Health versus Wealth: On the Distributional Effects of Controlling a Pandemic*

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Abstract

Many countries are shutting non-essential sectors of the economy to slow the spread of COVID-19. To quantify the welfare effects of such measures it is important to recognize that the gains and losses from these policies are very unequally distributed. Older individuals have most to gain from slowing virus diffusion. Younger workers in sectors that are shuttered have the most to lose. We extend a standard epidemiological model of disease progression to incorporate heterogeneity by age, and multiple sources of disease transmission. We incorporate a multi-sector economic model in which workers differ by sector (basic and luxury) as well as by health status, and value consumption, life, and health. We show that different types of individual favor very different economic policies, and that a utilitarian planner chooses a very different policy than would be suggested by a representative agent setting.

Keywords: Coronavirus; Economic Policy; Redistribution

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1 Introduction

The central debate about the appropriate economic policy response to the global COVID-19 pandemic is about how aggressively to slow down economic activity in order to slow down the spread of the virus. In this paper, we argue that one reason people disagree about the appropriate policy is that "lock-down" policies have very large distributional implications. These distributional effects mean that different groups prefer very different policies. Standard epidemiological models assume a representative agent structure, in which households face a common trade-off between restrictions on social interaction that slow virus transmission but which also depress economic activity. In practice, however, the benefits of slower viral transmission are not shared uniformly, but accrue disproportionately to older households, who face a much higher risk of serious illness or death conditional on becoming infected. At the same time, the costs of reduced economic activity are disproportionately born by younger households, who bear the brunt of lower employment. A second very important dimension of heterogeneity is between younger workers employed in different sectors. Sensible lock-down policies designed to reduce viral spread will naturally focus on reducing activity in sectors in which there there is a social aspect to consumption and sectors that produce goods or services perceived to be luxuries. For example, restaurants and bars are likely to be the first to be closed. Because workers cannot easily reallocate across sectors, this implies that lock-down policies will imply extensive redistribution between young households specialized in different sectors. Thus, different groups in the economy (old versus young, workers in different sectors, healthy versus sick) will likely have very different views about the optimal mitigation strategy.

We build a model to explore some of these trade-offs. We differentiate between three types of people: young workers in a basic sector, young workers in a luxury sector, and old people. The luxury sector is the one potentially subject to lock-down.

The health side of the model builds on a standard epidemiological diffusion framework. We label our variant a *SAFER* model, reflecting the progression of possible health states. Model individuals start out as susceptible S (heathy, but vulnerable), can then become infected but asymptomatic A, infected with flu-like symptoms F, infected and needing emergency hospital care E, recovered R (healthy and immune), or dead.

The three different types of model agent face differential infection risk (workers face more exposure than non-workers) and differential health outcomes conditional on infection (the old are more likely to end up in emergency care).

On the production side, output of workers in the two sectors is combined to produce a single final consumption good. Workers are immobile across sectors. There are two policy levers. First, at each date the planner chooses what fraction of activity in the luxury sector to shut down. We call this policy the extent of mitigation. Mitigation reduces income of workers in the luxury sector, but also slows the spread of the virus. Second, the planner chooses how much to redistribute income away from workers and toward those who are not working, either because they are old, because they are unwell, or because their workplaces have been closed. A key assumption is that this redistribution is costly, so that perfect insurance is not optimal. Conditional on a given path for mitigation, the optimal redistribution problem is static. Harsher mitigation policies will imply greater inequality between different types.

We compute optimal paths for mitigation, where the path for mitigation is a simple parametric function of time. The optimal path for mitigation is highly sensitive to the relative weights the planner attaches to the three types of people in the model. A planner that prioritizes the old chooses extensive and prolonged mitigation. A planner that prioritizes workers in the risky sector chooses much weaker mitigation.

We also consider how the optimal policy for a utilitarian equal-weights planner varies with the cost of redistribution across worker types. We find that the larger is this cost, the more moderate is optimal mitigation. Thus our economy which features key dimensions of inequality implies a more modest shutdown than a representative agent analogue, and the cost of higher mortality.

There is an extraordinary set of papers being written about the pandemic. To cite the ones that we are aware as of now: Atkeson (2020) perhaps the galvanizer of the economists interest on the coronavirus pandemia developed The SIR model with special attention if and when the fraction of active infections in the population exceeds 1% (at which point the health system is forecast to be severely challenged) and 10% (which may result in severe staffing shortages for key financial and economic infrastructure) as well as the cumulative burden of the disease over an 18 month horizon. Greenstone and Nigam (2020) compare using the state of the art Imperial College Epidemiological model (Flaxman et al. (2020)) the paths under moderate distancing and no action and using the statistical value of life asses the social cost of no action. They calculate 1.7 million lives saved between March 1 and October 1, 37% of them due to the avoidance of overcrowding in hospitals. They impute a benefit of such action of 58% of per caput yearly consumption.

Eichenbaum, Rebelo, and Trabandt (2020) extend the canonical epidemiology model to study the interaction between economic decisions and epidemics consider how equilibrium without interventions suboptimally leads to more severity of the pandemia becaus einfected people do not fully internalize the effect of their economic decisions on the spread of the virus. Moll, Kaplan, and Violante (2020) pose a version of a Hank model where agents choose to engage in economic activities depending on their personal circumstances including economic sector. Bayer and Kuhn (2020) explore how differences in living arrangements of generations within families contribute to the cross country differences in terms of case-fatality rates. They document a strong positive correlation between this variable and the share of working-age families living with their parents. Berger, Herkenhoff, and Mongey (2020) extend the baseline Susceptible-Exposed-Infectious-Recovered (SEIR) infectious disease epidemiology model to explore the role of tersting and henceforth get a better idea of how to implement selective social separation policies. Fang, Wang, and Yang (2020) quantify the causal impact of human mobility restrictions using the Chincese experience and find that the lockdown was very effective providing estimates of diffusion under different scenarios. Hall et al. (2020) provide a simple calculation to assess how much would we pay to have never had the virus (the answer is about a quarter of one year worth of consumption).

We start describing how we model the joint evolution of the economy and the population in Section 2. We turn then to what economic policy is in this environment (something that goes beyond social distancing) and how to think of the best policy Section 3. We then move to describe how we specify the quantitative details of our environment in Section 4.

2 The Model

We start describing the demographic structure in our *SAFER* extension of the standard *SIR* epidemiologic model in Section 2.1. We then introduce multisector production on this demographic structure in Section 2.2, describing how *mitigation* shapes the pattern of production, and how redistribution shapes the pattern of consumption.

2.1 Demographics and Ages

Agents can be young or old, which we denote y and o respectively. We think of the young as being below 65 and making up the vast majority of the population. For simplicity, and given the short horizon that we have in mind, we will assume no births and no aging, but some non-COVID-19 deaths of the old at constant rate γ^d.

Within each age group, agents are differentiated by health status that can take six different values: susecptible s, asmyptomatic a, miserable with flu-symptoms f, requiring emergency care e, recovered r, or dead d. The first group have no immunity and are susceptible to infection. The a, f, and e groups all carry the virus and can shed it onto others. However, they differ in terms of their symptoms. The asymptomatic have no symptoms or very mild ones, and thus spread the virus unknowingly. Those with flu-like symptoms are sufficiently sick to know they are likely contagious and they stay at home and avoid the workplace. Those requiring emergency care are hospitalized. The recovered are well, no longer contagious, and are immune from future infection. A worst case virus progression is from susceptible to asymptomatic to flu to emergency care to dead. However recovery is possible from the asymtomatic, flu and emergency-care states. We model three sources of possible virus contagion: people can catch the virus from colleagues at work, from family or friends outside work, and from taking care of the sick in hospitals.

