Labor Share and Productivity Dynamics

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Abstract

We present a novel way to model technological shocks, with the feature that it can be biased towards more recently installed production units. We show that at one extreme, the shock is like a neutral technological shock, while at the other end of the spectrum, it resembles the spirit of investment specific technological shocks, since it affects newly created machines the most. To make these ideas operational, we embed our proposed shocks in a model with putty-clay technology (where the notion of new and old firms is clear) and estimate the shock process using salient cyclical features of U.S. labor markets and its labor share. Our estimates favor investment specific technological-type shocks as the main driver of business cycles. In addition, it provides a feasible explanation of the cyclical behavior of the labor share.

Keywords: Factor Shares, Technology shocks, Putty-Clay, Business Cycles
JEL Codes: **E01, E13, E25, E32**

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1 Introduction

In this paper we present a new way of modelling aggregate productivity shocks, where we introduce a "time" bias with respect to installed capital: shocks may affect newer firms in a stronger way than older firms. On the one end of the spectrum, when shocks do not exhibit time bias, these shocks are like standard TFP shocks. If the time bias is extreme (only new machines are affected by it), the shocks point to behavior akin to that of investment specific technological shocks.[1]

To make this notion operational we embed the technology shocks into a model with putty-clay technology where new and old firms have a sharp distinction, as in Johansen (1959), Atkeson and Kehoe (1999), Gilchrist and Williams (2000), Wei (2003) and Gourio (2011). Under this technology assumption, individual productive units consisting of one worker each are indexed by their installation dates and differ in the "quality" (or size of its capital). The pre-installation size menu available to firms is Cobb-Douglas (i.e., 'putty'), but once installed, plants remain in place without the possibility of changing its quality (i.e., 'clay'). This assumption also creates a Leontief productive structure in the short-run between capital and labor: workers must be attached to machines in order for the firm to reap the benefits of positive productivity innovations. This technological assumption calls for natural additional frictions: vacant machines have to wait one period to become operational (matched with a non-employed worker) and unit specific wages are not equalized (they are determined through bilateral Nash bargaining).

We then proceed to estimate our model using the method of simulated moments. We use a variety of economy responses to productivity innovations as moment targets, mainly employment and labor share data. We use these because they are at the core of the concerns of economists and because they exhibit behavior which has proved hard to explain. We also make sure that the model replicates the cyclical behavior of the Solow residual and measures of labor market fluctuations. We find that the parameter that controls the degree of time bias has a first order effect on the predictions of the model regarding the cyclicality of investment and factor shares. The answer that we find is sharp: the data leans very heavily towards significant time biased shocks.

In addition to making a sharp distinction between the alternative type of shocks, our estimated model

[1] Investment specific technology shocks have gained prominence as a source of aggregate fluctuations. The relative importance of these shocks has been studied extensively, using diverse methods. See for example Fisher (2006), Fernandez-Villaverde and Rubio-Ramirez (2007), Justiniano, Primiceri, and Tambalotti (2010),Justiniano, Primiceri, and Tambalotti (2011), Bai, Ríos-Rull, and Storesletten (2011), Schmitt-Grohe and Uribe (2011) and the survey in by Ramey (2016).

is able to reproduce the overshooting property of the labor share, as described in Choi and Ríos-Rull (2009) and Ríos-Rull and Santaeulalia-Llopis (2010) for the U.S. economy: After a positive technological shock, the share of output that corresponds to labor falls temporarily but it quickly rises, staying persistently above average for around thirty quarters. As noted by Choi and Ríos-Rull (2009), these dynamics are a challenge for a wide array of standard models of the business cycle. The *overshooting* of the labor share uncovers positive responses of *both* aggregate real wages and employment during an upturn, thus providing an alternative way of disciplining how we model the mechanisms through which productivity innovations affect labor market volatility. Our modeling choices make new workers more productive than the average labor force member but also hard to substitute with capital in the short run, producing the desired joint dynamics between productivity, real wages and (un)employment.

This paper is related to a strand of the labor-macro literature studying amplification mechanisms from productivity shocks to labor market variables in models with search frictions. In this vein we find Shimer (2005), Hall (2005), Blanchard and Galí (2007), Costain and Reiter (2008) and Hall (2009), among others. The main insight from this literature is clear: matching and the terms of bargaining in real wages matter for labor market outcomes during the cycle.[2] The incentives for firms to post wages during favorable aggregate conditions depend on how much profits they can get from matched employees, or in model terms, in the wedge between labor productivity and real wages. Hence, (more) rigid wages give more incentives for employment creation by increasing the dynamic difference between labor productivity and real wages. We are complementary to this literature, by pinpointing the type of technology representation that is most supported by the data, and showing how labor market aggregates react to aggregate innovation shocks in our related setup.

The paper is also related to a literature that has studied the cyclical properties of the labor share in the U.S. Shao and Silos (2014) pose a model with frictional labor markets and costly entry of firms and highlight the low correlation of real interest rates and output. León-Ledesma and Satchi (2017) also generate the overshooting property of the labor share in a model where there are adjustment costs to technology choices that generate an elasticity of substitution between capital and labor that is lower in the short run than in the long run.

The structure of the rest of the paper is as follows: In section 2 we present our model and its variants. Section 3 discusses our calibration strategy while section 4 presents the numerical results of

[2] See for example Mortensen (1992), Caballero and Hammour (1996) and Rogerson and Shimer (2010).

our simulations. Section 5 concludes.

2 The Model

The model is a growth model in discrete time with infinitely lived agents. Production of the unique good of this economy takes place in independent plants that have one worker each and that may differ in the amount of capital installed. We denote this amount as k and we think of it as the "size" or "quality" of the plant. We assume that workers provide labor inelastically, thus k also represent capital/labor ratios at the plant level. A plant takes one period to become operational and once built, its capital cannot be changed and its scrap value is zero. Productive units face an exogenous break-down probability of δ.

