## Course in Heterogeneity: Econ 081

VIII: Wealth, Wages, and Employment

Jose-Victor Rios-Rull University College London Nov/Dec 2023

Based on joint work with Per Krusell and Jinfeng Luo

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  - Price of those skills (evolution of the skill premia that very recently seems to have switched back)

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  - Relate to Business Cycles Version of Aiyagari (1994)

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- We use the volatility of gross flows to estimate the extent of wage rigidity.

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• Especially Eeckhout and Sepahsalari (2018), Chaumont and Shi (2022), Griffy (2021).

### WHAT ARE THE USES?

- The study of Business cycles including gross flows in and out of employment, unemployment and outside the labor force
- Policy analysis where now risk, employment, wealth (including its distribution) and wages are all responsive to policy.
- Get some insights into the extent of wage rigidity
- Life-Cycle versions of these ideas (under construction) will allow us to assess how age dependent policies fare.

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- Outside Steady State Employers commit to a wage schedule w(z) that depends on the aggregate state.

### Key Findings

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- With partial wage rigidity the model fares reasonably well with the data. A few things still to improve. (Excessive Job-to-JOB transitions)
- Similar behavior to that in the Shimer/Hagedorn-Manowski debate. Here we can try to move towards an accommodation of both points of view.

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Investment	25	4.88	0.90	NIPA

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② Endogenous Quits using Extreme Value Shocks

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- General equilibrium: Workers own firms.

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- **4** Search: Firms and the unemployed choose wage w and tightness  $\theta$ .
- Job Matching : M(V, U) : Some vacancies meet some unemployed job searchers. A match becomes operational the following period.

Job finding and job filling rates  $\psi^h(\theta) = \frac{M(V,U)}{U}, \ \psi^f(\theta) = \frac{M(V,U)}{V}.$ 

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- Problem of the employed: (Standard)

$$V^{e}(a, w) = \max_{c, a'} u(c) + \beta \ [(1 - \delta)V^{e}(a', w) + \delta V^{u}(a')]$$
  
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• Problem of the unemployed: Choose which wage to look for

$$V^{u}(a) = \max_{c,a',w} u(c) + \beta \left\{ \psi^{h}[\theta(w)] \ V^{e}(a',w) + [1 - \psi^{h}[\theta(w)]] \ V^{u}(a') \right\}$$
  
s.t.  $c + a' = a(1 + r) + b, \quad a \ge 0$ 

 $\theta(w)$  is an equilibrium object

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$$\psi^{h}[\theta(w)] V_{w}^{e}(a',w) = \psi_{\theta}^{h}[\theta(w)] \theta_{w}(w) \left[V^{u}(a') - V^{e}(a',w)\right]$$

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• Up to a certain level of wealth, richer households apply to higher wages. After that, it seems not. Consistent with theory

# WORKER'S WAGE APPLICATION DECISION



# WORKER'S SAVING DECISION



• Value of wage-w job: uses constant  $\overline{k}$  capital that depreciates at rate  $\delta^k$   $(\Omega = \overline{k})$ 

$$\Omega(w) = z - \overline{k}\delta^{k} - w + \frac{1 - \delta^{f}}{1 + r} \left[ (1 - \delta^{h}) \Omega(w) + \delta^{h} \Omega \right]$$

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• Affine in w: 
$$\Omega(w) = \left[z + \overline{k}\left(\frac{1-\delta^f}{1+r}\delta^h - \delta^k\right) - w\right] \frac{1+r}{r+\delta^f + \delta^h - \delta^f \delta^h}$$

Block Recursivity Applies (firms can be ignorant of Eq)

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$$\Omega(w) = z - \overline{k}\delta^{k} - w + \frac{1 - \delta^{f}}{1 + r} \left[ (1 - \delta^{h}) \Omega(w) + \delta^{h} \Omega \right]$$

• Affine in w: 
$$\Omega(w) = \left[z + \overline{k}\left(\frac{1-\delta^f}{1+r}\delta^h - \delta^k\right) - w\right] \frac{1+r}{r+\delta^f + \delta^h - \delta^f \delta^h}$$

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Functions {V<sup>e</sup>, V<sup>u</sup>, Ω, g'<sup>e</sup>, g'<sup>u</sup>, w<sup>u</sup>, θ}, an interest rate r, and a stationary distribution x over (a, w), s.t.