Young agents in the model are further differentiated by the sector in which they can work. A fraction of the young work in the basic sector, denoted b, while the rest work in a luxury sector, denoted ℓ. The assumption will be that output of the basic sector is so vital that it will never make sense to ask b sector workers to not work. In contrast, it may be optimal to ask some or all of the workers in the ℓ sector to stay at home in order to reduce workplace transmission of the virus.[1] We will call this a mitigation policy, m. More precisely, m will be the fraction of luxury workers that are instructed to stay at home and not go to work. We assume that workers cannot change sectors: thus sector is a fixed individual characteristic.

We start at time t_0 so can start from any set of initial conditions and normalize so t_0+1 is one day later. Time is continuous and variables denoted with roman letters should be understood to be functions of time. Policy variables and technology parameters are denoted with greek letters. Those that depend on time are explicitly indicated to do so.

[1] We think of schools as part of the luxury sector.

We use x to denote measures with superindices denoting the subsets of the population that we refer to. At t_0, we have $x^{yb} + x^{y\ell} + x^o = 1$, where $x^{yb} = \sum_{i \in \{s,a,f,e,r\}} x^{ybi}$, $x^{y\ell} = \sum_{i \in \{s,a,f,e,r\}} x^{y\ell i}$ and $x^o = \sum_{i \in \{s,a,f,e,r\}} x^{oi}$. In the interests of more compact notation, we will also let $x^i = x^{ybi} + x^{y\ell i} + x^{oi}$ for $i \in \{s, a, f, e, r\}$ denote the total number of individuals in health state i. Finally, let $x = \sum_{i \in \{s,a,f,e,r\}} x^i = x^{yb} + x^{\ell b} + x^o$ denote the entire living population.

Health dynamics are described by the equations below. Equation (1) captures the flow of young suspectible basic sector workers into the asymptomatic state. The number of such workers who catch the virus is their mass, x^{ybs}, times the number of virus-transmitting interactions they have (the term in square brackets). The three terms in the bracket capture the three sources of infection: from co-workers, from caring for the sick, and from outside the home.

The rates of contagion in these different settings depend on how many contagious people a given suspectible basic worker can expect to meet and also on the extent to which people practice social distancing behaviors that reduce the spread of the virus. Those social distancing behaviors are captured via the β coefficients, and they are indexed by t to reflect the fact that changes in policy and in fear will lead to changes in behavior that reduce transmission rates. We allow the extent of social distancing to depend on the setting: social distancing at work is denoted $\beta_w(t)$, while social distancing outside of work is denoted $\beta_h(t)$. We assume that workers in hospitals always take maximum precautions, and that β_w^e is the associated contagion-mitigation parameter.

The risk of contracting the virus at work is assumed proportional to the fraction of coworkers who are contagious, c_w. We assume that people with symptoms always stay at home (a minimal precaution), and that basic and luxury workers mingle together at work. Thus the fraction of contagious co-workers c_w is the number of asymptomatic workers, $x^{yba} + (1 - m)x^{y\ell a}$ divided by the total number of workers. The social distancing parameter $\beta_w(t)$ defines the probability a suspectible worker will contract the virus if all his co-workers are asymptomatic.

The risk that a young basic worker contracts the virus at home depends on the share of contagious workers in the household, c_h. Note that both asymptomatic and flu-suffering workers are at home. We allow for a lower contact rate between young and old within the household via the parameter β_h^o. To the extent that people are especially careful to reduce contact with the old, $\beta_h^o < 1$

We assume that caring for those requiring emergency care is a task that falls entirely on basic workers. The risk of contracting the virus from this activity is proportional to the number of hospitalized people, x^e.

Equation (2) describes contagion for the suspectible population in the luxury sector. It differs from the basic sector only in the work terms. People in this sector work reduced hours when $m > 1$ and thus have fewer work interactions. In addition, people in this group do not take care of the very sick in hospitals.

Equation (3) is the counterpart for the old. The old do not work, and are cautious in all their interactions at home.

$$\dot{x}^{ybs} = -\left[\beta_w(t)\ c_w + \beta_h(t)\ c_h + \beta_e\ x^e\right]\ x^{ybs} \tag{1}$$

$$\dot{x}^{y\ell s} = -\left[\beta_w(t)\ c_w\ \ (1 - m(t)) + \beta_h(t)\ c_h\right]\ x^{y\ell s} \tag{2}$$

$$\dot{x}^{os} = -\beta_h(t)\beta_h^o\ \left[\frac{x^a + x^f}{x - x^e}\right]\ x^{os} \tag{3}$$

where

$$c_w = \frac{x^{yba} + (1 - m(t))x^{y\ell a}}{x^{ybs} + x^{ybr} + x^{yba} + (1 - m(t))\left[x^{y\ell s} + x^{y\ell r} + x^{y\ell a}\right]}, \tag{4}$$

$$c_h = \frac{\left[x^{yba} + x^{y\ell a} + x^{ybf} + x^{y\ell f}\right] + \beta_h^o\left[x^{oa} + x^{of}\right]}{x - x^s}. \tag{5}$$

The next set of equations describes the evolution of the population shares of asymptomatic, flu-suffering, hospitalized, and recovered individuals. All the relevant parameters of disease evolution depend on age.

Equations (6) to (8) describes the evolution of the asymptomatic. There is entry from the suspectible and there is exit to flu-suffering at rate σ^{yaf} and to recovered at rate σ^{yar}. Note that someone who recovers at this stage will never know they contracted the virus. Equations (9) to (11) are the movements of the flu-suffering. There is entry from the asymptomatic and there is exit to hospitalized at rate σ^{yfe} and to recovered at rate σ^{yfr}. Equations (12) to (14) are the movements of those in emergency care. There is entry from the flu-suffering, and there is exit to death at rate $\sigma^{yed} + \varphi$ and to recovered at rate $\sigma^{yer} - \varphi$, where φ, described below, is related to hospital overuse. Equations (15) to (17) displays the evolution of the recovered population, which features only entry.

