Demand Shocks as Technology Shocks*

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Abstract

We provide a macroeconomic theory where *demand* for goods has a productive role. A search friction prevents perfect matching between producers and potential customers. Larger demand induces more search, which in turn increases GDP and measured TFP. We embed the product-market friction in a standard neoclassical model and estimate it using Bayesian techniques. Business cycles are driven by preference shocks, true technology shocks, and investment-specific shocks. Preference shocks have qualitatively similar effects as true productivity shocks. These shocks account for a large share of the fluctuations in consumption, GDP, and measured TFP and can be identified using shopping time data.

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1 Introduction

In the standard neoclassical model, output is a function of inputs such as labor and capital. There is no explicit role for demand because potential consumers are always available and Walrasian prices adjust so that all produced goods become used. In reality, customers and producers must meet in order for the produced good to be consumed, so value added depends on how well they are matched. As an example, consider a restaurant. According to the neoclassical view, the value added of a restaurant should be a function of its inputs (employees, tables, etc.), irrespective of the number of patrons and how hungry they are. Moreover, the restaurant owner would set prices so that all tables were in use. However, actual production takes place only when customers show up. The more customers demand the restaurant's meals, the larger the value added will be. The idea that the demand for goods plays a direct role extends to many forms of production: dentists need patients, car dealers need shoppers, all producers need buyers.

This paper provides a theory where search for goods —which we, with some abuse of terminology, refer to as *demand*— has a productive role. The starting point is that customers search for producers, and a standard matching friction prevents Walrasian market clearing in the sense that all potential productive capacity necessarily translates into actual value added.[1] Allowing an explicit role for demand has implications for business cycle analysis, especially for our understanding of the driving forces of business cycles. In our model, changes in search effort affect output even if conventional inputs remain constant. Demand shocks therefore influence the measured aggregate TFP. This paper quantifies how important this mechanism is for aggregate fluctuations, relative to more standard business-cycle shocks.

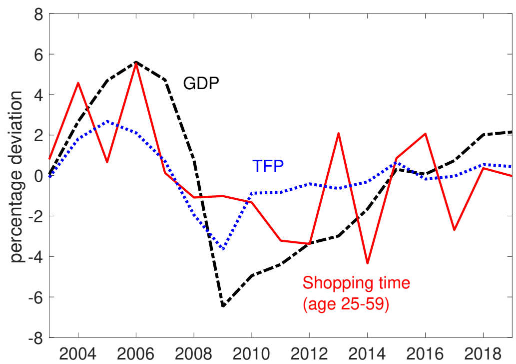
Our study is motivated by the empirical observation that search effort for goods is highly procyclical. This property can be illustrated by looking at the average time households spend shopping for goods and services. While search effort is a broader concept than just time spent shopping, we believe this measure is informative about the true search effort for goods. Figure 1 plots the cyclical component of shopping time using annual data from the American Time Use Survey (ATUS). The correlations of shopping time with GDP and the Solow residual are 0.56 and 0.52, respectively.[2]

We pose a model of competitive search in the market for goods. Firms post prices and customers trade off good prices versus congestion when searching for goods. In equilibrium, firms charge higher prices in markets where they have a lower chance of realizing a sale, and it is easier for customers to find goods. Search effort is determined by a familiar condition: the marginal rate of substitution between search costs and consumption (i.e., the ratio of the marginal disutility of search to the marginal utility

[1] The National Income and Product Accounts (NIPA) measures value added only when goods are purchased. Indeed, a lost opportunity, such as an empty seat on a bus, is not value-added. In the neoclassical model, value added is the price times the quantity of goods and services, consistent with NIPA.

[2] Procyclical search effort is in line with the findings of Petrosky-Nadeau et al. (2016). They use cross-state and individual regressions analysis to argue that the decline in aggregate shopping time during the Great Recession reflects that consumer search in the goods market is procyclical.

Figure 1: Shopping Time versus GDP and TFP



Note: The figure plots average shopping time for the working-age population (25-59) against GDP and the Solow residual. All series are linearly detrended. Shopping time is measured as average time spent shopping for goods, professional and personal care services, and household services, plus travel time associated with this shopping. Data source: American Time Use Survey (ATUS) and NIPA.

of consumption) must equal the marginal transformation of search, i.e., the marginal increase in the probability that a match will occur when search effort is increased marginally.

We embed this search friction in an otherwise standard stochastic neoclassical growth model. On the one hand, the search friction is a source of shocks which we interpret as a particular implementation of the demand shocks. On the other hand, it is a mechanism that amplifies more traditional shocks. To assess the role of the search friction for business cycles and TFP, we model search shocks as a preference shock to shopping effort.[3] The model also includes shocks that are more standard in the business cycle literature such as a shock to the marginal rate of substitution (MRS) between consumption and leisure, a standard economy-wide technology shock, and an investment-specific productivity shock (see e.g., Justiniano et al. (2010) and references therein).

The model is estimated using Bayesian estimation techniques targeting time series for output, labor productivity, investment, the relative price of investment goods, and a measure of aggregate shopping effort (time spent shopping). This allows us to quantify the contribution of each shock to the variance of aggregate variables. The Bayesian estimation chooses shocks to maximize the probability of observing the realized aggregate time series. One of our main findings is that it is necessary to attribute a large role to demand shocks in order to match the data: preference shock to shopping accounts for between 39% and 60% of the variance of the Solow residual and output and up to half the variance of consumption. The endogenous search amplifies the effect of the true technology shocks and the labor preference shock because it is optimal to search more when output is larger. This in turn generates procyclical search effort and contributes to procyclical productivity.

We compare our implementation of demand shocks – i.e., search-effort preference shocks – to more

[3]We take a broad view of shocks to the disutility of search effort. For example, these shocks could alternatively be interpreted as a shock to the search efficiency.

traditional implementations of demand shocks. We find that incorporating our search shock in the model allows us to fit U.S. data substantially better than alternative formulations based on more traditional demand shocks. In particular, the likelihood of the various models suggest that there is decisive evidence against models that replace our search shock with a discount factor shock or a preference shock to the marginal utility of consumption.

The business-cycle literature has emphasized a different mechanism for how fluctuations in demand transmits to productivity, namely varying capacity utilization (cf. Corrado and Mattey (1997)). Under this view, NIPA mismeasures factor inputs because it ignores variations in the intensity of factor use. A demand shock will be accommodated by producers via an increase in the intensity of use of available factors of production. This results in an increase of measured TFP, inducing procyclical TFP (cf. Basu (1996)). This begs the question of whether the significant role our estimation assigns to demand shocks through shopping utility preference shocks is simply a stand-in for varying capacity utilization. To address this issue, we extend our benchmark model to allow for variable capacity utilization of capital along the lines of Christiano et al. (2005). We find that the quantitative role of the shopping preference shocks is robust to this extension in the sense that our search/demand shocks continue to account for a large share of the aggregate fluctuations. In a version of the model allowing shocks to both capacity utilization and shopping effort, and where we target empirical measurements of both shopping time (from ATUS) and capacity utilization (from the Board of Governors) in the Bayesian estimation, we find that our shopping friction shocks are orders of magnitude more important than standard capacity utilization shocks in accounting for aggregate fluctuations. We interpret this result as evidence that our search-based mechanism is quantitatively relevant for the propagation of US business cycles, while varying capacity utilization is less relevant. This suggests that the potential effect of demand shocks on TFP might stem through a mechanism of product market frictions and endogenous variation in search effort rather than a mechanism of varying intensity in the use of capital.

As a robustness exercise we relax the assumption of just-in-time production in line with the restaurant example above. If firms could store produced goods as inventories, then demand shocks might —ceteris paribus— matter less for business cycles. To address this issue, we extend the baseline model to incorporate storable goods. Our main results hold up: demand shocks continue to be a major driving force for consumption, TFP, and GDP, even when goods can be stored. This is because storable goods represents a relatively small share of aggregate output.

Summing up, we see our paper as an implementation of Keynes' central idea that shocks to demand can give rise to business cycle fluctuations. This is done by extending a standard neoclassical framework with a search friction for goods. The role of demand is intrinsic to the process of production and is not arbitrarily imposed: markets clear, and no agent has incentives to deviate. There is a long tradition of attributing a role for demand in business cycle analysis, starting with Keynes' seminal contributions. However, in none of the earlier approaches, demand had a direct productive role. Michaillat and Saez

(2015) also study a model with product-market frictions. They show that when prices and wages are predetermined (i.e., exogenous), aggregate demand can increase output and employment. They abstract from a mechanism that can determine search intensity and their equilibrium is inefficient. In a flexible-price version of their model, the allocation is efficient and aggregate demand has no effect on output and employment. In contrast, our dynamic model is embedded in a standard dynamic business cycle framework, it assumes flexible prices, and poses a theory of determination of search effort that yields a unique and efficient equilibrium as in Moen (1997). We pursue neither the fixed-price tradition of the New Keynesian literature nor the coordination-problem tradition that sees a recession as a bad outcome within environments susceptible to multiple equilibria. Instead, we follow a tradition where a fall in demand generates a recession via the infra-utilization of productive capacity.

Our model allows us to study the propagation of various preference and technology shocks in the presence of frictions in the matching between buyers and sellers of goods and services, and quantify the potential role this friction plays for understanding business cycles. We view it as a virtue that demand fluctuations show up as fluctuations in measured TFP in our model. Our starting point is that the measured aggregate TFP – the Solow residual – does indeed fluctuate and that these fluctuations are strongly positively correlated with GDP and shopping time both at high and low frequencies (see Figure 1). Galí (1999) and others have challenged the role of TFP fluctuations, identifying true TFP shocks as permanent shocks to measured TFP and assigning non-permanent shocks to demand shocks. Our model provides a theory for endogenous TFP fluctuations without the need to rule out transitory true TFP shocks. We find that, across specifications, our estimated model assigns about equal roles for demand shocks and technology shocks in accounting for GDP and measured TFP.

Our paper is also related to Petrosky-Nadeau and Wasmer (2015). They also model costly search for goods in final goods markets, although their focus is on how this search interacts with search in the labor market and influences the business cycle properties of the model. In particular, they do not focus on aggregate demand effects. Rudanko and Gourio (2014) study a business cycle model with a search friction in the market for consumption goods. Firms form long-lasting relationships with customers, and the authors focus on the role of customers as capital. Our contribution is also related to several papers emphasizing the effects of search frictions in shaping TFP (Alessandria (2005), Faig and Jerez (2005), and Lagos (2006)), although none of these focus on business cycles.

Some papers examine, as we do, how demand changes affect productivity and capacity utilization, although through very different mechanisms. In Fagnart et al. (1999), monopolistic firms with putty-clay technology are subject to idiosyncratic demand shocks, which causes fluctuations in capacity utilization. In Floetotto and Jaimovich (2008), changes in the number of firms cause changes in markups and, hence, changes in the measured Solow residual. Swanson (2006) shows that government expenditure shocks can increase aggregate output, consumption, and investment in a model with heterogeneous sectors. Finally, a number of papers build directly on our approach of consumption demand shocks in models with a search friction in product markets, including Huo and Ríos-Rull (2020), Petrosky-Nadeau and Wasmer (2015), Duras (2015, 2016), Bai and Ríos-Rull (2015), Qiu and Ríos-Rull (2022), and a variety of papers studying price dispersion using versions of the Burdett and Judd (1983) structure (see for instance Nord (2022) and references therein).

The paper is organized as follows. Section 2 lays out the main mechanism in a tractable economy where we show how preference shocks that increases demand are partially accommodated by an increase in productivity via more search. The full production economy is presented in Section 3. We then map the model to data in Section 4. Section 5 estimates the model and derive the main results. Section 6 extends the model to allow for varying capacity utilization of capital and storable goods. Section 7 concludes. Appendices A to G provide proofs, computational and data details, and some additional tables and figures.

2 Competitive search for goods in a tractable setting

We start our analysis of goods search and the business cycle by laying out our main argument in a simple and transparent version of our model. We first study a static endowment economy with competitive search for goods and then extend it to a dynamic production environment with shocks.

Consider a static economy where a measure one of identical households have preferences $u(c, d)$ over consumption c and search effort d. There is a continuum of suppliers with measure $T = 1$ owned by the households. Each supplier is endowed with z units of the consumption good. A standard search friction makes it difficult for consumers to find suppliers and the household makes search effort $d \in [0, 1]$ to overcome this friction.

Following Moen (1997), we assume a competitive search protocol where households search in markets indexed by price and market tightness. The measure of matches is given by the matching function $M(D, T) \to [0, T)$. M is constant returns to scale and increasing, strictly concave in both arguments, and satisfies the Inada conditions. Market tightness q is defined as search effort per supplier, $q = D/T$, where D is search effort and T is the measure of suppliers in the specific market. The rate at which shoppers finds suppliers is then $M/D \equiv \Psi_d(q)$ and the probability that a supplier is matched is $M/T \equiv \Psi_T(q) = q \cdot \Psi_d(q)$. Once a match is formed, goods are traded at the posted price p per unit. The real quantity of consumption goods purchased by the household is $c = d\Psi_d(q)z$.[4]

Consider first the problem of a household who has an endowment y and who shops at a market offering a pair $(\widehat{p}, \widehat{q})$. The budget constraint for this household is given by

$$\widehat{p}\; d\; \Psi_d(\widehat{q})\; z \;\leq\; y. \tag{1}$$

[4]The underlying assumption is that households are large and the search efforts of its members eliminate any idiosyncratic uncertainty in the measure of matches obtained.

Using the budget constraint (1), search effort d can be expressed as $d = y/[\widehat{p}\Psi_d(\widehat{q})z]$. Moreover, given expenditure y and price $\widehat{p}$, the the real consumption purchased is $c = y/\widehat{p}$ in equilibrium.[5] Given the expressions for c and d, the indirect utility $\widehat{V}(y, \widehat{p}, \widehat{q})$ is defined as

$$\widehat{V}(y, \widehat{p}, \widehat{q}) \equiv u\left(\frac{y}{\widehat{p}}, \frac{y}{\widehat{p}\Psi_d(\widehat{q})z}\right). \tag{2}$$

Let $V(y)$ be the indirect utility for the household once it has incorporated the optimal market choice. V is determined by the best market; $V(y) = \max_{(p,q)\in\Phi} \widehat{V}(y, p, q)$, where Φ is the set of available markets in equilibrium, yet to be determined.

Consider now the problem for the suppliers. Each supplier offers a market bundle $(\widehat{p}, \widehat{q})$. To attract shoppers, it must offer bundles no worse than the most attractive one available to shoppers. Let Y denote the endowment of the representative household. The firm's objective is to maximize profits, $\max\{\widehat{p}\Psi_T(\widehat{q})z\}$, subject to the participation constraint for shoppers, i.e., subject to $(\widehat{p}, \widehat{q})$ satisfying

$$\widehat{V}(Y, \widehat{p}, \widehat{q}) \geq V(Y). \tag{3}$$

The solution to the firm's problem is characterized by the following first-order condition[6]

$$-\frac{\frac{\partial u}{\partial d}}{\frac{\partial u}{\partial c}} = z\frac{\partial \Psi_T(q)}{\partial q}. \tag{4}$$

A *competitive search equilibrium* is defined by indirect utility functions $(V, \widehat{V})$, individual decision rules (c, d), aggregate allocations (D, C, Y), and a set of markets $\Phi = \{(p_j, q_j)\}_{j\in J}$ so that (i) the individual decision rules and value functions solve the household's problems of choosing the best market and maximizing u subject to Equation (1); (ii) for every active market $j \in J$, the price and market tightness $(p_j, q_j) \in \Phi$ satisfy the firms' first-order condition (4); (iii) the household endowment Y equals aggregate profits, and (iv) individual decisions and market tightness are consistent with aggregate allocations.

For convenience, we let the consumption good be the numéraire good. The competitive search equilibrium and its efficiency properties can be characterized by the following proposition.

Proposition 1. *There exists a unique competitive search equilibrium with a unique active market featuring $p = 1$ and $q = D$. Equilibrium allocations are given by $C = Y = z \cdot \Psi_T(D)$, where the aggregate search D is determined by the functional equation*

$$-\frac{\partial u\left(z\,\Psi_T(D), D\right)/\partial D}{\partial u\left(z\,\Psi_T(D), D\right)/\partial C} = z\frac{\partial \Psi_T(D)}{\partial D}. \tag{5}$$

[5] We focus on economies where the disutility of search effort is sufficiently low that the entire budget is spent in equilibrium.

[6] To see this, take first-order conditions w.r.t. $\widehat{p}$ and $\widehat{q}$ in the constrained supplier problem and exploit that $\Psi_T(q) = q \cdot \Psi_d(q)$.

The competitive equilibrium is efficient.

The proof follows from the first-order condition (4) and the equilibrium conditions. The efficiency result follows from considering the planning problem $\max_{C,D} \{u(C, D)\}$ subject to the aggregate resource constraint $C = z\ \Psi_T(D)$. The solution to equation (5) solves this planning problem, which establishes efficiency. See Appendix A for details.

Intuitively, the equilibrium condition (5) states that the marginal rate of substitution between shopping and consumption equals the marginal rate of transformation of shopping, i.e., the marginal increase in the probability that the suppliers are matched times the amount of goods per supplier. The reason why there can only be one active market in equilibrium is that households are identical and the equilibrium is unique. It follows that the set of available markets Φ is a singleton.

In this economy, *potential* output is z whereas the *realized* output is smaller than z. Some units are not found, and hence some potential output is lost due to insufficient search. Nevertheless, the economy is efficient: finding additional goods is not worth the additional search effort. Total factor productivity (TFP) is a function of both search effort D and z, TFP $\equiv z\ \Psi_T(D)$, and will respond to changes in preferences.

Note that we measure realized consumption consistently with the way value added is measured in NIPA: a lost opportunity does not contribute to value added and it is only when it is purchased that potential production translates into consumption. In our model, search effort can be interpreted as an input to production. Note that when constructing measures for factor inputs in NIPA, the national statistical offices do not include search effort as an input. The reason why fluctuations in search can give rise to fluctuations in measured TFP is precisely because we (mis-)measure inputs in the same way as in NIPA. Indeed, if search were properly accounted for as an input factor, TFP would be given by z.

2.1 Introducing shocks and dynamics

We now analyze how shocks to preferences —which we interpret as demand shocks— affect the allocations in economies with search for goods. To this end, we extend the static endowment economy above to a dynamic production economy that features preference shocks and endogenous labor supply. This simple economy will illustrate how preference shocks that affect the disutility of search will have qualitatively similar effects as a standard productivity shock, generating joint comovements between consumption, labor supply, savings, and productivity. As we shall see, this form of demand shock is qualitatively different from more traditional sources of demand shocks that affect consumption and labor supply.

Consumption goods are produced by a unit measure of firms. Each firm resides in a location, equivalent to a supplier in the section above. Firms transform labor input n into the consumption good via a production function $f = z\ n^{\alpha}$, where z is a *true* productivity shock.

Households search for suppliers and trade with the firms they find. Let the number of matches between suppliers and consumers be given by a Cobb-Douglas matching function

$$M = \mu \ D^{\varphi} \ T^{1-\varphi}, \tag{6}$$

where φ is a matching technology parameter and μ is a parameter capturing search efficiency. This implies $\Psi_d(q) = \mu \ q^{\varphi-1}$ and $\Psi_T(q) = \mu \ q^{\varphi}$.

The definition of a competitive search equilibrium is similar to the definition in the endowment model above, with two differences. First, the market for labor must clear. Second, since production is endogenous, the competitive search equilibrium requires indexing goods markets not only by price and market tightness (p, q) but also by the quantity of the good offered for sale by the firm, F. In Appendix B we show that with endogenous production optimality requires that markets are indexed by the triplet (p, q, F) of price, market tightness, and the quantity of the good produced. We conjecture that this property applies to all competitive search models where consumers search for firms and firms can decide the magnitude of the available goods.

Since households are identical and firms are identical, there will be only one active market where firms offer goods for sale and where households search for goods. This market has $T = 1$ and $M = \mu \ D^{\varphi}$ number of matches between firms and consumers. Equilibrium also requires that all firms offer the same number of goods for sale, $F = z \ N^{\alpha}$, where N is aggregate labor supply and F is potential output. The aggregate value added is therefore

$$Y = \mu \ z \ D^{\varphi} \ N^{\alpha}, \tag{7}$$

Let $A \equiv Y/N^{\alpha} = \mu \ z \ D^{\varphi}$ denote what the measured TFP would be when search effort is being ignored as a factor input. We impose $\varphi + \alpha \leq 1$, which ensures that aggregate production does not feature increasing returns to scale.

