+r}+\left[1-v(\bar{c}) \psi^{f}[\theta(w)]\right] \frac{\bar{k}\left(1-\delta_{k}\right)}{1+r}-\bar{c}-\bar{k}\right\},
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$$

- With FOC given by

$$
v_{\bar{c}}(\bar{c})\left\{\psi^{f}[\theta(w)]\left[\frac{\Omega(w)}{1+r}-\frac{\bar{k}\left(1-\delta_{k}\right)}{1+r}\right]\right\}=1
$$

How to make it consistent with the current steady state

- If $v(\bar{c})=\frac{v_{1} \bar{c}^{2}}{2}+v_{2 \bar{c}}$, we have

$$
\left(v_{1} \bar{c}+v_{2}\right)\left\{\psi^{f}[\theta(w)]\left[\frac{\Omega(w)}{1+r}-\frac{\bar{k}\left(1-\delta_{k}\right)}{1+r}\right]\right\}=1
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$$

- By Choosing $v$ so that for the numbers that have now

$$
\left\{\left[\frac{v_{1} \bar{c}^{2}}{2}+v_{2} \bar{c}\right] \psi^{f}[\theta(w)] \frac{\Omega(w)}{1+r}+\left[1-\frac{v_{1} \bar{c}^{2}}{2}-v_{2} \bar{c}\right] \psi^{f}[\theta(w)] \frac{\bar{k}\left(1-\delta_{k}\right)}{1+r}\right\}=\bar{c}+\bar{k}
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$$

- Solving for $\left\{v_{1}, v_{2}\right\}$ that satisfy both equations given our choice of $\bar{c}$ we are done


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## Steady-States

|  | m 1 | m 2 | m 3 | m 4 | m 4 (low xi) |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\beta$ | 0.975 | 0.972 | 0.975 | 0.976 | 0.976 |
| interest rate | 0.030 | 0.030 | 0.030 | 0.030 | 0.030 |
| avg consumption | 0.686 | 0.682 | 0.691 | 0.684 | 0.680 |
| avg wage | 0.707 | 0.719 | 0.696 | 0.689 | 0.690 |
| avg wealth | 2.789 | 2.763 | 2.361 | 3.041 | 2.919 |
| stock market value | 2.971 | 2.692 | 3.065 | 2.953 | 2.931 |
| avg labor income | 0.659 | 0.655 | 0.668 | 0.654 | 0.652 |
| consumption to wealth ratio | 0.246 | 0.247 | 0.293 | 0.225 | 0.233 |
| labor income to wealth ratio | 0.236 | 0.237 | 0.283 | 0.215 | 0.223 |
| quit ratio | 0.090 | 0.088 | 0.090 | 0.090 | 0.092 |
| unemployment rate | 0.129 | 0.165 | 0.076 | 0.097 | 0.106 |
| job losers | 0.117 | 0.115 | 0.117 | 0.117 | 0.119 |
| wage of newly hired unemployed | 0.707 | 0.719 | 0.656 | 0.677 | 0.689 |
| std consumption | 0.013 | 0.010 | 0.011 | 0.011 | 0.011 |
| std wage | 0.000 | 0.000 | 0.003 | 0.002 | 0.001 |
| std wealth | 2.989 | 2.715 | 2.624 | 3.606 | 3.677 |
| mean-min consumption | 2.057 | 2.045 | 2.072 | 2.051 | 2.039 |
| mean-min wage | 1.012 | 1.001 | 1.094 | 1.058 | 1.042 |
| UE transition | 0.121 | 0.114 | 0.128 | 0.125 | 0.126 |
| total vacancy | 0.544 | 0.308 | 0.704 | 0.578 | 0.707 |
| avg unemp duration | 1.062 | 1.449 | 0.589 | 0.773 | 0.745 |
| avg emp duration | 7.228 | 7.335 | 7.228 | 7.228 | 7.131 |
| OJS move rate | 0.000 | 0.000 | 0.420 | 0.395 | 0.292 |
| avg job duration | 7.228 | 7.335 | 1.814 | 1.898 | 2.342 |

## Wage Distributions



## Derive the Idle Value

- Value of an idle firm is

$$
\Omega^{0}=-\delta^{k} k+\frac{1-\delta^{f}}{1+r}\left[-c^{\vee}+\psi^{f} \Omega+\left(1-\psi^{f}\right) \Omega^{0}\right]
$$

