An Aggregate Economy with Different Size Houses[*]

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Abstract

We build an aggregate model with different size houses and liquid assets. Typical households are born, are subject to idiosyncratic earnings risk and save for both life cycle reasons and housing reasons. Typically, a subset of these households, after accumulating some assets, put a down payment and buy a small, starter's, house or flat. As time passes, some households upgrade to a larger and nicer house. Households with houses may also eventually downgrade to a flat or even to no house and flat owners may sell. Our specification attempts to replicate some important features of modern aggregate economies: The distribution of earnings and of housing and non-housing wealth as well as some macroeconomic aggregates, including features of the mortgage issuing sector.

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1 Introduction

There is a tradition of work in modern macroeconomics using models with a large number of households subject to idiosyncratic earnings shocks where savings are used to smooth consumption across time (see Aiyagari (1994), Castañeda, Díaz-Giménez, and Ríos-Rull (1998), Krusell and Smith (1997) to cite a few). This work has studied the determinants of wealth inequality as the outcome of uninsurable income shocks where households have access to a perfectly liquid asset that is used to smooth consumption across time. In the model economies of this type households are continually adjusting their asset level. In this paper we extend this work to environments where there are various assets, financial assets that are perfectly divisible and can be costless adjusted and other assets that come in predetermined but quite large sizes, that can be traded only at a considerable cost, that can be partially purchased on credit, and that give the owner some advantages (due perhaps to the tax system or to moral hazard reasons). We take these assets to be houses.

The specific aim of hour work is to build blocks for the study of asset price changes. In this regard, an important specificity of our work that departs from standard macroeconomic models is that the stock of assets is not capital which is essentially lagged output. Instead, we take the opposite extreme and model the stock of assets as Lucas trees that are in limited supply and where the model determines their prices instead of their quantities. As we will see, this structure is equally well posed as the standard growth model to map to a modern aggregate economy.

There are several paper in the literature related to us. Gruber and Martin (2003) study an economy with iliquid durable consumption goods. They show that the decision rules display areas of inaction, and that financial wealth displays more dispersion than wealth held in the form of consumer durables. They also compare economies with identical parameterizations varying transaction costs (finding that higher transaction costs yield more savings) and the relative size of down payments (the bigger the down payment the higher savings). Martin (2005) is quite an interesting paper that explores in the context of a representative household model what are the implications of changes in characteristics over time (like those matching some of the demographics of the last half century) for asset prices, in particular housing prices, and interest rates. Ortalo-Magne and Rady (2003) is a slightly different paper that made interesting claims about the possibility of chain effects in the changes of the price of houses of different sizes due to the multiplying effects of capital gains –small changes in the price of small houses induce large increases in the equity of their highly leveraged owners that may want to switch to larger houses pushing their prices even further. In a way our work aims in part to find out the quantitative possibility of this channel if any. Nakajima (2005) studies the response of of housing prices to an increase in the volatility of individual earnings and finds that the level of housing prices moves quite a bit in response to what is a pure second moment change without any change in the level of economic activity. Heathcote and Davis (2005) is interested in the business cycle properties of housing construction, but they worry about housing quantities not prices. Díaz-Giménez and Puch (1998) documents how the properties of model economies relate to the down payment requirements. Chambers, Garriga, and Schlagenhauf (2005) connects the increase in housing ownership to reductions

in the down payment. Finally, Diaz and Luengo-Prado (2004) studies the determinants of housing tenure choice. There are other papers that also work on housing prices, but there notion of houses lacks some of the features that we think are more crucial to capture their essence, namely that they are big relative to its purchasers finances, that they are very costly to buy and sell, and that they provide an advantage to the owner that prevents the appearance of a rental market as a good substitute of ownership. A fourth feature, is, we think, new, that we both have different sizes and that houses cannot be built from scratch, in fact ours is more a model of hosing lots rather than a model of structures.