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  - 2 Zero profit condition holds for active markets

$$\bar{c} + \bar{k} = \psi^{f}[\theta(w)] \frac{\Omega(w)}{1+r} + [1 - \psi^{f}[\theta(w)]] \frac{\bar{k}(1 - \delta - \delta_{k})}{1+r}, \quad \forall w \text{ offered}$$

- Functions {V<sup>e</sup>, V<sup>u</sup>, Ω, g'<sup>e</sup>, g'<sup>u</sup>, w<sup>u</sup>, θ}, an interest rate r, and a stationary distribution x over (a, w), s.t.
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3 An interest rate r clears the asset market

$$\int_{A\times (W\cup 0)} a \ dx = \int_{A\times (W\cup 0)} \Omega(w) \ dx + \mu^0 \ \overline{k}.$$

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- When solved with aggregate shocks ...
- It is almost like a two-agent model (employed, unemployed) of Pissarides despite curved utility and savings

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- Adds a (smoothed) quitting motive so that conditional on wealth, high wage workers quit less often.
- Firms may want to pay high wages to retain workers.

# ENDOGENOUS QUITS MODEL: TIME-LINE

**1** Workers enter period with or without a job:  $\{e, u\}$ .

**2** Production occurs and consumption/saving choice ensues:

**3** Exogenous job/firm destruction happens.

#### Quitting:

• The employed, e, draw shocks  $\{\epsilon^e,\epsilon^u\}$  and make quitting decision. Job losers cannot search this period.

**5** Search: New or Idle firms post vacancies. Choose  $\{w, \theta\}$ .

6 Matches occur

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- Value of the employed right before receiving those shocks:

$$\widehat{V}^{e}(a',w) = \int \max\{V^{e}(a',w) + \epsilon^{e}, V^{u}(a') + \epsilon^{u}\} dF^{e}$$

 $V^e$  and  $V^u$  are values after quitting decision as described before.

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- The higher the wage the higher the difference bw V<sup>e</sup> and V<sup>u</sup>, so longer job durations.
- Firms could pay more to keep workers longer.

#### **QUITTING MODEL: WORKERS PROBLEM**

• Problem of the employed: just change  $\widehat{V}^e$  for  $V^e$ 

$$V^{e}(a, w) = \max_{c, a'} u(c) + \beta \left[ (1 - \delta) \widehat{V}^{e}(a', w) + \delta V^{u}(a) \right]$$
  
s.t.  $c + a' = a(1 + r) + w, \quad a \ge 0$ 

- We let  $\mu = -\alpha\gamma \ln(2)$  so that  $E\{\max[\epsilon_1^u, \epsilon_2^u]\} = 0$ . To avoid the option value of working we have also add  $E\{\max[\epsilon_1^u, \epsilon_2^u]\}$  to the utility of the unemployed
- Alternatively we could accept the fact that a job is an option to get utility.

#### **QUITTING MODEL:** VALUE OF THE FIRM

• Free entry condition requires that for all offered wages

$$\bar{c} + \overline{k} = \frac{1}{1+r} \left\{ \psi^f[\theta(w)] \Omega^0(w) + [1 - \psi^f[\theta(w)]] \Omega \right\},$$

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Probability of retaining a worker with tenure j at wage w is l<sup>j</sup>(w).
 (One to one mapping between wealth and tenure)

$$\ell^{j}(w) = 1 - q[g^{e,j}(a,w),w]$$

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Firm's value

$$\Omega^{j}(w) = z - \overline{k}\delta^{k} - w + \frac{1 - \delta^{f}}{1 + r} \left\{ \ell^{j}(w)\Omega^{j+1}(w) + \left[1 - \ell^{j}(w)\right]\Omega \right\}$$

#### **QUITTING MODEL: SOLVING FORWARD FOR THE VALUE OF THE FIRM**

$$\Omega^{0}(w) = (z - w - \delta^{k}k) Q^{1}(w) + (1 - \delta^{f} - \delta_{k})k Q^{0}(w),$$

$$Q^{1}(w) = 1 + \sum_{\tau=0}^{\infty} \left[ \left( \frac{1-\delta^{f}}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^{i}(w) \right],$$
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- New equilibrium objects  $\{Q^0(w), Q^1(w)\}$ . Rest is unchanged.
- Except for when there are agents in a decreasing part of the wage applying function, it is Block Recursive because wealth can be inferred from *w* and *j*. (No need to index contracts by wealth (as in Chaumont and Shi (2022)) ).