Firms can find workers after paying a hiring cost c^v that can be thought of as due to vacancy posting and training. This cost partly replicates the hiring frictions in a standard search and matching framework (except for the increased difficulties of finding a worker in expansions). Moreover, given the required "incubation" period for plants, aggregate employment in the economy moves with some lag, as if there were matching frictions (because the capital that the worker will use has to be installed), adding a mechanism to stall the fast expansion of employment right after a positive shock in aggregate conditions. Once a worker is hired her wage is set by bilateral Nash bargaining between the firm (owner of the plant) and (the family of) the worker.

A plant of size k produces an amount $e^z k^\alpha$ of goods per period, where z is an aggregate productivity shock. Its evolution is given by

$$z = \begin{cases} \widetilde{z} & \text{if new machine,} \\ \lambda\widetilde{z} & \text{otherwise,} \end{cases} \tag{1}$$

with

$$\widetilde{z}' = \rho\widetilde{z} + \epsilon', \tag{2}$$

where $\rho \in (0,1)$ and $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. *New machine* denotes productive units which were installed one period ago and will start production in the current period. Note that when $\lambda = 1$, the shock reverts to a classic factor neutral aggregate shock, which in our case, it could be understood also as machine-cohort-neutrality. On the other extreme, if $\lambda = 0$, the shock affects only the newest productive units installed which makes it look similar to an investment specific technical shock (in Appendix C we provide a discussion of how the two shocks are related).[3]

[3]We thank Joachin Hubmer, who pointed inaccuracies in our comparison to investment specific technological shocks in

The aggregate state of the economy S, is summarized by the aggregate shock $\widetilde{z}$ and $X(k,t)$, the measure of installed plants of capital amount k, and age or type $t \in \{0,1\}$, one if it is a new plant and zero otherwise.

2.1 Households

The economy is populated by a unit measure of households (families), each of them consisting of a mass of identical consumers/workers (normalized to one). Each household has preferences over consumption and leisure streams of their members. Every household member is part of the labor force, but employment is limited to the number of machines operating in the economy. We assume perfect capital markets, so there is no distinction in terms of consumption between employed or unemployed household members.

We write the per period utility function of the family as $u(c) + v(n) = \log(c) + (1-n)b$, where c is the common consumption level for everyone inside the family, n is the fraction of household members that work this period and b is the additional value of leisure to the non-working members of the family (we normalize the leisure value to those working at zero). In each family, its working members do so in a variety of firms or plants, each one characterized by size k and type t. Family employment then satisfies $n = \Sigma_{t\in\{0,1\}} \int x(k,t)\ dk$, where x is the size/type distribution of firms where the family members are employed at. The state of the household is comprised also of its assets, $s = \{a, x\}$. In addition, households discount the future at rate β and take the appropriate expectations whenever necessary. Their value function is

$$W(S,s) = \max_{c,a'} \quad \log(c) + (1-n)b + \beta\ E\{W(S',s')\} \qquad \text{s.t.} \tag{3}$$

$$c + a' = R(S)\ a + \sum_{t=\{0,1\}} \int w(k,t,S,s)\ x(dk,t) + \pi(S), \tag{4}$$

$$x'(k,1) = q^k(S), \quad \forall i, \tag{5}$$

$$x'(k,0) = (1-\delta) \sum_{t=\{0,1\}} x(k,t) \quad \forall i, \tag{6}$$

and the law of motion of the aggregate state S. Here $R(S)$ is the aggregate gross rate of return, $w(k,t,S,s)$ is the real wage a household member can command in a plant with characteristics (k,t), $\pi(S)$ represents current period profits of firms owned by the household and $q^k(S)$ represent the measure

a previous draft of our paper.

of hires of household members into $(k, 1)$ firms. The last two equations in the problem of the household above, show that households need to forecast the measure of employment in different types of firms. We avoid the description of the evolution of the aggregate state because the evolution of the aggregate measure of firms X is the same than that of the employers of the workers of the representative household.

The optimization of the household involves the standard Euler equation

$$1 = \beta \, E \, \left\{ \frac{c(S, s)}{c(S', s')} \, R(S') \right\}. \tag{7}$$

2.2 Firms and Investment Decisions

The value of an active plant of size k and type t, attached to a worker from a household with characteristics in s is given by[4]

$$\Pi(k, t, S, s) = e^z k^\alpha - w(k, t, S, s) + (1 - \delta) \, E \left\{ R(S') \, \Pi \left(k, 0, S', s'\right) \right\}, \tag{8}$$

where $w(k, t, S, s)$ is the wage paid to the worker attached to this firm. Once a plant is installed, the firm does not make any decisions: plants get destroyed at the exogenous rate δ and their productivity is given by z (depending whether they are new or old) and the installed capacity k.

We can define the value of marginally increasing plant size k:

$$\Pi_k(k, t, S, s) = e^z \alpha k^{\alpha - 1} - w_k(k, t, S, s) + (1 - \delta) \, E \left\{ R(S') \, \Pi_k \left(k, 0, S', s'\right) \right\}, \tag{9}$$

This expression is useful to calculate the optimal installation size, which solves the following problem

$$\max_i \quad -k + E \left\{ R(S') \, \Pi \left(k, 1, S, s\right) \right\}, \tag{10}$$

where the solution $k^*(S, s)$ satisfies the first order condition

$$1 = E \left\{ R(S') \, \Pi_k \left(k^*(S, s), 1, S, s\right) \right\}. \tag{11}$$

On the other hand, the number of plants $Q^*(S, s)$ that are installed is determined by a zero profit

[4]With this notation, we are assuming that firms and workers get matched before the investment decision. As we show in the next section, in equilibrium, this timing is irrelevant.

condition:

$$k^*(S,s) + c^v = E\left[R(S')\,\Pi\left(k^*(S,s),1,S,s\right)\right] \tag{12}$$

$Q^*(S,s)$ and $k^*(S,s)$ also determine overall capital investment in the current period, since total investment equals Q^*k^*.

