A Theory of Credit Scoring and Competitive Pricing of Default Risk[1]

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Abstract

We propose a theory of unsecured consumer credit where: (i) borrowers have the legal option to default; (ii) defaulters are not exogenously excluded from future borrowing; (iii) there is free entry of lenders; and (iv) lenders cannot collude to punish defaulters. In our framework, limited credit or credit at higher interest rates following default arises from the lender's optimal response to limited information about the agent's type and earnings realizations. The lender learns from an individual's borrowing and repayment behavior about his type and encapsulates his reputation for not defaulting in a credit score. We take the theory to data estimating the parameters of the model to match key data moments such as the overall and subprime delinquency rates. We test the theory by showing that our underlying framework is broadly consistent with the way credit scores affect unsecured consumer credit market behavior. The framework can be used to shed light on household consumption smoothing with respect to transitory income shocks and to examine the welfare consequences of legal restrictions on the length of time adverse events can remain on one's credit record.

1 Introduction

It is well known that lenders use credit scores to regulate the extension of consumer credit. People with high scores are offered credit on more favorable terms. People who default on their loans experience a decline in their scores and, therefore, lose access to credit on favorable terms. People who run up debt also experience a decline in their credit scores and have to pay higher interest rates on new loans. While credit scores play an important role in the allocation of consumer credit, credit scoring has not been adequately integrated into the theoretical literature on consumption smoothing and asset pricing. This paper attempts to remedy this gap.[1]

We propose a theory of unsecured consumer credit where: (i) borrowers have the legal option to default; (ii) defaulters are not exogenously excluded from future borrowing; (iii) there is free entry of lenders; and (iv) lenders cannot collude to punish defaulters. We use the framework to try to understand why households typically face limited credit or credit at higher interest rates following default and why this changes over time. We show such outcomes arise from the lender's optimal response to limited information about the agent's type and earnings realizations. The lender learns from an individual's borrowing and repayment behavior about his type and encapsulates his reputation for not defaulting in a credit score.

The legal environment surrounding the U.S. unsecured consumer credit market is characterized by the following features. Individual debtors have can file for bankruptcy under Chapter 7 which permanently discharges net debt (liabilities minus assets above statewide exemption levels). A Chapter 7 filer is ineligible for a subsequent Chapter 7 discharge for 6 years. During that period, the individual is forced into Chapter 13 which is typically a 3-5 year repayment schedule followed by discharge. Over two-thirds of household bankruptcies

[1] One important attempt to remedy this deficiency in the consumption smoothing literature is Gross and Souleles ¡cite¿GSQJE¡/cite¿. That paper empirically tests whether consumption is excessively sensitive to variations in credit limits taking into account a household's risk characteristics embodied by credit scores.

in the U.S. are Chapter 7. The Fair Credit Reporting Act requires credit bureaus to exclude the filing from credit reports after 10 years (and all other adverse items after 7 years).

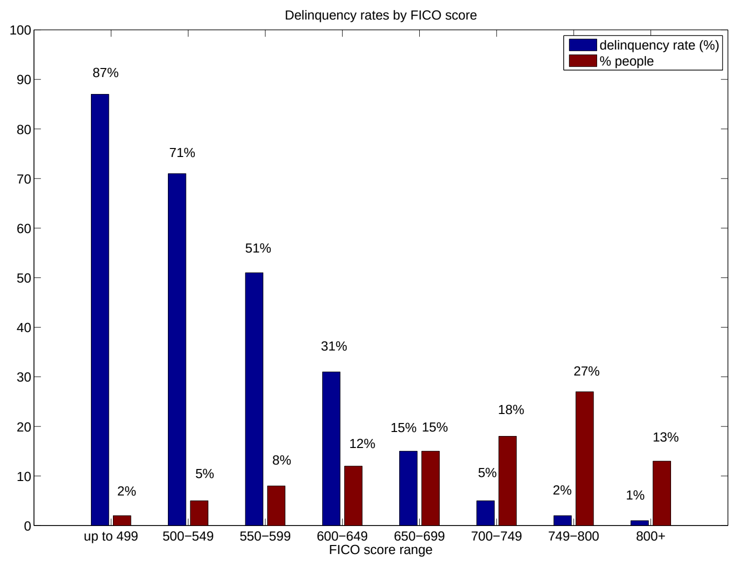
Beginning with the work of Athreya [1], there has been a growing number of papers that have tried to understand bankruptcy data using quantitative, heterogeneous agent models (for example Chatterjee, et. al. [4], Livshits, et. al. [14]). For simplicity, these models have assumed that an individual is exogenously excluded from borrowing while a bankrutpcy remains on his credit record. This exclusion restriction is often modelled as a Markov process and calibrated so that on average the household is excluded for 10 years, after which the Fair Credit Reporting Act requires that it be stricken from the household's record. This assumption is roughly consistent with the findings by Musto [16] who documents the following important facts: (1) households with low credit ratings face very limited credit lines (averaging around \$215) prior to and \$600 following the removal of a bankruptcy flag; (2) for households with medium and high credit ratings, their average credit lines were a little over \$800 and \$2000 respectively prior to the year their bankruptcy flag was removed from their record; and (3) for households with high and medium credit ratings, their average credit lines jumped nearly doubled to \$2,810 and \$4,578 in the year that the bankruptcy flag was removed from their record.[2]

While this exogenous exclusion restriction is broadly consistent with the empirical facts, a fundamental question remains. Since a Chapter 7 filer is ineligible for a subsequent Chapter 7 discharge for 6 years (and at worst forced into a subsequent Chapter 13 repayment schedule), why don't we see more lending to those who declare bankruptcy? If lenders believe that the Chapter 7 bankruptcy signals something relatively permanent about the household's unobservable characteristics, then it may be optimal for lenders to limit future credit. But if the circumstances surrounding bankruptcy are temporary (like a transitory, adverse income shock), those individuals who have just shed their previous obligations may be a good future credit risk. Competitive lenders use current repayment and bankruptcy status to try to infer

[2]These numbers are actually drawn from Table III, panel A of Musto's Wharton working paper #99-22.

an individual's future likelihood of default in order to correctly price loans. There is virtually no existing work embedding this inference problem into a quantitative, dynamic model.

Figure 1: Delinquency Rates in the Population



Given commitment frictions, it's important for a lender to assess the probability that a borrower will fail to pay back – that is, assess the risk of default. In the U.S., lenders use *credit scores* as an index of the risk of default. The credit scores most commonly used are produced by a single company, the Fair Isaac and Company, and are known as FICO scores.[3] These scores range between 300 and 850, where a higher score signals a lower probability of default. Scores under 620, which account for roughly one quarter of the population with scores, are called "subprime".[4] There is ample empirical evidence that households with subprime credit scores are more likely to default. Figure 1 provides one such example. As discipline on our theory, we require our framework to match key credit market facts like that in Figure 1.

[3] Over 75% of mortgage lenders and 80% of the largest financial institutions use FICO scores in their evaluation and approvals process for credit applications.

[4] http://www.privacyrights.org/fs/fs6c-CreditScores.htm.

A FICO score aggregates information from an individual's credit record like his payment history (most particularly the presence of adverse public records such as bankruptcy and delinquency) and current amounts owed.[5] It's also worth noting the kinds of information that are not used in credit scores. By law, credit scores cannot use information on race, color, national origin, sex, and marital status. Further, FICO scores do not use age, assets, salary, occupation, and employment history.

These scores appear to affect the extension of consumer credit in four primary ways.

1. Credit terms (e.g. interest rates) improve with a person's credit score.

2. The presence of adverse public records (e.g. a bankruptcy) lowers an individual's score and removal can substantially raise it.

3. Taking on more debt (paying off debt) tends to lower (raise) credit scores.

4. Credit scores are mean reverting.

The Fico website (http://www.myfico.com/myfico/Credit Central/LoanRates.asp) documents the negative relationship between FICO scores and average interest rates on loans. Item 2 is consistent with evidence provided in Musto [16], as well as Fisher, et. al. [9]. Using data from the PSID and SCF, Fisher, et. al. document that a higher percentage of post-bankruptcy households were denied access to credit. Musto found (p.735) "there is a strong tenth year effect for the best initial credits...these consumers move ahead of 19% of the nonfiler population in apparent creditworthiness when their flags are removed." Furthermore, he states (p.740) "...the boost translates to significant new credit access for these filers over the ensuing year". Items 1 and 2 taken together imply that an individual who fails to pay back an unsecured loan will experience an adverse change in the terms of (unsecured) credit. Thus, a failure to pay back a loan adversely impacts the terms of credit and may

[5]The score also takes into account the length of a person's credit history, the kinds of credit accounts (retail credit, installment credit etc.) and the borrowing capacity (or line of credit) on each account.

result in outright denial of credit. Item 3 is consistent with the advice given by FICO for improving one's credit score.[6] Item 3 in conjunction with item 1 indicates that even absent default, the terms of credit on unsecured credit worsen as an individual gets further into debt – people face a rising marginal cost of funds. Item 4 is documented by Musto [16].

These facts suggest the following characterization of the workings of the unsecured consumer credit market. Given the inability of borrowers to commit to pay back, lenders condition the terms of credit (including whether they lend at all) on an individual's credit history encapsulated by a credit score. Individuals with higher scores are viewed by lenders as less likely to default and receive credit on more attractive terms. A default may signal something about the borrower's future ability to repay and leads to a drop in the individual's credit score. Consequently, post-default access to credit is available on worse terms and may not be available at all. Even absent default, greater indebtedness may signal something about the borrower's future ability to repay which subsequently leads to a lower credit score and worse terms of credit.