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**BUT** we will see a problem

#### VALUE OF THE FIRM AS WAGE VARIES: THE POOR

• For the poorest, employment duration increases when wage goes up.



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- For the poorest, employment duration increases when wage goes up.
- Firms value is increasing in the wage



#### VALUE OF THE FIRM AS WAGE VARIES: THE RICH

- For the richest, employment duration increases but not fast enough.
- Firm value is slowly decreasing in wages (less than static profits).



#### VALUE OF THE FIRM: ACCOUNTING FOR WORKER SELECTION

- Large drop from below to above equilibrium wages.
- In Equilibrium wage dispersion COLLAPSES due to selection.



• Related to the Diamond dispersion paradox but for very different reasons

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  - However, by paying higher wages, firms attract workers with more wealth.
- Wealthy people quit more often, shrink employment duration.
- In equilibrium, the wage gap is narrow (disappears?) and the effect of wealth dominates. graph

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• Direct search with noise.

# MODEL 3: AIMING AND QUITTING SHOCKS TIME-LINE

- **()** Workers enter period with or without a job: {e,u}.  $V^e$ ,  $V^u$  defined here.
- **2** Production & Consumption:
- Exogenous Separation.
- **4** Quitting  $\widehat{V}^{e}(a', w)$ , determined here.
- Search: Firms choose {w, θ}. The unemployed asses the value of all wage applying options, receive match specific aiming shocks {e<sup>w'</sup>} and choose the wage level w' to apply. Those who successfully find jobs become e', otherwise become u'.
- **(b)**  $\widehat{V}^{u}(a'), \{\Omega^{j}(w)\}$  are determined with respect to this stage.

#### Matching

• After saving, the unemployed problem is

$$\widehat{V}^{u}(a') = \int \max_{w'} \left[ \psi^{h}(w') V^{e}(a', w') + [1 - \psi^{h}(w')] V^{u}(a') + \epsilon^{w'} \right] dF^{\epsilon}$$

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• h(w'; a') is now the logit choice density of wage for wealth level a'

$$h(w';a') = \frac{\exp\left\{\alpha^{w}\left[\psi^{h}(w')V^{e}(a',w') + (1-\psi^{h}(w'))V^{u}(a')\right]\right\}}{\int \exp\left\{\alpha^{w}\left[\psi^{h}(\widetilde{w})V^{e}(a,\widetilde{w}) + (1-\psi^{h}(\widetilde{w}))V^{u}(a')\right]\right\} d\widetilde{w}}$$

no longer FOC for which wage to apply.

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no longer FOC for which wage to apply.

• After saving, the employed choose whether to quit as before

$$\widehat{V}^{e}(a',w) = \int \max\{V^{e}(a',w) + \epsilon^{e}, V^{u}(a') + \epsilon^{u}\}dF^{e}$$

 $V^{e}(a, w)$  and  $V^{u}(a)$  are as before beginning of period values.

• The employed solve

$$V^{e}(a, w) = \max_{c, a' \ge 0} u(c) + \beta \left[ (1 - \delta) \widehat{V}^{e}(a', w) + \delta V^{u}(a') \right]$$
  
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• The unemployed face the problem

$$V^{u}(a) = \max_{c,a' \ge 0} u(c) + \beta \widehat{V}^{u}(a')$$
  
s.t.  $c + a' = a(1+r) + b$ 

#### Aiming and Quitting Shocks Model: Value of the Firm

• The value of the firm is again given like in the Quitting Model

$$\Omega^0(w) = (z - w - \delta_k k) Q^1(w) + (1 - \delta - \delta_k) k Q^0(w),$$

$$Q^{1}(w) = 1 + \sum_{\tau=0}^{\infty} \left[ \left( \frac{1-\delta}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^{i}(w) \right],$$
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• Explicitly Not Block Recursive unless contracts were indexed by wealth which is illegal.