2.3 Wage Determination

Wages are determined by Nash bargaining between the firm and the household. To determine the value of the match note that in our economy the only job market friction is the hiring cost c^v. We can think then, that there are no dynamic loses if there is no agreement between the worker and the firm as the worker can be recalled without paying again the hiring cost. Hence, the value of the worker for the firm is $e^z k^\alpha - w(k,t,S,s)$, while the value of the firm for the household is just its wage net of the value of being unemployed, $w(k,t,S,s) - b\ c(S,s)$. Firms and workers bargain over the match surplus, and if the bargaining power of the household is μ, the wage is just fraction μ of the surplus which is

$$w(k,t,S,s) = \mu\ e^z\ k^\alpha + (1-\mu) b\ c(S,s) \tag{13}$$

Given this wage function, plant specific profits each period are given by $(1-\mu)\left[e^z k^\alpha - b\ c(S,s)\right]$. This expression can be used to rewrite equation (11). Note that in this setting, all firms have the same information and installation costs are the same for them. Thus, only one size k^* is installed each period.

2.4 Aggregation and a simplified problem

This model can be simplified dramatically (following Gourio (2011)) to get rid of the measure $X(k,t)$ of plants and substitute it by a few aggregates. Define aggregate installed capacity by type 0 (old) firms as

$$\overline{Y}^0 = \int k^\alpha\ X(dk,0). \tag{14}$$

In a similar same vein, capacity of new firms the following period is

$$\overline{Y}^{1'} = Q^*\ (k^*)^\alpha. \tag{15}$$

These two expressions give a recursive characterization that allows to avoid using the measure of firms:

$$\overline{Y}^{0'} = (1-\delta)\left[\overline{Y}^{0} + \overline{Y}^{1}\right]. \tag{16}$$

Given the shock process described in equation (1), total output is given by

$$Y = e^{\lambda\tilde{z}}\ \overline{Y^0} + e^{\tilde{z}}\ \overline{Y}^1. \tag{17}$$

Aggregate employment can also be defined recursively with the aid of the number of new plants

$$N' = (1-\delta)N + Q^*. \tag{18}$$

Similarly, the entire wage bill for the economy is just

$$\omega = \mu\ Y + N\ (1-\mu)\ b\ C. \tag{19}$$

Note that the aggregate feasibility constraint in the economy is now written as

$$Y = C + Q^*\ \ [k^* + c^v]. \tag{20}$$

Given these simplifications, the aggregate state of the economy reduces to $\{\tilde{z}, \overline{Y}^0, \overline{Y}^1, N, \}$. The nature of the shock process that distinguishes between new and old installed capacity requires us to add one state variable to those posed by Gourio (2011). The model in turn simplifies to a set of equilibrium equations, which we solve using standard approximation methods. These equilibrium conditions can be summarized as the Euler equation of the household, the equations for optimal current plant size and optimal number of plant installations by firms and aggregate market consistency: equation (7), equations (10) and (11) and equations (2), (13), (16), (18) and (20) respectively.

The state space for the individual household also collapses, since similar arguments can be used to show that average employment and wage bills are needed to solve the problem of the household. Savings and consumption decisions depend on aggregate interest rates and the total wage bill that the household receives for its labor.

3 Mapping the Model to U.S. Data

The model period is a month. We set β to 0.9966 so that the interest rate is four percent per annum. We set another group of parameters, $\{b, \delta, \alpha, c^v, \mu\}$, (value of home production, firm destruction rate, coefficient of capital in the production function, hiring cost, and worker's bargaining weight) to match a set of long-term statistics for the US economy. The statistics that we match are a labor share of output of 0.6514 (reported by Koh, Santaeulalia-Llopis, and Zheng (2016)); a share of consumption out of output of 0.75; an unemployment rate of 5.8%; a total value of leisure per period that amounts to .70 of consumption (as calculated by Hall and Milgrom (2008));[5] and a flow cost of a vacancy in terms of days of pay per hire of 0.446 which closely matches an aggregate vacancy expenditure relative to annual GDP of 0.50%, reported by Cheron and Langot (2004).[6] The implied values for the parameters are reported in Table 1.

Parameter	Description	Value
b	value of leisure	0.6459
δ	plant destruction rate	0.0084
α	curvature of production menu	0.5383
c^v	vacancy cost	0.3090
μ	bargaining weight workers	0.3596

Table 1: Parameter values.

3.1 Estimation of the TFP Shock

We estimate the rest of the parameters that we use, those controlling the process for the shock in the model, $\{\lambda, \rho, \sigma_\epsilon\}$, using simulated method of moments. Two of the moments that we use are obtained from an AR(1) representation of the Solow residual, after linearly log-detrending a TFP time series. More specifically, we follow Ríos-Rull and Santaeulalia-Llopis (2010) and use their definition of the Solow residual as

$$\widehat{z}_t = y_t - \overline{LSH}\ n_t + (1 - \overline{LSH})\ k_t,$$

where $\overline{LSH}$ is the average labor share, y_t is detrended output, n_t is detrended employment and k_t is detrended capital. From here, a regression of the form $\widehat{z}_t = \gamma\ \widehat{z}_{t-1} + \eta_t$ produces a linear least square

[5] In their discussion this value amounts to their value of z and includes not only the value of unemployment but also the direct value of leisure

[6] This is close to the value of 0.433 used by Hall and Milgrom (2008) based on Silva and Toledo (2009).

estimate for the persistence, $\hat{\gamma}$ and the volatility of the Solow residual, $\hat{\sigma}_\eta$. These two moments are closely related to the parameters ρ and σ_ϵ of the model. We use various alternatives as the candidate for the third moment that has to be matched, the most likely to be informative about λ, the time bias parameter. Because this parameter controls how profitable is to construct new productive units, and hence, the overall volatility of employment and unemployment during the business cycle, we target different employment-related moments. We target in succession: the relative volatility of the hp-filtered log of unemployment to hp-filtered log output (**U vol**), the relative volatility of unemployment plus employment (**U+E vol**), the relative volatility of employment (**E vol**), and the Impulse Response of labor share (**IRF LSH**). For comparison we use the first two moments alone to target the autocorrelation and standard deviation of the shock process under the assumptions that λ is either one, a neutral TFP shock, (**No bias**) or zero, akin to an investment specific TFP shock (**Full bias**). The estimation based on the dynamics of labor share uses the whole impulse response (depicted in the right panel of Figure 4) as moments to be matched by simulated data of our model.[7] To match the *overshooting* property, we put more weight in the estimation to responses between quarters 10 and 40 of the impulse response function in the figure. The estimated parameters are in Table 2 where the standard errors are computed using standard methods.[8]

The estimates of λ are either zero or close to zero in all cases, overwhelmingly pointing to a time biased TFP shock rather than to a neutral one. The differences between the estimates across the moments that we use are quite small, indicating that it does not really matter which one we use. This is the main finding of the paper: with a different identification strategy than what is usually the case, we find that the nature of the stochastic process for TFP seems to be more related to new plants than to established plants.