Equation (18) shows the extent of overuse of the health care system that has a normalized size Θ. The probability of death conditional on being sick depends on the extent of hospital overuse. In particular, φ is the amount by which the death rate of the sick rises (and the recovery rate falls) once hospital capacity is exceeded.

$$\dot{x}^{yba} = -\dot{x}^{ybn} - \left(\sigma^{yaf} + \sigma^{yar}\right) x^{yba} \tag{6}$$

$$\dot{x}^{y\ell a} = -\dot{x}^{y\ell n} - \left(\sigma^{yaf} + \sigma^{yar}\right) x^{y\ell a} \tag{7}$$

$$\dot{x}^{oa} = -\dot{x}^{on} - \left(\sigma^{oaf} + \sigma^{oar} + \gamma^{d}\right) x^{oa} \tag{8}$$

$$\dot{x}^{ybf} = \sigma^{yaf} x^{yba} - \left(\sigma^{yfe} + \sigma^{yfr}\right) x^{ybf} \tag{9}$$

$$\dot{x}^{y\ell f} = \sigma^{yaf} x^{y\ell a} - \left(\sigma^{yfe} + \sigma^{yfr}\right) x^{y\ell f} \tag{10}$$

$$\dot{x}^{of} = \sigma^{oaf} x^{oa} - \left(\sigma^{ofe} + \sigma^{ofr} + \gamma^{d}\right) x^{of} \tag{11}$$

$$\dot{x}^{ybe} = \sigma^{yfe} x^{ybf} - \left(\sigma^{yed} + \sigma^{yer}\right) x^{ybe} \tag{12}$$

$$\dot{x}^{y\ell e} = \sigma^{yfe} x^{y\ell f} - \left(\sigma^{yed} + \sigma^{yer}\right) x^{y\ell e} \tag{13}$$

$$\dot{x}^{oe} = \sigma^{ofe} x^{of} - \left(\sigma^{oed} + \sigma^{oer} + \gamma^{d}\right) x^{oe} \tag{14}$$

$$\dot{x}^{ybr} = \sigma^{yar} x^{yba} + \sigma^{yfr} x^{ybf} + (\sigma^{yer} - \varphi) x^{ybe} \tag{15}$$

$$\dot{x}^{y\ell r} = \sigma^{yar} x^{y\ell a} + \sigma^{yfr} x^{y\ell f} + (\sigma^{yer} - \varphi) x^{y\ell e} \tag{16}$$

$$\dot{x}^{or} = \sigma^{oar} x^{oa} + \sigma^{ofr} x^{of} + (\sigma^{oer} - \varphi) x^{oe} \tag{17}$$

$$\varphi = \lambda_o \max\{x^{e} - \Theta, 0\}. \tag{18}$$

We can of course keep track of the dead, noting that $\dot{x}^{ybd} = (\sigma^{yed} + \varphi)x^{ybe}$, $\dot{x}^{y\ell d} = (\sigma^{yed} + \varphi)x^{y\ell e}$ and $\dot{x}^{od} = (\sigma^{oed} + \varphi)x^{oe} + \gamma^{d} x^{o}$. We write them separately given that they are an absorbing state with no further role in the model.

2.2 Economic Block

We start describing how production takes place given a mitigation policy (Section 2.2.1) and then move to describe the preferences of agents that take into account longevity and the utility of being both health and alive (Section 2.2.2).

2.2.1 Activity: Technology and Mitigation

There are two sectors that we label basic and luxury. Every worker is attached to one of them. The difference between the sectors is that the luxury goods sector's output depends on the mitigation policy in such a way that more mitigation (a higher m) reduces the amount of output from the luxury sector that can be produced. Output in the basic sector is

$$y^b = x^{ybs} + x^{yba} + x^{ybr}. \tag{19}$$

Here we are assuming no mitigation of those without symptoms in the basic sector.[2]

Output in the luxury sector does depend on the mitigation policy and is given by

$$y^\ell = [1 - m(t)] \left(x^{y\ell s} + x^{y\ell a} + x^{y\ell r}\right). \tag{20}$$

We assume that both sectors are perfect substitutes. This is, total available output using the basic good as numerator is

$$y = y^b + y^\ell. \tag{21}$$

We also pose that a fixed amount of output, Θ, is spent on health care.

2.2.2 Preferences

The old care about consumption and being alive:

$$E\left\{\int e^{-\rho t} \left[u^o(c_t^o) + \bar{u} + \widehat{u}_t^j\right] dt\right\} \tag{22}$$

where the expectation is taken over the being alive, where $\bar{u}$ is the value of being alive, and where $\widehat{u}_t^j$ is the intrisic utility of being in state health j (here having flu is bad, and being in hospital is very bad).

[2]One could imagine a policy of tracing contacts of infected people, which would allow the planner to keep some portion of exposed workers at home.

The young also care about consumption and being alive, perhaps differently,

$$E\left\{\int e^{-\rho t}\ \left[u^{y}\left(c_{t}^{y}\right)+\bar{u}+\widehat{u}_{t}^{j}\right]\ dt\right\}, \tag{23}$$

Note that workers who are in the flu or hospitalized states do not work, and neither do a random fraction m of workers in the luxury sector whose workplaces have been shut down. In equilibrium young workers will experience difference consumption values depending on whether they work or not. Thus, expected utility will depend on sector both because sector affects the likelihood of being shut-down, and also because sector will affect the distribution of health outcomes.

3 The Public Sector

We start describing the government policy tools in Section 3.1, we then show how public transfers are determined statically to yield a social welfare function. Moroever we characterize the utilitarian welfare function in Section 3.2 and we end by posing the Ramsey problem (with discounting) which maximizes the integral over time of social welfare.

3.1 Transfers

The public sector is responsible for two choices: mitigation (shutdowns) $m(t)$ and redistribution to individuals that currently do not or cannot work. We assume that the degree of social interaction within the workplace and outside the workplace (what we called the β's) are determined exogenously outside the model. Or, alternatively, that least costly measures of social separation are already in place. What the government chooses is the extent to which it imposes a shutdown of economic activity, via $m(t)$, and how much to transfer to those hurt by shutdowns, those that have fallen sick, and those that have retired. In each instant individuals either work (those healthy enough, not subject to mitigation and not old and thus retired) or do not work. We assume that all workers share a common consumption level c^{w} and all individuals not working share a common consumption level c^{n}; this would also be the efficient allocation implemented by a government that values all individuals equally (same Pareto weights) and can transfer resources costlessly within both groups. The second policy choice beyond mitigation is redistribution between working and non-working individuals, and we assume that this redistribution is costly. Specifically, denote by $T(c^n)$ the per-capita cost of transfering consumption c^n. We assume that $T(.)$ is increasing and differentiable.

To simplify notation, denote by $(\mu^n(m,x), \mu^w(m,x))$ the mass of non-working and working people, respectively, as a function[3] of the health population distribution x and current mitigation $m = m(t)$. These are defined as

$$\begin{aligned}
\mu^n(m,x) &= x^{y\ell f} + x^{y\ell e} + x^{ybf} + x^{ybe} + m\left(x^{y\ell s} + x^{y\ell a} + x^{y\ell r}\right) && (24)\\
&+ x^{os} + x^{oa} + x^{of} + x^{oe} + x^{or} && (25)\\
\mu^w(m,x) &= x^{ybs} + x^{yba} + x^{ybr} + [1-m]\left(x^{y\ell s} + x^{y\ell a} + x^{y\ell r}\right) && (26)\\
\nu^w(m,x) &= \frac{\mu^w(m,x)}{\mu^w(m,x) + \mu^n(m,x)} && (27)
\end{aligned}$$

where $\nu^w(m,x)$ is the share of working individuals in the population. The aggregate resource constraint can then be written as

$$\mu^w c^w + \mu^n c^n + \mu^n T(c^n) = y - \Theta = \mu^w - \Theta \tag{28}$$

where we have exploited that $y = \mu^w$ since each productive workers produces one unit of output (and outputs in both sectors are perfect substitutes).

Notice that there are no dynamic consequences of the transfer choice c^n. In particular, this choice has no impact on any health transitions. We can therefore solve a static optimal transfer problem at each t, that delivers a maximum period utility value which we denote $W(m,x)$. We turn to derive this expression.