- Free entry

$$
k=\frac{1}{1+r}\left[-c^{\vee}+\psi^{f} \Omega+\left(1-\psi^{f}\right) \Omega^{0}\right]
$$

- Newly entered firms do not receive the destruction shock immediately
- Vacancy posting cost is paid immediately before searching
- Combine the above

$$
\Omega^{0}=\left(1-\delta^{f}-\delta^{k}\right) k
$$

## M4 Low Ave J-2-J 1\% Productivity Shock ( $\rho=9$ ) [IRF]



Figure 5: Wages


Figure 6: Unemployment Rate

- Similar Wage Responses
- $70 \%$ more unemployment volatility: J: mainly comes from more responsive quits


## M4 Low Ave J-2-J 1\% Productivity Shock ( $\rho=.9$ ) IRF



Figure 7: Quits


Figure 8: Job-to-job Moves

- More quitting
- Similar (excessive) J-2-J transitions


## M4 Low Ave J-2-J 1\% Delta Shock ( $\rho=.95$ )



Figure 9: Wages


Figure 10: Unemployment Rate

- Similar Wage Response
- $16 \%$ more unemployment response
- Note wage is not pegged to the delta shock


## M4 Low Ave J-2-J 1\% Delta Shock ( $\rho=.95$ )



Figure 11: Quits


Figure 12: Job-to-job Moves

- More Quit similar (excessive) volatility for job-to-job transitions


## M4 Low Ave J-2-J: Business Crcle Statistics

- Two ways to aggregate shocks

|  | shock corr $=0.95$ |  | shock corr $=0$ |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Std |  | corr | Std | corr |  | 1.00 | 1.00 | 1.00 | 1.00 |
| :--- | :--- | :--- | :--- | :--- |
| output | 0.41 | 0.93 | 0.41 | 0.90 |
| avg wage | 1.69 | 0.76 | 1.38 | 0.52 |
| new wage | 2.59 | -0.73 | 2.80 | -0.63 |
| unemployment | 29.85 | 0.77 | 26.72 | 0.38 |
| quits + j2j movers | 36.30 | 0.79 | 32.51 | 0.41 |

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| :--- | :--- | :--- | :--- | :--- |
|  | Std | corr | Std | corr |
| output | 1.00 | 1.00 | 1.00 | 1.00 |
| avg wage | 0.41 | 0.93 | 0.41 | 0.90 |
| new wage | 1.69 | 0.76 | 1.38 | 0.52 |
| unemployment | 2.59 | -0.73 | 2.80 | -0.63 |
| quits + j2j movers | 29.85 | 0.77 | 26.72 | 0.38 |
| J2J movers | 36.30 | 0.79 | 32.51 | 0.41 |

- Not too successful in reducing volatility of quits and J2J movers.


## M4 Low Ave J-2-J: Business Crcle Statistics

- Two ways to aggregate shocks

|  | shock corr $=0.95$ |  | shock corr $=0$ |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Std | corr | Std | corr |
| output | 1.00 | 1.00 | 1.00 | 1.00 |
| avg wage | 0.41 | 0.93 | 0.41 | 0.90 |
| new wage | 1.69 | 0.76 | 1.38 | 0.52 |
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| J2J movers | 36.30 | 0.79 | 32.51 | 0.41 |

- Not too successful in reducing volatility of quits and J2J movers.
- Need to look for alternatives.


## M4 Higher Wage Peg: $1 \%$ Productivity Shock ( $\rho=.95$ )



Figure 13: Quits


Figure 14: OJS Searchers

- Higher wage peg lowers the reponse of on-the-job search and quit.
- Workers find it less so attractive to move/quit as existing wages now comove more with the productivity shock


## M4 Higher Wage Peg: $1 \%$ Productivity Shock ( $\rho=.95$ )



Figure 15: Job-to-job transitions


Figure 16: Unemployment

- Job-to-job transition rate also lowers: from $12 \%$ to $9 \%$. This is from
- less search on the job (see Fig 14)
- less improvement of job finding rate due to smaller s-s firm profits
- Also less persistence of the unemployment response (less turnover).
- However the j2j transition rate is still far more responsive than the unemployment


## M4 Higher Wage Peg: Business Cycle Statistics

|  | Wage Peg $=0.5$ |  |  | Wage Peg $=0.8$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Output | Mean | Std | Corr | Mean | Std | Corr |
|  | 1 | 1 | 1 | 1 | 1 | 1 |