Another feature of the estimates is that, the lower the estimated value of λ, the lower the value of our estimate of ρ and the higher the value for σ_ϵ. Notice that low values of λ demand very high values of σ_ϵ, as the movements in total TFP are carried out by a small fraction of the operating plants.

[7] This is similar to the estimation procedure in Christiano, Eichenbaum, and Evans (2005), who estimate a set of parameters to match empirical impulse response functions.

[8] We follow Duffie and Singleton (1993): the standard errors are the square roots of the diagonal elements of matrix $\{G'SG\}^{-1}$, where G is the matrix of partial derivatives and S is a diagonal matrix with the inverse of the variance of the data moments in its diagonal.

Table 2: Estimated Process for the TFP Shock

	No bias	U vol	U+E vol	E vol	IRF LSH	Full bias
λ	1.0000	0.0334	0.0249	0.0038	0.0025	0.0000
	–	(0.0119)	(0.0075)	(0.0036)	(0.0013)	–
ρ	0.9740	0.9725	0.9720	0.9659	0.9614	0.9600
	(0.0109)	(0.0108)	(0.0100)	(0.0106)	(0.0047)	(0.0131)
σ_ϵ	0.0048	0.1151	0.1444	0.3928	0.4414	0.5686
	(0.0002)	(0.0317)	(0.0310)	(0.1127)	(0.0398)	(0.0273)

Standard errors in parenthesis. The columns of the table represent the moment used to estimate the parameters: **U vol** represents model simulations from an estimation where relative standard errors (with respect to output) are targeted. **E vol** represents a model where standard errors of employment are targeted, while in **U+E vol**, both are targeted at the same time. **IRF LSH** refers to a model where the values of the impulse response function in the right panel of Figure 4 are targeted in the estimation. **No bias** and **Full bias** refer to models where we set $\lambda = 1$ and $\lambda = 0$ respectively.

4 Quantitative Properties of the Model Economy

The implied properties of the model economy are shown in Tables 3 and 4 where we report relative standard deviations (columns σ_x/σ_y) and correlations (columns $\rho_{x,y}$ between several U.S. aggregates (variables x) with output (y). In the tables, we take logs of each variable and detrend them using the Hodrick-Prescott filter, with smoothing parameter set to 1600. For the model simulated data, we aggregate from monthly to quarterly simulated data by averaging three month periods.

Table 3: Cyclical volatility of U.S., No Bias ($\lambda = 1$) and Full Bias ($\lambda = 0$)

	US data		**No bias** $\lambda = 1$		**Full bias** $\lambda = 0$	
x	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$
Employment	0.487	0.792	0.210	0.341	0.579	0.612
Unemployment	6.479	-0.840	3.944	-0.316	14.006	-0.493
Labor Share	0.502	-0.261	0.277	-0.790	0.848	-0.540
Wages	0.619	0.781	0.710	0.999	0.287	0.504
Consumption	0.571	0.832	0.476	0.926	1.202	-0.350
Investment	3.418	0.885	2.743	0.980	6.369	0.838

Note: US data is taken from the Bureau of Labor Statistics (see appendix for details) for the period 1947:QI-2017:QI. y represents output; σ_x/σ_y is the relative standard deviation of variable x with respect to output; $\rho_{x,y}$ is the correlation of variable x with respect to y. All series are logged and HP-filtered.

Table 3 displays results for the two extreme cases we consider: Neutral and investment specific technology shocks, in columns **No bias** and **Full bias** respectively. Their differences are large. With neutral technology shocks employment (and unemployment) and movements in labor share display too low volatility while wage are too large movements. On the other hand, consumption and investment move a little less than the data (as they should given that there are no net exports) and they are strongly correlated. The economy with biased shocks does the opposite: overpredicts volatility of employment, unemployment and labor share but underpredicts movements in wages. It also overpredicts consumption and investment movements but with a negative correlation between consumption and output.

The economies with a estimated bias are displayed in Table 4. They present a more nuanced comparison with the data. In all economies, simulated relative volatilities and correlations with output are in the ballpark of the US data, with the exception of the correlation between consumption and output which in some cases turns negative (economies **E vol** and **IRF LSH**). As discussed above, the **No bias** and **Full bias** versions of the model represent two extremes in terms of implied volatilities in labor market variables. Not surprisingly, in Table 4 we observe that the various estimated models sit between the results of the two models in Table 3, with model **U vol** being the closest to the data using as a metric the simple sum of squared difference (between US data and model moments).

Table 4: Cyclical volatility of Models with Various Estimated Process for the Shock

	US data		**U vol**		**U+E vol**		**E vol**		**IRF LSH**	
x	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$
Employment	0.487	0.792	0.298	0.405	0.321	0.423	0.487	0.545	0.520	0.570
Unemployment	6.479	-0.840	6.478	-0.339	7.271	-0.348	12.258	-0.427	12.965	-0.455
Labor Share	0.502	-0.261	0.396	-0.745	0.427	-0.732	0.683	-0.619	0.744	-0.597
Wages	0.619	0.781	0.582	0.999	0.549	0.999	0.316	0.920	0.286	0.822
Consumption	0.571	0.832	0.384	0.571	0.393	0.417	0.815	-0.282	0.949	-0.329
Investment	3.418	0.885	3.491	0.961	3.689	0.955	5.310	0.892	5.697	0.875

Note: y represents output; σ_x/σ_y is the relative standard deviation of variable x with respect to output; $\rho_{x,y}$ is the correlation of variable x with respect to y. All series are logged and HP-filtered. **U vol** uses the estimates from targeting unemployment volatility. The estimates in **E vol** use employment, while those in **U+E vol** use the sum of employment and unemployment. **IRF LSH** refers to the model that uses the impulse response function of labor share to estimate the shock.