In particular, our work is closely related to Gruber and Martin (2003) and to Diaz and Luengo-Prado (2004) and we have slightly different calibration properties. For example we manage to get a larger group of people with no housing; but our wealth Gini Index while being closer to that in the data than the former paper is further than the one in the latter paper; we generate too much indebtedness and this two paper too little. So what is our value added? First, we are building the blocks of a structure capable of addressing price changes by having a finite number of housing (or perhaps better of lots) sizes and what is more important, units since we have an economy of the Lucas trees variety. Second, we are interested in the dynamics of the purchases and upgrades of houses and in this regard we provide information of effective down payments of first and repeated home buyers. Third, our structure is designed to be expanded in the direction of aggregate uncertainty and we give some information of how to do this, which is a challenge both theoretically and computationally.

2 The Stationary model economy

The model economy analyzed in this article is a modified version of a Lucas trees' economy with uninsured idiosyncratic risk and in this version no aggregate uncertainty (although it is lurking at the end of the tunnel). Until we say otherwise we are only looking at stationary equilibrium. The key features of our model economy are the following: *(i)* it includes a large number of households that die exponentially, have identical preferences and face an uninsured, household-specific shock to their endowments of efficiency labor units;[1] *(ii)* there are three type of assets in fixed supply (Lucas trees) in the economy, a standard type of tree with a constant dividend in terms of the consumption good that we label bond, and two other types of trees (houses and flats) that provide their holders with a utility shift that we take to be dwellings. Trades of the standard tree do not require resources and can be done in any size while trades of the dwellings are expensive and each household can hold at most one dwelling which comes in a fixed size.

Preferences and Shocks Households are born and die exponentially with probability π. Preferences are given by $u_d(c)$ where d indicates the type of dwelling a household may be associated with and that can be either none, a flat or a house, $d \in \{0, f, h\}$. Houses are better than flats which is better than nothing $u_h(c) > u_f(c) > u_0(c)$. Individuals are heterogeneous in their earnings ability that is uncertain. We consider 3 different earnings groups so that

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[1]Actually there is also population growth but we abstract from it in the presentation of the model to reduce the burden of notation.

we reproduce observed earnings inequality. The earnings group of each individual evolves according to a Markov process. Furthermore, individuals face another uninsurable shocks to earnings. Households subject to earnings shocks ϵ drawn from $F(\epsilon, e)$ with $e \sim \Gamma_{ee'}$ and

$$F(\epsilon, e) = \left[\frac{\epsilon - \underline{\epsilon}}{\overline{\epsilon} - \underline{\epsilon}} \right]^{\chi}$$

Markets There are various markets in this economy. There are spot labor market that (essentially irrelevant), and bonds and dwellings markets, are traded also in spot markets at prices $\{p_\ell, p_f, p_h\}$ respectively. There is also an annuity market so that no assets disappear. This means that the assets of the dead are share proportionally by the holders of that asset.

There are borrowing constraints and dwellings can be used as collateral. Individuals can borrow a fraction $1 - \alpha$ of dwellings value. Dwellings are traded with costs that we pose on the buyer and that we write as $\phi(d, d') = p_{d'}(1 + \delta)$ if $d = 0$ and $\phi(d, d') = p_{d'}(1 + \delta) - p_d$ otherwise.

Household's Problem To write the problem of household in a convenient way given that it is non concave we use two different functions. Function $V_{e,d}(y)$ denotes the value function of an household that belongs to an earnings class e, has dwelling d and financial savings y before the realization of the earnings shock ϵ and after realization of the earning class shock e. Function $W_{e,d}(a)$ is the value function of a household in earnings class e, has dwelling d and cash in hand a. Consequently their relation is

$$V_{e,d}(y) = \int_{\epsilon} W_{e,d}[(p_l + r)y + \epsilon] \, F(d\epsilon, e)$$