3.2 The Instantaneous Social Welfare Function

We now derive the instantaneous social welfare function $W(x,m)$, a necessary ingredient into the optimal mitigation problem of the government. The function $W(x,m)$, assuming that all individuals have log-utility and the same welfare weights, is given by

$$W(x,m) = \max_{c^n, c^w} \left[\mu^w \log(c^w) + \mu^n \log(c^n)\right] \tag{29}$$

[3] We will suppress the dependence on these variables when there is no room for confusion.

subject to the aggregate resource constraint (28). Combining the first order conditions with respect to (c^n, c^w) yields

$$\frac{c^w}{c^n} = 1 + T'(c^n). \tag{30}$$

Thus the optimal solution to the government transfer problem is given by the solution to the following system (where we have expressed the resource contraint in per-capita terms by dividing by $\mu^w + \mu^n$

$$\begin{aligned} c^w &= c^n(1 + T'(c^n)) & (31) \\ \nu c^w + (1-\nu)(c^n + T(c^n)) &= \nu - \frac{\Theta}{\mu^w + \mu^n} := \tilde{y} & (32) \end{aligned}$$

where $\tilde{y}$ is the net output per capita. Note that the share of workers in the population $\nu = \nu(x, m)$ is strictly decreasing in the extent of mitigation m. Since the size of the population is indepent of mitigation m, so is the last term $\frac{\Theta}{\mu^w+\mu^n}$, which will be important from the comparative statics. Solving these two equations yields optimal consumptions and period welfare. Assume that the transfer cost function is given by $T(c^n) = \frac{\tau}{2}(c^n)^2$. Then

$$\begin{aligned} c^n &= \frac{(1 + 2\tau(1+\nu)\tilde{y})^{0.5} - 1}{\tau(1+\nu)} & (33) \\ c^w &= c^n(1 + T'(c^n)) & (34) \\ W(x, m) &= \mu^w \log(c^w) + \mu^n \log(c^n) + (\mu^w + \mu^n)\bar{u} + \sum_{i,j} x^{i,j}\widehat{u}^j & (35) \end{aligned}$$

It is instructive to compare these results with linear transfer costs $T(c^n) = \tau c^n$. In that case

$$\begin{aligned} \frac{c^w}{c^n} &= 1 + \tau. \\ \nu c^w + (1-\nu)c^n(1+\tau) &= \tilde{y} \end{aligned}$$

and thus

$$\begin{aligned} c^w &= \tilde{y} \\ c^n &= \frac{\tilde{y}}{1+\tau} \end{aligned}$$

and thus there is no impact of mitigation m on the relative consumption of both groups, just on net per capita income $\tilde{y}$. Since we want to allow for this interaction, we require a transfer cost that is strictly convex. The quadratic form is chosen for anallytical convenience, but not central for our qualitative arguments.

3.3 Optimal Policy

We now assume there is government/planner (we use these names as synonymous as there is no time consistency problem) that chooses optimal policy over time by choosing mitigation $m(t)$ (the choice of redistribution $T(t)$ is embodied in the social welfare functino $W(x)$. The actual problem to solve is

$$\max_{m(t)} \int_0^\infty e^{-\rho t} W(x)\ dt. \tag{36}$$

subject to the laws of motion of the population Equation (1) to Equation (18).

We could approximate for instance the optimal mitigation function by having one with the following generalized logistic function of time:

$$m(y) = \frac{\alpha_0}{1 + \exp(-\alpha_1(t - \alpha_2))}$$

Here the parameter α_0 would control the level of mitigation at $t = 0$. The parameter α_2 would control when mitigation is reduced, and the parameter α_1 commands how swiftly mitigation is reduced. Note that as $t \to \infty$, $m(t) \to 0$.

In general though the proper characterization of the solution derives from a formal maximization process.

3.3.1 The Hamiltonian

We have derived the period return function $W(\mathbf{x}, m)$. In addition, the evolution of the state (the distribution of the population by health status $\boldsymbol{x} = (x^{i,j})$) evolves according to the vector-valued equation (summarizing Equations (1) to (17) the paper in a compact form):

$$\dot{\boldsymbol{x}} = G(\boldsymbol{x}, m) \tag{37}$$

To solve for the optimal time path of the scalar mitigation variable is then a starightforward optimal control problem with a mutli-dimensional state vector and a one-dimensional control variable. Define the current value Hamiltonian as

$$\mathcal{H}(\boldsymbol{x}, m, \boldsymbol{\mu}) = W(\boldsymbol{x}, m) + \boldsymbol{\mu} G(\boldsymbol{x}, m) \tag{38}$$

where $\boldsymbol{\mu}$ is the vector of co-state variables associated with the population state vector $\boldsymbol{x}$. Necessary conditions at an interior solution for mitigation m are the optimality condition for m

$$\begin{aligned} W_m(\boldsymbol{x}, m) &= -\boldsymbol{\mu} \cdot G_m(\boldsymbol{x}, m) && (39) \\ \dot{\boldsymbol{\mu}} &= \rho\boldsymbol{\mu} - [W_{\boldsymbol{x}}(\boldsymbol{x}, m) + \boldsymbol{\mu} \cdot G_{\boldsymbol{x}}(\boldsymbol{x}, m)] && (40) \\ \dot{\boldsymbol{x}} &= G(\boldsymbol{x}, m) && (41) \end{aligned}$$

The key tradeoffs with mitigation efforts m are encoded in equation (39). A marginal increase in m entails static economic costs of $W_m(\boldsymbol{x}, m)$ stemming from the loss of output and thus consumption of all individuals in the economy, as encoded in $y^n(m)$. The dynamic benefit is a better change in the population health distribution, as encoded in the vector $G_m(\boldsymbol{x}, m)$. Concretely, as is clear from equations (1 – 3) an increase in m reduces the outflow of indviduals from the suspectible to the asymptomatic state. The value (in units of the objective function) are given by the costate vector $\boldsymbol{\mu}$.

It should be kept in mind that since $(\boldsymbol{x}, \boldsymbol{\mu})$ are vectors, so are the entities $G_m(\boldsymbol{x}, m) = (G_m^{i,j}(\boldsymbol{x}, m))$ and $W_{\boldsymbol{x}}(\boldsymbol{x}, m) = (W_{x^{i,j}}(\boldsymbol{x}, m))$ and $G_{\boldsymbol{x}}(\boldsymbol{x}, m) = (G^k_{x^{i,j}}(\boldsymbol{x}, m))$ so that equation (39) reads explicitly

$$W_m(\boldsymbol{x}, m) = -\sum_{i,j} \mu^{i,j} G_m^{i,j}(\boldsymbol{x}, m) \tag{42}$$

and a specifc row of the vector-valued Section 3.3.1 is given by

$$\dot{\mu}^{i,j} = \rho\mu^{i,j} - \left[W_{\,\boldsymbol{x}^{i,j}}(\boldsymbol{x}, m) + \sum_k \mu^k G^k_{x^{i,j}}(\boldsymbol{x}, m) \right]. \tag{43}$$

4 Calibration

There is a long list of parameters to specify, most of them epidemiologic, and we start with them. We set $\gamma^d = 1/3,650$, implying a remaining life expectancy of 10 years for the old. This is because life expetancy at 65 may be 18, and we take as an approximation that the average

age of the population over 65 is 72.

There are 12 σ parameters to calibrate, describing transition rates for disease progression, 6 for each age. These describe the chance of moving to the next worse health status and the chance of recovery at the three infectious stages: asymptomatic, flu-suffering, and hospitalized. We assume that young and old exit each stage at the same rate, but potentially differ in terms of the share of these exits that are into recovery. In particular, the old will be much more likely to require hospital care conditional on developing flu, and slightly more likely to die conditional on being hospitalized.