Next, we ask whether the model can replicate some salient facts of the cyclicality of the U.S. labor share in recent decades. More specifically, we are interested in the *overshooting* of the labor share which is summarized by the its impulse response with respect to output. Details of our estimates are in

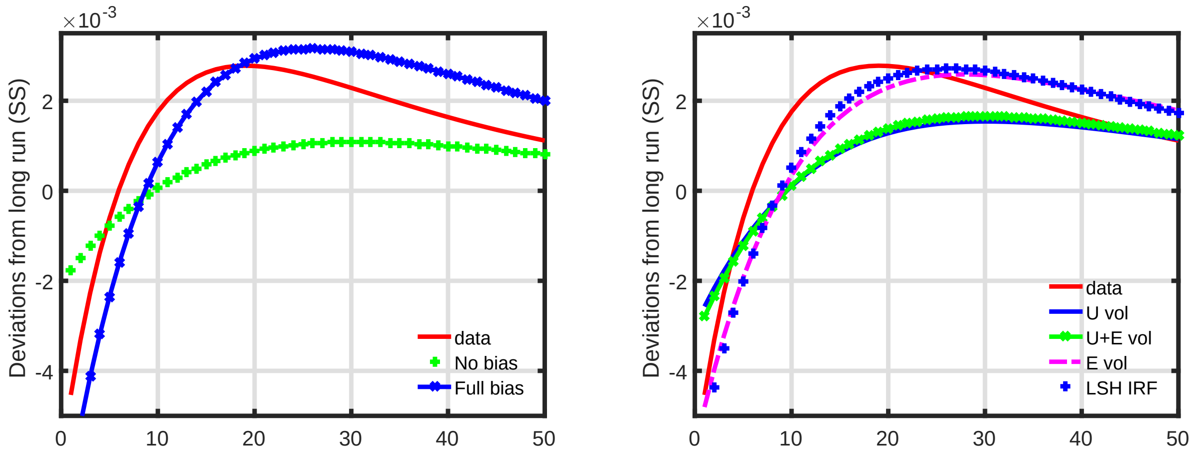

Figure 1: Impulse response functions of labor share: US data and different model under different estimation strategies. See text and table table 2.

Appendix B.

4.1 The Overshooting Property of Labor Share

After an initial positive shock to TFP, labor share drops but subsequently goes up a lot with a positive deviation that is larger than the initial drop, a feature that has been described as the *overshooting* property of the labor share. It is associated to positive responses of *both* aggregate real wages and employment, which provides an alternative way of disciplining how we model the mechanisms through which productivity innovations affect labor market volatility. Note that our model makes new workers more productive than incumbent ones (especially when $\lambda = 0$) but also hard to substitute with capital in the short run, producing the desired joint dynamics between productivity, real wages and aggregate hours. Interestingly, rigid wages are not needed to produce high job creation during cycles, a common prescription for similar models with frictions in the labor market.[9]

In both panels of Figure 1 we compare empirical and model implied *overshooting*, where we treat all model simulated data the same way as the real data. The left panel displays the data, the **No bias** and the **Full bias** economies, while the right panel displays the different variants of the model, defined by the calibration targets used.

All the economies display some form of overshooting. This feature has been difficult to obtain. See for example Choi and Ríos-Rull (2009), Ríos-Rull and Santaeulalia-Llopis (2010), Shao and Silos (2014)

[9]See Shimer (2005) and Hall (2005) for a discussion.

and León-Ledesma and Satchi (2017). This indicates that putty-clay technology plus wage bargaining go a long way in achieving this property. Putty-clay implies that productivity takes some time to pick up and wage bargaining makes the wage of workers in productive firms to also go up generating the appropriate co-movements. Still, not all models fare equally well with respect to the impulse response features of the labor share. The **Full bias** economy does clearly better than the **No bias** economy: it is bigger both at impact (time zero) and later in time (around quarter 25 after the initial shock).

As discussed above, the lower the value of λ, the larger the implied time bias and the more the shock that we incorporate in the model resembles a pure investment specific technological shock. This is also consistent with a more pronounced response of the labor share during the business cycle. This can be seen in the right panel of Figure 1: the model where we target unemployment volatility is the one with the lowest overshooting (relative to the other models in the panel); when we target either employment volatility (**E vol**) or the impulse response function itself (**IRF LSH**) we obtain lower values for λ and higher overshooting. Clearly, targeting the impulse response itself yields the results that are closest to the data, but the point we want to make here is that with very low values of λ the impulse response of the labor share is very close in the models to their data counterparts.

Table 5: Autocorrelation of variables

	US data	**No bias**	**Full bias**
Output	0.848	0.794	0.842
Employment	0.902	0.962	0.956
Unemployment	0.893	0.961	0.944
Labor Share	0.629	0.785	0.769
Wages	0.788	0.786	0.957
Consumption	0.811	0.856	0.805
Investment	0.807	0.776	0.759

Note: US data is taken from the Bureau of Labor Statistics (see appendix for details) for the period 1947:QI-2017:QI. All series are logged and HP-filtered. *No bias* and *Full bias* refer to models where we set $\lambda = 1$ and $\lambda = 0$ respectively.

In terms of propagation, Table 5 shows autocorrelation statistics for the two extreme (No bias and Full bias) economies, since as shown before, they encapsulate results within the range of estimated λ's. From the table is clear that there is a negative relationship between the value of λ and the implied autocorrelation in output from the model and thus, the endogenous propagation inside the model after an exogenous shock. However, the main takeaway from the table is the similarity in results when considering the rest of aggregates.