Furthermore, denote by $W_{e,d}^{d'}(a)$ the maximal utility of an household that belongs to earnings class e, starts the period with dwelling d, has dwelling tomorrow d' and has cash in hand a. Consequently,

$$W_{e,d}(a) = \max_{d'} \left\{ W_{e,d}^{d}(a) \right\}$$

We can now write the problem of an household conditional to not changing dwelling as

$$W_{e,d}^{d}(a) = \max_{y} u_d(c) + \pi \beta E \left\{ V_{e',d}(y)/e \right\} \qquad \text{subject to:} \qquad c + p_l y = a,$$

while that of an household that trades dwellings is

$$W_{e,d}^{d'}(a) = \max_{y} u_{d'}(c) + \pi \beta E \left\{ V_{e',d'}(y)/e \right\} \qquad \text{subject to:} \qquad c + p_l y - \phi(d, d') = a,$$

Stationary Equilibrium To describe this economy at a point in time (a set of initial conditions) we need a distribution of households over dwellings, assets, and earnings shocks, x defined over those variables. A stationary equilibrium is a specific distribution of households x^* together with a set of asset prices $\{p_\ell^*, p_f^*, p_h^*\}$ such that when the distribution of households is given by $x*$ and the households face constant prices forever given by $\{p_\ell^*, p_f^*, p_h^*\}$

then their choices *(i)* induce a distribution of households next period that is again x^*, and *(ii)* prices clear assets markets, that is

$$\int_{e,d,y} y\,dx = 1, \qquad \int_{e,f,y} dx = \mu_f, \qquad \int_{e,h,y} dx = \mu_h.$$

3 Mapping the Model to the data

To specify the model we have to choose some parameters and functional forms. Some details of the specification of the model are independent of the equilibrium and can be set beforehand. The equilibrium allocation depends on the parameter choices and our aim is to set parameter values so some statistics in the model economy have certain desired values, typically the same as their counterpart in the data. The process of parameter selection can be referred either as a calibration or as a method of moments estimator, depending on the scholar. We first describe the specification and then the targets.

3.1 Description of parameters

Parameters that can be set independently The parameters that can be include population turnover features, 1.5% per year, and 67 years of average adult life (which sets $\pi = 0.985$); we also set ex ante some features of the financial system such as a 1.% mortgage premium, this is borrowing rate minus lending rate, a 20.% down payment and a 10.% cost of buying a dwelling, as a stand in for real estate commissions, taxes, and the time and hassle for the households involved in the purchase of the dwelling.

Preferences (3) Preferences are time separable with discount rate β and a standard CRRA per period utility function with risk aversion parameter $\sigma = 2$ (one of those parameters that is hard to pin down) and utility shifters γ^d (two of them).

$$u_d(c) = \frac{c^{1-\sigma}}{1-\sigma}\gamma^d$$

Earnings Shocks (11) We choose an earnings process with three earnings classes and within each earnings class there is an interval of earnings with a continuous density. The cdf function is $F(\epsilon, e) = \left[\frac{\epsilon - \underline{\epsilon}}{\overline{\epsilon} - \underline{\epsilon}}\right]^\chi$. This gives 5 parameters for the intervals of earnings (one is normalization), 4 possible parameters of the transition matrix $\Gamma_{e,e'}$ since we assume zero the probabilities of going from the top group to the bottom group and vice-versa and the additional parameter χ that adds flexibility to vary the mean to median ratio within each earnings class. To achieve a life cycle earnings profile where households increase their earnings on average over time, we assume that all households are born as poor. In models without housing there is a lot of work to estimate these parameters (see for example Castañeda, Díaz-Giménez, and Ríos-Rull (1998) and Diaz, Pijoan-Mas, and Ríos-Rull (2003)), and in this paper we use a process quite similar to that in the latter paper.

Asset parameters (3) While we normalize the size of the Lucas tree to 1, the size of its dividend d has to be specified. Also the number of flats μ^f and the number of houses μ^h relative to the population has to be set.