To study dynamics, we assume there are two periods and that goods that have been found can be saved until (or borrowed from) the next period at zero interest. In this section, we abstract from productive capital. Given the storage technology, consumption is

$$\begin{aligned} c &= Y - s \\ c' &= Y' + s, \end{aligned} \tag{8}$$

where s is goods in storage and the notation x' denotes the second-period variable x. A representative household has time-additive preferences over consumption, search, and labor supply. The discounted utility is

$$V = u\left(c, d, n; \theta\right) + \beta\, u\left(c', d', n'; \theta'\right), \tag{9}$$

where the period felicity function is of the type introduced by Greenwood et al. (1988) (GHH preferences) and given by

$$u\left(c, d, n; \theta\right) = \frac{1}{1-\gamma}\left(\theta_c\, c - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta} - \theta_n \frac{n^{1+1/\nu}}{1+1/\nu}\right)^{1-\gamma}, \tag{10}$$

and $\theta \equiv \{\theta_c, \theta_d, \theta_n\}$ is a vector of preference shifters. The aim of this section is to understand how preference shocks to β and θ propagate in the economy. The GHH preferences allow us to derive sharp implications for how various shocks affect household choices. Moreover, as we shall see in Section 4.1.1, these preferences are also consistent with household-level micro data on search behavior.

The competitive equilibrium is efficient (see Appendix B). We derive the equilibrium allocation by solving a planning problem maximizing (9) subject to equations (7)-(8). Let x^* denote the equilibrium allocation when all preference shifters are equal to unity ($\beta = 1$ and $\theta_i = \theta_i' = 1$ for $i \in \{c, d, n\}$) and let the notation $\widehat{x}$ denote the log deviation from x^*, i.e., $\widehat{x} \equiv \ln x - \ln x^*$. We summarize the competitive equilibrium allocation in the following proposition.

Proposition 2. *The equilibrium allocation is given by*

$$\xi_1 \cdot \widehat{d} = \left(\frac{1}{\nu}+1\right)\ln z - \alpha \ln\left(\frac{\theta_n}{\theta_c}\right) - \left(\frac{1}{\nu}+1-\alpha\right)\ln\left(\frac{\theta_d}{\theta_c}\right) \tag{11}$$

$$\xi_1 \cdot \widehat{n} = \left(\frac{1}{\eta}+1\right)\ln z - \left(\frac{1}{\eta}+1-\varphi\right)\ln\left(\frac{\theta_n}{\theta_c}\right) - \varphi \ln\left(\frac{\theta_d}{\theta_c}\right) \tag{12}$$

$$\xi_1 \cdot \widehat{y} = \left(\frac{1}{\eta}+1\right)\left(\frac{1}{\nu}+1\right)\ln z - \left(\frac{1}{\eta}+1\right)\alpha \ln\left(\frac{\theta_n}{\theta_c}\right) - \left(\frac{1}{\nu}+1\right)\varphi \ln\left(\frac{\theta_d}{\theta_c}\right) \tag{13}$$

$$\xi_1 \cdot \widehat{A} = \left(\frac{1}{\eta}+1\right)\left(\frac{1}{\nu}+1-\alpha\right)\ln z - \varphi\alpha \ln\left(\frac{\theta_n}{\theta_c}\right) - \varphi\left(\frac{1}{\nu}+1-\alpha\right)\ln\left(\frac{\theta_d}{\theta_c}\right), \tag{14}$$

where $\xi_1 \equiv \left(1/\eta + 1 - \varphi\right)/\nu + \left(1/\eta+1\right)\left(1-\alpha\right) - \varphi > 0$*. Moreover, savings and consumption are given by*

$$s = \frac{\left(\frac{\theta_c'}{\theta_c}\right)^{\frac{1-\gamma}{\gamma}} \beta^{\frac{1}{\gamma}} \xi_2}{1+\left(\frac{\theta_c'}{\theta_c}\right)^{\frac{1-\gamma}{\gamma}} \beta^{\frac{1}{\gamma}}} \cdot y - \frac{\xi_2}{1+\left(\frac{\theta_c'}{\theta_c}\right)^{\frac{1-\gamma}{\gamma}} \beta^{\frac{1}{\gamma}}} \cdot y' \tag{15}$$

$$c = \left(\frac{\xi_2}{1+\left(\frac{\theta_c'}{\theta_c}\right)^{\frac{1-\gamma}{\gamma}} \beta^{\frac{1}{\gamma}}} + 1 - \xi_2\right) \cdot y + \frac{\xi_2}{1+\left(\frac{\theta_c'}{\theta_c}\right)^{\frac{1-\gamma}{\gamma}} \beta^{\frac{1}{\gamma}}} \cdot y', \tag{16}$$

where $\xi_2 \equiv 1 - \varphi\eta/(1+\eta) - \alpha\nu/(1+\nu)$ *and* $\xi_2 \in (0,1)$.

Proposition 2, proved in Appendix B, shows how the various shocks propagate in the economy. Consider first the search shock θ_d. A lower disutility of search leads the household to search more. This increases labor productivity and induces a larger n and a larger y. Moreover, a larger search effort increases the measured TFP A and the aggregate labor productivity y/n. The household distributes the increase in income y on both c and c', where the larger c' comes about through increased savings s. Thus, a negative θ_d shock will simultaneously increase output y, consumption c, labor supply n, savings s, the TFP A, labor productivity y/n, and search d. These comovements are qualitatively similar to the patterns in the data, where consumption, labor supply, output, investment, labor productivity, shopping time, and TFP are strongly positively correlated.

Consider now a shock to the discount factor β, i.e., a standard demand shock. A lower β will increase c and lower s and c'. However, due to the GHH preference assumption, this change has no effect on d and n. It follows that a β shock generates negative comovement between consumption and savings and zero correlation between consumption and the other variables (y, n, d, and z).

Next, consider another standard preference shock, the labor supply shock θ_n. A fall in the disutility of work induces more labor supply. This makes it more attractive to search, which in turn increases d and A. The resulting increase in output induces increases in c and s. Thus, a negative θ_n shock will simultaneously increase labor supply n, output y, consumption c, savings s, search d, and TFP A. However, it will decrease the aggregate labor productivity y/n. The reason is that the decreasing return to labor supply associated with the decrease in n outweighs the increase in A stemming from the increase in d. This result is guaranteed by the assumption of non-increasing return to scale, $\varphi + \alpha \leq 1$. We conclude that θ_n can generate positive comovements in (n, y, c, s, A) but implies countercyclical labor productivity. Moreover, note that endogenous search effort contributes to positive co-movements between labor supply and TFP. Any exogenous shocks that increase the search effort d will increase both the measured TFP and, in turn, the labor supply (since the return to working is higher).

Finally, consider the shock θ_c. Note that this shock can be replicated by a suitable adjustment of the triplet $(\beta, \theta_d, \theta_n)$. A positive shock to θ_c can therefore be interpreted as a combination of a proportional decrease in θ_d and θ_n and a change in β. An increase in θ_c will therefore increase search d, labor supply n, and output y. Thus, the consumption shock will imply that more output is squeezed out of the economy, even when controlling for the labor input. However, the effect of θ_c on labor productivity, consumption, and savings are ambiguous and depends on parameters. See Appendix B for details.

We conclude that the preference shocks θ_d, θ_n, and β can all induce an increase in current consumption and can therefore be interpreted as different types of demand shocks. However, the shocks differ in the comovements they induce across variables in the economy. In particular, the search shock θ_d

causes dynamics reminiscent of a standard TFP shock in real business-cycle models which create positive comovements between all variables.

2.1.1 A precursor of amplification

One insight from Proposition 2 is that the presence of endogenous search effort affects how the economy responds to all shocks, including shocks other than θ_d. The reason why endogenous search effort influences the propagation of for example a larger true productivity shock z or a larger willingness to work is that these (non-search) shocks make it optimal to simultaneously increase search effort d in order to capture the larger return to search effort. This in turn increases the measured TFP and output, see equations (11)-(14). In this sense, the presence of search effort amplifies the effects of the non-search shocks. In fact, the proposition shows that all shocks move output and search effort in the same qualitative direction. This implies that all shocks, including technology shocks, cause search effort to be procyclical. We view this as an attractive feature of our model in light of the fact that an empirical proxy measure for search effort – shopping time – is strongly procyclical (cf. Figure 1). Since search determines market tightness, this implies that market tightness is also procyclical, even conditional on the θ_d shock.[7]

To see this amplification effect, consider a slight rewriting of the expression for measured TFP in equation (14),

$$\hat{A} = \hat{z} + \underbrace{X\frac{1+\nu}{\nu}}_{\text{amplification of } z}\hat{z} - \underbrace{X\alpha}_{\text{ampl. of } \theta_n}\hat{\theta}_n - \underbrace{X\left(\frac{1+\nu}{\nu} - \alpha\right)}_{\text{ampl. of } \theta_d}\hat{\theta}_d + \underbrace{X\frac{1+\nu}{\nu}}_{\text{ampl. of } \theta_c}\hat{\theta}_c. \tag{17}$$

where $X \equiv \eta\varphi\left(\frac{1+\nu}{\nu}(1+\eta(1-\varphi)) - (1+\eta)\alpha\right) \geq 0$. Naturally, in the absence of endogenous search or search frictions, i.e., with $\eta = 0$ or $\varphi = 0$, all the amplification terms disappear. In this case, the elasticity $\mathcal{E}_{A,z}$ of measured TFP to an increase in true productivity is unity and the corresponding elasticities to any of the preference shocks are zero. In the presence of search frictions ($\eta > 0$ and $\varphi > 0$), the elasticity $\mathcal{E}_{A,z}$ becomes larger than unity, amplifying the propagation of the true TFP shock. This amplification extends to labor supply. As explained above, a decrease in θ_n that increases n makes it optimal to also increase search effort d. When everybody exerts more search effort, the TFP increases, inducing a further bump in output.

We conclude that search effort amplifies the response of TFP, labor supply, and GDP to technology shocks and preference shocks relative to a model without search effort. Absent variable search effort, the only mechanism for generating movements in measured TFP is true productivity shocks. It follows that the endogenous search effort will amplify the variance of measured TFP. Therefore, the search model will

[7] The mechanism in Michaillat and Saez (2015) is qualitatively different. There, all shocks except for the demand shock cause negative correlation between market tightness and output. Therefore, their model will by construction assign a large role to demand shocks.

require a lower variance of true productivity shocks to generate a certain volatility of TFP. We will later quantify these effects in the estimated model.

2.1.2 A precursor of identification

In the quantitative sections below we estimate versions of this model, simultaneously incorporating shocks to both θ_d and z. This requires a discussion of identification. We pursue two different ways to identify these processes.

Note first that when considering standard aggregate data on labor supply, output, consumption, and savings, fluctuations in search effort are observationally equivalent to fluctuations in true productivity z. To see this, note that a marginal increase in z has the same effect on the allocation (n, y, c, s) as a decrease of size $(1 + 1/\eta)/\varphi$ in θ_d. However, this equivalence does not extend to search effort d in the sense that z and θ_d have very different impacts on d. This can be seen by comparing the loadings on z and θ_d in equation (11) versus equations (12)-(14). This has implications for identification when estimating the model: if we had direct observations of search effort, it would be straightforward to identify θ_d and z using observations of (n, y, c, s) and d for the current period. This will be our first identification strategy when we pursue a Bayesian estimation exercise in Section 5.1.

In the analysis above, we model the search shock as a preference shock θ_d that affects search effort. However, one might expect product-market frictions to matter also though variations in the search process itself, summarized by the search efficiency parameter μ. As it turns out, in our model a shock to search efficiency has an equivalence with θ_d similar to the equivalence between θ_d and z. Namely, in equilibrium, a shock to μ has the same effects on c, s, n, y, and A as that of θ_d (but different implications for search effort d). It follows that up to observations of labor supply, output, consumption, and savings, a shock to search disutility θ_d could be interpreted as a shock to search efficiency. This observation has implications for how to interpret the shock θ_d in the quantitative analysis below. In particular, the θ_d shock captures broader aspects of the product market frictions than just the disutility of search effort. This insight suggests that it would be useful to also estimate the model without using direct observations on d. We therefore lay out an alternative identification strategy that does not rely on observing d. The crux is to make additional assumptions on the time-series properties of θ_d and z. In particular, we assume that $\ln \theta_d$ and $\ln z$ follow independent AR(1) processes with persistence ρ_d and ρ_z, respectively. Since current consumption c and savings s react to changes in next-period output y' through the Euler equation, the magnitude of these changes identify z and θ_d, provided that $\rho_z > \rho_d$. This will constitute our second identification strategy in Section 5.1.

3 The stochastic growth model version of the economy

We embed the matching frictions in product markets into an otherwise standard growth model suitable for quantitative business cycle analysis. There are two sectors, one for consumption goods and one for investment goods. Both sectors have matching frictions. Production uses capital and labor as inputs. We start by describing technology and preferences. We then analyze the problems faced by households and firms, and study price determination in the presence of the search friction.

3.1 Technology and Markets

There is a unit measure of firms in the economy, measure T_c of consumption firms and T_i of investment firms, $T_c + T_i = 1$. Each firm resides in a location, equivalent to a supplier in Section 2. Consumption firms have a technology for transforming capital k and labor n into the consumption good via a standard production function $zf(k, n)$, where f is differentiable and strictly concave. Investment firms are subject to an additional aggregate investment shock z_I. Their production function is $z_I \, zf(k, n)$. For convenience we index the shocks by subscripts c and i, respectively: $z_c \equiv z$ and $z_i \equiv z \, z_I$

Both consumption and investment goods are subject to search frictions with competitive search protocols. The matching technology for shoppers and firms is given by equation (6). The aggregate state vector is $\Lambda \equiv (\theta, Z, K)$, comprising a vector of preference shocks θ, a vector of technology shocks $Z = (z_c, z_i)$, and a vector of capital stocks $K = (K_c, K_i)$ installed in consumption and investment firms, respectively. To simplify the description of equilibrium we formulate the problem so that all dynamic choices are made by the household: it chooses how much capital to accumulate of each type.

3.2 Households

Households have GHH preferences over (c, d, n) as in equation (10), where $d = d_c + d_i$. A household owns k_c and k_i units of capital installed in consumption- and investment-firms, respectively. They also receive the net profits from the firm as dividends. The household's state vector includes the aggregate state Λ and individual capital holdings (k_c, k_i).

Households take aggregate variables as given, including prices, market tightness, and quantity given by the triplets (p_c, q_c, F_c) and (p_i, q_i, F_i) in the active consumption-good and investment-good market, respectively, the rental rates on capital invested in consumption- and investment-producing firms, R_c and R_i, respectively, aggregate dividends π, the wage W, and the laws of motion of aggregate capital $K_c' = G_c(\Lambda)$ and $K_i' = G_i(\Lambda)$. These aggregate equilibrium objects are functions of the state vector Λ.

We denote by $V(\Lambda, k_c, k_i)$ the value of a representative household and by $\widehat{V}^c\left(\Lambda, k_c, k_i, \hat{p}_c, \hat{q}_c, \hat{F}_c\right)$ the value that it would obtain if it went shopping in a $(\hat{p}_c, \hat{q}_c, \hat{F}_c)$ market for the consumption good and

$\widehat{V}^i\left(\Lambda, k_c, k_i, \hat{p}_i, \hat{q}_i, \hat{F}_i\right)$ the value for shopping in a $(\hat{p}_i, \hat{q}_i, \hat{F}_i)$ market for the investment good. Specifically, households in market $(\hat{p}, \hat{q}, \hat{F}) = \{(\hat{p}_c, \hat{q}_c, \hat{F}_c), (\hat{p}_i, \hat{q}_i, \hat{F}_i)\}$ choose consumption c, shopping efforts d_c and d_i, labor supply n, and future capital k_c' and k_i' so as to maximize

$$\widehat{V}\left(\Lambda, k_c, k_i, \widehat{p}, \widehat{q}, \widehat{F}\right) = \max_{d_c, d_i, n, c, k_c', k_i'} u\left(c, d_c + d_i, n; \theta\right) + \beta \, \mathrm{E}\left\{V\left(\Lambda', k_c', k_i'\right) \,\middle|\, \Lambda\right\}, \tag{18}$$

subject to the shopping constraints, the budget constraint, the capital accumulation, and the aggregate laws of motion of capital,

$$c = d_c \, \Psi_{d,c}[\hat{q}_c] \, \hat{F}_c, \tag{19}$$

$$i = d_i \, \Psi_{d,i}[\hat{q}_i] \, \hat{F}_i, \tag{20}$$

$$\hat{p}_c \, c + \hat{p}_i \, i = \pi + k_c \, R_c(\Lambda) + k_i \, R_i(\Lambda) + n \, W(\Lambda), \tag{21}$$

$$i = k_c' + k_i' - (1-\delta) \, (k_i + k_c), \tag{22}$$

$$K_c' = G_c(\Lambda), \qquad K_i' = G_i(\Lambda). \tag{23}$$

The value function V is determined by the best market;
$V(\Lambda, k_c, k_i) = \max_{(p,q,F)\in\Phi} \widehat{V}(\Lambda, k_c, k_i, p, q, F)$, where $(p, q, F) = \{(p_c, q_c, F_c), (p_i, q_i, F_i)\}$ and Φ is the set of available markets.

3.3 Firms

Representative firms rent capital and hire labor in spot markets. Given the state of the economy Λ, a firm in sector $j \in \{c, i\}$ makes two choices: how much labor and sector-specific capital to rent for producing output, and what market bundle (p_j, q_j, F_j) to offer.[8] The problem for the firm is

$$\pi_j(\Lambda) = \max_{k, n, \hat{p}_j, \hat{q}_j, \hat{F}_j} \hat{p}_j \, \hat{F}_j \, \Psi_{T,j}\left(\hat{q}_j\right) - W\left(\Lambda\right) \, n - R_j\left(\Lambda\right) \, k, \tag{24}$$

$$\text{subject to} \quad \widehat{V}\left(\Lambda, K, p_{-j}, q_{-j}, F_{-j}, \hat{p}_j, \hat{q}_j, \hat{F}_j\right) \geq V\left(\Lambda, K\right),$$

$$z_j \, f\left(k, n\right) \geq \widehat{F}_j.$$

The firm's problem (24) is static in the sense that current decisions do not influence the future value of the firm, and future variables do not matter for the firm's current problem. This determines the dividends π_j, the factor demands n_j^f and k_j^f, and the triplet (p_j, q_j, F_j), all expressed as functions of Λ.

[8]We abstract from firms' effort to overcome the search friction. We focus on one-sided search effort by the consumers because this search effort is not measured in NIPA and therefore contribute to mismeasurement of aggregate TFP. Incorporating firms' search effort to our model would not contribute to this mismeasurement because firms' efforts to overcome the matching frictions require inputs that are measured in NIPA. Therefore, the contribution of this effort to output is already taken into account.

3.4 Equilibrium

The competitive search equilibrium of the production economy consists of value functions and decision rules for the households $\{V, \widehat{V}, c, d_c, d_i, n, k'_c, k'_i\}$, all expressed as functions of the individual state (Λ, k), and for the firm, $\{n^f_c, n^f_i, k^f_c, k^f_i\}$, and a set of aggregate allocations $\{C, D_c, D_i, I, N, N_c, N_i, K'_c, K'_i\}$, prices $\{W, R_c, R_i, p_c, p_i\}$, market tightness q_c, q_i, production capacity F_c, F_i, dividends π, and profits π_i, π_c, where all aggregate variables are expressed as functions of Λ, such that

1. The consumption good is the numéraire, $p_c = 1$.

2. The decision rules $c(\Lambda, k_c, k_i)$, $d(\Lambda, k_c, k_i)$, $n(\Lambda, k_c, k_i)$, $k'_c(\Lambda, k_c, k_i)$, $k'_i(\Lambda, k_c, k_i)$, and the associated value functions $\widehat{V}(\Lambda, k_c, k_i, p, q, F)$ and $V(\Lambda, k_c, k_i)$ solve the household problem (18-23), taking as given prices $\{W, R_c, R_i\}$ and dividend income π. Moreover,

$$V(\Lambda, K_c, K_i) = \widehat{V}(\Lambda, K_c, K_i, 1, q_c(\Lambda), F_c(\Lambda), p_i(\Lambda), q_i(\Lambda), F_i(\Lambda)).$$

3. The triplet $(p_j(\Lambda),\ q_j(\Lambda),\ F_j(\Lambda))$, factor demands $n^f_j(\Lambda)$ and $k^f_j(\Lambda)$, and profit $\pi_j(\Lambda)$ solve the problem (24) of producers in sector $j \in \{c, i\}$, taking as given $\{W, R_j, V, \widehat{V}\}$, where $F_j(\Lambda) = z_j f(k^f_j(\Lambda), n^f_j(\Lambda))$.

4. Individual decision rules are consistent with aggregates $T_c k^f_c(\Lambda) = K_c$, $T_i k^f_i(\Lambda) = K_i$, $C(\Lambda) = c(\Lambda, K)$, $D_c(\Lambda) = d_c(\Lambda, K)$, $D_i(\Lambda) = d_i(\Lambda, K)$, $N(\Lambda) = n(\Lambda, K)$, $T_c n^f_c(\Lambda) = N_c(\Lambda)$, and $T_i n^f_i(\Lambda) = N_i(\Lambda)$.