• Higher wage dispersion

### Aiming and Quitting Shocks: Equilibrium Properties

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- Rich unemployed apply for higher wages (on average)
- But have more dispersion in its applications as utility differentials are lower


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# **ON THE JOB SEARCH:** HOUSEHOLD PROBL

• After saving, the unemployed problem is

$$\widehat{V}^{u}(a') = \int \max_{w'} \left[ \psi^{h}(w') V^{e}(a',w') + (1-\psi^{h}(w')) V^{u}(a') + \epsilon^{w'} \right] dF^{\epsilon}$$

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• The value of searching is

$$V^{s}(a',w) = \int \max_{w'} \left[ \psi^{h}(w') V^{e}(a',w') + [1-\psi^{h}(w')] V^{e}(a',w) + \epsilon^{w'} \right] dF^{\epsilon}$$

#### **ON THE JOB SEARCH:** HOUSEHOLD CHOICES

• The probabilities of quitting and of searching

$$\begin{aligned} q(a',w) &= \frac{1}{1 + \exp(\alpha [V^{e}(a',w) - V^{u}(a')]) + \exp(\alpha [V^{s}(a',w) - V^{u}(a') + \mu^{s}])}, \\ s(a',w) &= \frac{1}{1 + \exp(\alpha [V^{u}(a') - V^{s}(a',w)]) + \exp(\alpha [V^{e}(a',w) - V^{s}(a',w) - \mu^{s}])}. \end{aligned}$$

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• Households solve

$$V^{e}(a, w) = \max_{a' \ge 0} u[a(1+r) + w - a'] + \beta \left[ \delta V^{u}(a') + (1-\delta) \widehat{V}^{e}(a', w) \right]$$

$$V^{u}(a) = \max_{c,a' \ge 0} u[a(1+r) + b - a'] + \beta V^{u}(a')$$

### **OJS** QUITTING PROBABILITIES, VARIOUS WEALTHS & WAGE DENSITY



• The rich pursue often other activities (leisure?)

#### THE JOB SEARCH MODEL: VALUE OF THE FIRM

• The value of the firm is again given like in the Quitting Model

$$\Omega^{0}(w) = (z - w - \delta^{k}k) Q^{1}(w) + (1 - \delta - \delta_{k})k Q^{0}(w),$$

$$\begin{split} &Q^{1}(w) = 1 + \sum_{\tau=0}^{\infty} \left[ \left( \frac{1-\delta}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^{i}(w) \right], \\ &Q^{0}(w) = \sum_{\tau=0}^{\infty} \left[ \left( \frac{1-\delta}{1+r} \right)^{1+\tau} \left[ 1 - \ell^{\tau}(w) \right] \left( \prod_{i=0}^{\tau-1} \ell^{i}(w) \right) \right]. \end{split}$$

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• Except that now the probability of keeping a worker after j periods is

$$\ell^{j}(w) = 1 - \int h(w; a) \ q[g^{e,j}(a, w), w] \ dx^{u}(a) - \int h(w; a) \ s[w; g^{e,j}(a, w)] \left[ \int \hat{h}[\widetilde{w}; g^{e,j}(a, w), w] \xi \phi^{h}(\widetilde{w}) \ d(\widetilde{w}) \right] \ dx^{u}(a)$$

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• Not block recursive but  $Q^1$  and  $Q^2$  are sufficient.