There is now a fairly substantial literature (beginning with Kehoe and Levine [13]) on how and to what extent borrowing can occur when agents cannot commit to pay back. This literature typically assumes that a default triggers permanent exclusion from credit markets. A challenge for this literature is to specify a structure with free entry of lenders and where lenders cannot collude to punish defaulters that can make quantitative sense of the characterization of a competitive unsecured consumer credit market with on-the-equilibrium-path default offered in the previous paragraphs. This paper take steps toward meeting this challenge.[7] We consider an environment with a continuum of infinitely-lived agents who at any point in time may be one of two types that affect their earnings realizations and preferences. An agent's type is drawn independently from others and follows a persistent two-state

[6] To improve a score, FICO advises to "Keep balances low on credit card and 'other revolving credit"' and "[p]ay off debt rather than moving it around". Source:www.myfico.com/CreditEducation/ImproveYour Score

[7] In Chatterjee, et.al. [5] we show that credit can be supported even in a finite horizon model where trigger strategies cannot support credit.

Markov process. Importantly, a person's type and earnings realizations are unobservable to the lender.[8]

These people interact with competitive financial intermediaries that can borrow in the international credit market at some fixed risk-free rate and make one-period loans to individuals at an interest rate that reflects that person's risk of default.[9] Because differences in earnings distributions and preferences bear on the willingness of each type of agent to default, intermediaries must form some assessment of a person's type which is an input into his credit score. We model this assessment as a Bayesian inference problem: intermediaries use the recorded history of a person's actions in the credit market to update their prior probability of his or her type and then charge an interest rate that is appropriate for that posterior. The fundamental inference problem for the lender is to assess whether a borrower or a defaulter is a chronically "risky" type or just experiencing a temporary shortfall in earnings. A rational expectations equilibrium requires that a lender's perceived probability of an agent's default must equal the objective probability implied by the agent's decision rule. Incorporating this equilibrium Bayesian credit scoring function into a dynamic incomplete markets model is the main technical challenge of our paper.

We model the pricing of unsecured consumer loans in the same fashion as in our predecessor paper Chatterjee, *et.al.* [4]. As in that paper, all one-period loans are viewed as discount bonds and the price of these bonds depend on the size of the bond. This is necessary because the probability of default (for any type) will depend on the size of the bond (i.e., on the person's liability). If the bond price is independent of the size of the loan and other characteristics, as it is in Athreya [1], then large loans which are more likely to be defaulted upon must be subsidized by small loans which are less likely to be defaulted upon. But with competitive credit markets, such cross subsidization of pooling contracts will fail to be an equilibrium. This reasoning is corroborated by recent empirical work by Edelberg [8] who

[8]Ausubel [2] documents adverse selection in the credit market both with respect to observable and unobservable household characteristics.

[9]Our earlier paper Chatterjee, et. al. [4] shows that there is not a big gain to relaxing the fixed risk-free rate assumption.

finds that there has been a sharp increase in the cross-sectional variance of interest rates charged to consumers.

In Chatterjee, *et.al.* [4], we also assumed that the price of a one-period bond depended on certain observable household type characteristics like whether households were blue or white collar workers. Here we assume those characteristics are not observable but instead assume that the bond depends on the agent's probability of repayment, in other words, his credit score. The probability of repayment depends on the posterior probability of a person being of a given type *conditional* on selling that particular sized bond. This is necessary because the two types will not have the same probability of default for any given sized bond and a person's asset choice is potentially informative about the person's type. With this asset market structure, competition implies that the expected rate of return on each type of bond is equal to the (exogenous) risk-free rate.

This is possibly the simplest environment one could imagine that could make sense of the observed connection between credit history and the terms of credit. Suppose it turns out that, in equilibrium, one type of person, say type g, has a lower probability of default. Then, under competition, the price of a discount bond (of any size) could be expected to be positively related to the probability of a person being of type g. Further, default will lower the *posterior* probability of being of type g because type g people default less frequently. This provides the basis for a theory why people with high scores are offered credit on more favorable terms.This would explain the fact that people with high scores are offered credit on more favorable terms.

There are two strands of existing literature to which our paper is closely related. The first strand relates to Diamond's [7] well-known paper on acquisition of reputation in debt markets. Besides differences in the environment (e.g. preferences in his case are risk neutral), the main difference is that here the decision to default is endogenous while in Diamond it happens exogenously. The second strand relates to the paper of Cole, Dow and English [6] on sovereign debt. In their setting a sovereign who defaults is shut out of international credit markets until such time as the sovereign makes a payment on the defaulted debt. Chapter 7 bankruptcy law, which we consider here, results in discharge of uncollateralized debt.[10] Further, the law does not permit individuals to simultaneously accumulate assets during the discharge of debt granted by the bankruptcy court.[11]

Our framework has the ability to address an interesting question that arises from Musto's empirical work. What are the effects on consumption smoothing and welfare of imposing legal restrictions (like the Fair Credit Reporting Act), which requires adverse credit information (like a bankruptcy) to be stricken from one's record after a certain number of years (10 in the U.S.)? Specifically, Musto p. 726 states that his empirical "results bear on the informational efficiency of the consumer credit market, the efficacy of regulating this market with reporting limits, and the quality of postbankruptcy credit access, which in turn bears on the incentive to file in the first place." He finds p. 747 "the removal of the flag leads to excessive credit, increasing the eventual probability of default. This is concrete evidence that the flag regulation has real economic effects. This is market efficiency in reverse." We use our model to assess this efficiency concern. In a world of incomplete markets and private information, flag removal may provide insurance to impatient agents in our framework that competitive intermediaries may not be able to provide. Hence extending the length of time that bankruptcy flags remain on credit records may not necessarily raise ex-ante welfare. This issue echoes Hart's [11] examples where the opening of a market in a world of incomplete markets may make agents worse off and Hirschleifer's [12] finding regarding the potential inefficiency of revealing information.

The paper is organized as follows. Section 2 describes a model economy where there are no restrictions on information about asset market behavior, defines an equilibrium, and discusses existence. Section 3 describes a model economy where there are restrictions on what

[10] Given the choice between Chapter 7 and 13, individuals would choose to file Chapter 13 only if they wished to keep assets they would lose under a Chapter 7 filing. Since borrowers in our model have negative net worth (there is only one asset), Chapter 7 is always the preferred means to file for bankruptcy.

[11] This fact rules out the purchase of consumption insurance from savings in the period of discharge studied by Bulow and Rogoff [3].

information on asset market behavior can be kept in an agents credit history. In particular, we assume that information can be kept only for a finite amount of time and that there are partitions on what asset transactions are recorded. These restrictions on information are intended to capture the requirement that adverse events be stricken from an individual's credit history and the fact that credit scores are based on debt transactions rather than assets in the current system. Section 4 estimates parameters of the model of Section 3 to match certain key moments in the data. Section 5 studies the properties of the model. Section 6 assesses the welfare consequences of restrictions on asset market information used by credit scoring agencies like that in the model of Section 3 compares to the unrestricted case of Section 2. This exercise sheds some light on the impact of the Fair Credit Reporting Act.

2 Model Economy 1

2.1 People, Preferences and Endowments

Time is discrete and indexed by $t = 0, 1, 2, \ldots$ There is a unit measure of infinitely-lived people alive at each date. At each date, a person can be one of two types, denoted $i_t \in \{g, b\}$. An individual of type g (or b) at time t can become an individual of type b (or g) at the beginning of time $t+1$ with probability $\Gamma_{\{i_{t+1}=b, i_t=g\}} \in (0,1)$ (or $\Gamma_{gb} \in (0,1)$), respectively.[12] Let γ denote the unconditional probability that an individual is of type H. An individual of type i_t draws her endowment e_t independently (across time and agents) from a probability space $(E_i, \mathcal{B}(E_i), \Phi_i)$, where $E_i = [\underline{e}_i, \overline{e}_i] \subset \mathbb{R}_{++}$ is a strictly positive closed interval and $\mathcal{B}(E_i)$ is the Borel sigma algebra generated by E_i.

Denote the life-time utility from a non-negative stream of current and future consumption $\{c_t, c_{t+1}, c_{t+2}, \ldots\}$ of an individual who is of type i_t by $U_i(c_t, c_{t+1}, c_{t+2}, \ldots, \theta_t)$ where $\theta_t \in \Theta$ is an independent (across time and agents) time preference shock drawn at time t from a

[12] This is a similar assumption to Phelan [17], who studies reputation aquisition by a government.

finite set with probability mass function Λ. For each i, $U_i(c_t, c_{t+1}, c_{t+2}, \ldots, \theta_t)$ is defined by the recursion

$$U_i(c_t, c_{t+1}, c_{t+2}, \ldots, \theta_t) = u_i(c_t) + \beta_i \theta_t \sum_{j, \theta_{t+1}} \Gamma_{ji} U_j(c_{t+1}, c_{t+2}, c_{t+3}, \ldots, \theta_{t+1}) \Lambda(\theta_{t+1}) \tag{1}$$

where, for all i, $u_i(c_t) : \mathbb{R}_+ \to \mathbb{R}$ is a bounded, continuous, twice differentiable and strictly concave function and $\beta_i \in [0, 1)$.

Importantly, we assume that a person's type i_t, endowment e_t, and time preference shock θ_t are unobservable to others.

2.2 Default Option and Market Arrangement

There is a competitive credit industry that accepts deposits and makes loans to individuals. We assume that there is a finite set $L \subset \mathbb{R}$ of possible loans or deposits (L contains negative and positive elements as well as 0). If an individual takes out a loan $\ell_{t+1} < 0$ at time t there is some probability p_t that the individual will repay ℓ_{t+1} units of goods at time $t+1$. If $\ell_{t+1} > 0$ then the individual makes a deposit which we assume that the intermediary promises to pay back with probability $p_t = 1$ for simplicity.