5. Market clearing conditions are satisfied,

$$\begin{aligned}
C(\Lambda) &= T_c\ \Psi_{T,c}(q_c(\Lambda))\ F_c(\Lambda), & q_c(\Lambda) &= D_c(\Lambda)/T_c,\\
I(\Lambda) &= T_i\ \Psi_{T,i}(q_i(\Lambda))\ F_i(\Lambda), & q_i(\Lambda) &= D_i(\Lambda)/T_i,\\
I(\Lambda) &= G_c(\Lambda) + G_i(\Lambda) - (1-\delta)(K_c + K_i), & &\\
N(\Lambda) &= N_i(\Lambda) + N_c(\Lambda), & \pi(\Lambda) &= T_i\pi_i(\Lambda) + T_c\pi_c(\Lambda).
\end{aligned}$$

6. Aggregate laws of motion of capital are consistent with individual behavior, $K'_c = G_c(\Lambda) = k'_c(\Lambda, K)$ and $K'_i = G_i(\Lambda) = k'_i(\Lambda, K)$.

As we show in Appendix C, the welfare theorems apply so the competitive equilibrium is efficient. It is therefore convenient to solve for the equilibrium using a standard social planner approach where the planner faces the same search friction in the consumption and investment markets. Given an aggregate state variable Λ, the social planner's problem is

$$\mathcal{W}(\Lambda) = \max_{D_c, D_i, N_c, N_i, K_c', K_i'} \left\{ u\left(D_c\, \Psi_{d,c}\left[D_c\right] z_c f\left(\frac{K_c}{T_c}, \frac{N_c}{T_c}\right), D_c + D_i, N_c + N_i, \theta \right) + \beta\, \mathrm{E}\left\{ \mathcal{W}(\Lambda') \mid \Lambda \right\} \right\} \tag{25}$$

$$\text{subject to} \qquad D_i\, \Psi_{d,i}[D_i] z_i f\left(\frac{K_i}{T_i}, \frac{N_i}{T_i}\right) = K_c' + K_i' - (1-\delta)(K_c + K_i) \text{ and } \Lambda' = \left(\theta', Z', K_c', K_i'\right),$$

We define $K_j^f \equiv K_j / T_j$ and $N_j^f \equiv N_j / T_j$ for $j \in \{c, i\}$. The first-order conditions yield four optimality conditions (see Appendix C for derivations). The first is an intra-temporal first-order condition equating the marginal cost and the marginal gain of consumption search. Equation (26) equates the marginal rate of substitution between search and consumption to the marginal rate of transformation, i.e., the marginal increase in aggregate consumption from searching slightly harder,

$$-\frac{\frac{\partial u}{\partial D}}{\frac{\partial u}{\partial C}} = \frac{\partial M\left[D_c, T_c\right]}{\partial D_c}\, z_c\, f\left(K_c^f, N_c^f\right) \tag{26}$$

This corresponds to the optimality condition (5) in the endowment economy with the only difference that capacity, $z_c\, f\left(K_c^f, N_c^f\right)$, is now endogenous. The equivalent equilibrium condition for investment goods is $-\frac{\partial u}{\partial D} / \frac{\partial u}{\partial C} = \frac{\partial M[D_i, T_i]}{\partial D_i}\, p_i z_i\, f\left(K_i^f, N_i^f\right)$, where p_i is the implied relative price of investment goods (in terms of found consumption goods)., Given the equilibrium allocations it is straightforward to back out the p_i that obtains in a decentralized competitive equilibrium,

$$p_i \;=\; \frac{\Psi_{T,c}(D_c)}{\Psi_{T,i}(D_i)} \frac{z_c \partial f\left(K_c^f, N_c^f\right)/\partial N_c^f}{z_i \partial f\left(K_i^f, N_i^f\right)/\partial N_i^f}. \tag{27}$$

The investment price is declining in the investment technology shock z_i since higher investment productivity increases investment production and lowers the relative value of investment goods. Similarly, p_i is increasing in relative tightness of the consumption market $\Psi_{T,c}(D_c)/\Psi_{T,i}(D_i)$ and, hence, search effort in consumption good. The reason is that higher D_c lowers the relative price of consumption.

The second optimality condition is a standard intra-temporal first-order condition for labor supply, equating the marginal utility cost of working one additional hour to the marginal utility gain of the increased consumption production due to the additional hours worked,

$$-\frac{\frac{\partial u}{\partial N}}{\frac{\partial u}{\partial C}} = \Psi_{T,c}\left[D_c\right]\; z_c \frac{\partial f\left(K_c^f, N_c^f\right)}{\partial N_c^f}. \tag{28}$$

The third optimality condition equalizes the marginal gain of investing in each sector,

$$0 = \mathrm{E}\left\{ \frac{\partial u}{\partial C'} \left({\Psi'}_{T,i} z_i' \frac{\partial f\left(K_i^{f\prime}, N_i^{f\prime}\right)}{\partial K_i^{f\prime}} - {\Psi'}_{T,c} z_c' \frac{\partial f\left(K_c^{f\prime}, N_c^{f\prime}\right)}{\partial K_c^{f\prime}} \right) \middle| \Lambda \right\}. \tag{29}$$

Finally, the planner program implies a standard Euler equation in terms of found goods,

$$p_i \frac{\partial u}{\partial C} = \beta \ \mathrm{E}\left\{ \frac{\partial u}{\partial C'} p_i' \left({\Psi'}_{T,i} z_i' \frac{\partial f\left(K_i^{f\prime}, N_i^{f\prime}\right)}{\partial K_i^{f\prime}} + 1 - \delta \right) \middle| \Lambda \right\}. \tag{30}$$

The left-hand side is the utility cost of producing one more unit of the investment good. The right-hand side is the equivalent next-period utility gain of a marginal increase in $K_i^{f\prime}$ taking into consideration future matching frictions. Note that the Euler equation (30) is similar to the standard Euler equation but it incorporates the probability of matching a customer in the marginal return of capital.

4 Mapping the Model to Data

We now choose functional forms for preferences and technology. We maintain the GHH formulation of preferences as in equation (10). This rules out wealth effects in search efforts and ensures that the model generates a procyclical search effort, in line with the empirical evidence on shopping time documented by Petrosky-Nadeau et al. (2016). We first abstract from consumption shocks θ_c. We set the weight on labor supply to $\chi\theta_n$, where $\theta_n = 1$ in steady state and the parameter χ determines average hours worked. The remaining preference parameters are the elasticity of shopping effort with respect to variation in the return to search η, the discount rate β, the inverse of the intertemporal elasticity of substitution, γ, and the Frisch elasticity of labor supply ν.

Firms have decreasing returns to scale. This is a natural assumption in a model with frictions in the matching between consumers and firms.[9] The production function is Cobb-Douglas, $f(k, n) = k^{\alpha_k}\, n^{\alpha_n}$.

4.1 Calibration

We calibrate some parameters here and estimate the rest of the model in Section 5. As far as possible, our calibration targets the steady state, with parameter values that are standard in business cycle research. For the parameters specific to our search economy, we exploit cross-sectional data. Table 1 reports the calibration targets and the parameters most closely associated with each target. The targets are defined in yearly terms even though the model period is a quarter.

The first group of parameters are set exogenously: the intertemporal elasticity of substitution is 1 and

[9] Note that if production were constant return to scale in capital and labor, the search friction would become irrelevant as one firm would produce all goods and all search would focus on finding the firm with available goods.

Table 1: Calibration Targets, Implied Aggregates, and (Quarterly) Parameter Values

Targets	Value	Parameter	Calibration
First Group: Parameters Set Exogenously			
Risk aversion	1	γ	1
Real interest rate	4%	β	0.997
Average growth rate	3%	$\bar{g}$	0.74%
Frisch elasticity	0.72	ν	0.72
Second Group: Normalizations			
Steady-state output	1	$E(z)$	1.22
Relative price of investment	1	$E(z_i)$	1.00
Fraction of time spent working	30%	χ	7.99
Capacity utilization	81%	μ	0.86
Consumption share of output	0.80	T_c	0.80
Third Group: Standard Targets			
Investment share of output	0.20	δ	1.08%
Physical capital to output ratio	2.75	α_k	0.23
Labor share of output	0.67	α_n	0.45
Fourth Group: Targets Specific to This Economy			
Ratio cross-sectional st. dev. of goods prices to st.dev. of cons. expenditure	28.6%	φ	0.32
Shopping time expenditure elasticity	7.5%	η	0.20

the rate of return is 4 percent. We set the Frisch elasticity to 0.72 based on Heathcote et al. (2010), who take into account the response of hours worked for both men and women in a model with joint labor supply decisions.

The second group of parameters are normalizations that are immaterial to the cyclical behavior of the economy. We target values that have a direct interpretation. The average adult works about 1600 hours per year, so 30% of available time is spent working. Moreover, the share of potential goods that are found is set equal to the average capital utilization in the Capacity Utilization series published by the Board of Governors (81%). In the model this is captured by the search efficiency parameter μ. The measure of firms in each sector, T_c and T_i, is calibrated to match the share of consumption and investment, respectively.

The third group of parameters are determined simultaneously through specified steady-state targets. The physical capital to output ratio, the investment share of output, and the labor share allow us to identify the depreciation rate and the capital share and labor shares α_k and α_n, respectively. In particular, the quarterly capital depreciation rate δ is set to 1.08%, pinned down by the annual investment-output ratio of 20% and annual growth rate of 3%. A capital-output ratio of 2.75 then generates a capital share parameter $\alpha_k = 0.23$. Finally, note that setting $\alpha_n = 0.67 * (1 - \varphi)$ ensures that the labor share target is met, given a value of φ. See Appendix D for derivations of the expressions for α_k and α_n.

4.1.1 Identifying the Search Friction with Cross-sectional Data

The fourth group of calibration targets concerns the magnitude of the search friction and how willing households are to vary their search effort. Kaplan and Menzio (2016) argue that the search friction for goods is manifest in the fact that there is a cross-sectional dispersion of prices for identical goods. The price dispersion is therefore a useful moment to identify the search-related parameters of our model. Moreover, search effort for households with different levels of expenditure is informative about the elasticity of search effort.

The representative-agent version of our economy does not feature any dispersion in neither prices nor search effort. However, a version of our model with cross-sectional heterogeneity in consumption expenditure generates such price dispersion in a very a natural way. In particular, if expenditure were unevenly distributed across households, our search model would induce endogenous dispersion in prices and search effort. This insight allows us to use US micro data on shopping time, consumption expenditure, and prices to discipline the parameters specific to our shopping theory, namely the weight of shopping in the matching function, φ, and the elasticity of shopping effort η.

To study the cross-sectional implications of our search model, consider a version of our economy where there are J types of households. The types differ in their consumption expenditure, denoted y_j.

The search equilibrium can be characterized as follows (proof in Appendix D.1.)

Proposition 3. *The competitive equilibrium in the stochastic growth model with cross-sectional heterogeneity is a vector of prices and allocations where all firms offer the same capacity F and the posted price of the consumption good in the market targeting type j households is*

$$p_j = \frac{C^{1-\varphi}}{F} \cdot (y_j)^{\varphi} (D_j)^{-\varphi}, \tag{31}$$

where C is aggregate real consumption. The real consumption of type j households is $c_j = D_j^{\varphi} (y_j/C)^{1-\varphi} F$. The measure of firms selling to type j households is equal to the households' expenditure share, $T_j = s_j y_j / C$, where s_j denotes the measure of type j households. Finally, the search effort is given by the solution to the first-order condition

$$-\frac{\partial u}{\partial d} = \varphi (D_j)^{\varphi-1} (y_j)^{1-\varphi} \frac{\partial u}{\partial c} F. \tag{32}$$

The equilibrium price relationship in equation (31) has the attractive feature that it is consistent with cross-sectional micro evidence on prices, search effort, and expenditure. First, prices are falling in search effort D_j. Sorensen (2000) documents that more search effort is associated with lower prices in retail markets for prescription drugs. Second, prices are increasing in expenditure y_j. Using product-level data from the Kielts-Nielsen Consumer Panel, Aguiar and Hurst (2007) and Nord (2022) document that rich households pay more for goods than poor households.

Evaluating the first-order condition (32) using the GHH preferences implies that the shopping effort for type j households is given by

$$\ln(D_j) = \frac{1-\varphi}{1-\varphi+1/\eta} \ln(y_j) + \frac{1}{1-\varphi+1/\eta} \ln\left(\varphi F \frac{\theta_n}{\theta_d}\right). \tag{33}$$

Substitute this expression for D_j in equation (31) in log form and take the standard deviation on each side of the equation. This yields an expression linking the search parameters to empirical moments:

$$m \equiv \frac{std\,(\log\,(p_j))}{std\,(\log\,(y_j))} = \frac{\varphi}{\eta(1-\varphi)+1}. \tag{34}$$

Thus, our first source of identification for the parameters φ and η is through the dispersion of prices for identical goods, relative to the dispersion of expenditures across households. Note that this calibration target does not require that we have direct measurements of search effort. Kaplan and Menzio (2016) estimate that the cross-sectional standard deviation of household price indexes is 15% when using data from the Kielts-Nielsen Consumer Panel Data and standardized bar codes to identify goods. With a standard deviation of log consumption expenditures on services and non-durables of 0.524 (cf. Heathcote et al. (2020)), the ratio m takes the value $m = 0.286$.

For the second source of identification the parameters φ and η we rely on direct measurements of search effort D_j. Take the difference of logs in equation (33) to obtain an expression for how search effort varies across households with different spending levels;

$$\Delta \log (D_j) = \frac{\eta (1-\varphi)}{\eta (1-\varphi) + 1} \Delta \log (y_j) . \tag{35}$$

We assume that the time spent shopping is a good stand-in for actual search effort.[10] Note first that search effort in equation (35) is positively related to expenditure. This property, which is due to the lack on income effects under GHH preferences, is consistent with the empirical evidence on shopping time in the cross section from ATUS: Petrosky-Nadeau et al. (2016) documents that shopping time is *increasing* in household income when controlling for observable household characteristics. For example, households with income between \$100,000 and \$150,000 spend 3.6 minutes more on shopping per day than households with income between \$25,000 and \$50,000 (controlling for households' demographic characteristics, labor force status, and state and time fixed effects). With an average of 42 minutes shopping per day, this difference amounts to $3.6/42 = 8.6\%$ of average shopping time. The expenditures of the \$100,000-\$150,000 group is approximately twice as large as that of the \$25,000-\$50,000 group. This implies an elasticity $\Delta \log (D_j) / \Delta \log (y_j) \approx 0.12$. Equations (34)-(35) then imply $\varphi = 0.32$ and $\eta = 0.20$.

5 Quantitative results

The main purpose of our quantitative exercise is to assess the role of search-related demand shocks in accounting for aggregate fluctuations in the U.S. (Section 5.1). To this end we estimate the model with Bayesian methods using two different sets of data—one with a narrow view of search effort utilizing data on shopping time as a direct measure of search effort and one with a broader view of search effort where we do not target shopping time data.

5.1 Estimating the model with U.S. data

We estimate the model using U.S. aggregate data. We consider four types of shocks, two preference shocks and two technology shocks. The preference shocks are the disutility to shopping θ_{dt} and the disutility to work θ_{nt}. We explore alternative preference shock specifications in Section 5.1.3. We assume that the neutral technology shock is a shock to the trend of z, whose growth rate follows an AR(1)

[10] We acknowledge that actual search effort is a broader concept than just the time spent shopping. Since the parameters φ and η are specific to our search economy and crucial for our quantitative exercise, we consider also alternative parameterizations where we estimate φ as part of the Bayesian estimation, without relying on direct observations of search effort and shopping time data.

process. Let $g_t = z_t/z_{t-1}$ be the growth rate of the technology,

$$\log g_t = (1-\rho_g)\log \bar{g} + \rho_g \log g_{t-1} + \sigma_g \varepsilon_{gt}, \quad \varepsilon_{gt} \sim N(0,1).$$

There is also a standard investment-specific technology shock z_I, implying that TFP for consumption and investment ($z_c = z$ and $z_i = z\ z_I$) are correlated. The investment-specific shock z_I follows an AR(1) process with persistence ρ_i and standard deviation of innovation σ_i. We assume that all four shocks follow AR(1) processes with respective persistence $\{\rho_d, \rho_n, \rho_g, \rho_i\}$. The innovations to the shocks are independent and normally distributed with standard deviations denoted by $\{\sigma_d, \sigma_n, \sigma_g, \sigma_i\}$. To ensure stationarity of the problem when the economy is growing and preferences are GHH, we do a standard adjustment of the preference weights, making them proportional to z_{t-1}. Preferences are then,

$$u_t(c_t, n_t, d_t; \theta_{dt}, \theta_{nt}) = \frac{1}{1-\gamma}\left(c_t - \theta_{dt} z_{t-1}\frac{d^{1+1/\eta}}{1+1/\eta} - \theta_{nt} z_{t-1}\chi\frac{n^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right)^{1-\gamma}.$$

Summing up, we assume that aggregate TFP is a random walk while the preference shocks and z_I are stationary. We estimate jointly the autocorrelations $\{\rho_d,\ \rho_n,\ \rho_g,\ \rho_i\}$ and the standard deviation of innovations $\{\sigma_d,\ \sigma_n,\ \sigma_g,\ \sigma_i\}$ using Bayesian methods. In line with standard practice in the business-cycle literature, we estimate the model in growth rates.

We use four data series in the estimation. Three of these—common for all estimations—are output growth, investment growth, and labor productivity growth. These are based on real data from the National Income and Product Accounts at a quarterly frequency. Since our model is a closed economy without a government sector, we measure investment as private investment and construct output from the sum of private consumption and private investment. Labor productivity is the ratio of our measure of output to total working hours.

For the fourth data series, recall from Section 2.1.2 that the model can be identified in two ways – either by targeting direct observations of search efforts or by making assumptions about time-series properties of shocks to z and θ_d.

Our first estimated benchmark model pursues the direct-observation identification scheme, relying on ATUS time-use data on shopping time as a direct measure of search effort. These data provide annual survey data on shopping time from 2003 to 2019. We measure shopping time as the average time spent shopping for goods, professional and personal care services, and household services, plus the travel time associated with this shopping. Appendix E describes the shopping time data and documents that alternative assumptions about how to measure ATUS shopping time yield very similar dynamics (cf. Appendix Figure 2). In particular, we show that our shopping time measure is robust to excluding travel time and, following the measurement proposed by Petrosky-Nadeau et al. (2016), excluding time used for shopping for gas and groceries. To handle the different frequency on annual shopping time data and quarterly NIPA data, we apply a mixed frequency approach (see Schorfheide et al. (2018) for details).

For the second estimated benchmark model we drop shopping time as an empirical target. This is motivated by two concerns with using shopping time as a proxy for d. First, true search effort for goods is broader than simply the time used shopping. Second, our theory of search frictions encompasses shocks and fluctuations beyond what is captured by search effort. Recall that in the tractable model in Section 2, shocks to θ_d are equivalent to shocks to match efficiency μ. This property extends to the production economy. θ_d should therefore be expected to capture also variations in match efficiency. To allow such broader interpretation of search shocks and capture aspects of the search friction and search effort beyond what is manifested in shopping time, we consider an alternative estimation procedure where we drop shopping time as a direct measure of search and replace this series with the quarterly series on the quality-adjusted price of investment goods from Ríos-Rull et al. (2012), updated to 2019Q4.[11] We choose this series because it is closely tied to the investment-specific technology shock z_I. We then rely on the time-series identification of shocks to θ_d.

5.1.1 Bayesian estimation with data on shopping time

Table 2 lays out the estimation of the benchmark model. The upper panel shows the priors and posteriors for all shock parameters. We assume that autocorrelations follow a Beta distribution and that standard deviation of innovations follow an inverse Gamma distribution. We assume an initial prior of equal autocorrelation and equal volatility for the shocks z_I, θ_d, and θ_n and let the estimation tell us which one is more persistent. The estimated autocorrelation of θ_n is close to unity while θ_d is less persistent.

The main exercise of our estimated model is to quantify the role of search-related demand shocks with the use of a standard variance decomposition, reported in the lower panel of Table 2. The preference shock to the disutility of search, θ_d, is a major driver of GDP and TFP, accounting for 39% of the variance of GDP and TFP. Moreover, search shocks also matter for consumption, accounting for 29% of the variance of C. The shock to θ_n is the main driver of labor dynamics but it accounts for just 12% of the fluctuations in GDP.

Note that while the previous business cycle literature has often treated fluctuations in the relative investment price P_i as reflecting investment-specific technology shocks (cf. Greenwood et al. (1997)), we find that θ_d accounts for 20% of the variance in the relative price of investment goods. Finally, recall the result from Section 2.1.1 that the contribution of search to aggregate fluctuations is not limited to the direct effect through θ_d but include also the amplification of the non-search shocks. We defer quantifying this contribution to Section 5.2 below.

[11] Ríos-Rull et al. (2012) construct this investment-price series as a Tornquist aggregate of the price index of quality-adjusted equipment investment and the price index of structures investment.