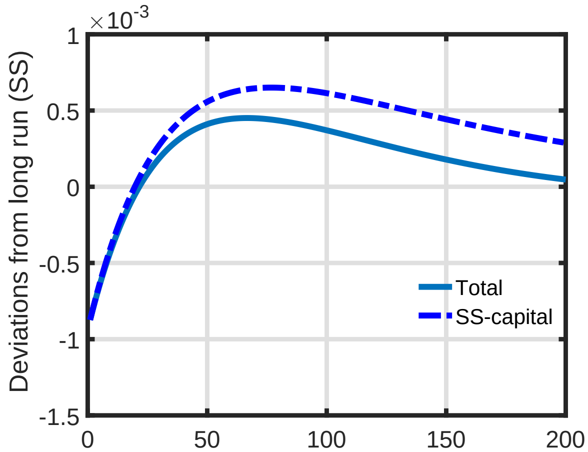

Figure 2: Impulse response functions of labor share in baseline model. Left panel: comparison between total and labor share in machines with steady state level of capital. Right panel: response of labor share for plants of different ages: **cohort Xm** is the response of the labor share in plants created X months after the initial shock to productivity.

4.2 Reasons for the Overshooting

Why does our baseline model reproduce the overshooting of the labor share? As it is well known, under Cobb-Douglas technology and perfect factor markets (which includes neoclassical settings such as the real business cycle) factor shares are constant. Thus, we have to depart from at least one of these two assumptions.

To explore this issue, we use the *No bias* version of our model to trace out the response of aggregate labor share for different cohorts of firms. On the left panel of Figure 2, we show the response of labor share for the whole economy and for those firms who, at the start of the expansion, possess exactly the capital level of a firm that chooses capital when aggregate conditions are neutral (the ones in the theoretical steady state).[10] As seen from the figure, the reaction of the labor share in those firms is more pronounced than in the aggregate economy. The complete picture is formed by looking at the right panel: labor share at firms who are *born* at different time months after the initial positive shock (different firm *cohorts*), exhibit smaller responses than the aggregate.

In our baseline, and during periods in which productivity is higher than average (200 months shown in the figures), each new firm chooses their size, and at the same time, its wage/labor share. The simple fact that not all firms are bound to change their idiosyncratic capital-output ratios in the exact same

[10]For an individual firm, we compute its labor share as $w(S,k)/(e^z k^\alpha)$.

way, delays the flattening of the labor share response. This is in stark contrast to a model with standard Cobb-Douglas technology: to realize the benefits of positive productivity innovations, firms hire more labor. Since the technology and the installed capital-labor ratios are the same for all operating firms, the aggregate labor share moves dictated by the parameter of the Cobb-Douglas function, leaving little room for more nuanced dynamics. This is exactly the finding in Choi and Ríos-Rull (2009).

5 Conclusions

In this paper we propose a flexible way to introduce TFP shocks into a framework with putty clay technology with an additional parameter whose value tells us whether we should think of technology shocks as being neutral or as being akin to investment specific shocks. The main insight from our analysis, is the fact that the latter can be thought of as neutral shocks which have a *time* bias, which favors recently created productive units. Our results show that, when targeting volatility of labor market aggregates in the US economy, our model implies a strong time bias of the aggregate shock making a strong case for technology shocks being of the investment specific type. The model can also replicate the business cycle fluctuations of labor share, showing a novel way to introduce propagation from technology shocks to the rest of the economy and how rigid wages are not needed to produce volatility.

References

Atkeson, A., and P. Kehoe (1999): "Models of Energy Use: Putty-Putty versus Putty-Clay," *American Economic Review*, 89(4), 1028–1043.

Bai, Y., J.-V. Ríos-Rull, and K. Storesletten (2011): "Demand Shocks as Productivity Shocks," manuscript, Federal Reserve Bank of Minneapolis.

Blanchard, O., and J. Galí (2007): "Real Wage Rigidities and the New Keynesian Model," *Journal of Money, Credit and Banking*, 39(S1), 35–65.

Caballero, R. J., and M. L. Hammour (1996): "On the Timing and Efficiency of Creative Destruction," *Quarterly Journal of Economics*, 111(3), 805–852.

Cheron, A., and F. Langot (2004): "Labor Market Search and Real Business Cycles: Reconciling Nash bargaining with the Real Wage Dynamics," *Review of Economic Dynamics*, 7(2), 476–493.

Choi, S., and J.-V. Ríos-Rull (2009): "Understanding the Dynamics of Labor Share: The Role of Noncompetitive Factor Prices," *Annales d'Economie et Statistique*, 95/96, 251–277.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Costain, J., and M. Reiter (2008): "Business Cycles, Unemployment Insurance and the Calibration of Matching Models," *Journal of Economic Dynamics and Control*, 32(4), 1120–1155.

Duffie, D., and K. J. Singleton (1993): "Simulated Moments Estimation of Markov Models of Asset Prices," *Econometrica*, 61(4), 929–952.

Fernandez-Villaverde, J., and J. F. Rubio-Ramirez (2007): "Estimating Macroeconomic Models: A Likelihood Approach," *The Review of Economic Studies*, 74(4), 1059–1087.

Fisher, J. D. M. (2006): "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks," *Journal of Political Economy*, 114(3), 413–451.

Gilchrist, S., and J. Williams (2000): "Putty-Clay and Investment: A Business Cycle Analysis," *Journal of Political Economy*, 108(5), 928–960.

Gourio, F. (2011): "Putty-Clay Technology and Stock Market Volatility," *Journal of Monetary Economics*, 58(2), 117–131.

Hagedorn, M., and I. Manovskii (2008): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 98(4), 1692–1706.

Hall, R. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95(1), 50–65.

——— (2009): "Reconciling Cyclical Movements in the Marginal Value of Time and the Marginal Product of Labor," *Journal of Political Economy*, 117(2), 281–323.

Hall, R. E., and P. R. Milgrom (2008): "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review*, 98(4), 1653–74.

Johansen, L. (1959): "Substitution versus Fixed Production Coefficients in the Theory of Economic Growth: A Synthesis," *Econometrica*, 27(2), 157–176.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010): "Investment shocks and business cycles," *Journal of Monetary Economics*, 57(2), 132 – 145.

——— (2011): "Investment shocks and the relative price of investment," *Review of Economic Dynamics*, 14(1), 102 – 121, Special issue: Sources of Business Cycles.

Koh, D., R. Santaeulalia-Llopis, and Y. Zheng (2016): "Labor Share Decline and Intellectual Property Products Capital," mimeo.