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# Mapping the Model to Data

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- Searching while on the job is slightly more inefficient than while unemployed.
- Workers hired from the ranks of unemployment require some training

$$\Omega^{0}(w) = z - \overline{k}\delta^{k} - w - CT + \frac{1 - \delta^{f}}{1 + r} \left\{ \ell^{0}(w)\Omega^{1}(w) + \left[1 - \ell^{0}(w)\right] \Omega \right\}$$

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Matching function	$m=\chi u^\eta v^{1-\eta}$ , OJS	$\chi = 0.3$
		$\eta=$ 0.5

# Steady State Allocations in Yearly Units: Endog Quits & OJS $% \left( {{{\rm{C}}} \right) = {{\rm{C}}} \right)$

interest rate	0.030
avg consumption	0.652
avg wage	0.683
avg wealth	2.938
stock market value	3.015
avg labor income	0.653
consumption to wealth ratio	0.222
labor income to wealth ratio	0.222
quit ratio	0.061
unemployment rate	0.087
job losers	0.089
wage of newly hired unemp	0.619
std consumption	0.013
std wage	0.004
std wealth	3.875
mean-min consumption	1.956
mean-min wage	1.153
UE transition	1.152
total vacancy	0.826
avg unemp duration	0.531
avg emp duration	9.108
avg job duration	0.317
OJS move rate	2.368

#### JOB FINDING PROBABILITY CURVES



#### WAGE DISTRIBUTIONS: BASELINE



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#### WAGE DISTRIBUTIONS: COMPARING WITH LOWER OJS



#### WAGE APPLICATIONS OF THE UNEMPLOYED BY WEALTH



48

#### Wage Applications of U and $\overline{w}$ and densities of all



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• But it can deliver gross flows (3% per month OJS and a bit less for quits).

### **Aggregate Fluctuations**

**1** Productivity shocks  $z_t$ : Output = EmpRate  $\times$  (1 +  $z_t$ )

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  - We use the Boppart et al. (2018) way of solving aggregates

## Baseline: IRF to z shock: Typical Response when wages sufficiently flexible



Figure 1: Wages



- Obviously New wages move more than average wages
- Some response of unemployment

#### BASELINE: IRF TO Z SHOCK



- Too much responsive j2j transitions
- Due to improved job finding probabilities, not more searchers

### Assessing Performance in terms of standard hp-filtered 2nd moments

• 1st order data moments are from standard database: CPS, JOLTS, LEHD and NIPA.

• 2nd order data moments are from Haefke et al. (2013), Campolmi and Gnocchi (2016), Brown et al. (2017) and Fujita and Nakajima (2016).

	Model	Data	
Output	1	1	

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Table 1: Standard Deviation Relative to Output: Only Productivity Shock

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• Unemployment moves way way too little

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Table 2: Correlation with Contemprary Output: Only Productivity Shock

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• Correlations are too large but appropriate
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- Useful for business cycle analysis: We are getting procyclical
  - Quits
  - Employment
  - Investment and Consumption
  - Wages

- Exciting set of continuation projects:
  - ① Incorporate movements in and out of the labor force.
  - Endogenous Search intensity on the part of firms and in general abandon the constant zero profit entry condition (Qiu (2022))
  - 3 Aiming Shocks to soften correlation between wages and wealth
  - Efficiency Wages: Endogenous Productivity (firms use different technologies with different costs of idleness)
  - Move towards more sophisticated household structures (more life cycle movements, multiperson households).



# **Outside the Labor Force**

#### **OUTSIDE THE LABOR FORCE MODEL: TIME-LINE**

- 1 Workers enter period with or without a job:  $V^e, V^u$ .
- Production & Consumption:
- 8 Exogenous Separation
- In the beginning of the period non Workers get a shock to the utility of either searching or not searching. They then choose whether to sit out and not search or to search. It is an extreme value shock. Workers get a utility injection equal to the expected utility of the maximum of those two shocks to get no bias in the value of working versus not.
- **5** Quitting? Searching? Neither?:
- 6 Search
- $\widehat{V}^{u}(a'), \{\Omega^{j}(w)\}$  are determined with respect to this stage.
- 8 Match

• The number of vacancies posted is chosen by firms

• Easy to implement

• Slightly Different steady state

#### FREE ENTRY WITH VARIABLE RECRUITING INTENSITY

• Let  $v(\overline{c})$  be a technology to post vacancies where  $\overline{c}$  is the cost paid.