Table 2: Bayesian Estimation: Benchmark Model, Mixed Frequency with Shopping Data

Priors and Posteriors for the Shock Parameters likelihood = 603.7

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.90	0.05	0.866	[0.789, 0.942]
ρ_g	Beta	0.10	0.05	0.211	[0.064, 0.354]
ρ_i	Beta	0.90	0.05	0.881	[0.800, 0.995]
ρ_n	Beta	0.90	0.05	0.969	[0.947, 0.992]
σ_d	Inv Gamma	0.01	Inf	0.097	[0.081, 0.114]
σ_g	Inv Gamma	0.01	Inf	0.008	[0.006, 0.009]
σ_i	Inv Gamma	0.01	Inf	0.012	[0.009, 0.014]
σ_n	Inv Gamma	0.01	Inf	0.016	[0.014, 0.018]

	Variance Decomp			
	θ_d	g	z_I	θ_n
Y	38.60	44.21	5.47	11.71
Y/N	23.40	40.67	6.92	29.01
Investment	21.32	11.33	62.12	5.23
P_i	19.66	6.14	70.30	3.89
TFP	39.09	48.43	8.73	3.75
Consumption	28.50	41.19	20.82	9.49
Labor	10.34	23.34	0.57	65.75
Shopping	88.86	1.17	9.69	0.28

The estimation targets the data series of quarterly GDP growth, labor productivity growth, investment growth, and annual shopping time. U.S. data, 2003 to 2019.

5.1.2 Bayesian estimation without shopping-time data

Table 3 reports the estimation results for the benchmark model when replacing the annual shopping data series with the quarterly relative price of investment series. The four quarterly data series used in this estimation are GDP, labor productivity, investment, and relative price of investment from 1967 to 2019, i.e., for the longest possible time span. Note that the assumption that z is a random walk guarantees that TFP is more persistent than preference shocks. This in turn ensures identification of θ_d and z.

Table 3: Bayesian Estimation: Benchmark Model with P_i, no shopping data

Priors and Posteriors for the Shock Parameters likelihood = 2405.0

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.900	0.05	0.9269	[0.9139, 0.9379]
ρ_g	Beta	0.100	0.05	0.6026	[0.5973, 0.6066]
ρ_i	Beta	0.900	0.05	0.9542	[0.9473, 0.9624]
ρ_n	Beta	0.900	0.05	0.9985	[0.9973, 0.9997]
σ_d	Inv Gamma	0.010	Inf	0.2399	[0.2204, 0.2578]
σ_g	Inv Gamma	0.010	Inf	0.0107	[0.0098, 0.0116]
σ_i	Inv Gamma	0.010	Inf	0.0143	[0.0131, 0.0154]
σ_n	Inv Gamma	0.010	Inf	0.0167	[0.0153, 0.0179]

	Variance Decomp			
	θ_d	g	z_I	θ_n
Y	65.99	28.18	2.17	3.66
Y/N	41.84	45.83	3.03	9.31
Investment	54.91	9.02	34.56	1.51
P_i	47.62	0.09	51.63	0.66
TFP	57.92	38.03	3.02	1.02
Consumption	53.66	35.10	7.49	3.74
Labor	36.69	19.70	0.39	43.22
Shopping	96.74	1.33	1.92	0.01

The estimation targets the data series of quarterly GDP growth, labor productivity growth, investment growth, and relative price of investment data, 1967 to 2019.

In this case, the preference shock to the disutility of search θ_d is the main driver of business cycles, accounting for around 60% of the variance of GDP and TFP and more than half the variance of consumption. For comparison, this is a larger contribution —for every variable—than that of the two technology shocks z and z_I combined.

The Bayesian estimation procedure chooses shocks so as to maximize the probability of the observed aggregate time series. We conclude that to match the data it is necessary to attribute a significant role to θ_d. This holds true regardless of whether we assume that search effort is narrowly defined as shopping time or whether we allow a broader notion of search effort.

5.1.3 Search shocks versus more traditional Keynesian demand shocks

The preference shock θ_d is a key innovation of our paper relative to the business-cycle literature. The previous literature has emphasized more traditional "Keynesian" demand shocks as the source of demand shocks. For example, Michaillat and Saez (2015) argue that discount-factor demand shocks are needed to account for the aggregate data in a fixed-price model with product-market frictions. This begs the question whether our θ_d shock could be replaced by more traditional demand shocks.

To address this question, we estimate a range of models with alternative preference shock structures involving either a θ_d shock, a β discount factor shock, or a consumption weight shock θ_c (as in equation 10). All models include the labor shock θ_n and technology shocks $\{z, z_I\}$. Moreover, all alternative models have the same calibrated parameters and priors with the same standard deviation and autoregressive parameters as the benchmark economies.

Table 4 reports the results. Consider first models targeting shopping time (rows 1-6). The log likelihood falls sharply when we replace a shock to θ_d with shock to θ_c or β (compare rows 1 and 4 with rows 2, 3, 5, and 6). The log Bayes factors suggests that there is decisive evidence against the models replacing θ_d with a shock to θ_c or β (Kass and Raftery (1995)).

The reason why the β model performs so poorly is that a β shock does not affect the trade-off between search effort and labor supply (see Proposition 2). The model therefore struggles with generating fluctuations in shopping time separately from movements in labor supply. The reason why the θ_c model has low likelihood is that the quantitative effects of θ_c and θ_n are very similar – both shocks induce small effects on search effort d and large effects on labor supply n.[12] To generate an increase in labor productivity, this model requires a combination of a large increase in θ_c combined with a contractive shock θ_n to counteract the large increase in n generated by θ_c. Since the Bayesian estimation assumes that all shocks are i.i.d., the likelihood of the implied sequence of θ_c and θ_n shocks is low.

To illustrate the poor fit of the models without a θ_d shock, consider the volatility of aggregates at the estimated parameter values. Models without a θ_d shock have a volatility of labor supply growth which is an order of magnitude larger than in the data. Moreover, as we anticipated in Section 2, shocks to θ_c generate negative correlation between output growth and labor productivity growth.

[12]The reason is that the search elasticity η is small compared to the Frisch elasticity ν. At the calibrated parameters, shocks to θ_c and θ_n that would increase output by 1% would increase N by 1.7% and 2.1%, respectively. However, the shocks would increase measured TFP by just 0.2% and 0.05%, respectively.

Table 4: Robustness: alternative preference shocks

	LML	ΔLML	$std(\theta_d)$	$std(\theta_n)$	$std(\theta_c)$	$std(\beta)$	$std(\widehat{n})$	$corr(\widehat{y}, \widehat{y} - \widehat{n})$
Use shopping time data 2003-2019								
GHH preferences								
(1) θ_d benchmark	603.7	-25.5	0.097	0.016	-	-	1.05	0.51
(2) θ_c	487.8	-141.4	-	0.089	0.086	-	7.05	-0.92
(3) β	78.9	-550.3	-	0.090	-	0.024	7.26	0.86
Additively separable preferences								
(4) θ_d	629.2	0	0.104	0.014	-	-	0.84	0.60
(5) θ_c	470.9	-158.3	-	0.096	0.100	-	5.05	-0.83
(6) β	223.3	-405.9	-	0.081	-	0.027	7.68	0.34
Drop shopping time data, 1967-2019								
GHH preferences								
(7) θ_d benchmark	2405.0	-196.2	0.240	0.017	-	-	1.34	0.80
(8) θ_c	2257.3	-343.9	-	0.058	0.017	-	5.16	-0.84
(9) β	2510.3	-90.9	-	0.017	-	0.001	1.07	0.32
Additively separable preferences								
(10) θ_d	2601.2	0	0.141	0.018	-	-	0.91	0.64
(11) θ_c	2513.0	-88.2	-	0.020	0.019	-	1.24	0.27
(12) β	2562.2	-39.0	-	0.017	-	0.001	1.06	0.35
U.S. data, 2003-2019 quarterly							0.65	0.57

The table documents the estimation results for alternative specifications of preference shocks. LML denotes the log marginal likelihood and ΔLML denotes the log marginal likelihood differentials (or log Bayes factors). For each data set, ΔLML is calculated relative to the specification with the highest marginal likelihood. Rows (1) and (7) refer to the two benchmark economies with a θ_d shock (cf. Tables 2-3).

Consider now models that do not target shopping time data (rows 7-12). In this case, the case for our benchmark GHH model is more mixed as the models with additively separable preferences are significantly better than the benchmark GHH model. However, the main point holds true even for this data set: the model in row 10, with a θ_d shock and additively separable preferences, clearly outperforms all models with shocks to β or θ_c.[13]

We conclude that models containing our benchmark preference shock θ_d offer a substantially better fit than alternative models replacing θ_d with traditional demand shocks in the form of preference shock to β or θ_c.

5.1.4 Further robustness analysis

Table 5 documents a range of sensitivity analyses with the aim of establishing that the quantitative importance of the θ_d shocks is robust to a alternative specifications and data. The first six rows document robustness analyses for models targeting aggregate shopping time data. The last four rows refer to estimations that do not target shopping time data. The rows (1) and (6) restate the results from the benchmark model (cf. Tables 2-3).

The search-specific parameters φ and η are important for our quantitative exercise. Rows (2) and (3) of Table 5 consider estimations where we drop the calibration target based on cross-sectional data on search effort, i.e., the requirement that the model capture the cross-sectional variation in shopping time across households (equation 35). We maintain, however, the price dispersion target of equation (34). Row (2) considers a calibration where we simply set the search elasticity η equal to the labor supply elasticity ν. The results point to a slightly larger role for θ_d shocks in accounting for fluctuations in GDP and TFP. Relative to the benchmark calibration (row 1), the main difference is that the volatility of θ_d is substantially smaller when η is large.

The third row considers an estimation where we impose the price dispersion target and estimate η as part of the Bayesian estimation. The resulting elasticities φ and η are slightly smaller than in the benchmark, and the contribution of θ_d to output and TFP is approximately unchanged.[14]

The fourth row assumes that there is no search friction for investment goods and households search only for consumption goods. This economy has a somewhat smaller role for θ_d, accounting for around one quarter of the variance of GDP and TFP. Unsurprisingly, preference shocks to search effort matter

[13] The model with additively separable preferences outperforms the benchmark GHH model even when we target shopping time, see rows 1 versus 4. We nevertheless prefer the GHH model because of its analytical tractability (see Section 2) and because this model is able to match cross-sectional data for households' shopping time (see Section 4.1.1).

[14] We could in principle ignore all cross-sectional evidence on prices and shopping time and estimate both the elasticity parameters (φ, η) as part of the Bayesian estimation. However, we find that the parameter φ is not well identified by the aggregate data. In particular, the resulting estimates of φ vary widely across data sets, ranging from 0.09 to 0.44 depending on whether we include or omit shopping time data. This would involve large differences in the scope of the search friction relative to the benchmark calibration, making the economies less comparable.

Table 5: Further robustness analysis

	φ	η	LML	$std(\theta_d)$	Var Decomp Y			Var Decomp TFP		
					θ_d	g	z_I	θ_d	g	z_I
Use shopping time data 03-19, mixed frequency										
(1) Benchmark: impose (34)-(35)	0.32	0.20	603.7	0.097	38.60	44.21	5.48	39.09	48.43	8.74
(2) Alternative: $\eta = \nu$ & (34)	0.41	0.72	588.2	0.034	40.09	47.30	3.86	42.17	48.73	6.38
(3) Impose only eq. (34)	0.31	0.10	604.9	0.213	42.32	41.42	5.07	43.15	45.09	8.15
(4) Search only for C & (34)-(35)	0.32	0.20	609.3	0.096	28.04	40.45	20.70	23.44	42.65	30.98
(5) Estimation in levels & (34)-(35)	0.32	0.20	594.4	0.120	33.44	57.51	5.68	33.28	57.21	8.45
(6) Separable pref. & (34)-(35)	0.32	0.20	629.2	0.104	41.28	26.54	24.84	45.76	43.08	8.99
Drop shopping time data, 1967-2019, quarterly, GHH(c,d,n)										
(7) Benchmark: impose (34)-(35)	0.32	0.20	2405.0	0.240	65.99	28.18	2.17	57.92	38.03	3.02
(8) Alternative: $\eta = \nu$ & (34)	0.41	0.72	2349.1	0.084	64.91	31.51	1.19	58.96	38.66	1.72
(9) Impose only eq. (34)	0.30	0.07	2410.9	0.682	67.60	26.25	2.36	59.13	36.49	3.32
Drop shopping time data, 2003-2019 quarterly, GHH(c,d,n)										
(10) Benchmark: impose (34)-(35)	0.32	0.20	767.7	0.207	69.86	25.90	1.84	62.04	34.66	2.62

The table documents the estimation results for under alternative calibrations and data choices. All estimations target the quarterly U.S. data series of GDP growth, labor productivity growth, and investment growth. In addition, rows (1)-(6) target aggregate ATUS shopping time, while rows (7)-(10) target the real price of investment goods. LML denotes the log marginal likelihood and The benchmark model (rows 1 and 8) has GHH(c,d,n) preferences and uses the benchmark calibration in Table 1, imposing conditions (34)-(35). Row (2) uses the alternative calibration in Table 1, assuming $\eta = \nu$ and imposing eq. (34). Row (3) imposes only eq. (34) and estimates φ. Row (4) assumes there is a search friction only for consumption goods. Row (5) estimates the benchmark model in levels (as opposed to growth rates), Row (6) assumes additively separable preferences. Rows (7)-(9) correspond to rows (1)-(3), using investment price data instead of the series for shopping time. Row (10) is the same as row (7) but uses a shorter time sample.

less when search frictions apply to a smaller share of the economy.

Row (5) considers an estimation of the model in levels as opposed to the estimation in growth rates in the benchmark estimation. The parameter estimates are similar, but the variance decomposition reveals a significantly larger importance for θ_d, accounting for two thirds of the variance of GDP and TFP.

Row (6) considers an estimation with preferences that are additively separable between C, D, and N. The preferences are calibrated to be as similar as possible to the GHH preferences, with a risk aversion of 1 for consumption and a Frisch elasticity of 0.72 for labor supply. When estimating this version of the model we find a somewhat larger contribution of θ_d shocks for TFP and GDP.

The next set of robustness analysis (rows 7-10) reports the results when the Bayesian estimation targets investment price data instead of shopping time. Rows (8)-(9) correspond to rows (2)-(3), i.e., when we drop one of the cross-sectional calibration targets. The main message is that when we do not target aggregate shopping time data, shocks to θ_d account for a larger share of fluctuations – around 60% of the variance of GDP and TFP. This thanks to a somewhat larger estimated volatility of θ_d than in the economies that target shopping time. For example, under the benchmark calibration (rows (1) and (7)) the standard deviation increases from 9.7% to 24%. One should expect that the estimated variance of θ_d would be larger once we allow this variable to capture all shocks to the search friction, including sources of variation that affect search effort beyond what can be inferred from shopping time.

In rows (7)-(9) we use the longest possible time series (1967:1 to 2019:4). Row (10) restricts the time frame to be the same as under the estimation targeting shopping time data (2003:1 to 2019:4). This illustrates that it does not make much difference if we use the shorter data series versus going back to 1967.

All in all, we conclude that our main finding is robust: θ_d accounts for a large share of fluctuations in GDP and TFP. This share ranges from 40% to 60%, depending on whether or not we target shopping time in the estimation.

5.2 Amplification revisited

Our model differs from the existing literature in that shoppers affect productivity via their endogenous search effort. So far our quantitative analysis has established that search-related shocks, which we model as preference shocks θ_d, account for a significant share of fluctuations in output and TFP. However, search effort has an impact on business cycles beyond the direct impact of search shocks. The reason is that, as we demonstrated in Section 2.1.1, the presence of endogenous search effort amplifies the propagation of other shocks to output and measured TFP. We now revisit this issue and quantify the amplification of non-search shocks in the benchmark version of our estimated search model. To this end, we compare the effects of shocks in our benchmark endogenous search model relative to a model where we shut down fluctuations in search effort ($\eta = 0$). This alternative "Fixed Search" economy has constant search effort at a level equal to the steady-state search in the benchmark economy. All other parameters are as in the benchmark economy.

As a first exercise we calculate the contemporaneous elasticity – i.e., the passthrough – of output and TFP to shocks z, z_I, θ_n, and θ_d with and without endogenous search effort. The upper panel of Table 6 shows that search effort amplifies the passthrough of true technology shocks z by about 8% for both TFP and output. The amplification of the investment-specific shock z_I and the labor supply shock θ_n is quantitatively smaller. A quantitatively small amplification of θ_n is in line with the findings of the analytical model, see equation (17).[15] The search shock θ_d has obviously no effect without endogenous search.

Table 6: Search-Effort Amplification

Measure 1: Instantaneous Impact on TFP and GDP				
	TFP		Output	
	Search	Fixed Search	Search	Fixed Search
g	1.075	1.000	1.339	1.259
z_I	0.207	0.203	0.233	0.237
θ_n	-0.015	0	-0.254	-0.239
θ_d	-0.059	0	-0.074	0

Measure 2: Variance from Each Shock Alone						
	Variance of TFP			Variance of Output		
	Search	Fixed Search	% increase	Search	Fixed Search	% increase
g	0.423	0.360	17.5	0.624	0.548	14.1
z_I	0.063	0.063	0.0	0.078	0.078	0.0
θ_n	0.0004	0	–	0.160	0.144	11.1
θ_d	0.348	0	–	0.533	0	–
Total	0.828	0.420	97.1	1.416	0.774	83.0

The upper panel reports the contemporaneous elasticities of TFP and GDP to the shocks z, z_I, θ_n, and θ_d in economies with endogenous search ($\eta > 0$) and with fixed search ($\eta = 0$), respectively. The lower panel reports the variance of TFP and output attributable to each shock in the benchmark economy ("Search") and in and the fixed-search economy. The "% increase" column reports how much the variance contribution of each shock is amplified when going from constant search to endogenous search effort.

To put these results in perspective, we quantify how the search effort amplifies the variance of aggregate fluctuations attributable to each shock on its own. The lower panel of Table 6 reports how much the variance of GDP and TFP increases when going from a fixed-search economy to our benchmark endogenous search economy. The main finding is that the variance of TFP and output attributable to non-search shocks increases. For TFP the lion's share of the effect comes through the amplification of the

[15] A log-linearization of our quantitative model implies the same expression for the elasticities of z and θ_n as in equation (17), plus terms reflecting aggregate capital and the investment shock z_I which Section 2 abstracts from. The analysis in Section 2.1.1 is therefore informative about the amplification in our estimated model.

productivity shock z (17.5% larger contribution), while for output both z and θ_n provide quantitatively relevant amplification.

We conclude that search effort is a quantitatively important propagation mechanism of shocks. One implication of this findings it that fluctuations in measured TFP will be larger than the fluctuations in true productivity both because search-related shocks (θ_d) induce fluctuations in search effort and, hence, in measured TFP, and because search effort interacts with non-search shocks in a way that influences measured TFP. This amplifies the effect of true technology shocks.

6 Extensions: Variable capacity utilization and Storable Goods

We now provide two substantial extensions of our model, each of whom has the potential to mitigate our mechanism.

6.1 Varying capacity utilization

In our model demand shocks induce endogenous change in search effort and, hence, a change in the share of the production capacity that is realized as actual output. Since shopping effort is an unmeasured factor, fluctuations in search influence the measured Solow residual, as illustrated in Section 2. This point is related to the well known measurement concern that any mismeasurement of factor inputs will show up as movements in the Solow residual. It is a common practice in DSGE models to allow for varying capacity utilization, i.e., changes in the intensity of factors of production, which in turn induce movements in the Solow residual (see for instance Basu (1996) and Francis and Ramey (2005)). This is often motivated as a mechanism for how aggregate demand shocks pass through to aggregate productivity (cf. Corrado and Mattey (1997)).

The purpose of this section is to demonstrate that the significant role our estimation attributes to (shopping utility) preference shocks is robust to allowing for varying capacity utilization of capital and shocks to this utilization. To this end we extend our benchmark model to allow for varying capacity utilization of capital along the lines of Christiano et al. (2005). In particular, we assume that to utilize a fraction h of pre-installed capital, households have to pay the variable cost $\psi(h)$ given by

$$\psi(h) \; = \; \xi \, \frac{h^{1+\sigma_a} - 1}{1 + \sigma_a}, \tag{36}$$

where $1/\sigma_a$ captures the elasticity of depreciation of capital with respect to how intensively it is used. Households can choose a separate capacity utilization for each capital stock, denoted h_c and h_x for utilization of capital for consumption and investment sector, respectively. The budget constraint becomes

$$c + P_i \, i \; = \pi + (h_c k_c) \, R_c + (h_i k_i) \, R_i + n \, W,$$

where the expression for investment i —the equivalent to equation (22)— now incorporates the additional cost of using capital more intensively,

$$i = k_c' + k_i' - (1 - \delta - \psi(h_c))\, k_c - (1 - \delta - \psi(h_i))\, k_i.$$

We first estimate this version of model with the same shocks and data series as in the benchmark model, but allowing the value for σ_a to be estimated. We assume that σ_a follows an inverse Gamma distribution. Christiano et al. (2005) calibrate σ_a to be 0.1. We therefore use this value as the prior for σ_a. Table 7 presents the estimated results. In line with the findings of Christiano et al. (2005) the estimated value for σ_a implies a large elasticity.