A probability of repayment $p_t < 1$ reflects the possibility of default on the part of the individual. We model the default option to resemble, in procedure, a Chapter 7 bankruptcy filing. If an individual defaults, the individual's beginning of period liabilities are set to zero (i.e., the individual's debt is discharged) and the individual is not permitted to enter into new contracts in the period of default.

There is a competitive market in financial contracts. The unit price of a financial contract (ℓ_{t+1}, p_t) is $q(\ell_{t+1}, p_t)$. For $\ell_{t+1} < 0$, $q(\ell_{t+1}, p_t) \cdot (-\ell_{t+1})$ is the amount received by an indvidual at time t who promises to pay ℓ_{t+1} next period with probability p_t. For $\ell_{t+1} > 0$, $q(\ell_{t+1}, 1) \cdot \ell_{t+1}$ is the amount handed over by the individual at time t in return for the certain promise to receive ℓ_{t+1} next period.

As noted earlier, there are two types of people in this economy. Let $s_t \in [0, 1]$ be the prior probability at time t that a person is of type g. Beliefs about an individual's type are important to lenders because the probability of repayment on a consumer loan may (and will) vary across types. An important part of the market arrangement is the existence of an agency that collects information on financial transactions of every individual and, using this information, estimates the probability s_t that a given individual is of type g at time t. We call an individual's estimated repayment probability the individual's *credit score*. We call the agency that computes this score the *credit scoring agency*. And, we call the type probability s on which the credit score (or the repayment probability) is based an individual's *type score*.[13]

Thus the existence of the credit scoring agency implies the presence of two functions that are part of the market arrangement. First, there is a *credit scoring function* $p(\ell_{t+1}, \psi)$ which gives the estimated probability of repayment on a loan $\ell_{t+1} < 0$ taken out by an individual with type score ψ. And, second, there is a *type score updating function* $\psi(d_t, \ell_{t+1}, \ell_t, s_t)$ which gives an individual's type score at the start of next period conditional on having begun the current period with asset ℓ_t and type score s_t and choosing (d_t, ℓ_{t+1}) – a choice of default corresponds to the 2-tuple $(1, 0)$ and choice of loan/deposit ℓ_{t+1} corresponds to the 2-tuple $(0, \ell_{t+1})$ (the precise definitions of these functions will be given in the next section).

[13]Nothing depends on the assumption that there are only two types. With $N > 2$ types, we could let s_t be a $N - 1$ length vector (and correspondingly ψ be a vector valued function). Even in this case, the credit score p_t is just the probability of repayment on a loan.

2.3 Decision Problems

2.3.1 People

Let a current variable, say a_t, be denoted a and let next period's variable a_{t+1} be denoted a'. In the special case of assets/liabilities we will let ℓ_{t+1} be denoted y and ℓ_t be denoted x. Let $\mathbb{Y} = \{(d,y) : (d,y) \in (0 \times L) \text{ or } (d,y) = (1,0)\}$ be the set of possible (d,y) choices (recall that a person can borrow or save only if she does not default and if she defaults then she cannot borrow or save).

Each individual takes as given

- the price function $q(y,p) : \{L_{--} \times [0,1]\} \cup \{L_{+} \times \{1\}\} \rightarrow R$,
- the credit scoring function $p(y,s') : L_{--} \times [0,1] \rightarrow [0,1]$, and
- the type scoring function $\psi(d,y,x,s) : \mathbb{Y} \times L \times [0,1] \rightarrow [0,1]$.

We can now develop the recursive formulation of an individual's decision problem. The state variables for an individual are (i,e,x,s). We begin with the definition of the set of feasible actions.

Definition 1 Given (e,x,s), the *set of feasible actions* is a finite set $B(e,x,s;\, q,p,\psi) \subset \mathbb{Y}$ that contains (i) all $(0,y)$ $y < 0$ such that $c = e + x - q(y,p(y,s')) \cdot y \geq 0$, where $s' = \psi(d,y,x,s)$, (ii) all $(0,y)$ $y \geq 0$ such that $c = e + x - q(y,1) \cdot y \geq 0$ and (iii) if $x < 0$ it also contains $(1,0)$.

Observe that the feasible action set does not depend on i nor θ since these are not directly known either to financial intermediaries or to the credit scoring agency. Of course, the credit scoring agency assigns probabilities to the individual being of a good type, ψ, and the set of feasible actions does depend on these probabilities. The dependence of the feasible action set on the functions p, q and ψ is noted.

We permit randomization so individuals choose probabilities over elements in the set of feasible actions. We will use $m(d, y) \in [0, 1]$ to denote the probability mass on the element $(d, y) \in \mathbb{Y}$ and m as the choice probability vector.

Definition 2 Given (i, e, θ, x, s) the *feasible choice set* $M_i(e, \theta, x, s; q, p, \psi)$ is the set of all $m \geq 0$ such that (i) $m(d, y) = 0$ for all $(d, y) \notin (B(e, x, s; q, p, \psi))$ and (ii) $\sum_{(d,y)\in\mathbb{Y}} m(d, y) = 1$.

In order to keep the type score updating function well defined across all actions (thereby avoiding having to supply an exogenous set of off-the-equilibrium-path beliefs) , we will assume each probability vector m assigns some (very small) probability $\epsilon > 0$ on every feasible action (i.e. $m \geq \epsilon$). This is similar to the "trembling hand" assumption made in Selten [18] and can be interpreted as "tiny mistakes".

Given (i, e, θ, x, s) and the functions p, q and ψ, the current-period return of a type i individual from choosing a feasible action $(0, y)$ is

$$R_i(0, y; e, x, s, q, p, \psi) = \begin{cases} u_i(e + x - q(y, p(y, \psi(0, y, x, s)) \cdot y) & \text{if } y < 0 \\ u_i(e + x - q(y, 1) \cdot y) & \text{if } y \geq 0 \end{cases}$$

and the current-period return from choosing $(1, 0)$ (if this choice is feasible) is

$$R_i(1, 0; e, x, s, q, p, \psi) = u_i(e).$$

Denote by $W_i(x, s) : L \times [0, 1] \to R$ the value of a type i individual *before* learning the current realization of their endowment, time preference, and type shocks. Then, a currently type-i individual's recursive decision problem is given by

$$V_i(e, \theta, x, s; q, p, \psi, W_i) = \max_{m\in M_i(e,x,s;q,p,\psi), m\geq\epsilon} \sum_{(d,y)} [R_i(d, y; e, x, s, q, p, \psi) + \beta_i\theta W_i(y, \psi(d, y, x, s))] \cdot m(d, y).$$

A solution exists if there exists a unique set of value functions $W_i^*(\cdot)$, $i \in \{g, b\}$ such that

$$W_i^*(x, s; q, p, \psi) = \sum_{j \in \{g,b\},\theta} \Gamma_{ji} \int_E V_j(e, \theta, x, s; q, p, \psi, W_j^*)\Phi_j(de)\Theta(\theta) \; \forall i \in \{g, b\}.$$

Denote the set of optimal decision rules by $M_i^*(e, \theta, x, s; q, p, \psi)$ and let $m_i^{\epsilon *}(e, \theta, x, s, q, p, \psi) \in M_i^*(e, \theta, x, s; q, p, \psi)$.

2.3.2 Financial Intermediary

The (representative) financial intermediary has access to an international credit market where it can borrow or lend at the risk-free interest rate $r \geq 0$. The intermediary operates in a competitive market and takes the price function $q(y, p)$ as given. The profit $\pi(y, p)$ on financial contract of type (y, p) is:

$$\pi(y, p) = \begin{cases} (1 + r)^{-1} p \cdot (-y) - q(y, p) \cdot (-y) & \text{if } y < 0 \\ q(y, 1) \cdot y - (1 + r)^{-1} \cdot y & \text{if } y \geq 0 \end{cases} \tag{2}$$

Let $\mathcal{B}(L \times [0, 1])$ be the Borel sets of $L \times [0, 1]$. Let $\mathcal{A}$ be the set of all measures defined on the measurable space $(L \times [0, 1], \mathcal{B}(L \times [0, 1]))$. For $\alpha \in \mathcal{A}$, $\alpha(y, P)$ is the measure of financial contracts of type $(y, P) \in \mathcal{B}(L \times [0, 1])$ sold by the financial intermediary. The decision problem of the financial intermediary is:

$$\max_{\alpha \in \mathcal{A}} \int \pi(y, p) \, d\alpha(y, p).$$

2.3.3 Credit Scoring Agency

We do not explicitly model the process by which the credit scoring agency computes type scores and credit scores. Instead, we impose restrictions on the outcome of this process. Specifically we assume that (i) $p(y, s')$ is the actual fraction of people with loan y and type score s' who repay and (ii) $\psi(d, y, x, s)$ is the actual fraction of g type among people who start with assets x, type score s, and choose (d, y).

Condition (i) implies

$$\begin{aligned} p(y, s') \quad &= \quad s' \cdot \left[\sum_{\theta'} \int [1 - m_g(1, 0; e', \theta', y, s', q, p, \psi)] \Phi_g(de') \Lambda(\theta') \right] \\ &\quad + (1 - s') \cdot \left[\sum_{\theta'} \int [1 - m_b(1, 0; e', \theta', y, s', q, p, \psi)] \Phi_b(de') \Lambda(\theta') \right]. \end{aligned} \tag{3}$$

Condition (ii) implies

$$\begin{aligned} &\psi(d, y, x, s; q, p, \psi) = \\ &(1 - \Gamma_{bg}) \left[\frac{P_g(d, y; x, s, q, p, \psi) s}{P_g(d, y; x, s, q, p, \psi) s + P_b(d, y; x, s, q, p, \psi)(1 - s)} \right] \\ &+ \Gamma_{gb} \left[\frac{P_b(d, y; x, s, q, p, \psi)(1 - s)}{P_g(d, y; x, sp, q, \psi) s + P_b(d, y; x, s, q, p, \psi)(1 - s)} \right] \end{aligned} \tag{4}$$

where

$$P_i(d, y; x, s, q, p, \psi) = \sum_{\theta} \int m_i(d, y; e, \theta, x, s, q, p, \psi) \Phi_i(de) \Lambda(\theta). \tag{5}$$

2.4 Equilibrium

We can now give the definition of a recursive competitive equilibrium.