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- Then the free entry condition requires that for all offered wages

$$0 = \max_{\overline{c}} \left\{ \upsilon(\overline{c}) \ \psi^f[\theta(w)] \ \frac{\Omega(w)}{1+r} + \left[1 - \upsilon(\overline{c}) \ \psi^f[\theta(w)]\right] \ \frac{\overline{k}(1-\delta_k)}{1+r} - \overline{c} - \overline{k} \right\},$$

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• With FOC given by

$$\nu_{\overline{c}}(\overline{c}) \left\{ \psi^{f}[\theta(w)] \left[ \frac{\Omega(w)}{1+r} - \frac{\overline{k}(1-\delta_{k})}{1+r} \right] \right\} = 1.$$

• If 
$$v(\overline{c}) = \frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_2 \overline{c}$$
, we have

$$(\upsilon_1 \overline{c} + \upsilon_2) \left\{ \psi^f[\theta(w)] \left[ \frac{\Omega(w)}{1+r} - \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

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 $\bullet\,$  By Choosing  $\upsilon$  so that for the numbers that have now

$$\left\{ \left[ \frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_2 \overline{c} \right] \psi^f[\theta(w)] \frac{\Omega(w)}{1+r} + \left[ 1 - \frac{\upsilon_1 \overline{c}^2}{2} - \upsilon_2 \overline{c} \right] \psi^f[\theta(w)] \frac{\overline{k}(1-\delta_k)}{1+r} \right\} = \overline{c} + \overline{k}$$

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$$v(\overline{c}) = \frac{v_1 \overline{c}^2}{2} + v_2 \overline{c}$$
, we have

$$(\upsilon_1 \overline{c} + \upsilon_2) \left\{ \psi^f[\theta(w)] \left[ \frac{\Omega(w)}{1+r} - \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

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• Solving for  $\{v_1, v_2\}$  that satisfy both equations given our choice of  $\overline{c}$  we are done

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## STEADY-STATES

	m1	m2	m3	m4	m4 (low xi)
β	0.975	0.972	0.975	0.976	0.976
interest rate	0.030	0.030	0.030	0.030	0.030
avg consumption	0.686	0.682	0.691	0.684	0.680
avg wage	0.707	0.719	0.696	0.689	0.690
avg wealth	2.789	2.763	2.361	3.041	2.919
stock market value	2.971	2.692	3.065	2.953	2.931
avg labor income	0.659	0.655	0.668	0.654	0.652
consumption to wealth ratio	0.246	0.247	0.293	0.225	0.233
labor income to wealth ratio	0.236	0.237	0.283	0.215	0.223
quit ratio	0.090	0.088	0.090	0.090	0.092
unemployment rate	0.129	0.165	0.076	0.097	0.106
job losers	0.117	0.115	0.117	0.117	0.119
wage of newly hired unemployed	0.707	0.719	0.656	0.677	0.689
std consumption	0.013	0.010	0.011	0.011	0.011
std wage	0.000	0.000	0.003	0.002	0.001
std wealth	2.989	2.715	2.624	3.606	3.677
mean-min consumption	2.057	2.045	2.072	2.051	2.039
mean-min wage	1.012	1.001	1.094	1.058	1.042
UE transition	0.121	0.114	0.128	0.125	0.126
total vacancy	0.544	0.308	0.704	0.578	0.707
avg unemp duration	1.062	1.449	0.589	0.773	0.745
avg emp duration	7.228	7.335	7.228	7.228	7.131
OJS move rate	0.000	0.000	0.420	0.395	0.292
avg job duration	7.228	7.335	1.814	1.898	2.342

## WAGE DISTRIBUTIONS



#### DERIVE THE IDLE VALUE

• Value of an idle firm is

$$\Omega^{0} = -\delta^{k}k + \frac{1-\delta^{f}}{1+r} \left[ -c^{v} + \psi^{f}\Omega + (1-\psi^{f})\Omega^{0} \right]$$

• Free entry

$$k = \frac{1}{1+r} \left[ -c^{\mathsf{v}} + \psi^{\mathsf{f}} \Omega + (1-\psi^{\mathsf{f}}) \Omega^{\mathsf{0}} \right]$$

- Newly entered firms do not receive the destruction shock immediately
- Vacancy posting cost is paid immediately before searching
- Combine the above

$$\Omega^0 = (1 - \delta^f - \delta^k)k$$

## M4 Low Ave J-2-J 1% Productivity Shock ( $\rho = .9$ ) [IRF]



- Similar Wage Responses
- 70% more unemployment volatility: J: mainly comes from more responsive quits