Table 7: Bayesian Estimation: with Capacity Utilization on Capital

Priors and Posteriors for the Shock Parameters likelihood = 655.1

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.900	0.05	0.9571	[0.9293, 0.9851]
ρ_g	Beta	0.900	0.05	0.8018	[0.7163, 0.8920]
ρ_i	Beta	0.900	0.05	0.8550	[0.7684, 0.9449]
ρ_n	Beta	0.900	0.05	0.9710	[0.9502, 0.9929]
σ_d	Inv Gamma	0.010	Inf	0.0703	[0.0591, 0.0815]
σ_g	Inv Gamma	0.010	Inf	0.0031	[0.0024, 0.0038]
σ_i	Inv Gamma	0.010	Inf	0.0112	[0.0093, 0.0130]
σ_n	Inv Gamma	0.010	Inf	0.0118	[0.0101, 0.0136]
σ_a	Inv Gamma	0.100	Inf	1.4960	[0.4196, 2.7162]

	Var Decomp			
	θ_d	g	z_I	θ_n
Y	44.62	28.62	11.60	15.16
Y/N	19.92	42.70	10.33	27.04
Investment	13.17	6.11	76.51	4.21
P^i	11.33	1.51	83.72	3.44
TFP	36.98	44.28	14.78	3.96
Consumption	27.91	26.40	35.99	9.70
Labor	11.29	5.32	1.22	82.18
Capacity utilization	80.48	3.03	16.14	0.34

The estimation targets the data series of GDP, labor productivity, investment, and shopping time. Quarterly U.S. data, 2003:I to 2019:IV. The inverse of σ_a is the elasticity of capital depreciation rate to capital usage.

Capacity utilization on capital gives the model an additional channel for labor productivity and Solow residual movements. This in turn generates smaller estimated fluctuations for the demand shock θ_d and productivity shock z. The standard deviation of θ_d drops from 0.097 to 0.070 and the standard deviation of g drops in half, from 0.76 percent to 0.31 percent. The variance decomposition, however, shows that even when allowing for a standard mechanism of capital capacity utilization, our shopping mechanism remains quantitatively important (cf. Table 7). In particular, the demand shock θ_d accounts for about 45 percent of GDP variability, about 37 percent of TFP variability and 28 percent of consumption variability (versus 39, 39, and 29 percent, respectively, in the benchmark economy).

Table 8: Bayesian Estimation: with Capacity Utilization Shock

Priors and Posteriors for the Shock Parameters likelihood = 849.9

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.900	0.05	0.9343	[0.9101, 0.9590]
ρ_z	Beta	0.900	0.05	0.8576	[0.8383, 0.8829]
ρ_i	Beta	0.900	0.05	0.8919	[0.8536, 0.9256]
ρ_n	Beta	0.900	0.05	0.9268	[0.9082, 0.9435]
ρ_h	Beta	0.900	0.05	0.9489	[0.9279, 0.9744]
σ_d	Inv Gamma	0.010	Inf	0.0732	[0.0612, 0.0847]
σ_z	Inv Gamma	0.010	Inf	0.0031	[0.0024, 0.0037]
σ_i	Inv Gamma	0.010	Inf	0.0114	[0.0096, 0.0132]
σ_n	Inv Gamma	0.010	Inf	0.0124	[0.0105, 0.0142]
σ_h	Inv Gamma	0.005	Inf	0.0097	[0.0075, 0.0118]
σ_a	Inv Gamma	0.100	Inf	0.9400	[0.7665, 1.1106]

	Var Decomp				
	θ_d	g	z_I	θ_n	θ_h
Y	42.59	32.34	10.24	14.80	0.03
Y/N	17.68	48.81	8.65	24.85	0.01
Investment	14.11	9.55	70.36	4.94	1.04
P_i	13.01	1.38	78.89	4.60	2.12
TFP	33.73	50.13	12.44	3.69	0.00
Consumption	27.14	27.09	35.08	9.39	1.30
Labor	11.27	4.89	1.13	82.67	0.05
Capacity utilization	79.44	4.05	15.49	0.51	0.50

The estimation targets the data series of GDP, labor productivity, investment, shopping time, and total capacity utilization. Quarterly U.S. data, 2003:I to 2019:IV. The inverse of σ_a is the elasticity of capital depreciation to capital usage. ρ_h and σ_h are the autocorrelation and standard deviation of innovation of the cost for capacity utilization of capital.

Ideally we would like to quantify the role of varying capacity utilization versus the varying shopping effort. Fortunately, there exists aggregate measurements of capital capacity utilization, namely the quarterly series Total Capacity Utilization in Manufacturing from the Board of Governors. By incorporating empirical data for both shopping time and capacity utilization explicitly in the estimation of the model, we can undertake a horse-race comparison between the two mechanisms. Since we target an additional data series we must introduce an additional shock. In a way parallel to the shock to the disutility of shopping effort we introduce a shock θ_h to the cost of varying capacity utilization, affecting both capital stocks. The cost of capacity utilization h then becomes

$$\psi(h, \theta_h) \; = \; \theta_h \, \xi \, \frac{h^{1+\sigma_a} - 1}{1 + \sigma_a}, \tag{37}$$

We assume that the shock θ_h follows an AR(1) process with an autocorrelation of ρ_h and a standard deviation of innovation of σ_h. Table 8 provides the results. The estimation and variance decomposition are similar to the previous estimation without the extra shock. In particular, the role of θ_d is unaffected by the existence of the new shock in the capacity utilization model. Here, θ_d accounts for 43% of the variance of output, 34% of the variation of TFP, and 27% of consumption volatility.

Consider now the quantitative role for the capacity utilization shock θ_h relative to the other shocks. The lower panel in Table 8 shows that θ_d and shocks to technology account for the lion's share of fluctuations in output and TFP and are of approximately equal importance, while capital capacity utilization shocks are quantitatively irrelevant for business cycles. In particular, θ_h accounts for less than 0.1% of the fluctuations in TFP and output.

We conclude that our shopping effort mechanism remains important regardless of whether or not we allow for varying capacity utilization and shocks to capacity utilization costs in the model. Moreover, capital capacity utilization and shocks to such utilization play a quantitatively negligible role in accounting for the aggregate variables in our model — orders of magnitude smaller than the contribution of the shopping shock. This suggest that whereas variation in search effort is quantitatively relevant for the propagation of US business cycles, varying intensity in the use of capital is less relevant.

6.2 Storable Goods

In our benchmark model newly produced consumption goods can either be found by a shopper (and consumed instantly) or not found, in which case the goods are permanently lost. This property seems like a good description of services, which are consumed the moment they are produced. However, for many goods —especially for durables— producers have a third alternative: goods that are not instantly found can be stored as inventories and offered for sale later. As it turns out, the ratio of inventories to annual sales for total business (i.e., manufacturing, retail, and wholesale) is 11.5% (source: Bureau of Labor Statistics (BLS)).

The possibility to store goods could in principle change the mechanism for how shocks to shopping utility propagates to the aggregate economy. To investigate this possibility, we extend our shopping environment to model explicitly the distinction between services and consumer goods, allowing goods to be stored as inventories for firms and as a stock of consumer goods for households. Investment goods and services are modeled exactly as in the benchmark model in Section 3.

6.2.1 A Shopping Model with Storable Goods

We now extend the baseline model with intermediate goods and services, assuming that there are search frictions in the markets for both goods and services. An unmatched service location produces zero output. Goods, however, can be stored. Goods are produced in two stages. First, there is a manufacturing stage where a physical object is produced using capital and labor. This object is added to the firm's stock of inventories and these goods become available in a location that may or may not be matched with a shopper. If there is a match, a sale is produced that adds to output. If there is no match, the goods become inventories that may in turn be traded across firms in a secondary frictionless market.[16]

Aggregate consumption C and investment I are produced with intermediate inputs of goods and services according to CES aggregators for C and I,

$$C = \left[\omega_c \left(M_c\right)^{\frac{\zeta-1}{\zeta}} + (1-\omega_c)(S_c)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \quad I = \left[\omega_i \left(M_i\right)^{\frac{\zeta-1}{\zeta}} + (1-\omega_i)(S_i)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}} \tag{38}$$

where ζ is the elasticity of substitution between goods and services and (ω_c, ω_i) is the share of goods in the CES aggregators for consumption and investment. Moreover, (M_c, M_i) and (S_c, S_i) are the intermediate inputs of goods and services in the production of consumption and investment, respectively. Output is the sum of consumption, investment, and the change in firms' inventories. This procedure for measuring output is consistent with the way GDP is measured in NIPA. The aggregate state of the economy is $\Lambda = (\theta, X, K)$ where $X = (X_c, X_i)$ are the inventories in goods producing firms for consumption and investment, respectively, and K is a vector of the capital stocks, $K = (K_{mc}, K_{mi}, K_{sc}, K_{si})$, with a capital stock K_{mc} in the production of intermediate goods for consumption, K_{sc} capital in services for consumption, K_{mi} capital in goods for investment, and K_{si} capital in services for investment.

Markets are differentiated by what is traded, and by price, tightness, and quantities. There are four different markets, indexed with a triplet $(p_\iota, q_\iota, F_\iota)$ where p_ι is the price, $q_\iota = D_\iota / T_\iota$ is market tightness, and F_ι is the capacity. The market subscript $\iota = \{m_c, s_c, m_i, s_i\}$ denotes intermediate goods for consumption, services for consumption, goods for investment, and services for investment. The price of inventories are given by p_x and p_{xi} in intermediate goods production for consumption and investment, respectively. Each sector has a fixed measure of firms with $T_{mc} + T_{sc} + T_{mi} + T_{si} = 1$.

[16] In a previous version of this paper we allowed also households to store consumption goods. This extension has only a minor quantitative effect on the results.

Households Households have preferences over consumption goods m_c, services s_c, hours worked n, and total search effort, defined as the sum of search in each market, $d = d_{mc} + d_{sc} + d_{mi} + d_{si}$. Current utility is given by $u(m_c, s_c, d, n; \theta)$ where $\theta = (\theta_d, \theta_n)$ is the same vector of preference shocks as in the benchmark model.

Each period, households start with an initial holding capital in each sector $k = (k_{mc}, k_{sc}, k_{mi}, k_{si})$. Households choose consumption goods m_c, services s_c, shopping efforts $(d_{mc}, d_{sc}, d_{mi}, d_{si})$, hours worked n, and investments in each sector $(i_{mc}, i_{sc}, i_{mi}, i_{si})$, taking as given the aggregate state Λ, the wage W, and rental returns $R = (R_{mc}, R_{sc}, R_{mi}, R_{si})$, to maximize

$$V(\Lambda, k) = \max \; u(m_c, s_c, d, n; \theta) + \beta \; E\{V(\Lambda', k')\},$$

subject to the budget constraint

$$m_c \, p_{mc}(\Lambda) + s_c \, p_{sc}(\Lambda) + \sum_{\iota = \{mc,sc,mi,si\}} p_\iota(\Lambda) i_\iota \leq \pi(\Lambda) + n \, w(\Lambda) + \sum_{\iota = \{mc,sc,mi,si\}} R_\iota(\Lambda) K_\iota,$$

the shopping constraints for goods and services, $m_c = d_{mc} \; \Psi_d \left[q_{mc}(\Lambda)\right] \; F_{mc}(\Lambda)$, $s_c = d_{sc} \; \Psi_d \left[q_{sc}(\Lambda)\right] \; F_{sc}(\Lambda)$, $m_i = d_{mi} \; \Psi_d \left[q_{mi}(\Lambda)\right] \; F_{mi}(\Lambda)$, $s_i = d_{si} \; \Psi_d \left[q_{si}(\Lambda)\right] \; F_{si}(\Lambda)$, total investment, $i_{mc} + i_{sc} + i_{mi} + i_{si} = i$, and capital accumulations for each sector $\iota = \{mc, sc, mi, si\}$, $k'_\iota = (1 - \delta) k_\iota + i_\iota$, and investment i is assembled with m_i and s_i using the CES aggregator (38).

Let the temporary value function $\widehat{V^\iota}(\Lambda, k, \hat{p}_\iota, \hat{q}_\iota, \hat{F}_\iota)$ denote the utility attained by the household if the market $\iota \in \{mc, sc, mi, si\}$ is given by the triplets $(\hat{p}_\iota, \hat{q}_\iota, \hat{F}_\iota)$ as opposed to the equilibrium triplets $(p_\iota(\Lambda), \; q_\iota(\Lambda), \; F_\iota(\Lambda))$.

Firms As in the benchmark model, each sector has a fixed measure of firms and a Cobb-Douglas production function f. Firms in the services sector solve the same problem as in the benchmark model (cf. equations (24)) with the participation constraints in the service sector given by $\widehat{V^\iota}(\Lambda, K, \hat{p}_\iota, \hat{q}_\iota, \hat{F}_\iota) \geq V(\Lambda, K)$ for $\iota = \{sc, si\}$.

Consider now the problem of a good producer for either consumption or investment, $\iota = \{mc, mi\}$. Firms carry inventories x_ι from last period that depreciate at rate δ_x. Firms then choose inputs of labor n_ι and capital k_ι for production and add the current production $z \, f(k_\iota, n_\iota)$ to their net inventories $(1 - \delta_x) x_\iota$. This sum yields current capacity F_ι. Firms offer the triplet $(p_\iota, q_\iota, F_\iota)$ to customers. After the matching process, firms choose how much extra investment of inventory to purchase or sell in the frictionless market for inventories. The unmatched firms purchase $i_{x,u}$ units of inventory and the matched firms purchase $i_{x,m}$. This in turn gives the next period's inventories x'_u and x'_m for the unmatched and matched firms, respectively,

$$x'_u = z\,f(n_\iota, k_\iota) + (1-\delta_x)x_\iota + i_{x,u}, \tag{39}$$

$$x'_m = z\,f(n_\iota, k_\iota) + (1-\delta_x)x_\iota - F_\iota + i_{x,m}. \tag{40}$$

To summarize, a producer in sector $\iota = \{mc, mi\}$ chooses $\{n_\iota, k_\iota, p_\iota, q_\iota, F_\iota, i_{x,u}, i_{x,m}, x'_m, x'_u\}$ to solve the following recursive problem, where $\Psi_{T,\iota}(q_\iota)$ is the probability that the firm will be matched with a shopper, $\Omega(\Lambda, x_\iota)$ is the value of the firm, and $\Upsilon(\Lambda, \Lambda')$ is the stochastic discount factor.

$$\begin{aligned}\Omega_\iota(\Lambda, x_\iota) = \max_{n_\iota, k_\iota, i_{x,m}, i_{x,u}, p_\iota, q_\iota, F_\iota} \quad & -W(\Lambda)n_\iota - R_\iota(\Lambda)k_\iota \\ & + \Psi_{T,\iota}(q_\iota)\left\{p_\iota F_\iota - p_x(\Lambda)i_{x,m} + E\left[\Upsilon(\Lambda,\Lambda')\,\Omega_\iota(\Lambda', x'_m)|\Lambda\right]\right\} \\ & + (1-\Psi_{T,\iota}(q_\iota))\left\{-p_{x\iota}(\Lambda)i_{x,u} + E\left[\Upsilon(\Lambda,\Lambda')\,\Omega_\iota(\Lambda', x'_u)|\Lambda\right]\right\}\end{aligned}$$

subject to the capacity constraint

$$F_\iota \leq zf(n_\iota, k_\iota) + (1-\delta_x)x_\iota,$$

the dynamics of inventories for non-matched (39) and matched state (40), and the participation constraint for shoppers $\widehat{V}_\iota(\Lambda, K, \hat{p}_\iota, \hat{q}_\iota, \hat{F}_\iota) \geq V(\Lambda, K)$. Average dividends are

$$\pi_\iota(\Lambda) = \Psi_{T,\iota}(q_\iota)p_\iota F_\iota - W(\Lambda)n_\iota - R_\iota(\Lambda)k_\iota - \Psi_{T,\iota}(q_\iota)p_{x\iota}(\Lambda)i_{x,m} - (1-\Psi_{T,\iota}(q_\iota))p_{x\iota}(\Lambda)i_{x,u}.$$

Finally, in Appendix F we show that firms with identical initial inventories choose the same inventory holdings for the following period, i.e., $x'_u(\Lambda, x) = x'_m(\Lambda, x)$.

Equilibrium Let the consumption good be the numéraire, so $p_{mc} = 1$. The competitive equilibrium consists of allocations $\{S,\ M_c,\ N,\ D_\iota,\ I_\iota,\ N_\iota,\ K'_\iota,\ X'_c,\ X'_i,\ x'_c,\ x'_i\}$, dividend and profits $\{\pi,\ \pi_\iota\}$, values V and Ω, prices $\{W, R_\iota, p_{xc}, p_{xi}, \Upsilon\}$, market tightness and capacity $p_\iota, q_\iota, F_\iota$ such that

1. Households choose $\{M_c,\ S,\ D_\iota,\ I_\iota,\ N,\ \pi,\ K'_\iota,\ V\}$ to solve their problem taking as given prices $\{W, R_\iota\}$ and dividends π. The stochastic discount factor Υ satisfies $\Upsilon(\Lambda, \Lambda') = \beta u_s(\Lambda')/u_s(\Lambda)$.
2. The allocation $\{p_\iota, q_\iota, F_\iota, N_\iota/T_\iota, K_\iota/T_\iota\}$, profit π_ι, inventory choices $X'_\iota/T_\iota = x'_{m\iota} = x'_{u\iota}$, $i_{x,u,\iota}$, $i_{x,m,\iota}$, and value function Ω_ι for $\iota = \{mc, mi\}$ solve the problem of producers in sector ι, taking as given $\{\Upsilon, W, R_\iota, p_{x\iota}\}$.
3. The allocation $\{p_\iota, q_\iota, F_\iota, N_\iota, K_\iota\}$ and the profit π_ι solve the problem of the service producers, taking as given $\{W, R_\iota\}$ for $\iota = \{sc, si\}$.

4. Market clearing conditions are satisfied

$$
\begin{aligned}
M_c &= T_{mc}\Psi_{T,mc}(q_{mc})F_{mc} & S_c &= T_{sc}\Psi_{T,sc}(q_{sc})z_{sc}f(N_{sc},K_{sc}) \\
M_i &= T_{mi}\Psi_{T,mi}(q_{mi})F_{mi} & S_i &= T_{sc}\Psi_{T,si}(q_{si})z_{si}f(N_{si},K_{si}) \\
I &= I_{mc}+I_{sc}+I_{mi}+I_{si} & & \\
N &= N_{mc}+N_{sc}+N_{mi}+N_{si} & \pi &= T_{mc}\pi_{mc}+T_{sc}\pi_{sc}+T_{mi}\pi_{mi}+T_{si}\pi_{si} \\
X_c' &= T_{mc}[1-\Psi_{T,mc}(q_{mc})]F_{mc} & X_i' &= T_{mi}[1-\Psi_{T,mi}(q_{mi})]F_{mi} \\
F_{mc} &= (1-\delta)X_{mc}/T_{mc}+z_{mc}f(K_{mc}/T_{mc},N_{mc}/T_{mc}) & & \\
F_{mi} &= (1-\delta)X_{mi}/T_{mi}+z_{mi}f(K_{mi}/T_{mi},N_{mi}/T_{mi}). & &
\end{aligned}
$$

The equilibrium is optimal (we omit the proof since it is similar to the one in Section 3).

6.2.2 Calibration and Quantitative Analysis

We assume that consumption is a constant-elasticity-of-substitution (CES) composite of services and consumption goods. Households' preferences are otherwise similar to those in the benchmark model and are given by

$$
u(m_c,s_c,d,n)=\frac{1}{1-\gamma}\left\{\left[\omega_c(m_c)^{\frac{\zeta-1}{\zeta}}+(1-\omega_c)(s_c)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}-\frac{(d)^{1+1/\eta}}{1+1/\eta}-\chi\frac{n^{1+1/\nu}}{1+1/\nu}\right\}^{1-\gamma},
$$

where ω_c is the share of storable consumption goods and ζ is the elasticity of substitution between services and the storable good.

Most parameters are calibrated using the same moments as in the benchmark. The parameter values and targets are summarized in Table 13 in Appendix G. Following Herrendorf et al. (2013), we set ω_c and ω_i so that the model matches the average value-added share of services in consumption and investment of 87% and 52%, respectively, and set the elasticity between goods and services to $\zeta = 0.85$. Richardson (1995) argues that the annual cost of holding inventories is between 25 percent and 55 percent of the value of the stored goods. Taking out interest rate costs, the cost of holding inventories is between 19 percent to 43 percent per year. For simplicity, we set $\delta_x = 19$ percent.

We also match the observed ratio of inventories to sales since the ratio is directly linked to the market tightness D_{mc}/T_{mc} and D_{mi}/T_{mi}. Given we choose all prices equal to 1 at the steady state, the inventory-sales ratio helps the model pin down the mean productivities in sectors. In the data, the average annual real inventories to sales ratio for total business (manufacturing, wholesale and retail trade) is 11.5 percent (Source: BLS). We also normalize the steady-state investment goods price to 1 as in the benchmark calibration and equalize the steady-state productivity of goods and service production. These normalizations pin down the steady state productivities z_{mc}, z_{mi}, z_{sc}, and z_{si}.

As in the benchmark, we estimate the model using Bayesian methods. As in our benchmark, the model has four shocks: the disutility shock of shopping θ_d, the disutility shock of labor θ_n, the growth rate shock for the neutral technology, and the investment-specific shock z_I. The priors and distributions are the same as in the benchmark estimation.