Putting aside these differences by age for a moment, the six σ's are identified from the following six target moments: the average duration of time individuals spend in the asymptomatic (contagious but asymptomatic), flu-suffering (relatively mild symptoms), and emergency-care states, and the relative chance of recovery (relative to disease progression) in each of the three states. Following the literature on COVID-19 models we set the three durations to 5.2, 10, and 8 days, with these durations common across age groups. The exit rate to recovery from the asymptomatic state defines the number of asymptomatic cases of COVID-19 and is an important but highly uncertain parameter. We assume that asymptomatic recovery and progression to the flu-suffering state are equally likely.

We let the relative recovery rates from the flu-suffering and emergency care states vary with age, to reflect the fact that infections in older individuals are more much likely to require hospitalization, and hospitalizations are also more likely to lead to death. We set the recovery rate from flu-suffering to 96% for the young, and to 75% for the old, based on evidence from Table 1 of the Imperial College study. Similarly, given evidence on differential mortality rates, we set the recovery rates from the emergency care state to 95% for the young and to 80% for the old (assuming no hospital overuse).

Given the σ parameters, the β parameters determine the rate at which contagion grows over time. We set $\beta_e = 0.01$, implying that a very small share of overall transmission occurs in hospitals. The values of β_w and β_h determine the overall basic reproduction number R_0 value for COVID-19 and the share of disease transmission that occurs at work versus in non-work settings. At the time of writing, with fairly severe social distancing measures in place, R_0 appears to be around 1.1. In addition, the Imperial College study suggests that roughly 1/3 of transmission happens in workplaces and schools, with the rest in the home and social settings.

We use these targets to pin down choices for β_w and β_h as follows.

The basic reproduction number R_0 is the number of people infected by a single asymptomatic person. For a single young person, assuming everyone else in the economy is suspectible (a close approximation to the initial condition we will use) and zero mitigation ($m = 0$), R_0 is given by

$$R_0^y = \frac{\beta_w + \beta_h}{\sigma^{yar} + \sigma^{yaf}} + \frac{\sigma^{yaf}}{\sigma^{yaf} + \sigma^{yar}} \frac{\beta_h}{\sigma^{yfr} + \sigma^{yfe}} + \frac{\sigma^{yaf}}{\sigma^{yaf} + \sigma^{yar}} \frac{\sigma^{yfs}}{\sigma^{yfe} + \sigma^{yfr}} \frac{\beta_e}{\sigma^{yer} + \sigma^{yed}}$$

The logic is that this individual will spread the virus while asymptomatic, flu-suffering, and sick – the three terms in the expression. They expect to be asymptomatic for $(\sigma^{yar} + \sigma^{yaf})^{-1}$ days, flu-suffering (conditional on reaching that state) for $(\sigma^{yfr} + \sigma^{yfe})^{-1}$ days, and hospitalized (conditional on reaching that state) for $(\sigma^{yer} + \sigma^{yed})^{-1}$ days. The chance they reach the flu-suffering state is $\frac{\sigma^{yaf}}{\sigma^{yaf}+\sigma^{yar}}$ and the chance they reach the emergency room is the product $\frac{\sigma^{yaf}}{\sigma^{yaf}+\sigma^{yar}} \frac{\sigma^{yfs}}{\sigma^{yfe}+\sigma^{yfr}}$. While asymptomatic, they spread the virus at both work and at home, and pass the virus on to $\beta_w + \beta_h$ suspectible individuals per day. While flu-suffering, they stay at home and pass the virus to β_h people per day. While sick they pass it to β_e people per day in hospital.

The reproduction number for an old asymptomatic person is

$$\begin{aligned} R_0^o \quad = \quad & \frac{\beta_h}{\gamma^d + \sigma^{oar} + \sigma^{oaf}} + \frac{\sigma^{oaf}}{\gamma^d + \sigma^{oaf} + \sigma^{oar}} \frac{\beta_h}{\gamma^d + \sigma^{ofr} + \sigma^{ofe}} \\ & + \frac{\sigma^{oaf}}{\gamma^d + \sigma^{oaf} + \sigma^{oar}} \frac{\sigma^{ofs}}{\gamma^d + \sigma^{ofe} + \sigma^{ofr}} \frac{\beta_e}{\gamma^d + \sigma^{oer} + \sigma^{oed}} \end{aligned}$$

where this formula is similar to the one for the young, except that it recognizes the old pass the virus on less because they do not work and they die of natural causes, which reduces the time they are around to spread the virus. At the same time, however, because the old are less likely to recover once infected, they potentially carry the virus for a longer time, inducing more transmission in hospitals.

For the population as a whole, the overall R_0 is a weighted average of these two group-specific values

$$R_0 = \mu_y R_0^y + (1 - \mu_y) R_0^o$$

where μ^y is the fraction of the population that is young.

The share of total transmission that occurs in the workplace from a randomly drawn newly asymptomatic individual is then given by

$$\frac{\text{workplace transmission}}{\text{all transmission}} = \frac{\mu^y \frac{\beta_w}{\sigma^{yar} + \sigma^{yaf}}}{R_0}$$

Given these two equations, we set β_w and β_h to hit our targets for R_0 and this share, implying ...

To set the value of life $\bar{u}$ we follow the value of a statistical life VSL approach. The Environmental Protection Agency and the Department of Transportation assume a value of \$11.5 million for an 18 year old (see Greenstone and Nigam). Assuming 64 residual years discounted at a 3 percent rate, this translates to an annual flow value of $\$406,000$, which is 9.03 times yearly per capita consumption in the United States.

To translate this into a value for $\bar{u}$ we use the standard value of a statistic life calculation,

$$VSL = \frac{dc}{dr}|_{E[u]=k} = \frac{\ln(\bar{c}) + \bar{u}}{\frac{1-r}{\bar{c}}}$$

where $\bar{c}$ is average per capita model consumption, and r is the risk of death. Setting $VSL = 9.03\bar{c}$ and $r = 0$ gives $\bar{u} = 9.03 - \ln \bar{c}$. Note that this implies an easily interpretable tradeoff between mortality risk and consumption. For example, we can ask what reduction in consumption leads an invidual indifferent to facing a 1 percent risk of death. The answer is the solution m to

$$\ln(\bar{c}\,(1 - m)) + 9.03 - \ln \bar{c} = 0.99\,(\ln(\bar{c}) + 9.03 - \ln \bar{c})$$

which is $m = 0.086$.

For the disutility of having flu or being hospitalized, we define $\hat{u}^f = \hat{u}^e$ as

$$\hat{u}^e = -0.3\,(\ln(\bar{c}) + \bar{u})$$

following Hong, Pijoan-Mas, and Rios-Rull (2018).

The remaining parameters have to do with the economic side of the model. We set the discount rate ρ equal to 3 percent per year. We assume that the basic sector is 30 percent of the economy, corresponding to the size of the agriculture, groceries, utilities, financial services and healthcare. We set the cost of transfers τ to 10 percent. [MORE DETAILS][4]

For the time path of mitigation, our baseline simulation will assume $m = 0.5$ at $t = 0$, and that m declines to zero around $t = 100$. This is implemented given $\alpha_0 = 0.5$, $\alpha_1 = -0.3$, and $\alpha_2 = 100$.

