TIME–CONSISTENT OPTIMAL FISCAL POLICY*

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This paper studies the properties of optimal fiscal policy in a stochastic growth model when the government cannot commit itself beyond the next period's capital income tax rate. We find that the results contrast markedly with those under full commitment. First, capital income tax rates are very high (65% on average versus close to zero on average under full commitment). Second, labor income taxes are rather low on average (about 12% versus a value of around 31% under full commitment). Finally, labor income taxes are quite volatile, while under full commitment their standard deviation is essentially zero.

Keywords: Optimal taxation, Time-consistency, Markov perfect equilibrium.
JEL classification: E62, H21.

1 Introduction

The properties of optimal taxation in the growth model under full commitment are well understood. The seminal ideas of Ramsey (1927) were first applied to the growth

*Submitted: 7 December 2001. This version: October 1, 2002.

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1 We thank Marco Bassetto, Michael Ben-Gad, V. V. Chari, Larry Christiano, Pat Kehoe, George Mailath, Per Krusell, Stephanie Schmitt-Grohé, Torsten Persson, Kjetil Storesletten, Martín Uribe and participants at the 1999 Texas Monetary Conference, Dallas, Texas, the 1999 CEPR European Summer Symposium in Macroeconomics in Sintra, the IIES, the 1999 NBER Summer Institute, and the 1999 Northwestern University Summer Workshop in Economics. Ríos-Rull thanks the National Science Foundation for Grant SES–0079504 and the University of Pennsylvania Research Foundation for their support. We are particularly grateful for the helpful comments of two anonymous referees.

model by Chamley (1986) and Judd (1985), who showed that capital income should not be taxed in a deterministic steady state. Following up on these results, Zhu (1992), Chari, Christiano, and Kehoe (1994), and Chari, Christiano, and Kehoe (1995) characterized optimal taxation in the context of the stochastic growth model. Stockman (2001) studied the properties of optimal fiscal policy in a stochastic growth model in the presence of a balanced budget constraint. All of these authors assumed that the government can commit itself to the policies that will be in place arbitrarily far into the future. This assumption is an important one given the time–inconsistency of the policies that turn out to be optimal.

It is generally recognized that no actual government has access to a perfect commitment technology. This leaves us with two plausible approaches to analyzing optimal fiscal policy. One is to find mechanisms that might substitute for commitment. Several such mechanisms have been discussed in the literature. In a model without capital, Lucas and Stokey (1983) showed how to render the optimal fiscal policy time-consistent through a suitable choice of maturity structure for public debt. Persson, Persson, and Svensson (1987) used nominal debt in order to make optimal monetary policy time–consistent. Finally, Chari and Kehoe (1990) showed how to use reputational mechanisms such as trigger strategies that may substitute for commitment in a fully-fledged stochastic growth model.

The alternative possibility, the one we explore in this paper, is to see what optimal fiscal policy looks like when no commitment technology is available and mechanisms that substitute for commitment are inoperative. The way to do this is to force the government to base its policy on fundamentals only. Formally, this means that we focus on the properties of Markov perfect equilibria, which are time-consistent by construction. We contrast the properties of these Markov perfect equilibria with those of an economy where everything is the same except that the government *can* commit itself into the infinite future. We find dramatic differences between these two economies.

The main contribution of the present paper is to provide a quantitative treatment of the positive theory of factor taxation. In doing this, it turns out that we can account for the order of magnitude of empirical capital income taxes even in the absence of any heterogeneity in the wealth distribution. We also give a quantitative assessment of how various constitutional aspects affect the equilibrium level of factor income tax rates.

The notion of Markov perfect equilibrium is defined and defended in Maskin and Tirole (2001) as well as in Krusell and Rios-Rull (1997) and Mailath and Samuelson (1998). Krusell, Quadrini, and Rios-Rull (1996), Krusell and Rios-Rull (1997), and Krusell, Quadrini, and Rios-Rull (1997) made operational the notion of Markov perfect equilibrium in a growth model setting and used this to study environments with heterogeneous agents and with policies being set by majority vote.[2]

[2]In a representative agent setting, Ambler and Paquet (1996) look at the dynamics of beneficial government spending over the business cycle when it is set by a benevolent government under complete

Throughout the paper, we maintain the assumption that the government cannot issue debt. The main motivation for this is a practical one. Because there is no single government upon which we can impose a no-Ponzi-scheme constraint, the introduction of debt would require us to impose an upper limit on debt issue in any particular period. This would require us to use numerical methods that are dramatically different from the linear-quadratic approach that we adopt in this paper. Such methods exist, but are considerably more difficult to implement and their ability to solve for stochastic properties are as yet untested.[3] The work of Stockman (2001) shows that limits on the possibility of the government to run surpluses and deficits have no dramatic effects on the optimal policies when there is commitment, but there is no guarantee that this result would carry over to the no-commitment case. We leave this question for further research.