## M4 Higher Wage Peg: Business Cycle Statistics

|  | Wage Peg $=0.5$ |  |  | Wage Peg $=0.8$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | Std | Corr | Mean | Std | Corr |
|  | 1 | 1 | 1 | 1 | 1 | 1 |
| Output | 0.690 | 0.51 | 1.00 | 0.690 | 0.76 | 0.99 |

## M4 Higher Wage Peg: Business Cycle Statistics

|  | Wage Peg $=0.5$ |  |  | Wage Peg $=0.8$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | Std | Corr | Mean | Std | Corr |
| Output | 1 | 1 | 1 | 1 | 1 | 1 |
| Avg Wage | 0.690 | 0.51 | 1.00 | 0.690 | 0.76 | 0.99 |
| New Wage | 0.689 | 0.95 | 1.00 | 0.689 | 1.04 | 0.99 |

## M4 Higher Wage Peg: Business Cycle Statistics

|  | Wage Peg $=0.5$ |  |  | Wage Peg $=0.8$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | Std | Corr | Mean | Std | Corr |
| Output | 1 | 1 | 1 | 1 | 1 | 1 |
| Avg Wage | 0.690 | 0.51 | 1.00 | 0.690 | 0.76 | 0.99 |
| New Wage | 0.689 | 0.95 | 1.00 | 0.689 | 1.04 | 0.99 |
| Unemp Rate | $10.6 \%$ | 0.35 | -0.48 | $10.6 \%$ | 0.42 | -0.64 |

## M4 Higher Wage Peg: Business Cycle Statistics

|  | Wage Peg $=0.5$ |  |  | Wage Peg $=0.8$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | Std | Corr | Mean | Std | Corr |
| Output | 1 | 1 | 1 | 1 | 1 | 1 |
| Avg Wage | 0.690 | 0.51 | 1.00 | 0.690 | 0.76 | 0.99 |
| New Wage | 0.689 | 0.95 | 1.00 | 0.689 | 1.04 | 0.99 |
| Unemp Rate | $10.6 \%$ | 0.35 | -0.48 | $10.6 \%$ | 0.42 | -0.64 |
| Quits+J2J moves | $38.4 \%$ | 8.94 | 0.99 | $38.4 \%$ | 6.65 | -0.99 |

## M4 Higher Wage Peg: Business Cycle Statistics

|  | Wage Peg $=0.5$ |  |  | Wage Peg $=0.8$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | Std | Corr | Mean | Std | Corr |
| Output | 1 | 1 | 1 | 1 | 1 | 1 |
| Avg Wage | 0.690 | 0.51 | 1.00 | 0.690 | 0.76 | 0.99 |
| New Wage | 0.689 | 0.95 | 1.00 | 0.689 | 1.04 | 0.99 |
| Unemp Rate | $10.6 \%$ | 0.35 | -0.48 | $10.6 \%$ | 0.42 | -0.64 |
| Quits+J2J moves | $38.4 \%$ | 8.94 | 0.99 | $38.4 \%$ | 6.65 | -0.99 |
| J2J moves | $29.2 \%$ | 10.66 | 0.99 | $29.2 \%$ | 8.50 | -0.99 |

Table 3: M4 Compare Wage Pegs: Productivity Shock ( $\rho=0.95$ )

- Higher wage pegs lower the j 2 j transition volatility while raise the unemployment volatility
- However even we make the existing wages comove with productivity closely, the j 2 j transition volatility is still much higher than the unemployment volatility
- In the next several pages we take a closer look at this problem


## M4 Higher Wage Peg: Business Cycle Statistics

|  | Wage Peg $=0.5$ |  |  | Wage Peg $=0.8$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | Std | Corr | Mean | Std | Corr |
| Output | 1 | 1 | 1 | 1 | 1 | 1 |
| Avg Wage | 0.690 | 0.51 | 1.00 | 0.690 | 0.76 | 0.99 |
| New Wage | 0.689 | 0.95 | 1.00 | 0.689 | 1.04 | 0.99 |
| Unemp Rate | $10.6 \%$ | 0.35 | -0.48 | $10.6 \%$ | 0.42 | -0.64 |
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## Quitting Makes a Big Difference

- Job finding Rates back