Definition 3. An ϵ-constrained recursive competitive equilibrium is a (i) a pricing function $q^*(y, p)$, (ii) a credit scoring function $p^*(y, s')$, (iii) a type scoring function $\psi^*(d, y, x, s; p^*, q^*, \psi^*)$ and (iv) a set of decision rules $m_i^{\epsilon *}(e, \theta, x, s, q^*, p^*, \psi^*)$ such that

1. $m_i^{\epsilon *}(e, \theta, x, s, q^*, p^*, \psi^*) \in M_i^*(e, \theta, x, s; q^*, p^*, \psi^*)$, $\forall i$,
2. $q^*(y, p)$ is such that $\pi(y, p; q^*(y, p)) = 0 \;\forall\; y$ and p,

3. $p^*(y, s')$ satisfies condition (3) for $m_i^{\epsilon *}(e, \theta, x, s, q^*, p^*, \psi^*)$, $i \in \{g, b\}$,

4. $\psi^*(d, y, x, s)$ satisfies (4) for $m_i^{\epsilon *}(e, \theta, x, s, q^*, p^*, \psi^*)$, $i \in \{g, b\}$.

2.5 Existence

From condition (2) in the definition of equilibrium, we know that prices q simply depend on p and a primitive r so the equilibrium problem reduces to finding a pair of functions $p^*(y, s')$ and $\psi^*(d, y, x, s)$ such that $(1) - (4)$ hold. To prove existence we need to establish the following steps.

1. Since both functions share the same domain, we can extend p in the following way. Let $\Omega = \{0, 1\} \times L \times L \times [0, 1]$ and let ω be an element of Ω. For $d = 0$ and $y < 0$, $p(d, y, x, s') = p(y, s')$ for all x;for $d = 0$ and $y \geq 0$, $p(d, y, x, s') = 1$ for all x and s'; for $d = 1$ and $y = 0$, $p(d, y, x, s') = 0$ for all x and s'.

2. Stack the functions to create the continuous vector valued function

$$f : \Omega \to [0, 1]^2 \equiv \begin{bmatrix} f^1(\omega) \\ f^2(\omega) \end{bmatrix} \equiv \begin{bmatrix} \psi(\omega) \\ p(\omega) \end{bmatrix}.$$

   Let F be the set of all such functions. Let $\|f\| = \max\{\sup_\omega f^1, \sup_\omega f^2\}$. Let $K \subset F$ be the set of functions for which (i) $f^1(\omega) \in B \subset [0, 1]$ where B is a closed interval in $[0, 1]$ with strictly positive elements, and (ii) $f^2(\omega) = 1$ for all $d = 0$, $y \geq 0$, x and s' and $f^2(d, y, x, s') = 0$ for all $d = 1, y = 0$, x and s'. Then K is a closed (in the max-sup norm), convex subset of F.

3. Define an operator $T(f) : K \to F$ in the following way. Given $f \in K$, solve the individuals problem to get $m_i(d, y; e, x, s, f)$. Then use (4) and (3) to get $T^1(f)$ and $T^2(f)$, respectively (to get $T^2(f)$ we need to extend the the "output" function over to Ω as in step 1 above).

4. Prove the following properties regarding T and K

   (a) $T(K) \subset K$

   (b) K is compact (i.e., if f_n is a sequence in K then there exists a subsequence $f_{n_k} \to \overline{f} \in K$).

   (c) T is continuous (i.e., if $f_n \to \overline{f}$ implies $T(f_n) \to T(\overline{f})$).

5. Use the following version of the Schauder Fixed Point Theorem to assert the existence of a fixed point. Let F be a normed vector space and let $K \subset F$ be a non-empty, convex, compact set. Then given any continuous mapping $T : K \to K$ there exists f such that $T(f) = f$.

To establish the conditions for the Schauder Fixed Point Theorem, we use the fact that a subset of F is compact if and only if it is bounded and forms an equicontinuous family. Further, a sufficient condition for any subset of F to be an equicontinuous family is that each component function of f (namely, f^1 and f^2) be Lipschitz with respect to the continuous variable (s and s',respectively) and the same Lipschitz constant applies to all functions in the family.

3 Model Economy 2

Now we describe a model economy where there are restrictions on what information on asset market behavior can be kept in an agents credit history. In particular, we assume that information can be kept only for a finite amount of time and that there are partitions on what asset transactions are recorded. These restrictions on information are intended to capture the requirement that adverse events be stricken from an individual's credit history and the fact that credit scores are based upon data on liabilities rather assets. We also assume that there are regulatory and technological reasons that restrict what credit scoring agencies and intermediaries can observe about an individual's priors.[14]

An individual's history of asset market actions (asset choices and default decisions) at the beginning of period t is given by (ℓ_t, h_t^T) where $h_t^T = (d_{t-1}, \ell_{t-1}, d_{t-2}, ..., \ell_{t+1-T}, d_{t-T}) \in \{0,1\} \times L \times \{0,1\} \times ... \times L \times \{0,1\} \equiv \mathcal{H}^T$, the set of possible histories of finite length $T \geq 1$. This definition directly incorporates the restriction that information can only be kept for a finite number, denoted T, periods. We formalize the restrictions on observability of asset transactions via partitions on $L \times \mathcal{H}^T$. Because all feasible actions are taken with at least probability ϵ, all feasible (ℓ_t, h_t^T) are possible along the equilibrium path. Let the particular subsets (or blocks) of the partition of $L \times \mathcal{H}^T$ be denoted $\Xi^T = \{H_1, ..., H_k\}$ which by the assumptions that L and T are finite is itself a finite set. The restriction that data on assets (which we take as $\ell_t \in L_{++}$) are not included in the credit scoring agency's information set is modelled by a measurability assumption that (ℓ_t, h_t^T) is constant on each block of Ξ^T. As an example, suppose that $T = 1$ and $L = \{\ell_-, 0, \ell_+^1, \ell_+^2\}$ with $\ell_- < 0 < \ell_+^1 < \ell_+^2$. Then $L \times \mathcal{H}^{T=1} = \{(0,1), (\ell_-, 0), (0,0), (\ell_+^1, 0), (\ell_+^2, 0)\}$ and the measurability assumption requires $H_1 = \{(0,1)\}$, $H_2 = \{(\ell_-, 0)\}$, $H_3 = \{(0,0)\}$ and $H_4 = \{(\ell_+^1, 0), (\ell_+^2, 0)\}$. To conserve on notation, let $H(\ell_t, h_t^T)$ denote one of the partition blocks $H_1, ..., H_k$. We use a similar notation, that is $A(\ell_{t+1}, d_t)$ is a partition block, to denote what an intermediary can observe regarding an individual's current actions (ℓ_{t+1}, d_t). For the case where $L = \{\ell_-, 0, \ell_+^1, \ell_+^2\}$, the partition block is given (coincidentally) by $A_1 = \{(0,1)\}$, $A_2 = \{(\ell_-, 0)\}$, $A_3 = \{(0,0)\}$ and $A_4 = \{(\ell_+^1, 0), (\ell_+^2, 0)\}$.

How does this change in the environment affect decision problems? Since these informational restrictions are only on the credit scoring agency (as well as the financial intermediary since it uses credit scores as an input into its pricing calculations), the individual's problem is basically identical to what we had in section 2.3.1. In particular, we simply substitute h^T for s in the individual state (i, e, θ, ℓ, s). Note that since h^T is a finite object, the state space

[14]For instance, prices which incorporate priors are considered proprietary and are excluded from standard credit histories.

is now finite except for exogenous earnings. We can also define the endogenous measure of individuals across the state space by $\mu_i(e, \theta, \ell, h^T)$.

The informational restrictions on the credit scoring agency affect both the credit scoring and type scoring functions in (3)-(5). In particular now the type scoring function is given by

$$\begin{aligned} \psi(\ell', d, \ell, h^T) &= (1 - \Gamma_{bg}) \left[\frac{\hat{P}_g(\ell', d, \ell, h^T) \cdot s^T}{\hat{P}_g(\ell', d, \ell, h^T) \cdot s^T + \hat{P}_b(\ell', d, \ell, h^T) \cdot (1 - s^T)} \right] \\ &+ \Gamma_{gb} \left[\frac{\hat{P}_b(\ell', d, \ell, h^T) \cdot (1 - s^T)}{\hat{P}_g(\ell', d, \ell, h^T) \cdot s^T + \hat{P}_b(\ell', d, \ell, h^T) \cdot (1 - s^T)} \right] \end{aligned} \tag{6}$$

where the prior of an agent's type is calculated from the population distribution

$$s^T = \sum_{(\ell, h^T) \in H(\ell, h^T)} \left[\int_{E_g} \sum_{\theta} \mu_g(e, \theta, \ell, h^T) \Phi_g(de) \Lambda(\theta) \right] \tag{7}$$

and

$$\hat{P}_i(\ell', d, \ell, h^T) = \sum_{(\ell', d) \in A(\ell', d), (\ell, h^T) \in H(\ell, h^T)} P_i(\ell', d, \ell, h^T) \tag{8}$$

Then the credit scoring function is just as before

$$\begin{aligned} p(\ell', \psi) &= \psi(\ell', d, \ell, h^T) \cdot \left[\sum_{\theta'} \int [1 - m_g(1, 0; e', \theta', \ell', h^{T\prime}, q, p, \psi)] \Phi_g(de') \Lambda(\theta') \right] \\ &+ (1 - \psi(\ell', d, \ell, h^T)) \cdot \left[\sum_{\theta'} \int [1 - m_b(1, 0; e', \theta', \ell', h^{T\prime}, q, p, \psi)] \Phi_b(de') \Lambda(\theta') \right]. \end{aligned} \tag{9}$$

As can be easily seen, the key difference from (4)-(5) simply arises from the measurability restrictions in (7)-(8) and we use information on the distribution of agents in the economy μ to construct the "prior" likelihood that an agent with (ℓ, h^T) is of type g..