León-Ledesma, M., and M. Satchi (2017): "Appropriate Technology and Balanced Growth," forthcoming, Review of Economic Studies.

Mortensen, D. (1992): "Search Theory and Macroeconomics: A Review Essay," *Journal of Monetary Economics*, 29(1), 163–167.

Ramey, V. (2016): "Chapter 2 - Macroeconomic Shocks and Their Propagation," vol. 2 of *Handbook of Macroeconomics*, pp. 71 – 162. Elsevier.

Ríos-Rull, J.-V., and R. Santaeulalia-Llopis (2010): "Redistributive Shocks and Productivity Shocks," *Journal of Monetary Economics*, 57(8), 931–948.

Rogerson, R., and R. Shimer (2010): "Search in Macroeconomic Models of the Labor Market," *Handbook of Labor Economics*, 4.

Schmitt-Grohe, S., and M. Uribe (2011): "Business cycles with a common trend in neutral and investment-specific productivity," *Review of Economic Dynamics*, 14(1), 122 – 135, Special issue: Sources of Business Cycles.

Shao, E., and P. Silos (2014): "Accounting for the Cyclical Dynamics of Income Shares," *Economic Inquiry*, 52, 778–795.

Shimer, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95(1), 25–49.

Silva, J. I., and M. Toledo (2009): "Labor Turnover Costs and the Cyclical Behavior of Vacancies and Unemployment," *Macroeconomic Dynamics*, 13(S1), 76–96.

Wei, C. (2003): "Energy, the Stock Market, and the Putty-Clay Investment Model," *American Economic Review*, 93(1), 311–323.

Appendix

A Data

All data is taken from the Bureau of Labor Statistics (BLS)'s website and the FRED site of the Saint Louis Fed. The series we use are Output (BLS series PRS85006043), Labor Share Index (BLS series PRS85006173), Employment (BLS series LNS12000000Q), Unemployment (BLS series LNS13000000Q), Real Hourly Compensation (BLS series PRS85006153), Personal consumption expenditures (FRED series PCECC96) and Real Gross Private Domestic Investment (FRED series GPDIC96). All variables refer to non-farm business sector and are seasonally adjusted. We further normalize all variables by the non-institutionalized population in the US (BLS series LNU00000000Q).

B Labor Share During the Business Cycle

To analyze the behavior of labor share and its components, we use aggregate quarterly US data from the first quarter of 1948 to the fourth quarter of 2011. For the labor share, we use the ratio of worker compensation to total value added in the non-farm business sector, as constructed by the Bureau of Labor Statistics.[11] Below we compare labor share to output (value added in the non-farm sector). Both variables are in logs and detrended. Following Ríos-Rull and Santaeulalia-Llopis (2010), we use a linear trend.

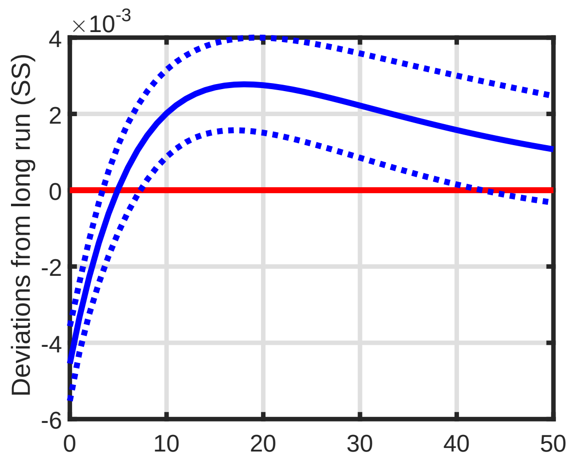

Figure 3: Impulse response function of labor share to a shock in output, from VAR(1) model. Source: Bureau of Labor Statistics, 1948Q1-2017Q1.

We estimate a vector autoregression of order 1 and compute the orthogonal impulse response of the labor share to an innovation in real output. This is presented in appendix B with its corresponding 95% confidence intervals.

The figure summarizes various facts with respect to the labor share which have been previously discussed in Choi and Ríos-Rull (2009) and Ríos-Rull and Santaeulalia-Llopis (2010): Upon impact,

[11] For extensive robustness exercises and different definitions of the labor share, see Ríos-Rull and Santaeulalia-Llopis (2010). Detailed information on the construction of this variable, as well as information on sources of data can be found in appendix appendix A.

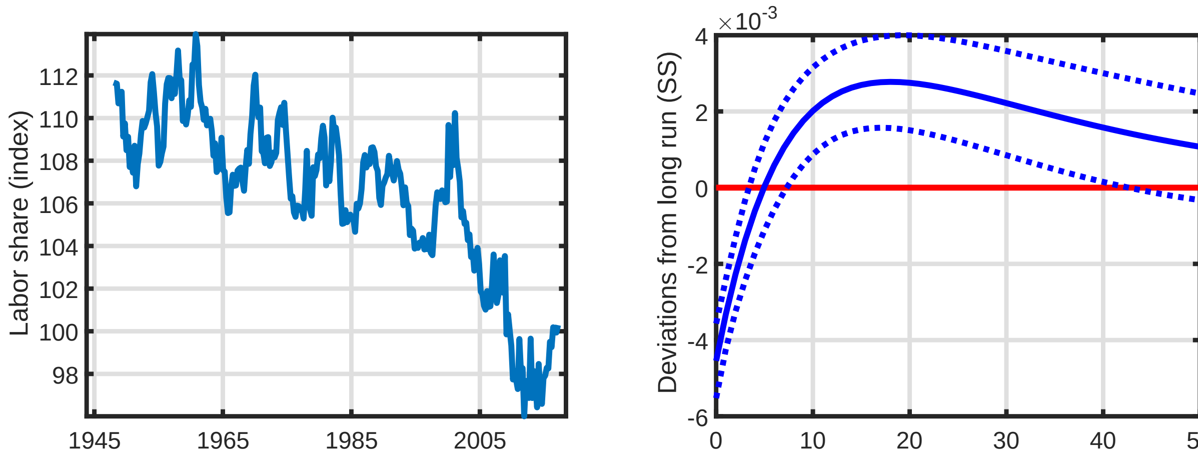

Figure 4: Left panel is a labor share Index for non-farm business US, 1948:Q1 to 2016:Q1. Right panel is an impulse response function of labor share to a shock in output, from VAR(1) model. Source: Bureau of Labor Statistics (details in the appendix).

labor share falls giving rise to the well known counter cyclical behavior of the series. In time, labor share starts to rise, and peaks around 15 to 20 quarters after the initial shock. This positive response is persistent and long lasting, making the aggregate gains of labor due to an expansion positive in the medium to long run, more than compensating the initial drop in this share (hence the overshooting label).[12] As seen in the figures, this response is statistically significant at the 95% level.