In the full commitment case, we assume, as did Chari, Christiano, and Kehoe (1994), that the capital income tax rate in the initial period is inherited from the past. In the case without full commitment, we proceed analogously, letting the capital income tax rate be inherited from the past in *every* period. As in Chari, Christiano, and Kehoe (1994), our motivation for this assumption is to avoid the rather trivial possibility of lump-sum taxation. It also has the added advantage of facilitating comparisons with

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discretion.

[3] Of course, this reasoning does not apply to standard environments where the policy of the government is not an endogenous object.

their work.

As we have stressed already, the optimal policies in the absence of full commitment are very different from those under full commitment. In particular, (*i*) the tax rates on capital are 65% on average under the benchmark parameterization (under full commitment they are close to zero on average), (*ii*) labor income taxes are rather low on average (about 12% versus a value of around 31% under full commitment), and (*iii*) labor income taxes are volatile (in some cases its coefficient of variation is higher than that of the capital income tax rate), while under full commitment their standard deviations are essentially zero.

Not surprisingly, we have found that the specification of the constitution matters for the equilibrium. In the baseline constitution, (contingency plans for) capital tax rates are committed to one year in advance. If instead the commitment period is two years, the average capital income tax is 47%.

Even apparently minor changes to the constitution may matter a lot. In particular, whether the commitment of the current government to previous government choices is through respecting a capital income tax rate or a labor income tax rate (both in the presence of a balanced budget constraint) changes the steady state capital income tax rate from 65% to 77%. The reason is that the perceived distortions by the current government are not the same. When the government inherits a capital income tax rate, then its choice of tomorrow's capital income tax rate will affect the rate of return on postponing consumption and leisure, and this will induce a distortionary increase in todays *labor* income tax. On the contrary, when what is inherited from the past are labor income tax rates, the corresponding deterrent is absent. When contemplating a lower labor income tax rate tomorrow the government is not so concerned about raising distortionary taxes today.

This paper has several forebears. In the first place, there is the literature that deals with optimal taxation under commitment that was discussed above. Secondly, there is the literature on time-consistent policies. Both Kydland and Prescott (1977) and Calvo (1978) noted that a large class of optimal policies (like the ones studied by Chari, Christiano, and Kehoe (1994) and Stockman (2001)) are time inconsistent. In addition to pointing to the time inconsistency of optimal policies, Kydland and Prescott (1977) also discussed the notion of a time-consistent equilibrium and how to calculate it in a seldomly referred to appendix.

In related work, Chang (1998) studies a monetary model and Phelan and Stacchetti (2001) look at an optimal taxation problem using the ideas of Abreu, Pearce, and Stacchetti (1990). Their interest in the entire set of subgame perfect equilibria (not only the Markov ones). While Chang (1998) studies some qualitative long-run properties of the model, Phelan and Stacchetti (2001) describe methods to find the entire set of equilibria, and in particular the best and the worst one. They also compute those equilibria for some economies. The methods developed there cannot be used to find the

Markov equilibria. Benhabib and Rustichini (1997) consider a very similar environment to ours and study optimal taxation without commitment. Their interpretation of the notion of "no commitment" is quite different from ours, however. In their work, a policy maker chooses a plan at time 0 subject to an implementability condition and a further constraint which is designed to capture the lack of commitment. This further constraint says that, at any node, it should be no worse to stay on the equilibrium path than to switch to a different path whose value for the representative consumer is $v(k)$ where k is the capital stock at the time of switching. The function v is exogenously specified. This contrasts with our approach, where any government only controls taxes in a single period, and the value of a deviation is derived (not imposed) by supposing that successor governments do not deviate from their equilibrium strategies.