Table 9: Bayesian Estimation: Model with Storable Goods, Shopping time, Short Sample

Priors and Posteriors for the Shock Parameters (Likelihood = 660.8)
Data used: Y, Y/N, Inv, shopping time

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.900	0.05	0.9546	[0.9269, 0.9856]
ρ_g	Beta	0.100	0.05	0.1734	[0.0784, 0.2738]
ρ_i	Beta	0.900	0.05	0.8581	[0.8949, 0.9991]
ρ_n	Beta	0.900	0.05	0.9657	[0.9429, 0.9922]
σ_d	Beta	0.010	Inf	0.0735	[0.0608, 0.0861]
σ_g	Inv Gamma	0.010	Inf	0.0042	[0.0034, 0.0050]
σ_i	Inv Gamma	0.010	Inf	0.0156	[0.0121, 0.0191]
σ_n	Inv Gamma	0.010	Inf	0.0118	[0.0100, 0.0134]

	Variance Decomposition			
	θ_d	g	z_I	θ_n
Y	36.76	28.73	17.17	17.33
Y/N	18.36	25.59	19.72	36.32
Investment	3.71	2.59	90.21	3.49
P_i	0.46	0.53	98.02	0.98
TFP	43.65	15.04	25.87	15.43
Consumption	34.77	27.20	25.53	12.50
Labor	8.93	14.92	1.11	75.04
Service	48.28	31.49	4.93	15.30
Goods	2.02	8.00	79.38	10.60
Serive price	27.97	49.12	21.21	1.70
goods/service	15.08	4.70	79.46	0.76
inventory/sale	91.75	1.19	6.15	0.90

The table shows the Bayesian estimation for the shopping model with storable goods and services. The estimation targets the data series of output, labor productivity, investment, and shopping time. The sample is quarterly U.S. data, from Q1 in 2003 to Q4 in 2019.

Tables 9 and 10 report the results from the estimation for our two benchmark economies with storage (i.e., with and without targeting shopping-time data). The variance decomposition shows that the demand shock θ_d still plays a significant role in accounting for business cycle fluctuations – 37 to 63 percent of the variance of output and 44 to 60 percent of TFP. Note also that the shopping friction is quantitatively more relevant for services than for durable consumption goods: according to the variance decomposition,

Table 10: Bayesian Estimation: Model with Storable Goods, Long Sample

Priors and Posteriors for the Shock Parameters (Likelihood = 2614.5) Data used: Y, Y/N, Inv, P^i					
Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.900	0.05	0.9923	[0.9880, 0.9966]
ρ_g	Beta	0.900	0.05	0.9165	[0.9019, 0.9301]
ρ_{zi}	Beta	0.900	0.05	0.0000	[0.8766, 0.9198]
ρ_n	Beta	0.900	0.05	0.9779	[0.9697, 0.9850]
σ_d	Beta	0.010	Inf	0.1754	[0.1609, 0.1903]
σ_g	Inv Gamma	0.010	Inf	0.0037	[0.0031, 0.0044]
σ_{zi}	Inv Gamma	0.010	Inf	0.0138	[0.0124, 0.0150]
σ_n	Inv Gamma	0.010	Inf	0.0000	[0.0110, 0.0129]

	Variance Decomposition			
	θ_d	g	z_I	θ_n
Y	63.11	27.06	4.33	5.50
Y/N	29.78	55.58	4.14	10.50
Investment	9.83	27.15	60.64	2.37
P_i	5.73	39.53	53.90	0.85
TFP	59.55	31.09	5.39	3.98
Consumption	54.95	34.86	6.72	3.48
Labor	36.60	3.62	0.90	58.88
Service	71.36	23.53	0.97	4.15
Goods	6.04	36.62	49.69	7.64
Service price	44.67	49.12	5.76	0.45
goods/service	45.72	20.17	33.80	0.31
inventory/sale	97.73	1.91	0.32	0.04

The table shows the Bayesian estimation for the shopping model with storable goods and services. The estimation targets the data series of output, labor productivity, investment, and investment price. The sample is quarterly U.S. data, from Q1 in 1967 to Q4 in 2019.

θ_d accounts for a share of the variance of services which is substantially larger than than for storable goods (cf. Table 9).

The extended model has rich predictions for the dynamics of goods, services, and the inventories-to-sales ratio. In the data, aggregate purchases of durable goods are much more volatile than services and non-durable goods, and the model has similar predictions. Moreover, the inventories-to-sales ratio in the model is countercyclical as in the data, although it is more volatile in the model than that in the data.

In our benchmark analysis firms have inventories of goods but households cannot store goods. However, many goods are durable and households do in fact keep inventories of durable goods. As a robustness analysis we assume that all consumption goods are storable (while services remain non-storable). We focus on the estimation targeting shopping time. Row 2 of Table 11 documents that the role of θ_d remains large in this economy, albeit slightly lower than in the benchmark economy (row 1).

Services are arguably more differentiated than goods and the search friction associated with services might therefore be larger than for goods. To explore this possibility we consider an extreme version of our economy where there are no search frictions for goods but the search friction for services remain in place. Rows 3-4 of Table 11 shows that the role for θ_d in these economies is approximately the same as in the economy with durables.

Table 11: Robustness analysis — Goods and Services

	Lhood	$std(\theta_d)$	Var Decomp Y			Var Decomp TFP			Rel. std goods
			θ_d	g	z_I	θ_d	g	z_I	
Use shopping time data 03-19, mixed frequency									
(1) Benchmark: non-durable	660.8	0.074	36.76	28.73	17.18	43.65	15.04	25.88	1.54
(2) Alternative: durable	648.9	0.075	34.64	26.21	23.60	39.70	13.33	34.01	1.63
(3) Goods no frictions, non-durable	652.8	0.076	33.49	29.81	21.00	40.36	15.55	30.31	1.70
(4) Goods no frictions, durable	657.6	0.075	33.30	29.89	19.88	39.77	15.82	30.24	1.57

The table documents the estimation results for under alternative calibrations and model assumptions with both goods and services. All estimations target the quarterly U.S. data series of GDP growth, labor productivity growth, investment growth, and aggregate shopping time. Row (1) and (3) assume non-durable goods, row (2) and (4) have durable goods with depreciation of 19% annually. Row (3) and (4) assume that goods are not subject to search friction, but services are.

Summing up, the main results from the benchmark model are robust to allowing firms to hold inventories of consumer durables and to let households store such goods: demand shocks are a major driver of TFP and output. In addition, the extension with inventories is consistent with stylized facts on cyclical properties of services, consumption goods, and inventories.

7 Conclusion

This paper provides a business cycle theory with an explicit productive role for the demand for goods. A search friction prevents perfect matching between producers and potential consumers. A larger demand for output is associated with more intense search for goods and, hence, a larger utilization of the potential production. Shocks that cause changes in search effort or search efficiency will therefore induce changes in aggregate output even if standard factors such as capital and labor are fixed. Thus, when applying a neoclassical production function that ignores search effort as an input, changes in demand and search effort generate procyclical movements in measured TFP. A competitive search protocol resolves the matching friction and the equilibrium outcome is efficient and unique. The framework is otherwise a standard neoclassical model with flexible prices. Krueger et al. (2016) illustrate how the ideas we develop in this paper could be implemented by means of a reduced-form expenditure externality in production.

Our main quantitative exercise is to estimate the model using standard Bayesian techniques. Business cycles are driven by preference shocks to search effort and labor supply, true technology shocks, and investment-specific shocks. Preference shocks affecting search effort account for a large share of the fluctuations in consumption, GDP, and the Solow residual. This finding holds true regardless of whether we use average shopping time from ATUS as a proxy for search effort and target this series in the Bayesian estimation or if we take a broader view on search effort and estimate the model without targeting direct observations of search effort. Our findings are also robust to extending the model to allow for varying capacity utilization of capital and to allow firms and households to store consumption goods as inventories and durables.

Our paper is consistent with Keynes' idea that consumer demand can have real effects. We show that this holds true even in a neoclassical model with flexible prices, amended with a product market matching friction.

We have modeled the product-market friction as a pure search friction. However, the idea that households contribute to productivity by extracting more output from a given productive structure is wider than the notion of narrow search for goods and services. When a product is not very popular one can choose the most desired specifications without any compromise. In an expansion, the economy is tight and the average quality of goods and services are lower. In such situations it might appear as if the firms have higher productivity, as more tables are busy and all varieties of a good can be sold. However, in these situations households' effort contributes to more matches and, hence, production.

In future work, we plan to extend this environment to contexts where the demand shocks are generated by financial frictions, government expenditures, or foreign demand shocks. It would also be interesting to consider additional frictions that could break the efficient outcome of the competitive search model, such as coordination failures or additional labor market frictions. Another promising direction is one where pure positive wealth effects increase, not decrease search efferts and productivity. This may help with understanding the great recession.[17] Finally, it is straightforward to embed our search friction for goods within other approaches to business cycles such as the New Keynesian tradition (eg. Qiu and Ríos-Rull (2022)), or the Mortensen-Pissarides view of search frictions in the labor market.[18] Ultimately, these two traditions build on technology shocks as a major source of fluctuations, and our findings provide a rationale for substituting productivity shocks for demand shocks in these models.

[17] Preliminary work in this direction is in Huo and Ríos-Rull (2020).

[18] Ritto (2024) uses our search friction to study a version of the search and matching monetary theory models in a way that can be estimated.

References

AGUIAR, M. AND E. HURST (2007): "Lifecycle Prices and Production," *American Economic Review*, 97, 1533–1559.

ALESSANDRIA, G. (2005): "Consumer Search, Price Dispersion, and International Relative Price Volatility," Working Paper No. 05-9 Federal Reserve Bank of Philadelphia.

BAI, Y. AND J.-V. RÍOS-RULL (2015): "Demand Shocks and Open Economy Puzzles," *American Economic Review Papers and Proceedings*, 105, 644–649, american Economic Review Papers and Proceedings.

BASU, S. (1996): "Procyclical Productivity: Increasing Returns or Cyclical Utilization?" *Quarterly Journal of Economics*, 111, 719–751.

BURDETT, K. AND K. L. JUDD (1983): "Equilibrium Price Dispersion," *Econometrica*, 51, 955–969.

CHRISTIANO, L., M. EICHENBAUM, AND C. EVANS (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113, 1–45.

CORRADO, C. AND J. MATTEY (1997): "Capacity Utilization," *Journal of Economic Perspectives*, 11, 151–167.

DURAS, J. (2015): "Goods Market Search and the Labor Wedge," Manuscript, Texas Tech University.

——— (2016): "A Search Model of Unemployment and Inventories," Manuscript, Texas Tech University.

FAGNART, J. F., O. LICANDRO, AND F. PORTIER (1999): "Firm heterogeneity, capacity utilization, and the business cycle," *Review of Economic Dynamics*, 2, 433–455.

FAIG, M. AND B. JEREZ (2005): "A theory of commerce," *Journal of Economic Theory*, 122, 60–99.

FLOETOTTO, M. AND N. JAIMOVICH (2008): "Firm Dynamics, Markup Variations and the Business Cycle," *Journal of Monetary Economics*, 55, 1238–1252.

FRANCIS, N. AND V. A. RAMEY (2005): "Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited," *Journal of Monetary Economics*, 52, 1379–1399.

GALÍ, J. (1999): "Technology, Employment and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?" *American Economic Review*, 89, 249–271.

GREENWOOD, J., Z. HERCOWITZ, AND G. W. HUFFMAN (1988): "Investment, Capacity Utilization and the Real Business Cycle," *American Economic Review*, 78, 402–417.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997): "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, 83, 342–362.

Heathcote, J., K. Storesletten, and G. L. Violante (2010): "The Macroeconomic Implications of Rising Wage Inequality in the United States," *Journal of Political Economy*, 118, 681–722.

——— (2020): "Presidential Address 2019: How Should Tax Progressivity Respond to Rising Income Inequality?" *Journal of the European Economic Association*, 18, 2715–2754.

Herrendorf, B., R. Rogerson, and Á. Valentinyi (2013): "Two Perspectives on Preferences and Structural Transformation," *American Economic Review*, 103, 2752–2789.

Huo, Z. and J.-V. Ríos-Rull (2020): "Demand induced fluctuations," *Review of Economic Dynamics*, 37, S99 – S117, the twenty-fifth anniversary of "Frontiers of Business Cycle Research".

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010): "Investment Shocks and Business Cycles," *Journal of Monetary Economics*, 57, 132?145.

Kaplan, G. and G. Menzio (2016): "Shopping Externalities and Self-Fulfilling Unemployment Fluctuations," *Journal of Political Economy*, 124, 771–825.

Kass, R. E. and A. E. Raftery (1995): "Bayes Factors," *Journal of the American Statistical Association*, 90, 773–795.

Krueger, D., K. Mitman, and F. Perri (2016): *Macroeconomics and Household Heterogeneity*, Elsevier, vol. 2 of *Handbook of Macroeconomics*, chap. 0, 843–921.

Lagos, R. (2006): "A Model of TFP," *Review of Economic Studies*, 73, 983–1007.

Michaillat, P. and E. Saez (2015): "Aggregate Demand, Idle Time, and Unemployment," *Quarterly Journal of Economics*, 130, 507–569.

Moen, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385–411.

Nord, L. (2022): "Shopping, Demand Composition, and Equilibrium Prices," Manuscript, European University Institute.

Petrosky-Nadeau, N. and E. Wasmer (2015): "Macroeconomic Dynamics in a Model of Goods, Labor and Credit Market Frictions," *Journal of Monetary Economics*, 72, 97–113.

Petrosky-Nadeau, N., E. Wasmer, and S. Zeng (2016): "Shopping Time," *Economic Letters*, 143, 52–60.

Qiu, Z. and J.-V. Ríos-Rull (2022): "Procyclical Productivity in New Keynesian Models," Working Paper 29769, National Bureau of Economic Research.

Richardson, H. (1995): "Control your costs then cut them," *Transportation and Distribution*, 36, 94–96.

Ríos-Rull, J.-V., F. Schorfheide, C. Fuentes-Albero, M. Kryshkoh, and R. Santaeulàlia-Llopis (2012): "Methods versus substance: Measuring the effects of technology shocks," *Journal of Monetary Economics*, 59, 826–846.

Ritto, J. (2024): "Doing Without Nominal Rigidities: Real Effects of Monetary Policy in a Monetary World," Job Market Paper, University of Pennsylvannia.

Rudanko, L. and F. Gourio (2014): "Customer Capital," *Review of Economic Studies*, 81, 1102–1136.

Schorfheide, F., D. Song, and A. Yaron (2018): "Identifying long-run risks: A Bayesian mixed-frequency approach," *Econometrica*, 86, 617–654.

Sorensen, A. T. (2000): "Equilibrium Price Dispersion in Retail Markets for Prescription Drugs," *Journal of Political Economy*, 108, 833–850.

Swanson, E. T. (2006): "The relative price and relative productivity channels for aggregate fluctuations," *The BE Journal of Macroeconomics*, 6, 10.

Appendix

Not Intended for Publication

A Proof of Proposition 1: Existence and Optimality of Equilibria in the Endowment Economy

Write the Lagrangian problem as $\mathcal{L} = p\Psi_T(q)z - \lambda\left(u(\frac{Y}{p}, \frac{Y}{p\Psi_d(q)z}) - \bar{u}\right)$. The first-order conditions are then

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial q} &= \Psi_T(q)z - \lambda\left(-\frac{\partial u}{\partial C}\frac{Y}{(p)^2} - \frac{\partial u}{\partial D}\frac{Y}{(p)^2}\frac{1}{\Psi_d(q)z}\right) = 0 \\
\frac{\partial \mathcal{L}}{\partial p} &= -pz\frac{d\Psi_T(q)}{dq} - \lambda\frac{\partial u}{\partial D}\frac{Y}{pz}\frac{1}{(\Psi_d(q))^2}\frac{d\Psi_d(q)}{dq} = 0
\end{aligned}$$

Substituting out λ and simplifying yields the following condition:

$$\frac{\partial u}{\partial C}z = \frac{\partial u}{\partial D}\frac{1}{\Psi_d(q)}\left(\frac{\Psi_T(q)}{\Psi_d(q)}\frac{\frac{d\Psi_d(q)}{dq}}{\frac{d\Psi_T(q)}{dq}} - 1\right)$$

Exploiting that $q^{-1}\Psi_T(q) = \Psi_d(q)$ and $\frac{d\Psi_d(q)}{dq} = -\frac{1}{q^2}\Psi_T(q) + \frac{1}{q}\frac{d\Psi_T(q)}{dq}$, we can rewrite the first-order conditions as

$$\frac{\partial u}{\partial C}z = -\frac{\partial u}{\partial D}\frac{1}{\Psi_d(q)}\frac{1}{q}\frac{\Psi_T(q)}{\frac{d\Psi_T(q)}{dq}} = -\frac{\partial u}{\partial D}\frac{1}{\frac{d\Psi_T(q)}{dq}}.$$

With $T = 1$ and $q = D$ in equilibrium, equation (5) follows. QED

B Proof of Proposition 2: Characterization of Equilibrium in the Dynamic Production Economy without Capital

Recall the potential output is given by zn^α and the number of matches is $M = (d)^\varphi$. Value added is $y = z\,(d)^\varphi\, n^\alpha$. We assume that $\alpha \geq \varphi$, which ensure that the production function does not have increasing returns to scale. Given the storage technology, consumption is $c = y - s$ and $c' = y' + s$. Substitute these constraints into a GHH-formulation of the objective function with four shocks:

$$\begin{aligned}
V &= \frac{1}{1-\gamma}\theta^\beta\left(\theta_c\left[z\,(d)^\varphi\, n^\alpha - s\right] - \theta_d\frac{d^{1+1/\eta}}{1+1/\eta} - \theta_n\frac{n^{1+1/\nu}}{1+1/\nu}\right)^{1-\gamma} \\
&\quad + \frac{1}{1-\gamma}\theta^{\beta\prime}\left(\theta^{c\prime}\left[z'\,(d')^\varphi\,(n')^\alpha + s\right] - \theta^{d\prime}\frac{(d')^{1+1/\eta}}{1+1/\eta} - \theta^{n\prime}\frac{(n')^{1+1/\nu}}{1+1/\nu}\right)^{1-\gamma}
\end{aligned}$$

The first-order conditions are,

$$\frac{\theta_d}{\theta_c} d^{\frac{1}{\eta}} = \varphi \frac{y}{d} \tag{B.1}$$

$$\frac{\theta_n}{\theta_c} n^{\frac{1}{\nu}} = \alpha \frac{y}{n} \tag{B.2}$$

$$\begin{aligned} & \left(A\left(d\right)^{\varphi} n^{\alpha} - s - \frac{\theta_d}{\theta_c} \frac{d^{1+1/\eta}}{1+1/\eta} - \frac{\theta_n}{\theta_c} \frac{n^{1+1/\nu}}{1+1/\nu} \right)^{-\gamma} \\ = \; & \frac{\theta^{\beta\prime}}{\theta^{\beta}} \left(\frac{\theta_c'}{\theta_c} \right)^{1-\gamma} \left(z' \left(d'\right)^{\varphi} \left(n'\right)^{\alpha} + s - \frac{\theta^{d\prime}}{\theta^{c\prime}} \frac{\left(d'\right)^{1+1/\eta}}{1+1/\eta} - \frac{\theta^{n\prime}}{\theta^{c\prime}} \frac{\left(n'\right)^{1+1/\nu}}{1+1/\nu} \right)^{-\gamma}. \end{aligned} \tag{B.3}$$

Combining (B.1)-(B.2) with (7) and production function, we can solve analytically for n, d, and y

$$\begin{aligned} \xi_1 \cdot \ln d &= \left(\frac{1}{\nu}+1\right) \ln z - \alpha \ln\left(\frac{\theta_n}{\theta_c}\right) - \left(\frac{1}{\nu}+1-\alpha\right) \ln\left(\frac{\theta_d}{\theta_c}\right) + \left(\frac{1}{\nu}+1-\alpha\right) \ln \varphi + \alpha \ln \alpha, \\ \xi_1 \cdot \ln n &= \left(1+\frac{1}{\eta}\right) \ln z - \left(\frac{1}{\eta}+1-\varphi\right) \ln\left(\frac{\theta_n}{\theta_c}\right) - \varphi \ln\left(\frac{\theta_d}{\theta_c}\right) + \varphi \ln\left(\varphi\right) + \left(\frac{1}{\eta}+1-\varphi\right) \ln\left(\alpha\right), \\ \xi_1 \cdot \ln y &= \left(\frac{1}{\eta}+1\right)\left(\frac{1}{\nu}+1\right) \ln z - \left(\frac{1}{\eta}+1\right) \alpha \ln\left(\frac{\theta_n}{\theta_c}\right) - \left(\frac{1}{\nu}+1\right) \varphi \ln\left(\frac{\theta_d}{\theta_c}\right) + const, \end{aligned} \tag{B.4}$$

where ξ_1 and *const* are constant with ξ_1 is positive (due to the assumption that $1 \geq \varphi + \alpha$) and given by $\xi_1 \equiv \left(\frac{1}{\eta}+1-\varphi\right)\left(\frac{1}{\nu}+1\right) - \left(\frac{1}{\eta}+1\right)\alpha$ and $const = \left(\frac{1}{\nu}+1\right)\varphi \ln\left(\varphi\right) + \left(\frac{1}{\eta}+1\right)\alpha \ln\left(\alpha\right)$.