Figures from these parameter values

[4]Normally the values estimated are much larger. See for instance Barrios et al. (2013) for labor taxes in the European Union who estimate a value between 30% and 140%.

Table 1: Epidemiological Parameter Values

		Behavior-Contagion (With Social Distancing)	
β_w	Interaction at Work	Share Infected at Work, No SD, Imperial	0.20
β_h	Interaction At Home	Share Infected Home or Community, SD	0.09
β_e	Interaction While Attending Ill People	Unsure	0.01
β_h^o	Lower Interaction Rate for the Old	Relative Share Infected Home or Community	1.0
		Evolution of the Disease	
σ^{yaf}	Rate for young asymptomatic into flu	50%, 5.2 days	$\frac{0.5}{5.2}$
σ^{yar}	Rate for young asymptomatic into recovered		$\frac{0.5}{5.2}$
σ^{oaf}	Rate for old asymptomatic into flu	50% 5.2 days	$\frac{0.5}{5.2}$
σ^{oar}	Rate for old asymptomatic into recovered		$\frac{0.5}{5.2}$
σ^{yfe}	Rate for young flu into emergency	4% Hospitalization	$\frac{0.04}{10}$
σ^{yfr}	Rate for young flu into recovered		$\frac{0.96}{10}$
σ^{ofe}	Rate for old flu into emergency	25% Hospitalization	$\frac{0.25}{10}$
σ^{ofr}	Rate for old flu into recovered		$\frac{0.75}{10}$
σ^{yed}	Rate for young emergency into dead	0.15% Mortality	$\frac{0.05}{8}$
σ^{yer}	Rate for young emergency into recovered		$\frac{0.95}{8}$
σ^{oed}	Rate for old emergency into dead	5% Mortality	$\frac{0.20}{8}$
σ^{oer}	Rate for old emergency into recovered		$\frac{0.80}{8}$
γ^d	Normal Mortality of the Old	Life Expectancy 10 years	$\frac{1}{3650}$

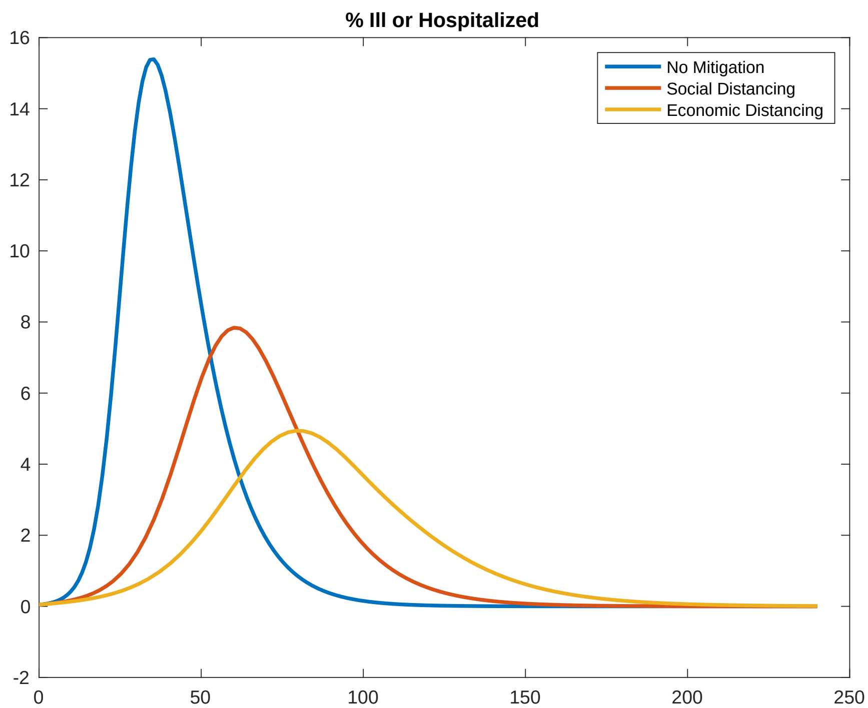
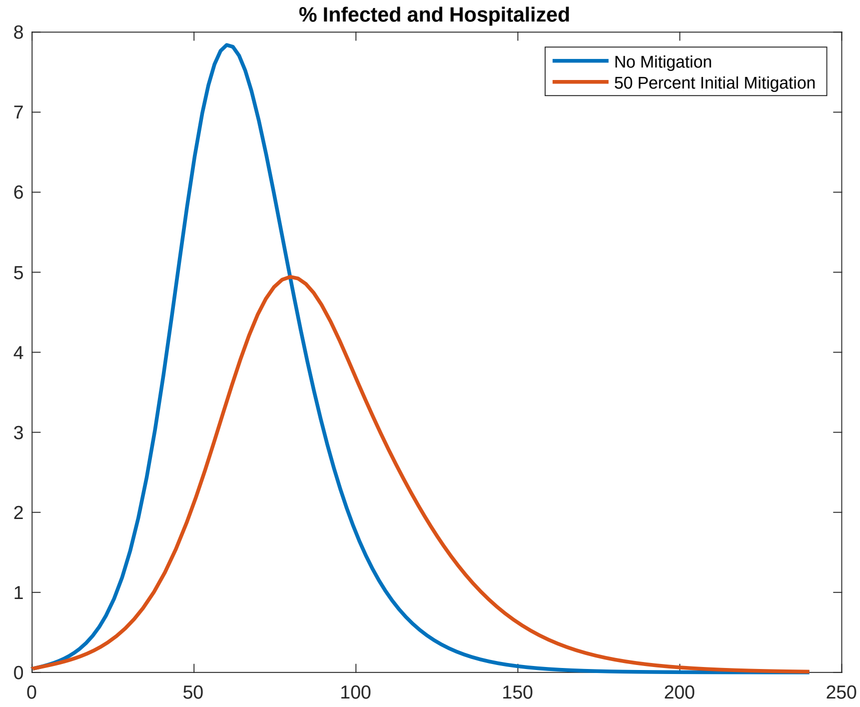