4 Moment Matching

According to the Fair Credit Reporting Act, a bankruptcy filing stays on an individual's credit record for 10 years. To keep the state space workable, we assume $T = 2$ so that a model period corresponds to 5 years. The discount rate β for both types is based on an annual discount rate of 0.96. The risk-free interest rate r is set to satisfy $\beta(1 + r) = 1$. We assume the time preference shock can take two values $\theta \in \{0, 1\}$ so that agents who receive the low shock are myopic for one period. This implies we need only pin down one probability for each type i, namely $\Lambda_i(0)$. The risk aversion coefficient σ is set to 2. We assume that the "tremble" parameter is $\varepsilon = 0.0001$. This is the probability that agents will play a suboptimal but feasible action by mistake. The remaining parameters are estimated, to which we now turn.

We assume an earnings process for low and high types that is similar to that in Chatterjee, et. al. [4]. It is assumed that each agent takes a random draw from an annual endowment distribution conditional on her type. The distribution of annual endowments for type i is assumed to have the following functional form:

$$\int_{\underline{e}_i}^{z} \Phi(e_i|\phi_i) = P(e_i \leq z|\phi_i) = \left[\frac{z - \underline{e}_i}{\bar{e}_i - \underline{e}_i}\right]^{\phi_i} \tag{10}$$

We use simulated method of moments to choose the parameters of the endowment distribution to match the earnings gini index, mean-to-median earnings ratio, coefficient of variation of earnings, autocorrelation of earnings, and the percentage of earnings for the second to fifth quintiles (see the Appendix 7.3 for our estimation algorithm). We used data from the PSID 2001-2005 to construct those statistics. We restrict our sample to households whose heads are between the age of 20 and 62. We calculate average annual earnings in the three survey years (2001, 2003, and 2005) and then multiply that number by five to get the average five year earnings estimate. The calculation of the autocorrelation of earnings takes advantage of the panel properties of the dataset. It uses earnings in 2001 and 2005 of the same households.

The estimated support of annual endowments for type b is [0.001,0.27] (we set $\underline{e}_b = 0.001$ exogenously since in the theory since we restrict the support to be strictly positive). The support of annual endowments for type g is [0.82,2.75]. The probability of type g switching to type b is estimated to be 0.01, while the probability of type b switching to type g is 0.02. This yields an invariant distribution where 2/3 of agents are type g. Table 1 summarizes the targeted earnings statistics and the parameter values chosen to match these statistics. As evident in the table, all parameters are statistically significant.

Table 1: Earnings Statistics (PSID 2001-2005) and Parameter Values

Statistics	Target	Model	Parameter	Estimate (s.e.)
Gini index	0.45	0.45	$\overline{e}_b$	0.27 (0.00048)
Mean/median	1.27	1.06	$\underline{e}_g$	0.82 (0.00140)
Coeff of variation	1.33	0.80	$\overline{e}_g$	2.75 (0.00460)
Autocorrelation	0.69	0.62	ϕ_g	0.50 (0.00005)
Percentage of 2nd quintile	8.67	2.55	ϕ_b	0.50 (0.00311)
Percentage of 3rd quintile	14.82	13.43	Γ_{gb}	0.02 (0.00008)
Percentage of 4th quintile	23.46	29.95	Γ_{bg}	0.01 (0.00017)
Percentage of 5th quintile	50.90	53.70		

Taking the earnings parameters as given, we then estimate the remaining parameters by matching data moments on delinquency and wealth statistics. The distribution of delinquency rates in Figure 1 allows us to construct the moments for overall and subprime delinquency rates. Other statistics, which include the debt to earnings ratio, asset to earnings ratio, and coefficient of variation of interest rates, are obtained from the 2004 SCF.

The set of asset choices L includes one borrowing level ($\underline{x}$), two saving levels ($\overline{x}_1$ and $\overline{x}_2$), and zero. The borrowing level is -0.03. The two saving levels are 0.01 and 2.31. Therefore, there are five elements in $\mathbb{Y}$, which are $\{(1,0),(0,-0.03),(0,0),(0,0.01),(0,2.31)\}$. The probability of the time preference shock is 4.07% for type g agents and 3.86% for type b agents. Table 2 summarizes the model statistics and the parameter values along with standard errors. As evident in the table, all parameters except for $\Lambda_b(0)$ are statistically significant.

Table 2: Model Statistics (TransUnion and SCF) and Parameter Values

Statistics	Target	Model	Parameter	Estimates (s.e.)
Targeted moments				
Overall delinquency rate	29.23%	37.07%	$\underline{x}$	-0.03 (0.0069)
Subprime (bottom 27%) del. rate	75.74%	47.83%	$\overline{x}_1$	0.01 (1.4250e-6)
Debt to earnings ratio	0.002	0.002	$\overline{x}_2$	2.31 (2.5775e-6)
Asset to earnings ratio	1.36	1.39	$\Lambda_g(0)$	0.0407 (0.0018)
Coeff. of var. of interest rates	0.43	0.59	$\Lambda_b(0)$	0.0386 (0.0778)
Untargeted moments				
Percentage in debt	6.7	6.6		
Percentage in zero assets	7.0	5.2		

5 Model Properties

Since credit scores are based on observed asset market decisions, we start by listing the equilibrium decision rules of agents. With $T = 2$, there are 13 possible (x, h^T) states that an individual will be in. If agents have the time preference shock (i.e. $\theta = 0$), they become perfectly myopic. In this case, they will default if they are in debt and will borrow if they are not in debt regardless of their earnings and any other characteristics. Tables 3 and 4 provide information on equilibrium decision rules for types g and b respectively, in the event the individual does not have a time preference shock. The tables provide the earnings levels for a given history tuple (x, h^T) that an agent finds the action (d, y) optimal. If an action is chosen by mistake, we list the earnings levels for which such an action is feasible (we denote this in the table as an interval following "ε for"). If an action is neither optimal nor feasible (i.e, agents can not choose this action even by mistake), then we place an empty set $\emptyset$ in that cell of the table. Finally, if the decision rule does not depend on history, we group all those (x, h^T) states into one row (which explains why there are not 13 rows in the tables. As can be seen in Tables 3 and 4, every action is taken by some agents in equilibrium (i.e.

there is no state that is infeasible for both types of agents).

It is clear from Table 3, that good type agents with debt $\underline{x}$ default when they have low earnings. They save to levels $\overline{x}_1$ and $\overline{x}_2$ the higher is their earnings. Next, good type individuals with zero assets choose to borrow if they receive low earnings or choose to save to levels $\overline{x}_1$ and $\overline{x}_2$ the higher is their earnings across histories h^T. When they choose to borrow, even if their asset positions are all zero, their history can affect their prices since the intermediary can use that information to better infer their types. Therefore, their feasibility sets are different and this affects their optimal actions given endowments. Next, good type individuals with asset level $\overline{x}_1$ choose to dissave if they receive low earnings or choose levels $\overline{x}_1$ and $\overline{x}_2$ the higher is their earnings across all histories. Finally, good type individuals with asset level $\overline{x}_2$ choose to stay at level $\overline{x}_2$ no matter what their earnings.

Table 3: Equilibrium Decision rules for type g agents with $\theta = 1$

$(x, h^T)\backslash(d, y)$	$(1, 0)$	$(0, \underline{x})$	$(0, 0)$	$(0, \overline{x}_1)$	$(0, \overline{x}_2)$
$(\underline{x}, (0, l_{-1}, d_{-2}))$	$[\underline{e}_g, 0.64]$	ε for $[\underline{e}_g, \overline{e}_g]$	ε for $[\underline{e}_g, \overline{e}_g]$	$[0.64, 2.16]$	$[2.16, \overline{e}_g]$
$(0, (1, \underline{x}, 0)$	NA	$[\underline{e}_b, 0.16]$	ε for $[\underline{e}_g, \overline{e}_g]$	$[0.16, 2.13]$	$[2.13, \overline{e}_g]$
$(0, (0, 0, 1)$	NA	$[\underline{e}_g, 0.16]$	ε for $[\underline{e}_g, \overline{e}_g]$	$[0.16, 2.13]$	$[2.13, \overline{e}_g]$
$(0, (0, \underline{x}, 0))$	NA	$[\underline{e}_g, 0.17]$	ε for $[\underline{e}_g, \overline{e}_g]$	$[0.17, 2.13]$	$[2.13, \overline{e}_g]$
$(0, (0, 0, 0))$	NA	$[\underline{e}_g, 0.16]$	ε for $[\underline{e}_g, \overline{e}_g]$	$[0.16, 2.13]$	$[2.13, \overline{e}_g]$
$(0, (0, \overline{x}_n, 0))$	NA	$[\underline{e}_g, 0.16]$	ε for $[\underline{e}_g, \overline{e}_g]$	$[0.16, 2.13]$	$[2.13, \overline{e}_g]$
$(\overline{x}_1, (0, l_{-1}, d_{-2})$	NA	$[\underline{e}_g, 0.15]$	ε for $[\underline{e}_g, \overline{e}_g]$	$[0.15, 2.12]$	$[2.12, \overline{e}_g]$
$(\overline{x}_2, (0, l_{-1}, d_{-2})$	NA	ε for $[\underline{e}_g, \overline{e}_g]$	ε for $[\underline{e}_g, \overline{e}_g]$	ε for $[\underline{e}_g, \overline{e}_g]$	$[\underline{e}_g, \overline{e}_g]$