Note that when search effort is constant ($\eta \to 0$), the elasticities of labor on z and θ_n become $\frac{1}{\frac{1}{\nu}+1-\alpha}$, as in a standard RBC model with GHH preference. Furthermore, the presence of endogenous search effort ($\eta > 0$) leads to a larger elasticity of n and y to z and θ_n shocks.

Let us examine the response of output to the shocks more closely. Let $\hat{x}$ denote the log deviation of variable x from a steady state. We can further write the output equation (B.4) as

$$\hat{y} = \frac{\frac{1+\nu}{\nu}}{\frac{1+\nu}{\nu} - \left(\alpha + \frac{1+\nu}{\nu}\frac{\eta}{1+\eta}\varphi\right)} \left[\hat{z} - \alpha \frac{\nu}{1+\nu}\hat{\theta}_n - \frac{\eta\varphi}{1+\eta}\hat{\theta}_d + \left(\alpha \frac{\nu}{1+\nu} + \frac{\eta\varphi}{1+\eta}\right)\hat{\theta}_c \right].$$

The equation again establishes that the presence of search friction $\eta > 0$ and $\varphi > 0$ leads to a larger response of output to shocks. Note that the propagation of y to a shock to z and/or θ_n is increasing in

φ, η, α, and ν:

$$\frac{\partial}{\partial\nu}\left(\frac{\frac{1+\nu}{\nu}}{\frac{1+\nu}{\nu}-\left(\alpha+\frac{1+\nu}{\nu}\frac{\eta}{1+\eta}\varphi\right)}\right)=\alpha\frac{(\eta+1)^2}{(-\nu-\eta+\alpha\nu-\nu\eta+\eta\varphi+\alpha\nu\eta+\nu\eta\varphi-1)^2}>0,$$
$$\frac{\partial}{\partial\nu}\left(\frac{\alpha}{\frac{1+\nu}{\nu}-\left(\alpha+\frac{1+\nu}{\nu}\frac{\eta}{1+\eta}\varphi\right)}\right)=\frac{\alpha(\eta+1)(\eta(1-\varphi)+1)}{(-\nu-\eta+\alpha\nu-\nu\eta+\eta\varphi+\alpha\nu\eta+\nu\eta\varphi-1)^2}>0,$$
$$\frac{\partial}{\partial\alpha}\left(\frac{\alpha}{\frac{1+\nu}{\nu}-\left(\alpha+\frac{1+\nu}{\nu}\frac{\eta}{1+\eta}\varphi\right)}\right)=\frac{(\eta(1-\varphi)+1)\nu(\nu+1)(\eta+1)}{(-\nu-\eta+\alpha\nu-\nu\eta+\eta\varphi+\alpha\nu\eta+\nu\eta\varphi-1)^2}>0.$$

Consider now the measured TFP A, i.e., the Solow residual when search effort is ignored. This can be defined as $A\equiv y/n^{\alpha}=z(d)^{\varphi}$, which implies

$$\begin{aligned}\xi_1\ln A &= \xi_1\ln z+\xi_1\varphi\ln d\\ &= \left(\frac{1}{\eta}+1\right)\left(\frac{1}{\nu}+1-\alpha\right)\ln z-\varphi\alpha\ln\left(\frac{\theta_n}{\theta_c}\right)-\varphi\left(\frac{1}{\nu}+1-\alpha\right)\ln\left(\frac{\theta_d}{\theta_c}\right)\\ &\quad+\varphi\left(\frac{1}{\nu}+1-\alpha\right)\ln\varphi+\varphi\alpha\ln\alpha.\end{aligned}$$

We now work on the solution for saving s and consumption c. Let us first calculate the terms inside the utility function:

$$\ln\left(\frac{\theta_d}{\theta_c}\right)+(1+1/\eta)\ln d-\ln(1+1/\eta)=\ln y+\ln\varphi-\ln\left(\frac{1+\eta}{\eta}\right)$$
$$\ln\left(\frac{\theta_n}{\theta_c}\right)+(1+1/\nu)\ln n-\ln(1+1/\nu)=\ln y+\ln(\alpha)-\ln(1+1/\nu)$$

It follows that we can write

$$z(d)^{\varphi}n^{\alpha}-\frac{\theta_d}{\theta_c}\frac{d^{1+1/\eta}}{1+1/\eta}-\frac{\theta_n}{\theta_c}\frac{n^{1+1/\nu}}{1+1/\nu}=\xi_2\cdot y,$$

where $\xi_2\equiv 1-\frac{\eta\varphi}{1+\eta}-\frac{\nu}{1+\nu}\alpha\in(0,1)$. Note that the assumption $1\geq\varphi+\alpha$ ensures that X is positive.

Consider now the Euler equation;

$$\begin{aligned}&\left[z(d)^{\varphi}n^{\alpha}-s\right]-\frac{\theta_d}{\theta_c}\frac{d^{1+1/\eta}}{1+1/\eta}-\frac{\theta_n}{\theta_c}\frac{n^{1+1/\nu}}{1+1/\nu}\\ =\;&(\beta)^{-\frac{1}{\gamma}}\left(\frac{\theta^{c\prime}}{\theta_c}\right)^{-\frac{1-\gamma}{\gamma}}\left(\left[z'(d')^{\varphi}(n')^{\alpha}+s\right]-\frac{\theta^{d\prime}}{\theta^{c\prime}}\frac{(d')^{1+1/\eta}}{1+1/\eta}-\frac{\theta^{n\prime}}{\theta^{c\prime}}\frac{(n')^{1+1/\nu}}{1+1/\nu}\right),\end{aligned}$$

or, equivalently,

$$\xi_2 \cdot y - s = (\beta)^{-\frac{1}{\gamma}} \left(\frac{\theta^{c\prime}}{\theta_c}\right)^{-\frac{1-\gamma}{\gamma}} (\xi_2 \cdot y' + s)$$

which implies savings s

$$s = \frac{\xi_2}{(\beta)^{-\frac{1}{\gamma}} \left(\frac{\theta_c'}{\theta_c}\right)^{-\frac{1-\gamma}{\gamma}} + 1} \cdot y - \frac{(\beta)^{-\frac{1}{\gamma}} \left(\frac{\theta_c'}{\theta_c}\right)^{-\frac{1-\gamma}{\gamma}} \xi_2}{(\beta)^{-\frac{1}{\gamma}} \left(\frac{\theta^{c\prime}}{\theta_c}\right)^{-\frac{1-\gamma}{\gamma}} + 1} \cdot y'$$

Consumption is then given by

$$c = \frac{(\beta)^{-\frac{1}{\gamma}} \left(\frac{\theta_c'}{\theta_c}\right)^{-\frac{1-\gamma}{\gamma}} + 1 - \xi_2}{(\beta)^{-\frac{1}{\gamma}} \left(\frac{\theta^{c\prime}}{\theta_c}\right)^{-\frac{1-\gamma}{\gamma}} + 1} \cdot y + \frac{(\beta)^{-\frac{1}{\gamma}} \left(\frac{\theta^{c\prime}}{\theta_c}\right)^{-\frac{1-\gamma}{\gamma}} \xi_2}{(\beta)^{-\frac{1}{\gamma}} \left(\frac{\theta^{c\prime}}{\theta_c}\right)^{-\frac{1-\gamma}{\gamma}} + 1} \cdot y'.$$

C Planner Problem, Competitive Equilibrium, and the First Welfare Theorem in the Stocahstic Growth Model Economy

In the text we appeal to a version of the First Welfare Theorem and focus on the planner problem. This Appendix characterizes the planner problem and the competitive equilibrium, and proves that the allocation implied by the decentralized competitive equilibrium is equivalent to the planner allocation.

C.1 Derivations of optimality condition for the planner

The optimality conditions follow from the first-order conditions of the planner problem (25), rewritten as

$$\max_{D_c, D_i, N_c, N_i, K_c'} \; u\left(D_c\, \Psi_{d,c}\left[D_c\right] z_c f\left(K_c/T_c, N_c/T_c\right), D_c + D_i, N_c + N_i, \Lambda\right) \\ + \; \beta \; \mathrm{E}\big\{W\left(\theta', K_c', D_i \Psi_{d,i}[D_i] z_i f\left(K_i/T_i, N_i/T_i\right) + (1-\delta)\left(K_c + K_i\right) - K_c'\right) \bigm| \Lambda\big\}.$$

The first-order conditions over D_c, D_i, N_c, N_i, and K_c' are given by

$$\begin{aligned}
\frac{\partial u}{\partial C}\frac{\partial \Psi_{T,c}[D_c]}{\partial D_c} z_c f(K_c/T_c, N_c/T_c) + \frac{\partial u}{\partial D} &= 0 && \text{(C.1)}\\
\frac{\partial u}{\partial D} + \frac{\partial \Psi_{T,i}[D_i]}{\partial D_i} z_i f(K_i/T_i, N_i/T_i)\,\beta\, \mathrm{E}\left\{\frac{\partial W(\Lambda')}{\partial K_i'}\,\middle|\,\Lambda\right\} &= 0 && \text{(C.2)}\\
\frac{\partial u}{\partial N} + \frac{\partial u}{\partial C}\Psi_{T,c}[D_c]\; z_c \frac{f(K_c/T_c, N_c/T_c)}{\partial N_c} &= 0 && \text{(C.3)}\\
\frac{\partial u}{\partial N} + \Psi_{T,i}[D_i]\, z_i \frac{\partial f(K_i/T_i, N_i/T_i)}{\partial N_i}\beta\, \mathrm{E}\left\{\frac{\partial W(\Lambda')}{\partial K_i'}\,\middle|\,\Lambda\right\} &= 0 && \text{(C.4)}\\
\beta\; \mathrm{E}\left\{\frac{\partial W(\Lambda')}{\partial K_c'} - \frac{\partial W(\Lambda')}{\partial K_i'}\,\middle|\,\Lambda\right\} &= 0. && \text{(C.5)}
\end{aligned}$$

And the envelope conditions are

$$\begin{aligned}
\frac{\partial W(\Lambda)}{\partial K_c} &= \frac{\partial u}{\partial C}\Psi_{T,c}[D_c]\, z_c \frac{\partial f(K_c/T_c, N_c/T_c)}{\partial K_c} - \frac{(1-\delta)}{\Psi_{T,i}[D_i] z_i \frac{\partial f(K_i/T_i, N_i/T_i)}{\partial N_i}}\frac{\partial u}{\partial N} && \text{(C.6)}\\
\frac{\partial W(\Lambda)}{\partial K_i} &= -\left(\Psi_{T,i}[D_i] z_i \frac{\partial f(K_i/T_i, N_i/T_i)}{\partial K_i} + 1 - \delta\right)\frac{1}{\Psi_{T,i}[D_i] z_i \frac{\partial f(K_i/T_i, N_i/T_i)}{\partial N_i}}\frac{\partial u}{\partial N}. && \text{(C.7)}
\end{aligned}$$

Optimality conditions (26)-(28) follow directly from equations (C.1)-(C.4). Combining equations (C.5)-(C.6)-(C.7) with (C.3) yields equation (29):

$$0 = \mathrm{E}\left\{\frac{\partial W(\Lambda')}{\partial K_c'} - \frac{\partial W(\Lambda')}{\partial K_i'}\,\middle|\,\Lambda\right\} = \mathrm{E}\left\{\frac{\partial u}{\partial N'}\left(\frac{\partial f(K_i'/T_i, N_i'/T_i)/\partial K_i'}{\partial f(K_i'/T_i, N_i'/T_i)/\partial N_i'} - \frac{\partial f(K_c'/T_c, N_c'/T_c)/\partial K_c'}{\partial f(K_c'/T_c, N_c'/T_c)/\partial N_c'}\right)\,\middle|\,\Lambda\right\}.$$

Euler equation (30) follows from plugging (C.7) into equation (C.4).

C.2 Competitive Equilibrium

Let $A_c = \mu T_c^{1-\varphi}$ and $A_i = \mu T_i^{1-\varphi}$. Consider the household problem in the benchmark economy, equations (18)-(22) in the text. Let the multiplier on the household's budget constraint (21) be denoted λ. The first-order conditions for the households can then be expressed as,

$$
\begin{aligned}
\lambda \hat{p}_c &= \frac{\partial u}{\partial c} + \frac{\frac{\partial u}{\partial d}}{\Psi_{d,c}(\hat{q}_c)\hat{F}_c} \qquad\qquad \text{(C.8)}\\
\lambda \hat{p}_i - \frac{\frac{\partial u}{\partial d}}{\Psi_{d,i}(\hat{q}_i)\hat{F}_i} &= \beta E\left\{\lambda' R_c(\Lambda') + (1-\delta)\left[\lambda' p_i(\Lambda') - \frac{\frac{\partial u}{\partial d'}}{\Psi_{d,i}(\hat{q}'_i)\hat{F}'_i}\right]\right\}\\
\lambda \hat{p}_i - \frac{\frac{\partial u}{\partial d}}{\Psi_{d,i}(\hat{q}_i)\hat{F}_i} &= \beta E\left\{\lambda' R_i(\Lambda') + (1-\delta)\left[\lambda' p_i(\Lambda') - \frac{\frac{\partial u}{\partial d'}}{\Psi_{d,i}(\hat{q}'_i)\hat{F}'_i}\right]\right\}\\
\lambda W &= \frac{\partial u}{\partial n}\\
\frac{\partial \hat{V}^c}{\partial \hat{p}_c} &= -\lambda d_c A_c \hat{q}_c^{\varphi-1}\hat{F}_c, \qquad \frac{\partial \hat{V}^c}{\partial \hat{q}_c} = (\varphi-1) d_c A_c \hat{q}_c^{\varphi-2}\hat{F}_c\left(\frac{\partial u}{\partial c} - \lambda \hat{p}_c\right)\\
\frac{\partial \hat{V}^c}{\partial \hat{F}_c} &= d_c A_c \hat{q}_c^{\varphi-1}\left(\frac{\partial u}{\partial c} - \lambda \hat{p}_c\right)\\
\frac{\partial \hat{V}^i}{\partial \hat{p}_i} &= -\lambda d_i A_i \hat{q}_i^{\varphi-1}\hat{F}_i, \qquad \frac{\partial \hat{V}^i}{\partial \hat{q}_i} = (\varphi-1) d_i A_i \hat{q}_i^{\varphi-2}\hat{F}_i\left[-\frac{\frac{\partial u}{\partial d}}{\Psi_{d,i}(\hat{q}_i)\hat{F}_i}\right]\\
\frac{\partial \hat{V}^i}{\partial \hat{F}_i} &= \frac{\partial u}{\partial c}\, d_i A_i \hat{q}_c^{\varphi-1}\left[-\frac{\frac{\partial u}{\partial d}}{\Psi_{d,i}(\hat{q}_i)\hat{F}_i}\right].
\end{aligned}
$$

Now let's consider the problem (24) of a firm $j = \{c, i\}$. Let the multiplier for the participation constraint be γ and for the production constraint be υ. We can write the firm's first-order conditions as

$$
A_j \hat{q}_j^{\varphi}\hat{F}_j + \gamma\frac{\partial \hat{V}^j}{\partial \hat{p}_j} = 0, \qquad \varphi A_j \hat{q}_j^{\varphi-1}\hat{p}_j\hat{F}_j + \gamma\frac{\partial \hat{V}^j}{\partial \hat{q}_j} = 0, \qquad \hat{p} A_j \hat{q}^{\varphi} - \upsilon + \gamma\frac{\partial \hat{V}^j}{\partial \hat{F}_j} = 0
$$

$$
W = \upsilon z_j \frac{\partial f(k,n)}{\partial n}, \qquad R_j = \upsilon z_j \frac{\partial f(k,n)}{\partial k}.
$$

Equilibrium In equilibrium, $p_c = \hat{p}_c = 1$, $p_i = \hat{p}_i$, $q_j = \hat{q}_j$, and $F_j = \hat{F}_j$ for each $j = \{c, i\}$. The competitive equilibrium consists of 16 functions, five functions from the households' problem, $\{C, D_c, D_i, N, I\}$, four functions from the firms' problems $\{k_c, k_i, n_c, n_i\}$, aggregate capital and labor in each sector $\{K_c, K_i, N_c, N_i\}$three functions from competitive search problem, $\{q_c, q_i, p_i\}$, and four price functions, $\{W, P_a, R_c, R_i\}$, which simultaneously satisfy the following functional equations.

First, the household problem yields the following functional equations,

$$\frac{\partial u}{\partial c}p_i = \beta\,\mathrm{E}\left\{\left[(1-\varphi)R_c(\Lambda')+p_i(\Lambda')(1-\delta)\right]\frac{\partial u}{\partial c'} \mid \Lambda\right\} \tag{C.9}$$

$$\frac{\partial u}{\partial c}p_i = \beta\,\mathrm{E}\left\{\left[(1-\varphi)R_i(\Lambda')+p_i(\Lambda')(1-\delta)\right]\frac{\partial u}{\partial c'} \mid \Lambda\right\} \tag{C.10}$$

$$(1-\varphi)\frac{\partial u}{\partial c} = \frac{\frac{\partial u}{\partial n}}{W} \tag{C.11}$$

$$-\frac{\partial u}{\partial d} = \frac{\partial u}{\partial c}\varphi A_c D_c^{\varphi-1} z_c f(k_c, n_c) \tag{C.12}$$

$$-\frac{\partial u}{\partial d} = \frac{\partial u}{\partial c}\varphi A_i D_i^{\varphi-1} p_i z_i f(k_i, n_i). \tag{C.13}$$

Second,the firms' problems yield four functional equations,

$$(1-\varphi)W = \Psi_{T,c}(q_c)z_c\frac{\partial f(k_c,n_c)}{\partial n_c} \tag{C.14}$$

$$\frac{W}{R_c} = \frac{\partial f(k_c,n_c)/\partial n_c}{\partial f(k_c,n_c)/\partial k_c} \tag{C.15}$$

$$(1-\varphi)\frac{W}{p_i} = \Psi_{T,i}(q_i)z_i\frac{\partial f(k,n_i)}{\partial n_i} \tag{C.16}$$

$$\frac{W}{R_i} = \frac{\partial f(k_i,n_i)/\partial n_i}{\partial f(k_i,n_i)/\partial k_i}. \tag{C.17}$$

Finally, market clearing yields

$$\begin{aligned}
q_c &= D_c, \quad q_i = D_i \\
C &= T_c\Psi_{T,c}(q_c)z_c f(k_c,n_c) \\
I &= T_i\Psi_{T,i}(q_i)z_i f(k_i,n_i) \\
I &= K_c' + K_i' - (1-\delta)(K_c+K_i) \\
N &= N_c + N_i \\
K_c &= T_c k_c, K_i = T_i k_i \\
N_c &= T_c n_c, N_i = T_i n_i.
\end{aligned}$$

Incidentally, by combining equations (C.14) and (C.16) we obtain the expression for the relative price of investment, equation (27) in the text.

C.3 A First Welfare Theorem in the Production Economy

We now show that the competitive equilibrium corresponds to the planner allocation. To this end, we show that the first-order conditions of the competitive equilibrium are the same as the ones in the planner problem.

First, equation (C.12) and (C.13) in the equilibrium is identical to equation (26) in the planner problem. Second, combining equation (C.11) and (C.14) yields the first-order condition on labor (28) in the planner problem. Finally, equation (29) in the planner problem holds in equilibrium. To see this, take the difference between (C.9) and (C.10) from the household's problem, and get

$$0 = \mathrm{E}\Big\{ \left(R_i(\Lambda') - R_c(\Lambda')\right)(1-\varphi)\frac{\partial u}{\partial c'} \mid \Lambda \Big\}$$

Now we replace u_c with u_n/W using equation (C.11) and obtain

$$0 = \mathrm{E}\Big\{ \left(R_i(\Lambda') - R_c(\Lambda')\right)\frac{\partial u/\partial n'}{W(\Lambda')} \mid \Lambda \Big\}.$$

Then replace R/W with firms' first-order conditions (C.15) and (C.17) to obtain the planner's Euler equation.