To understand the task of replicating these facts with standard macro models and to gain some insight, define aggregate labor share at time t as

$$Labor\ Share_t = \frac{W_t}{(Y_t/N_t)}$$

so movements in the labor share must come from persistent departures of the real wage (W_t) from average labor productivity (Y_t/N_t). In the classic business cycle framework, the combination of competitive pricing in the labor market alongside a Cobb-Douglas aggregate production function produces a constant labor share, given that real wages are proportional to average labor productivity.

On the other hand, Choi and Ríos-Rull (2009) study the inability of macro models with search frictions in the labor market and non-competitive wage setting to reproduce the labor share overshooting: wage rigidity introduced by Nash bargaining can produce a countercyclical labor share[13] but not its

[12] The estimates of the bivariate VAR show no effects whatsoever of lagged labor share on productivity. See Ríos-Rull and Santaeulalia-Llopis (2010) for details. Also, given that we are using a different time period and different variable definitions, this is also a sign of the robustness of the overshooting fact.

[13] Real wages react sluggishly to the productivity shock as opposed to average labor productivity, which reacts almost one to one. This is the result of using of Nash bargaining, since equilibrium wages in the model are a weighted average

subsequent dynamics. This is because the wedge between real wages and average labor productivity introduced by bilaterally bargained wages is not long lasting, given a strong feedback between productivity and equilibrium wages. Hence, after a productivity shock the model reverts quickly to a situation with constant factor shares. These findings are related to the mechanism pointed out by Shimer (2005) who shows that a calibrated version of the standard Mortensen-Pissarides search and matching model is unable to produce sizeable vacancy (and therefore employment) responses after a productivity shock. Shimer (2005) attributes this result to the excess sensibility of Nash bargained wages to productivity shocks: again, if the difference between average labor productivity and real wages imposed by a rigid wage is short lived, firms don't want to create employment, hence post few vacancies during a productivity induced upturn.[14]

between average labor productivity and worker's outside option.

[14] This view has been both underscored by Hall (2005) and challenged by Hagedorn and Manovskii (2008).

C Biased Shocks and Investment Specific Technological Innovations

To show the distinction between our time-biased shock and standard Investment Specific Technological (IST) shocks, in this section we depict the dynamics of both shocks. In table 6 we show the time biased shocks, while the dynamics of standard investment specific shocks are in table 7. For the latter, we imagine an economy where capital accumulation is given by $K_{t+1} = (1-\delta)K_t + e^{\widetilde{z}_t} Q_t$, where $\widetilde{z}$ evolves according to equation (2) and Q_t is the amount of capital installed in period t.

		machine age (in periods)				
time	shock	1	2	3	4	...
0	1	1	λ	λ	λ	...
1	ρ	ρ	$\lambda\rho$	$\lambda\rho$	$\lambda\rho$	...
2	ρ^2	ρ^2	$\lambda\rho^2$	$\lambda\rho^2$	$\lambda\rho^2$	...
3	ρ^2	ρ^3	$\lambda\rho^3$	$\lambda\rho^3$	$\lambda\rho^3$	...
4	ρ^3	ρ^4	$\lambda\rho^4$	$\lambda\rho^4$	$\lambda\rho^4$	...
...	...	...	...	...	...	...

Table 6: Representation of shock dynamics.

		machine age (in periods)				
time	shock	1	2	3	4	...
0	1	0	0	0	0	...
1	ρ	1	0	0	0	...
2	ρ^2	ρ	1	0	0	...
3	ρ^2	ρ^2	ρ	1	0	...
4	ρ^3	ρ^3	ρ^2	ρ	1	...
...	...	...	...	...	...	...

Table 7: Representation of investment specific shocks.

For both tables, we imagine an economy that is in steady state, and introduce a shock to productivity of size one (just for exposition). Given the AR(1) representation of the shocks, in both economies the value of the shock declines according to ρ.

In our time-biased representation of the shock (see equation (1) and equation (2)), machines that are 1 period old (installed just last period), enjoys productivity e^1 the first period that is operational (time 0 in the table), and enjoys the declining productivity sequence $\{\lambda\rho, \lambda\rho^2, \lambda\rho^3, ...\}$. Furthermore, all older machines enjoy the productivity shock, in amount λ, and as time passes, they also are exposed to a declining value of the shock.

On the other hand, in table 7 we see that capital/machines (Q) which is 1 period of age, enjoys IST for all of its existence (sequence of 1's in the table). In a model with IST, installed machines do not enjoy these shocks (thus the zeros in the table).

		machine age (in periods)				
time	shock	1	2	3	4	...
0	1	1	0	0	0	...
1	ρ	ρ	0	0	0	...
2	ρ^2	ρ^2	0	0	0	...
3	ρ^2	ρ^3	0	0	0	...
4	ρ^3	ρ^4	0	0	0	...
...	...	...	...	...	...	...

Table 8: Representation of shock dynamics with $\lambda = 0$

The extreme case of full biased shocks, i.e., $\lambda = 0$, is shown in table 8. There, we can see the similarities and differences between this type of shock and standard IST. On the one hand, both shocks give the same value of the shock to the newest machines. However, the $\lambda = 0$ case do not affect machines of any cohort beyond those 1 period old, which is the main difference with how IST affects dynamically investment. However, although our formulation removes the effect of the shock on older cohorts of machines/capital, these are depreciating at exponential rate δ, giving more weight to the newest cohorts.

Thus, although there are strong similarities between IST and our shock when $\lambda = 0$, they are not the same.