Moving to Table 4, bad type agents with debt $\underline{x}$ default if they have low earnings or choose to save $\overline{x}_1$ if they have high earnings. Bad type individuals with zero assets choose to borrow if they receive low earnings or choose no assets or to save to level $\overline{x}_1$ the higher is their earnings across histories h^T. Similar to the cases for type g agents, when agents with zero assets choose to borrow, their prices can affect their feasibility and therefore their optimal actions. Next, bad type individuals with asset level $\overline{x}_1$ choose to dissave if they receive low earnings or choose level $\overline{x}_1$ the higher is their earnings across all histories. Finally, bad type

Table 4: Equilibrium Decision rules for type b agents with $\theta = 1$

$(x, h^T)\backslash(d, y)$	$(1, 0)$	$(0, \underline{x})$	$(0, 0)$	$(0, \overline{x}_1)$	$(0, \overline{x}_2)$
$(\underline{x}, (0, l_{-1}, d_{-2}))$	$[\underline{e}_b, 0.09]$	ε for $[0.01, \overline{e}_b]$	ε for $[0.03, \overline{e}_b]$	$[0.09, \overline{e}_b]$	$\emptyset$
$(0, (1, \underline{x}, 0)$	NA	$[\underline{e}_b, 0.01]$	$[0.01, 0.03]$	$[0.03, \overline{e}_b]$	$\emptyset$
$(0, (0, 0, 1)$	NA	$[\underline{e}_b, 0.01]$	$[0.01, 0.03]$	$[0.03, \overline{e}_b]$	$\emptyset$
$(0, (0, \underline{x}, 0))$	NA	$[\underline{e}_b, 0.01]$	$[0.01, 0.03]$	$[0.03, \overline{e}_b]$	$\emptyset$
$(0, (0, 0, 0))$	NA	$[\underline{e}_b, 0.01]$	$[0.01, 0.03]$	$[0.03, \overline{e}_b]$	$\emptyset$
$(0, (0, \overline{x}_n, 0))$	NA	$[\underline{e}_b, 0.01]$	$[0.01, 0.03]$	$[0.03, \overline{e}_b]$	$\emptyset$
$(\overline{x}_1, (0, l_{-1}, d_{-2})$	NA	$[\underline{e}_b, 0.01]$	$[0.01, 0.02]$	$[0.02, \overline{e}_b]$	$\emptyset$
$(\overline{x}_2, (0, l_{-1}, d_{-2})$	NA	ε for $[\underline{e}_b, \overline{e}_b]$	ε for $[\overline{e}_b, \overline{e}_b]$	ε for $[\underline{e}_b, \overline{e}_b]$	$[\underline{e}_b, \overline{e}_b]$

individuals with asset level $\overline{x}_2$ choose to stay at level $\overline{x}_2$ no matter what their earnings are.

Given the earnings distributions Φ_i, the two decision rules imply certain properties for the type scoring function: an observation of default is more likely to come from a bad type individual; an observation of borrowing is more likely to come from a bad type individual too. Since the credit scoring function depends on the type scoring function via (9), this behavior translates into implications for credit scores. Figure 2 graphs the mapping between type scores and credit scores. The red dotted line plots the linear regression between the two and illustrates that there is not a perfect fit. This can be seen from equations (6)-(9). If the equilibrium borrowing actions are independent of the state/history tuple, then there is a direct mapping between s^T and ψ in (6). Hence, the higher is s^T, the higher is ψ. Because type g agents default less often, this translates via (9) into higher p. However, since the equilibrium borrowing action depends upon the state/history tuples (which can be seen from the equilibrium decision rules in Table 3 and 4, the type scores do not map perfectly into credit scores. We can nonetheless still clearly see from Figure 2 that type scores and credit scores are highly positively correlated. The correlation coefficient weighted by the distribution measure is 0.9327.

One way to test the model is to see if it can predict the four key properties of credit score facts stated in section 1. To do so, the dynamics of credit scores need to be constructed.

Let the credit score $p(\underline{x}, \Psi(\underline{x}, 0, \ell_t, h_t^T)) \equiv \rho(\underline{x}, 0, \ell_t, h_t^T)$ be the probability of repayment on a loan of size $\underline{x}$ by an agent in state (ℓ_t, h_t^T). It is possible to construct a credit score like this even if the agent does not choose (other than by mistake) to borrow in equilibrium. Then we can also use optimal decision rules to construct the agent's new credit score, denoted by $\rho'(\underline{x}, 0, \ell_{t+1}, h_{t+1}^T)$, according to his optimal actions last period and his updated history tuple.

1. Interest rates fall as a person's credit score rises.

   Since higher credit simply mean a higher probability of repayment and intermediaries earn zero profits, this implies a negative relation between credit score and interest rates as in the data (see Figure 3).

2. Default lowers a person's score, removal raises it.

   Default lowers an individual's type score because type b are more likely to default than type g. Specifically, type g agents default only if they get the time preference shock in this equilibrium, but type b agents also default if they receive low earnings. As Table 5 shows, for most histories (except for $(\underline{x}, 0, 0, 1)$) where there is debt and subsequent default, an individual's credit score falls.

   Table 5: Updates of credit scores after default

(ℓ_t, h_t^T)	$\mu(\ell_t, h_t^T)$	$\rho(\underline{x}, 0, \underline{x}, h_t^T)$	$\rho'(\underline{x}, 0, 0, 1, \underline{x}, 0)$
$(\underline{x}, 0, 0, 0)$	0.0059	0.4350	0.4240
$(\underline{x}, 0, 0, 1)$	0.0052	0.4194	0.4240
$(\underline{x}, 0, \underline{x}, 0)$	ϵ	0.7033	0.4240
$(\underline{x}, 0, l_{t-1} > 0, 0)$	0.0551	0.7063	0.4240

   From Table 6, we can see that agents have higher credit scores once their default history is erased in most cases except for when they have the state and history tuple $(l_t > 0, (0, 0, 1))$ and choose $(0, 0)$ because type g agents are a lot less likely to choose zero assets than type b agents (they either borrow when they receive a time preference shock or simply save).

Figure 2: Mapping between type scores and credit scores

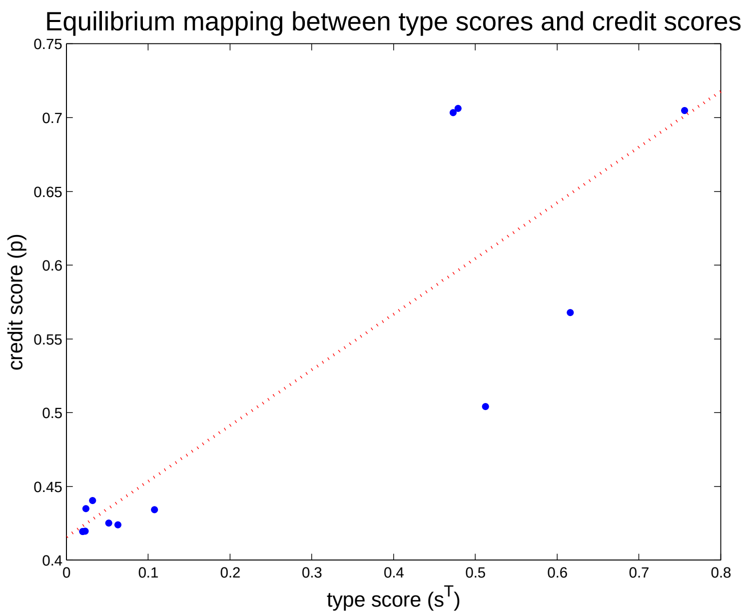


Figure 3: Credit scores and interest rates

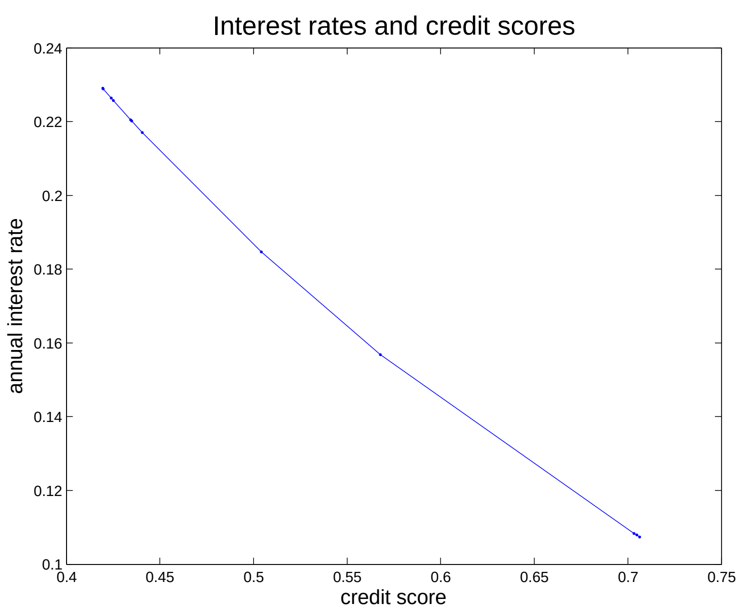


Table 6: Updates of credit scores after removal of default history

(ℓ_t, h_t^T)	$\mu(\ell_t, h_t^T)$	$\rho(\underline{x}, 0, l_t, d_{t-1}, 0, 1)$	$\rho'(\underline{x}, 0, \underline{x}, 0, l_t, d_{t-1})$ $\rho'(\underline{x}, 0, 0, 0, l_t, d_{t-1})$ $\rho'(\underline{x}, 0, l_{t+1} > 0, 0, l_t, d_{t-1})$
$(\underline{x}, (0, 0, 1))$	0.0052	0.4406	0.7033 0.5141 0.5677
$(0, (0, 0, 1))$	0.0029	0.4194	0.4350 0.5141 0.4252
$(l_t > 0, (0, 0, 1))$	0.0164	0.4344	0.7063 0.4197 0.7047

As in Musto [16], we can compute the changes in percentiles of the distribution of credit scores following a removal of the bankruptcy flag from one's record. Musto [16] categorized bankrupt households according to their initial post-default percentage in the distribution of credit scores and kept track of them for ten years (the length of time the bankruptcy record stays in their credit history by the FCRA). Since $T = 2$, there is not a lot of variation in state/history tuples after default; here it is simply $(0, (1, \underline{x}, 0))$ and this falls within the first quintile of the distribution at 5.16%. After two model periods when their default record is erased, an individual's new credit score on average increases 5% to around 13.85%. Musto found that for individuals in the first quintile of credit scores, they jumped ahead of 5% of households post default annually. However, these households are not the group in which people are mostly affected by the information restriction. If we raise $T > 2$ we should find more heterogeneity in post default scores which would map to Musto's dataset better.