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3.2 Description of Targets/Moments to match

1. We target a labor share out of income of 0.84. Note that the absence of depreciation in our model makes the labor share larger.

2. Average earnings of those aged 31-60 to those aged 20-30; 1.82.

3. Financial asset wealth relative to income: 2.18.

4. Owner occupied housing wealth times relative to income: 2.3.

5. Fraction of households that own a house: 0.35.

6. Fraction of people with flat: 0.30

7. House prices relative to flat prices $\frac{p_h}{p_f}$: 2.0.

8. Risk free interest rate of 5%.

9. Down payment the first time a household buys a dwelling: 16.3%.

10. Down payment of repeated buyers 26.5%.

11. Ratio of mortgage debt to income of 34%.

12. Fraction of people with debt in the model (those who have negative financial assets is 40.%.

13. Average ratio of financial debt to housing value is 50%. This number has been increasing in the last few years and our target is appropriate for the early nineties.

14. General Properties of the Lorenz Curve of earnings.

15. General Properties of the Lorenz Curve of assets.

3.3 Performance of the model economy

The model does in general quite well in matching our targets despite having 17 parameters for 12 specific targets plus the general properties of the Lorenz curves. Moreover, of those 17 parameters there are 4 that play little role (3 that define the bounds of the earning classes and the parameter χ that governs the curvature of the density of earnings within each earning class. The actual values of the model statistics relative to our targets are 18.% for the down payment at first buy and 27.9% for at all purchases. The ratio of mortgages to income is 33.2%. Also the actual fraction of people with debt is 41.6% and the the average ration of debt to housing value is 56.5%. The interest rate in the economy is a little bit too high, 7%.

Tables 1 and 2 shows selected statistics from the Lorenz curves for earnings and for wealth in both the model and data (as reported by Budría, Díaz-Gimenez, Quadrini, and Ríos-Rull (2001)). We think that they are close enough given the degrees of freedom that we operate with.

Table 1: The Distribution of Earnings: Model and Data (1998 SCF)

	Quintiles					Gini
	1st	2nd	3rd	4th	5th	
Model Earnings Share	3.5	v5.0	v7.7	11.1	72.3	.65
Data Earnings Share	−0.2	4.0	13.0	22.9	60.2	.61

Table 2: Total, Financial and Housing Wealth Distribution

		1st	2nd	3rd	4th	5th	Gini
Total	Model	0.24	1.30	2.27	9.92	86.27	.82
	US Eco	-0.29	1.35	5.14	12.37	81.43	.80
Finan	Model	-22.34	-17.43	-1.39	2.33	138.83	1.568
	US Eco	-9.53	-1.07	0.36	5.84	104.39	0.953
Housing	Model	0.00	5.69	20.92	31.56	41.84	0.457
	US Eco	0.22	2.92	12.78	22.45	61.63	0.626

4 Conclusion and a Preview of Aggregate Uncertainty

In this paper we have set the main building blocks to study housing prices in an economy where they are actively traded and that captures some of the important features of a modern economy with respect to what houses are. The next step is to study versions of this economy with aggregate uncertainty, on earnings, interest rates (dividends) mortgage premia, demoraphics and so on . The problem is that the state space includes the distribution of households across financial wealth, dwellings and earnings class. To avoid this problem we are using the approach pioneered by Krusell and Smith (1998) using the aggregate shocks and the current market clearing prices as the only state variables. This approach seems to be working and we will soon be able to report the stochastic properties of housing and equity prices.

A Appendix: Parameter Values

Preferences The preference parameters are $\beta = 0.892127$, while the utility modifiers are $\gamma^f = 0.19215$ and $\gamma^d = 0.0637$.

Earnings process We chose three earnings classes with earnings in the intervals $\{[0.5, 2.0], [1.5, 12, 5], [41.9, 51.2]\}$ respectively, and with Transition Matrix

$$\Gamma_{ee'} = \begin{bmatrix} 0.992 & 0.008 & 0.000 \\ 0.009 & 0.980 & 0.011 \\ 0.000 & 0.083 & 0.917 \end{bmatrix}$$

The value of the parameter χ is 0.5.