D Details about the Calibration of the Production Economy

We consider an economy with an annual growth rate of 3%, which implies a quarterly value $\bar{g} = 0.74\%$

Depreciation δ. With a constant growth rate, the capital accumulation is given by

$$(1+\bar{g})K' = (1-\delta)K + I.$$

Hence the steady state K and I satisfy $(\delta+\bar{g})K = I$. Let the investment share of output be $\kappa = p_I I/Y = 20\%$ and the ratio of aggregate capital to quarterly output $p_I K/Y = 2.75 * 4 = 11$. This allows us to pin down the depreciation rate at the quarterly frequency

$$\delta = \frac{p_I I/Y}{p_I K/Y} - \bar{g} = \frac{0.2}{11} - 0.74\% = 1.08\%$$

Labor's share α_n: Consider now the share of output for a consumption-producing firm that is paid to labor. Using the equilibrium condition (C.14) and (C.16), we can express the equilibrium price in sector $j = \{c, i\}$ as

$$p_j = (1-\varphi)\frac{W}{\Psi_{T,j}(q_j) z_j \frac{\partial f\left(k_j, n_j\right)}{\partial n_j}} = (1-\varphi)\frac{W N_j}{\alpha_n \Psi_{T,j}(q_j) F_j} \tag{D.1}$$

where the second equation comes from the assumption that f is Cobb-Douglas so $\alpha_n f\left(k_j, n_j\right) = n_j \frac{\partial f}{\partial n_j}$, and $K_j = T_j k_j$, $N_j = T_j n_j$. In each sector j, the wage bill is $W N_j$, and total (measured) value added is the sales $\Psi_{T,j}(q_j)F_j$, where $F_j = z_j f(k_j, n_j)$, so labor's share of output is

$$\frac{WN_c + WN_j}{Y} = \frac{\frac{\alpha_n}{1-\varphi} T_c \Psi_{T,c}(q_c) F_c + p_i \frac{\alpha_n}{1-\varphi} T_i \Psi_{T,i}(q_i) F_i}{Y} = \frac{\alpha_n}{1-\varphi} \frac{C + p_i I}{Y} = \frac{\alpha_n}{1-\varphi}$$

where the second equality follows from equation (D.1). We then find $\alpha_n = (1-\varphi) * \text{labor share}$.

Capital's share α_k. At the steady state, $R_c = R_i = R$. From households' first order conditions, steady state implies that

$$p_i = \beta(1+\bar{g})^{-\sigma}[(1-\varphi)R_i + (1-\delta)p_i] \quad \Rightarrow \quad (1-\varphi)\frac{R}{p_i} = \frac{1-\beta(1+\bar{g})^{-\sigma}(1-\delta)}{\beta(1+\bar{g})^{-\sigma}}. \tag{D.2}$$

Combining the investment firms' optimization (C.16) and (C.17), we have

$$(1-\varphi)\frac{R_i}{p_i} = \Psi_{T,i}(q_i) z_i \frac{\partial f(k_i, n_i)}{\partial k_i} = \alpha_k \frac{\Psi_{T,i}(q_i) z_i f(k_i, n_i)}{k_i} = \alpha_k \frac{I/T_i}{k_i} = \alpha_k \frac{I}{K_i}.$$

Similarly, from consumption firms' optimization, we have

$$(1-\varphi)\frac{R_c}{p_c} = \alpha_k \frac{C}{K_c}.$$

The share of output that goes to reproducible capital is

$$\frac{RK}{Y} = \frac{RK_c + RK_i}{Y} = \frac{\frac{\alpha_k}{1-\varphi} p_i I + \frac{\alpha_k}{1-\varphi} p_c C}{Y} = \frac{\alpha_k}{1-\varphi}.$$

Now let's check RK/Y,

$$\frac{\alpha_k}{1-\varphi} = \frac{RK}{Y} = \frac{R}{p_i}\frac{p_i K}{Y} = \frac{1}{1-\varphi}\frac{1-\beta(1+\bar{g})^{-\sigma}(1-\delta)}{\beta(1+\bar{g})^{-\sigma}}\frac{p_i K}{Y},$$

where the last equality comes from (D.2). Hence we can solve α_k from the above equation

$$\alpha_k = \frac{1-\beta(1+\bar{g})^{-\sigma}(1-\delta)}{\beta(1+\bar{g})^{-\sigma}}\frac{p_i K}{Y}.$$

With $r = 0.01$ quarterly, $\beta(1+\bar{g})^{-\sigma} = 1/(1+r)$, $\delta = 1.08\%$, $p_i K/Y = 11$, the above equation implies $\alpha_k = 0.23$.

Search efficiency μ**, measure** T_c**, and** T_i We first solve for D_j in each sector. Combining the households' FOCs (C.12) and (C.13), we have

$$\frac{C}{D_c} = \frac{p_i I}{D_i} \quad \Rightarrow \quad \frac{D_c}{D_i} = \frac{C}{p_i I}.$$

Recall the consumption share is $1-\kappa$. The above equation implies $D_c = (1-\kappa)(D_c + D_i) = (1-\kappa)D$. Under GHH preference and equation (C.12), we have

$$\varphi \frac{C}{D_c} = D^{\frac{1}{\eta}} \quad \Rightarrow \quad (1-\kappa)D^{1+1/\eta} = \varphi C = \varphi(1-\kappa).$$

Hence $D = \varphi^{\frac{\eta}{1+\eta}}$.

The capacity utilization in sector j is defined as the probability that a production unit gets matched, i.e., $\Psi_{T,j}[D_j] = A(D_j/T_j)^{\varphi}$. A capacity utilization of $\Psi_{T,j} = 81\%$ in each sector implies $T_c/T_i = D_c/D_i$. Given that $T_c + T_i = 1$, we can solve $T_c = 1-\kappa$ and $T_i = \kappa$. We can now evaluate $A = 0.81\varphi^{-\frac{\varphi\eta}{1+\eta}}$.

Mean TFP $Z_c = E(z_c)$ and $Z_i = E(z_i)$. To compute average TFPs, we first obtain the share of labor and capital in each sector. In steady state, the two sectors, consumption and investment, have the same ratio of wage to rental. Under the assumption of Cobb-Douglas of f, this implies $\frac{N_i}{K_i} = \frac{N_c}{K_c} = \frac{N}{K}$. In addition, we divided equation (C.14) by equation (C.16) and get

$$p_i = \frac{\Psi_T(q) z \partial f(k_c, n_c)/\partial n_c}{z z_i \partial f(k_i, n_i)/\partial n_i} = \frac{C/T_c}{I/T_i}\frac{n_i}{n_c} = \frac{C}{I}\frac{N_i}{N_c},$$

which implies

$$\frac{N_i}{N_c} = \frac{\kappa}{1-\kappa}. \tag{D.3}$$

Similarly $K_i/K_c = \kappa/(1-\kappa)$. We choose $p_i = 1$ in steady state, thus $I = \kappa$ and $K = 11$. We can then find $K_i = \kappa K$, $K_c = (1-\kappa)K$, $N_i = \kappa N$, and $N_c = (1-\kappa)N$, where N is chosen to be 0.3. We can now back up the mean productivities using the production function

$$z_c = \frac{C}{T_c \Psi_{T,c} f(k_c, n_c)} = \frac{1-\kappa}{(1-\kappa)0.81 \times (K_c/T_c)^{\alpha_k}(N_c/T_c)^{\alpha_n}},$$
$$z_i = \frac{I}{T_i \Psi_{T,i} f(k_i, n_i)} = \frac{\kappa}{\kappa 0.81 \times (K_i/T_i)^{\alpha_k}(N_i/T_i)^{\alpha_n}},$$

where we use the equilibrium condition that each firm in sector j uses $1/T_j$ fraction of total capital and labor in that sector.

D.1 Proof of Proposition 3: Characterization of Equilibrium in the Heterogeneous Agents Economy

Consider a static economy with J types of agents who differ in their consumption expenditure y_j, $j \in \{1, ..., J\}$. With a price level of $P = 1$, the aggregate expenditure must equal $\sum_j y_j s_j = C$, where s_j is the population share of type j households, and C is aggregate consumption. There is a unit measure of firms, each of whom has a production function $f(k,n) = (k)^{\alpha_k}(n)^{\alpha_n}$. We start by showing that in equilibrium all firms supply the same capacity F.

Lemma 1. *All firms supply the same capacity of consumption goods, given by $F = zf(K_c, N_c)$.*

Proof. Since there are no externalities or distortions, it is straightforward that the welfare theorems apply. Accordingly, we formulate the problem as a planner problem,

$$\begin{aligned} &\max \sum_j \xi_j u(C_j, D_j) \\ \text{s.t. } K_C &= \sum_j K_j \qquad N_C = \sum_j N_j \qquad 1 = \sum_j T_j, \qquad C_j = D_j^{\varphi} T_j^{1-\varphi} zf(K_j, N_j), \end{aligned}$$

where ξ_j denotes the planner weight on households of type j and K_j, N_j, and T_j denote the capital, labor, and share of firms allocated to production intended for group j. D_j and T_j represent the aggregate search effort in market j and the aggregate measure of firms catering to market j. We rewrite the problem as a Lagrange problem

$$\Theta = \sum_j \xi_j u\left(D_j^{\varphi}(T_j)^{1-\varphi} zf(K_j, N_j), D_j\right) - \lambda_K\left(\sum_j K_j - K_C\right) - \lambda_N\left(\sum_j N_j - N_C\right) - \Gamma\left(\sum_j T_j - 1\right)$$

Taking the first-order conditions yields,

$$\begin{aligned} -u_{d,j} &= u_{c,j} \cdot \varphi \left(\frac{D_j}{T_j}\right)^{\varphi-1} zf(K_j, N_j) \\ \frac{\Gamma}{\lambda_K} &= \frac{\xi_j u_{c,j} \cdot (1-\varphi)\left(\frac{D_j}{T_j}\right)^{\varphi} f(K_j, N_j)}{\xi_j u_{c,j} \cdot \left(\frac{D_j}{T_j}\right)^{\varphi} T_j \frac{\partial f(K_j,N_j)}{\partial K_j}} = (1-\varphi)\frac{f(K_j, N_j)}{T_j \frac{\partial f(K_j,N_j)}{\partial K_j}} \\ \frac{\lambda_K}{\lambda_N} &= \frac{\xi_j u_{c,j} \cdot \left(\frac{D_j}{T_j}\right)^{\varphi} T_j \frac{\partial f(K_j,N_j)}{\partial K_j}}{\xi_j u_{c,j} \cdot \left(\frac{D_j}{T_j}\right)^{\varphi} T_j \frac{\partial f(K_j,N_j)}{\partial N_j}} = \frac{\frac{\partial f(K_j,N_j)}{\partial K_j}}{\frac{\partial f(K_j,N_j)}{\partial N_j}}. \end{aligned}$$

When f is Cobb-Douglas, the last two conditions imply

$$\begin{aligned} \frac{\Gamma}{\lambda_K} &= (1-\varphi)\frac{(K_j)^{\alpha_k}(N_j)^{\alpha_n}}{T_j \alpha_k (K_j)^{\alpha_k-1}(N_j)^{\alpha_n}} = \frac{1-\varphi}{\alpha_k}\frac{K_j}{T_j} \\ \frac{\lambda_K}{\lambda_N} &= \frac{\alpha_k (K_j)^{\alpha_k-1}(N_j)^{\alpha_n}}{\alpha_n (K_j)^{\alpha_k}(N_j)^{\alpha_n-1}} = \frac{\alpha_k}{\alpha_n}\frac{N_j}{K_j}. \end{aligned}$$

It follows that all firms have the same capital-labor ratio, equal to K_c/N_c, and all firms have the same output per location, $F = zf(K_c, N_c)$. □

We now solve the decentralized problem. Conjecture that there will be J different markets open, which all provide F but differ in the offered pair (p_j, q_j). Let $\bar{\pi}$ represent the expected revenue for a firm operating in the most profitable market. Profit maximization then imposes the following arbitrage condition on any offered (p_j, q_j),

$$p_j = \frac{\bar{\pi}}{F} \cdot (q_j)^{-\varphi} = \frac{\bar{\pi}}{F} \cdot \left(\frac{D_j}{T_j}\right)^{-\varphi}.$$

The search technology implies that for households of type j, $c_j = D_j^{\varphi} (T_j)^{1-\varphi} F$. We can thus obtain income of a household $y_j = p_j c_j = \bar{\pi} T_j$. Thus, since $\int T_j dj = 1$, aggregate revenue equals aggregate expenditure, $\bar{\pi} = \int y_j dj = C$. It follows that the measure of firms catering to type j households is equal to j's expenditure share $T_j = s_j y_j / C$. This in turn implies equation (31). Finally, the intra-temporal first-order condition yields equation (32).

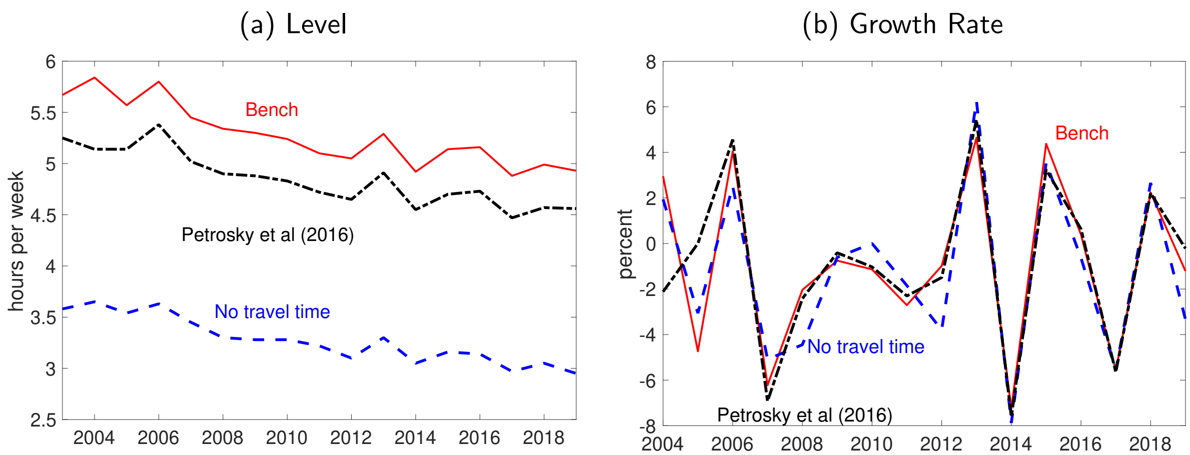
E Shopping Time Data and Robustness

Using the data from American Time Use Survey (ATUS), we construct "total shopping time" as the sum of the time spent shopping, professional and personal care service, household services, and the associated travel time. We specifically include the following categories: TU-7, TU-8, TU-9, TU-18-7, TU-18-8, and TU-18-9. We also consider two alternative measures of shopping time. The first one excludes travel time from the total shopping time, while the second measure uses a smaller set of categories as in Petrosky-Nadeau et al. (2016), which do not include security procedures or paid services like childcare, financial services, or personal care. To construct each of the three series, we take a weighted average across the working-age population (25-59), using the weights provided by ATUS. The data covers the period from 2003 to 2019.

Panel (a) presents the levels of the benchmark shopping time (black-solid line), shopping time excluding travel time (blue-dashed line), and shopping time constructed using Petrosky-Nadeau et al. (2016) (henceforth PNWZ, black-dotted-dashed line). Panel (b) shows the annual growth rates for each shopping series, with the average growth rate removed from each series. The results indicate that the three shopping series have different levels, with the benchmark measure being the highest. The average weekly shopping time is 5.27 hours using the benchmark construction, 4.85 hours using PNWZ method, and 3.27 hours without the travel time. Nonetheless, the growth rates of the three series show remarkable similarities.

To show the robustness of the benchmark estimation, we reestimate the model using the two alternative shopping series. Figure 2 presents a comparison of the results obtained from each of the three shopping series. Table 12 shows that the estimated θ_d shock and the variance decompositions are very similar across the three estimations.

Figure 2: Shopping Time



This figures shows the level and growth rates in shopping time between 2003 and 2019 for the working age group (25-59 years old). The data source is American Time Use Survey (ATUS). Panel (a) shows the benchmark shopping time (black -solid line), shopping time excluding travel time (blue-dashed line), and shopping time constructed using Petrosky-Nadeau et al. (2016) (black-dotted-dashed line). Panel (b) shows the growth rates for each shopping series.

Table 12: Robustness analysis — Alternative Measures of Shopping Time

	Lhood	$std(\theta_d)$	Var Decomp Y			Var Decomp TFP		
			θ_d	g	z_l	θ_d	g	z_l
Use shopping time data 03-19, mixed frequency								
(1) Benchmark	603.7	0.097	38.60	44.21	5.48	39.09	48.43	8.74
(2) No travel time	603.7	0.098	39.04	44.21	5.15	39.69	48.35	8.23
(3) Alternative 2	607.0	0.093	37.53	44.31	5.89	38.01	48.67	9.40

The table documents the estimation results for alternative shopping time. All estimations target the quarterly U.S. data series of GDP growth, labor productivity growth, investment growth, and growth of aggregate shopping time. Row (1) uses the benchmark shopping time, row (2) uses shopping time excluding travel time, and row (3) uses the shopping time constructed following Petrosky-Nadeau et al. (2016).

F Identical inventory choice at the storable good model

In this subsection, we show that consumption goods producers with identical initial inventory choose the same holdings of next period's inventory, i.e.

$$x'_u(\Lambda, x) = x'_m(\Lambda, x). \tag{F.1}$$

A good producer in sector $\iota = \{c, i\}$ chooses $\{n_\iota, k_\iota, p_\iota, q_\iota, F_\iota, i_{x,u}, i_{x,m}, x'_m, x'_u\}$ to maximize

$$\begin{aligned}\Omega(\Lambda, x) = \max \quad & -W(\Lambda)n_\iota - R_\iota(\Lambda)k_\iota \\ & + \Psi_{T,\iota}(q_\iota)\left\{p_\iota F_\iota - p_{x\iota}(\Lambda)i_{x,m} + E\left[M(\Lambda,\Lambda')\,\Omega(\Lambda', x'_m)|\Lambda\right]\right\} \\ & + (1-\Psi_{T,\iota}(q_\iota))\left\{-p_{x\iota}(\Lambda)i_{x,u} + E\left[M(\Lambda,\Lambda')\,\Omega(\Lambda', x'_u)|\Lambda\right]\right\}\end{aligned}$$

subject to the capacity constraint

$$F_\iota \le z_\iota f(n_\iota, k_\iota) + (1-\delta_x)x_\iota,$$

the inventory accumulation constraint for the unmatched state and for the matched state,

$$x'_u = z_\iota f(n_\iota, k_\iota) + (1-\delta_x)x + i_{x,u}, \tag{F.2}$$
$$x'_m = z_\iota f(n_\iota, k_\iota) + (1-\delta_x)x - F_g + i_{x,m}, \tag{F.3}$$

and the participation constraint of households. Let μ_u and μ_m be the multipliers on the unmatched constraint (F.2) and the matched inventory constraint (F.3) respectively. Taking the first order conditions on $i_{x,u}, i_{x,m}, x'_m, x'_u$, we have

$$\begin{aligned}\Psi_{T,\iota}(q_\iota)p_{x\iota}(\Lambda) &= \mu_m, \qquad (1-\Psi_{T,\iota}(q_\iota))p_{x\iota}(\Lambda) = \mu_u \\ \Psi_{T,\iota}(q_\iota)E[M(\Lambda,\Lambda')\Omega_x(\Lambda', x'_m)] &= \mu_m, \qquad (1-\Psi_{T,\iota}(q_\iota))E[M(\Lambda,\Lambda')\Omega_x(\Lambda', x'_u)] = \mu_u\end{aligned}$$

Combining these four equations, we have

$$\frac{\Psi_{T,\iota}(q_\iota)E[M(\Lambda,\Lambda')\Omega_x(\Lambda', x'_m)]}{(1-\Psi_{T,\iota}(q_\iota))E[[M(\Lambda,\Lambda')\Omega_x(\Lambda', x'_u)]} = \frac{\mu_m}{\mu_u} = \frac{\Psi_{T,\iota}(q_\iota)p_{x\iota}(\Lambda)}{(1-\Psi_{T,\iota}(q_\iota))p_{x\iota}(\Lambda)}$$

which implies

$$\frac{E[M(\Lambda,\Lambda')\Omega_x(\Lambda', x'_m)]}{E[M(\Lambda,\Lambda')\Omega_x(\Lambda', x'_u)]} = 1.$$

It is therefore $x'_m = x'_u$.

G Calibration of the storable good model

Table 13 reports the calibrated targets and parameter values.

Table 13: Calibration of the Shopping Model with Storable Goods

Targets	Value	Parameter	Value
First Group: Parameters Set Exogenously			
Risk aversion	1	γ	1
Real interest rate	4%	β	0.99
Frisch elasticity	0.72	ν	0.72
Elasticity of substitution	0.85	ζ	0.85
Inventory depreciation	0.185	δ_x	0.185
Second Group: Standard Targets			
Fraction of time spent working	30%	χ	6.50
Labor share of output	0.67	α_n	0.46
Consumption share	0.80	α_k	0.23
Capital-output ratio	2.75	δ	1.08%
Service share in consumption	0.90	ω_c	0.05
Service share in investment	0.50	ω_i	0.50
Inventory-sale ratio	11.5%	$E(z_{mc})$	0.70
Third Group: Normalization			
Steady-state output	1	$E(z_{mi})$	0.59
Relative price of investment	1	$E(z_{si})$	0.70
Relative price of service	1	$E(z_{sc})$	1.28
Capacity utilization of services	0.81	A_{sc}	0.92
Fourth Group: Targets Specific to This Economy			
Cross-sectional st. dev. of cons. good prices	9%	φ	0.32
Shopping time expenditure elasticity	7.5%	η_d	0.20