3. Taking on more debt (paying off debt) tends to lower (raise) credit scores.

In the model, agents may choose to be in debt because they have a time preference shock even if they are type g agents. Therefore, there is a lot of mixing for agents who go into debt. This makes it hard to match the prediction that increasing indebtedness always lowers scores. As Table 7 shows, in some cases this prediction holds, while in others it doesn't. For instance, when agents dissave from positive assets to zero assets, their credit scores drop. However, when agents actually go into debt from positive or zero assets, their credit scores may rise. As Table 8 shows, the model predicts the increase in credit scores after agents pay off debts for agents with state/history tuple $(\underline{x}, (0, 0, 1))$ but the model does poorly in other cases.

Table 7: Updates of credit scores after taking on more debt

(ℓ_t, h_t^T)	$\mu(l_t, h_t^T)$	$\rho(\underline{x}, 0, 0, h_t^T)$	$\rho'(\underline{x}, 0, \underline{x}, 0, 0, \{0, 1\})$
$(0, 0, 0, 0)$	0.0036	0.4194	0.4350
$(0, 0, 0, 1)$	0.0029	0.4194	0.4350
$(0, 0, \ell_{t-1} > 0, 0)$	5.2541e-6	0.4197	0.4350
$(0, 1, \underline{x}, 0)$	0.0245	0.4240	0.4406
(ℓ_t, h_t^T)	$\mu(l_t, h_t^T)$	$\rho(\underline{x}, 0, l_t > 0, h_t^T)$	$\rho'(\underline{x}, 0, \underline{x}, 0, \ell_t > 0, 0)$
$(\ell_t > 0, 0, 0, 0)$	0.0178	0.4252	0.7063
$(\ell_t > 0, 0, 0, 1)$	0.0164	0.4344	0.7063
$(\ell_t > 0, 0, \ell_{t-1} > 0, 0)$	0.8065	0.7047	0.7063
$(\ell_t > 0, 0, \underline{x}, 0)$	0.0416	0.5677	0.7063
(ℓ_t, h_t^T)	$\mu(l_t, h_t^T)$	$\rho(\underline{x}, 0, \ell_t > 0, h_t^T)$	$\rho'(\underline{x}, 0, 0, 0, \ell_t > 0, 0)$
$(\ell_t > 0, 0, 0, 0)$	0.0178	0.4252	0.4197
$(\ell_t > 0, 0, 0, 1)$	0.0164	0.4344	0.4197
$(\ell_t > 0, 0, \ell_{t-1} > 0, 0)$	0.8065	0.7047	0.4197
$(\ell_t > 0, 0, \underline{x}, 0)$	0.0416	0.5677	0.4197

4. Scores are mean reverting.

Figure 4 graphs the average credit score given current credit scores by using the equilibrium decision rules [15]. It can be seen that agents with lower (higher) credit scores tend to

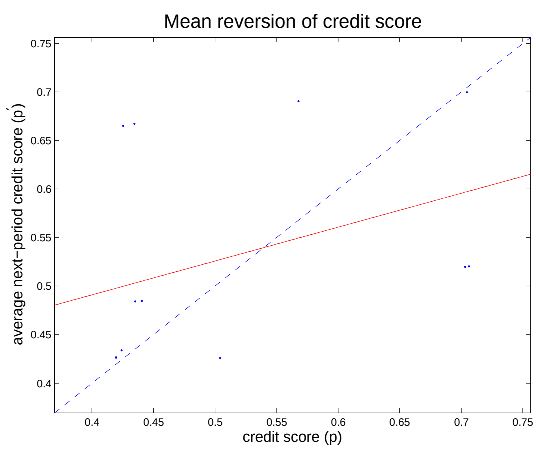
[15] The average next-period credit score given (x, h^T) is calculated as

$$\frac{\sum_i \left[\int_{E_i} \sum_{\theta,(d,y)} \rho(\underline{x}, 0, d, y, l, d_{-1}) m_i(d, y; e, \theta, x, h^T) \mu_i(e, \theta, l, h^T) \Phi_i(de) \Lambda(\theta)\right]}{\sum_i \left[\int_{E_i} \sum_\theta \mu_i(e, \theta, l, h^T) \Phi_i(de) \Lambda(\theta)\right]}$$

Table 8: Updates of credit scores after paying off debt

(ℓ_t, h_t^T)	$\mu(\ell_t, h_t^T)$	$\rho(\underline{x}, 0, \ell_t, h_t^T)$	$\rho'(\underline{x}, 0, 0, 0, \underline{x}, 0)$ $\rho'(\underline{x}, 0, l_{t+1} > 0, 0, \underline{x}, 0)$
$(\underline{x}, 0, 0, 0)$	0.0059	0.7033	0.5141 0.5677
$(\underline{x}, 0, 0, 1)$	0.0052	0.4406	0.5141 0.5677
$(\underline{x}, 0, \underline{x}, 0)$	ϵ	0.7033	0.5141 0.5677
$(\underline{x}, 0, l_{t-1} > 0, 0)$	0.0428	0.7063	0.5141 0.5677

Figure 4: Mean reversion of credit scores



have higher (lower) credit scores next period. Therefore, the (red) linear regression line has a flatter slope than the (blue dotted) 45 degree line.

6 Policy Experiment

Here we use the model to address a question about the welfare consequences of imposing legal restrictions (like the Fair Credit Reporting Act), which requires adverse credit information (like a bankruptcy) to be stricken from one's record after a certain number of years (10 in the U.S.). As discussed in the introduction, in a world of incomplete markets and private information, flag removal may provide insurance to impatient agents in our framework that competitive intermediaries may not be able to provide. Hence extending the length of time that bankruptcy flags remain on credit records may not necessarily raise ex-ante welfare. This issue is similar to Hart's [11] examples where the opening of a market in a world of incomplete markets may make agents worse off and Hirschleifer's [12] finding regarding the potential inefficiency of revealing information.

To assess this question, we compute consumption equivalents using the following formulas. Say the PDV of utility starting in state (i, e, θ, x, h^T) for a given T is given by

$$v(i, e, \theta, h^T; T) = E_i \left[\sum_{t=0}^{\infty} (\beta\theta)^t \frac{c_t(i, e, \theta, h^T; T)^{1-\gamma}}{1-\gamma} \right].$$

To assess how much an agent in history (i, e, θ, x, h^T) would be willing to pay forever to be in a regime where $T = \infty$ and there are no partitions, for each (i, e, θ, h^T) we compute $\lambda(i, e, \theta, x, h^T)$ such that

$$\begin{aligned} v(i, e, \theta, x, h^\infty; \infty) &= E_i \left[\sum_{t=0}^{\infty} (\beta\theta)^t \frac{\left[(1 + \lambda(i, e, \theta, x, h^T))c_t(i, e, \theta, x, h^T; T)\right]^{1-\gamma}}{1-\gamma} \right] \\ &= (1 + \lambda(i, e, \theta, x, h^T))^{1-\gamma} v(i, e, \theta, x, h^T; T) \end{aligned}$$

or

$$\lambda(i, e, \theta, x, h^T) = \left[\frac{v(i, e, \theta, x, h^\infty; \infty)}{v(i, e, \theta, x, h^T; T)} \right]^{1/(1-\gamma)} - 1.$$

Then the total welfare gain/loss is given by

$$\sum_{i, e, \theta, x, h^T} \lambda(i, e, \theta, x, h^T) \mu(i, e, \theta, x, h^T).$$

We use the same parameterization in the calibrated model for the $T = \infty$ world with no partitions. As a whole, the economy is better off without the legal restriction (specifically, the welfare figure is 0.0053). This small aggregate welfare gain, however, hides the fact that *not all* agents would be willing to pay to get rid of the restriction. Table **??** reports the average consumption equivalents by types and time preference shock.

Table 9: CE by types and shocks

$\theta \backslash i$	g	b
1	-0.0007	-0.0044
0	0.0000	-0.0013

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7 Appendix

7.1 Algorithm to compute T=∞ equilibrium with no partitions

1. Set grid points for endowments and scores.

    (a) There are 220 endowment grid points equally spaced between the bounds of the endowment distribution for each type.

    (b) There are twenty score grid points equally spaced between Γ_{LH} and $1-\Gamma_{HL}$.

2. Start iteration $j=1$ with a set of initial guesses for the price function $q^j(y,p)$, the credit scoring function $p^j(y,s')$, and the type scoring function $\psi^j(d,y,x,s)$.

3. Given the individual state (e,x,s), solve for the feasible actions set $B^j(e,x,s;q^j,p^j,\psi^j)$.

4. Solve for $V_i^j(e,\theta,x,s;q^j,p^j,\psi^j,W_i)$ by value function iteration. If s' is not on grids, linear interpolation is used for $W_i(y,s')$. The solution gives the set of optimal decision rule $m_i^j(d,y;e,\theta,x,s,q^j,p^j,\psi^j)\in M_i^j(e,\theta,x,s;q^j,p^j,\psi^j)$.