Figure 1: Number of Ill Hospitalized

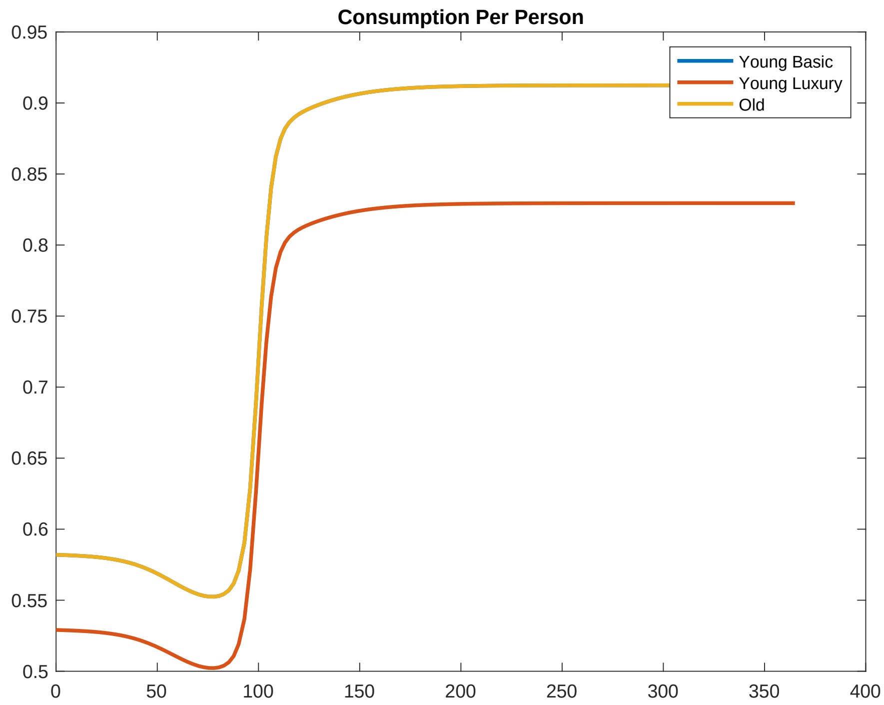
Table 2: Economic Parameters

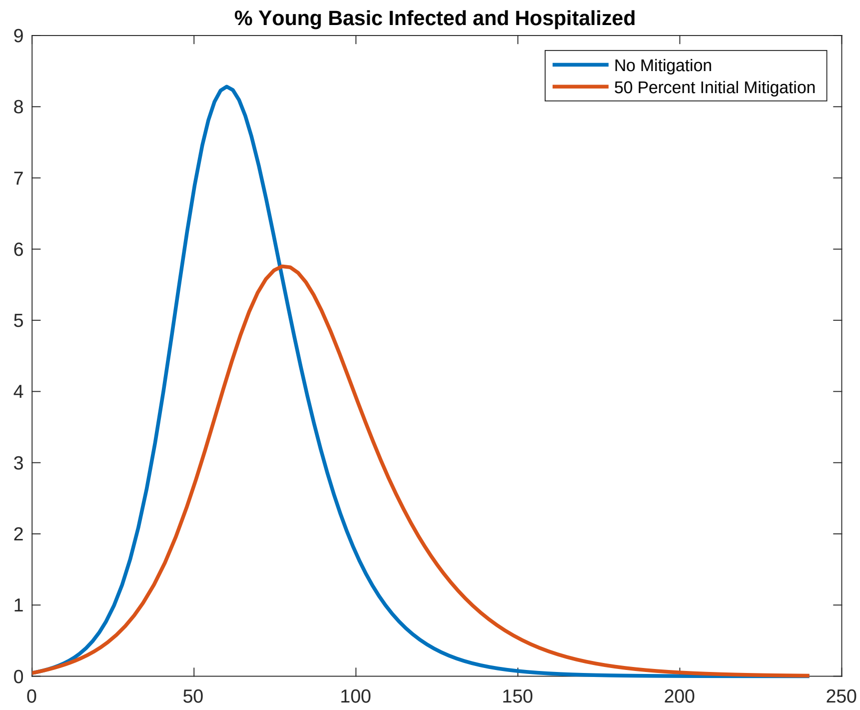
ρ	Discount Rate	3% per year	
$\bar{u}$	Value of life	3 Times Average Income	
$\widehat{u}^i$	Disutility of being infected	Times Average Income	
$\widehat{u}^s$	Disutility of being sick	Times Average Income	
$\frac{x^{yb\cdot}}{x^{yb\cdot}+x^{y\ell\cdot}}$	Relative Size of the Basic Sector	30%	.3
τ	Cost of transfers		0.1
χ	Relative value of luxuries		1
α_0	Eventual share mitigated		0.5
α_1	Speed of mitigation		-0.3
α_2	Time mitigation begins		100