B Computational Procedures

The problem that the households solve is very hard because it is not concave and first order conditions cannot be used to solve the model. Moreover, not only we have to solve the problem of the household and compute the stationary distribution for given parameters and prices, but we also have to do it many times to ensure that both the economy is in equilibrium (markets clear) and that the value of the model statistics are close to our targets.

There are three states in the household's problem: (i) financial assets, (ii) dwelling position and, (iii) earnings class and two decisions: savings, a continuous choice, and dwelling, that can take three values house, flat or nothing. This combination implies that in general the value function W is not concave which means that the first order conditions are not sufficient to find the optimum. We tried to get around this problem by using continuous earnings realizations and taking expectations over the following period earnings (and thus defining function V) hoping that the kinks would be smoothed out. While this approach works very often, it does not work always (the earnings cannot be ensure to be spread enough so that the integration enforces concavity) which causes serious problems when the value function has to be solved many times as is the case when we are estimating parameters and even more in stochastic environments. Consequently we opted for the discretization of the state space, that is computationally very costly but also very robust. Still, we maintained the continuous support of earnings because the distribution of households and the decision rules oh households become smoother which makes for an easier estimation process. We use 700 points in the grid for the financial asset for function V so there are 6300 possible individual states. The points in the grid are not equally spaced (they are closer for low values of wealth) and they are located in different locations for each dwelling. We then use successive approximations to find the solution to the household problem. Not that within each iteration the function W has to be constructed, and the decision rules found. The distribution of households over assets and earnings class is a now a vector and can be easily stored. We have done our work in a powerful Beowulf cluster with 26 processors.

References

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk, and aggregate saving. *Quarterly Journal of Economics 109*, 659–684.

Budría, S., J. Díaz-Gimenez, V. Quadrini, and J.-V. Ríos-Rull (2001). New facts on the U.S. distribution of earnings, income and wealth. Unpublished Manuscript.

Castañeda, A., J. Díaz-Giménez, and J. V. Ríos-Rull (1998). A general equilibrium analysis of progressive income taxation: Quantifying the trade-offs. Mimeo, University of Pennsylvania.

Chambers, M., C. Garriga, and D. E. Schlagenhauf (2005). Accounting for changes in the homeownership rate. Mimeo, Florida State University.

Diaz, A. and M.-J. Luengo-Prado (2004). Precautionary savings and wealth distribution with durable goods. http://www.eco.uc3m.es/ andiaz/pdfs/research/durables.pdf.

Diaz, A., J. Pijoan-Mas, and J.-V. Ríos-Rull (2003, September). Habit formation: Implications for the wealth distribution. *Journal of Monetary Economics 50*(6), 1257–1291.

Díaz-Giménez, J. and L. A. Puch (1998). Borrowing constraints in economies with indivisible household capital and banking: an application to the spanish housing market (1982-89). *ie XXII*(3), 469–499.

Gruber, J. W. and R. F. Martin (2003). Does housing wealth make us less equal? the role of durable goods in the distribution of wealth. Mimeo, Federal Reserve Board.

Heathcote, J. and M. Davis (2005). The price and quantity of residential land in the united states. Mimeo, Georgetown University.

Krusell, P. and A. Smith (1997). Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns. *Macroeconomic Dynamics 1*(2), 387–422.

Krusell, P. and A. Smith (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy 106*, 867–896.

Martin, R. F. (2005). The baby boom: Predictability in house prices and interest rates. Mimeo Federal Reserve Board.

Nakajima, M. (2005). Rising earnings instability, portfolio choice, and housing prices. Mimeo, University of Illinois.

Ortalo-Magne, F. and S. Rady (2003). Housing market dynamics: On the contribution of income shocks and credit constraints. http://research.bus.wisc.edu/fom/documents/hm-latest.pdf.