5. Given $m_i^j(e,\theta,x,s;q^j,p^j,\psi^j)$, calculate $\psi^{j+1}(d,y;x,s,q^j,p^j,\psi^j)$.

6. Given $\psi^{j+1}(d,y;x,s,q^j,p^j,\psi^j)$, calculate $p^{j+1}(y,s')$ and $q^{j+1}(y,p^{j+1})$.

7. Start iteration $j+1$ by using $q^{j+1}(y,p)$, $p^{j+1}(y,s')$, and $\psi^{j+1}(d,y,x,s)$ as the new set of initial guesses. Repeat until they converge.

8. (optional) Solve for the stationary distribution $\mu_i(e,\theta,x,s)$ according to $m_i(d,y;e,\theta,x,s,q,p,\psi)$ and $\psi(d,y,x,s)$. These distribution are defined recursively by

$$\mu_{i'}(e',\theta''(d,y,x,s))=\sum_{i,\theta}\left(\Gamma_{i'i}\cdot f(e'|i')\cdot\Lambda(\theta')\int_e m_i(d,y;e,\theta,x,s,q,p,\psi)\mu_i(\Phi(de),\theta,x,s)\right). \tag{11}$$

7.2 Algorithm to compute T=2 equilibrium with partitions

1. Set grids for endowments. There are 220 endowment grid points equally spaced between the bounds of the endowment distribution for each type.

2. Create history tuples $h^{T=2} = (d_{-1}, x_{-1}, d_{-2})$. The set of history tuples is denoted as $\mathcal{H} = \{(0,0,1), (0,\underline{x},0), (1,\underline{x},0), (0,0,0), (0,\overline{x}_1,0), (0,\overline{x}_2,0)\}$.

3. List all possible action/history pairs consistent with the partition that financial intermediaries can only observe default and borrowing. This will be useful in the later calculations of updating functions.

   We have now four tables and 43 applicable cells. A cell is marked NA if it is not an applicable action/history pair (for instance, a household can not default with non-negative assets which is why there are NAs in two rightmost columns in Table 1). Because there are partitions, one cell may include more than one possible action/history pair (for instance, in Table 1, the cell in the fifth row and first column includes the asset/history tuples $\{(\underline{x}, 0, \overline{x}_1, 0), (\underline{x}, 0, \overline{x}_2, 0)\}$). In that case, every action/history tuple in that same cell must have the same score due to the measurability restriction.

   - Table 1: For $(d, y) = (1, 0)$,the possible histories are

$h^{T=2} \backslash x$	$\underline{x}$	0	$\{\overline{x}_1, \overline{x}_2\}$
$(0,0,1)$	OK	NA	NA
$(0,\underline{x},0)$	OK	NA	NA
$(1,\underline{x},0)$	NA	NA	NA
$(0,0,0)$	OK	NA	NA
$\{(0,\overline{x}_1,0), (0,\overline{x}_2,0)\}$	OK	NA	NA

   - Table 2: For $(d, y) = (0, \underline{x})$,the possible histories are

$h^{T=2} \backslash x$	$\underline{x}$	0	$\{\overline{x}_1, \overline{x}_2\}$
$(0,0,1)$	OK	OK	OK
$(0,\underline{x},0)$	OK	OK	OK
$(1,\underline{x},0)$	NA	OK	NA
$(0,0,0)$	OK	OK	OK
$\{(0,\overline{x}_1,0),(0,\overline{x}_2,0)\}$	OK	OK	OK

- Table 3: For $(d, y) = (0, 0)$, the possible histories are

$h^{T=2} \backslash x$	$\underline{x}$	0	$\{\overline{x}_1, \overline{x}_2\}$
$(0,0,1)$	OK	OK	OK
$(0,\underline{x},0)$	OK	OK	OK
$(1,\underline{x},0)$	NA	OK	NA
$(0,0,0)$	OK	OK	OK
$\{(0,\overline{x}_1,0),(0,\overline{x}_2,0)\}$	OK	OK	OK

- Table 4: For $(d, y) = \{(0, \overline{x}_1), (0, \overline{x}_2)\}$,the possible histories are

$h^{T=2} \backslash x$	$\underline{x}$	0	$\{\overline{x}_1, \overline{x}_2\}$
$(0,0,1)$	OK	OK	OK
$(0,\underline{x},0)$	OK	OK	OK
$(1,\underline{x},0)$	NA	OK	NA
$(0,0,0)$	OK	OK	OK
$\{(0,\overline{x}_1,0),(0,\overline{x}_2,0)\}$	OK	OK	OK

4. Start iteration $j = 1$ with a set of initial guesses for the price function $q^j(y, p)$, the credit scoring function $p^j(y, h^{T=2})$, and the type scoring function $\psi^j(d, y, x, h^{T=2})$.

5. Given the individual state (e, x, s), solve for the feasible actions set $B^j(e, x, h^{T=2}; q^j, p^j, \psi^j)$.

6. Solve for $V_i^j(e, \theta, x, h^{T=2}; q^j, p^j, \psi^j, W_i)$ by value function iteration. The solution gives the set of optimal decision rule $m_i^j(e, \theta, x, h^{T=2}; q^j, p^j, \psi^j) \in M_i^j(e, \theta, x, h^{T=2}; q^j, p^j, \psi^j)$.

7. Given $M_i^j(e,\theta,x,h^{T=2};q^j,p^j,\psi^j)$, solve for stationary distribution $\mu_i^j(e,\theta,x,h^{T=2})$.

$$\mu_{i'}^j(e',y,d,x,d_{-1}) = \sum_{i,\theta,(x_{-1},d_{-2})} \left(\Gamma_{i'i} \cdot f(e'|i') \cdot \Lambda(\theta') \int_e m_i(d,y;e,\theta,x,d_{-1},x_{-1},d_{-2},q,p,\psi)\mu_i(\Phi(de),\theta,x,d_{-1},x_{-1},d_{-2})\right). \tag{12}$$

8. Given $m_i^j(e,\theta,x,h^{T=2};q^j,p^j,\psi^j)$ and $\mu_i^j(e,\theta,x,h^{T=2})$, calculate $\psi^{j+1}(d,y;x,h^{T=2},q^j,p^j,\psi^j)$ with respect to the partition blocks listed in step 3.

9. Given $\psi^{j+1}(d,y;x,h^{T=2},q^j,p^j,\psi^j)$, calculate $p^{j+1}(y,h^{T=2})$ and $q^{j+1}(y,p^{j+1})$.

10. Start iteration $j+1$ by using $q^{j+1}(y,p)$, $p^{j+1}(y,h^{T=2})$, and $\psi^{j+1}(d,y,x,h^{T=2})$ as the new set of initial guesses. Repeat until they converge.

11. With the distribution, the type score of an agent $s^T(x,h^T)$ can be calculated according to equation (7).

7.3 Algorithm to estimate the $T=2$ model parameters

1. Let the parameter vectors to be estimated be denoted η. In the case of estimating the parameters of the earnings process in Table 1, $\eta = \{\overline{e}_b, \underline{e}_g, \overline{e}_g, \phi_b, \phi_g, \Gamma_{gb}, \Gamma_{bg}\}$. In the case of estimating the asset grid and time preference shock parameters in Table 2, $\eta = \{\underline{x}, \overline{x}_1, \overline{x}_2, \Lambda_g(0), \Lambda_b(0)\}$. Let $\#(\eta)$ denote the number of parameters in η and $\#_M$ denote the number of data moments we consider.

2. Estimate η by Simulated Method of Moments (SMM). Specifically, the parameters η are chosen to minimize

$$g(\eta) = [f_D(\eta_0) - f_M(\eta)]W[f_D(\eta_0) - f_M(\eta)]', \tag{13}$$

where $f_D(\eta_0)$ is a vector of data moments given the true parameter vector η_0, $f_M(\eta)$ is a vector of model moments given parameters η, and W is any positive definite weighting matrix.

3. We start with an identity weighting matrix and solve for a first set of parameters $\hat{\eta}_1 = \arg\min_\eta g(\eta)$. These estimates are consistent but not efficient.

4. Given $\hat{\eta}_1$, simulate the economy with J agents for N periods, where J and N are large numbers ($J = 100,000$ and $N = 1000$).

   (a) Starting from the stationary distribution, draw an exogenous shock processes for type switching, time preference shock, and endowment process. Feed into the default and asset choice decision rules to get future state. Continue for each agent, N times to generate a sequence of exogenous shocks and decision rules.

   (b) For each period, calculate $\#_M$ model moments.

5. Given this $\#_M \times N$ matrix of moments, calculate the associated $\#_M \times \#_M$ variance-covariance matrix. Call it $\hat{\Sigma}_N$.

6. Obtain the optimal weighting matrix as the inverse of the variance-covariance matrix of moments, $W^* = \hat{\Sigma}_N^{-1}$.

7. Repeat the minimization problem to solve for parameters $\hat{\eta}_2$.

8. The estimates $\hat{\eta}_2$ are asymptotically normal for fixed number of simulations.

$$\sqrt{N}(\hat{\eta}_2 - \eta_0) \to N\left(0, \left(1 + \frac{1}{N}\right)\left(\frac{\partial g(\hat{\eta}_2)}{\partial \hat{\eta}_2}\hat{\Sigma}_N^{-1}\frac{\partial g(\hat{\eta}_2)'}{\partial \hat{\eta}_2}\right)^{-1}\right), \tag{14}$$

   where $\frac{\partial g(\hat{\eta}_2)}{\partial \hat{\eta}_2}$ is approximated by numerical derivatives.