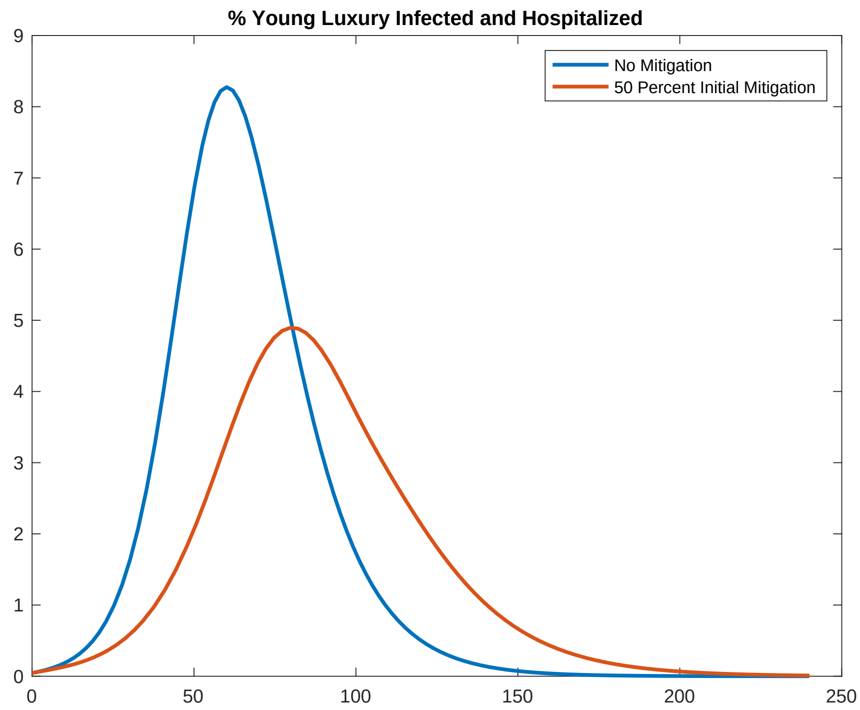
5 Findings

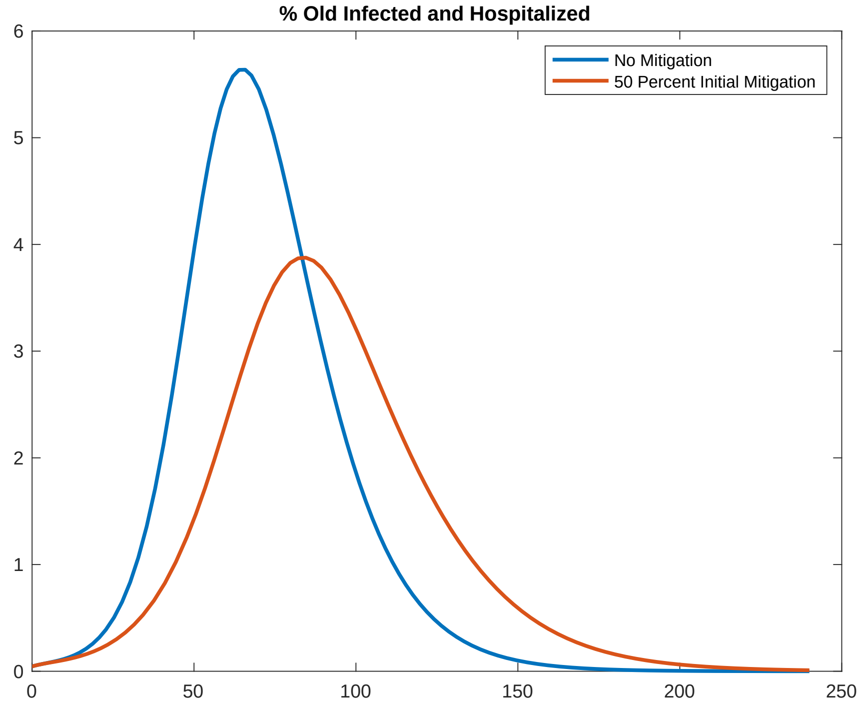


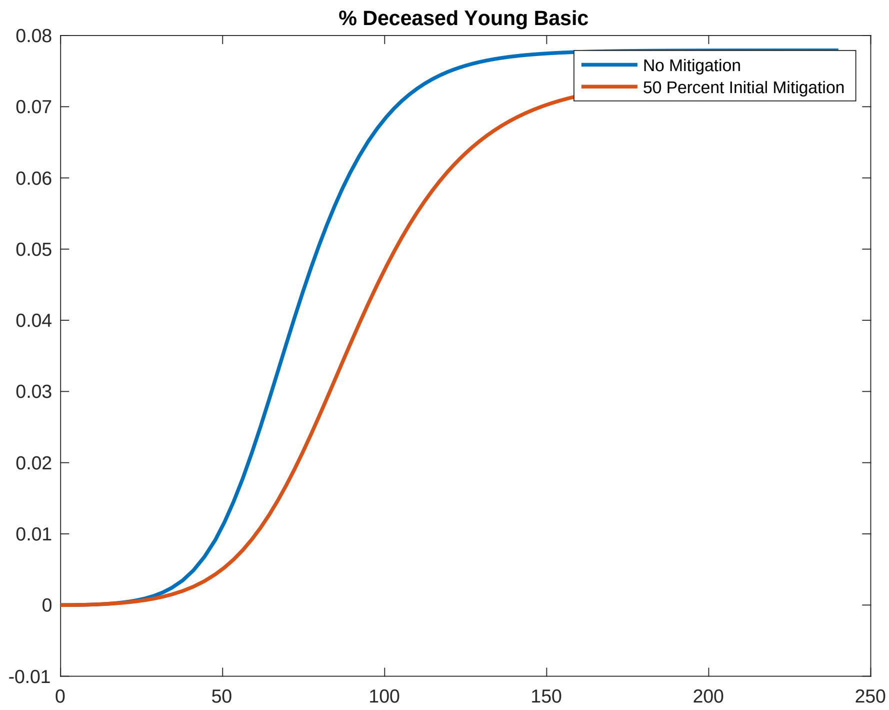
% Infected and Hospitalized
No Mitigation
50 Percent Initial Mitigation
0
1
2
3
4
5
6
7
8
0
50
100
150
200
250

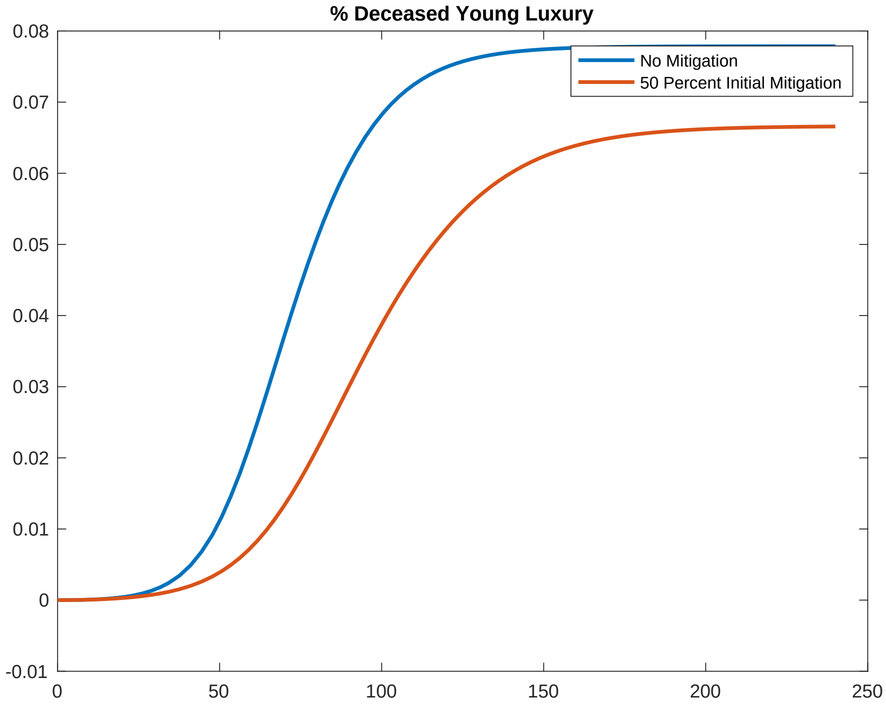

Consumption Per Person
Young Basic
Young Luxury
Old
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
50
100
150
200
250
300
350
400

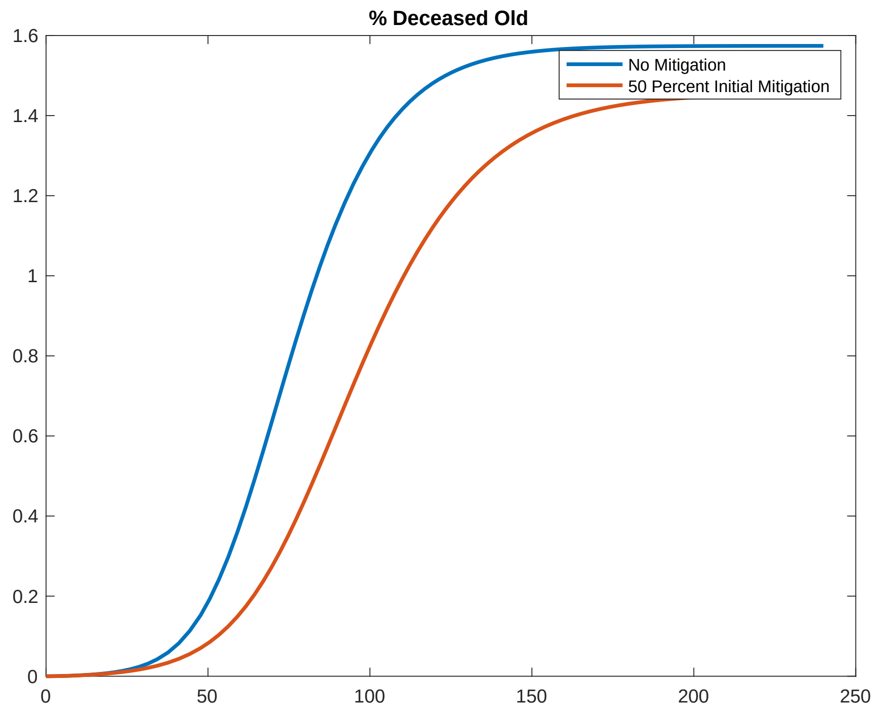

% Young Basic Infected and Hospitalized
No Mitigation
50 Percent Initial Mitigation
0
1
2
3
4
5
6
7
8
9
0
50
100
150
200
250


% Young Luxury Infected and Hospitalized
No Mitigation
50 Percent Initial Mitigation
0
1
2
3
4
5
6
7
8
9
0
50
100
150
200
250


% Old Infected and Hospitalized
No Mitigation
50 Percent Initial Mitigation
0
1
2
3
4
5
6
0
50
100
150
200
250


% Deceased Young Basic
No Mitigation
50 Percent Initial Mitigation
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
0
50
100
150
200
250


% Deceased Young Luxury
No Mitigation
50 Percent Initial Mitigation
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
0
50
100
150
200
250


% Deceased Old
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250
No Mitigation
50 Percent Initial Mitigation

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