#### M4 Low Ave J-2-J 1% Productivity Shock ( $\rho = .9$ ) IRF



Figure 7: Quits



- More quitting
- Similar (excessive) J-2-J transitions

#### M4 Low Ave J-2-J 1% Delta Shock ( $\rho = .95$ )



Figure 9: Wages



- Similar Wage Response
- 16% more unemployment response
- Note wage is not pegged to the delta shock

#### M4 Low Ave J-2-J 1% Delta Shock ( $\rho = .95$ )



• More Quit similar (excessive) volatility for job-to-job transitions

#### M4 Low Ave J-2-J: Business Cycle Statistics

• Two ways to aggregate shocks

	shock corr $= 0.95$		shock corr $= 0$	
	Std	corr	Std	corr
output	1.00	1.00	1.00	1.00
avg wage	0.41	0.93	0.41	0.90
new wage	1.69	0.76	1.38	0.52
unemployment	2.59	-0.73	2.80	-0.63
quits + j2j movers	29.85	0.77	26.72	0.38
J2J movers	36.30	0.79	32.51	0.41
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• Not too successful in reducing volatility of quits and J2J movers.

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- Not too successful in reducing volatility of quits and J2J movers.
- Need to look for alternatives.

#### M4 HIGHER WAGE PEG: 1% PRODUCTIVITY SHOCK ( $\rho = .95$ )



- Higher wage peg lowers the reponse of on-the-job search and quit.
  - Workers find it less so attractive to move/quit as existing wages now comove more with the productivity shock

# M4 HIGHER WAGE PEG: 1% PRODUCTIVITY SHOCK ( $\rho = .95$ )



Figure 15: Job-to-job transitions

Figure 16: Unemployment

- Job-to-job transition rate also lowers: from 12% to 9%. This is from
  - less search on the job (see Fig 14)
  - less improvement of job finding rate due to smaller s-s firm profits
- Also less persistence of the unemployment response (less turnover).
- However the j2j transition rate is still far more responsive than the unemployment

	Wag	e Peg =	= 0.5	Wage	e Peg =	= 0.8
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1

	Wag	ge Peg =	- 0.5	Wage $Peg = 0.8$		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.690	0.76	0.99

	Wag	Wage $Peg = 0.5$			e Peg =	: 0.8
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.690	0.76	0.99
New Wage	0.689	0.95	1.00	0.689	1.04	0.99

	Wag	Wage $Peg = 0.5$			Wage Peg =	
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.690	0.76	0.99
New Wage	0.689	0.95	1.00	0.689	1.04	0.99
Unemp Rate	10.6%	0.35	-0.48	10.6%	0.42	-0.64

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	Mean	Std	Corr	Mean	Std	Corr
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Avg Wage	0.690	0.51	1.00	0.690	0.76	0.99
New Wage	0.689	0.95	1.00	0.689	1.04	0.99
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Quits+J2J moves	38.4%	8.94	0.99	38.4%	6.65	-0.99

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Quits+J2J moves	38.4%	8.94	0.99	38.4%	6.65	-0.99
J2J moves	29.2%	10.66	0.99	29.2%	8.50	-0.99

**Table 3:** M4 Compare Wage Pegs: Productivity Shock ( $\rho = 0.95$ )

- Higher wage pegs lower the j2j transition volatility while raise the unemployment volatility
- However even we make the existing wages comove with productivity closely, the j2j transition volatility is still much higher than the unemployment volatility
- In the next several pages we take a closer look at this problem

	Wage $Peg = 0.5$			Wage $Peg = 0.8$		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.690	0.76	0.99
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Quits+J2J moves	38.4%	8.94	0.99	38.4%	6.65	-0.99
J2J moves	29.2%	10.66	0.99	29.2%	8.50	-0.99

**Table 3:** M4 Compare Wage Pegs: Productivity Shock ( $\rho = 0.95$ )

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### QUITTING MAKES A BIG DIFFERENCE

• Job finding Rates back