The paper is organized as follows. In section 2 we lay out the model, and in section 3 we describe the Ramsey equilibrium of the economy with commitment, where we state the problem in primal form in the sense of Atkinson and Stiglitz (1980), as applied in Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1991) and Stockman (2001). In section 4 we describe the details of the economy without commitment and the notion of Markov perfect equilibrium that we use. Section 5 describes the calibration of the model economies that we study, while section 6 describes our findings with respect to the properties of the optimal fiscal policies and the associated equilibrium allocations. It also includes some sensitivity analysis as well as a discussion of some properties of the constitutional arrangement in shaping the tax policies that are actually implemented. Section 7 concludes. Appendix A describes our computational methods.

2 The model

The model is a standard stochastic growth model with a government sector driven by a Markov process for useless government purchases g and for total factor productivity z. Both the support of government purchases and total factor productivity is finite. Let the set of pairs of possible shocks be denoted by $s \in S$, a set with n elements. Moreover, let Γ be its transition matrix with $\Gamma^{ss'}$ denoting the probability of the shock being s' tomorrow conditional on being s today. We write s_t to denote a particular realization in period t and $s^t = \{s_0, \ldots, s_t\}$ is a particular history. By some slight abuse of notation we write $\Gamma(s^t)$ to denote the unconditional probability of history s^t.

There is a representative consumer with standard preferences over streams of consumption and leisure that can be written as

$$(1) \qquad E\left\{\sum_t \beta^t \; u\left(c\left(s^t\right), h\left(s^t\right)\right)\right\} = \sum_t \beta^t \sum_{s^t} \Gamma\left(s^t\right) \; u\left(c\left(s^t\right), h\left(s^t\right)\right)$$

where $c(s^t)$ and $h(s^t)$ are consumption and hours worked after history s^t.

We assume competition in the factor markets that will allow us to abstract from the role of firms. The aggregate feasibility constraint is given by

$$(2) \qquad F\left(k\left(s^{t-1}\right), h\left(s^t\right), z\left(s^t\right)\right) + (1-\delta)\, k\left(s^{t-1}\right) = c\left(s^t\right) + k\left(s^t\right) + g\left(s^t\right)$$

where F is a neoclassical production function and where $k\left(s^{t-1}\right)$, is the capital stock available in period t, history s^t, that was chosen in period $t-1$, $z\left(s^t\right)$ is the productivity shock, and $g\left(s^t\right)$ is the level of government expenditures that is associated to shock s_t. Finally, δ is the depreciation rate.

Households accumulate assets and face capital and labor income taxes rates that are denoted by $\theta(s^t)$ and $\tau(s^t)$ respectively. Their budget constraint is then

$$c\left(s^t\right)+k\left(s^t\right)=\left(1-\tau\left(s^t\right)\right)w\left(s^t\right)h\left(s^t\right)+\left[1+r\left(s^t\right)\left(1-\theta\left(s^t\right)\right)\right]k\left(s^{t-1}\right) \tag{3}$$

where w and r are the respective rental prices of factors and r is defined net of depreciation.

The government's balanced budget constraint can be written as follows.

$$\theta\left(s^t\right)\,k\left(s^{t-1}\right)\,r\left(s^t\right)+\tau\left(s^t\right)w\left(s^t\right)\,h\left(s^t\right)=g\left(s^t\right) \qquad \text{for all } s^t. \tag{4}$$

A policy π is a stochastic process $\pi=\left\{\theta\left(s^t\right),\tau\left(s^t\right)\right\}$ for capital and labor income tax rates. Given π we can define a balanced budget competitive equilibrium.

Definition 1 *A balanced budget competitive equilibrium given an initial capital income tax rate θ_0 and an initial capital stock k_0 is a set of stochastic processes for aggregate consumption, $c(s^t)$, labor $h(s^t)$, capital $k(s^t)$, factor prices $r(s^t)$ and $w(s^t)$, and tax rates $\theta(s^t)$ and $\tau(s^t)$ such that (i) the allocation maximizes (1) subject to (3), a no–Ponzi–scheme constraint and initial capital k_0, (ii) factor prices are marginal productivities,* *i.e.* $r(s^t) = F_k[k(s^t), h(s^t), z(s^t)] - \delta$, *and* $w(s^t) = F_h[k(s^t), h(s^t), z(s^t)]$ *and (iii) the government satisfies its period by period budget constraint (4).*

Throughout, letter subscripts denote partial derivatives; *e.g.* F_k is marginal productivity of capital.

3 The full commitment economy

Now we are ready to characterize the policies chosen by a benevolent government under commitment (a Ramsey government).

A balanced budget Ramsey equilibrium is a balanced budget equilibrium where (1) is maximized by the choice of π over the set of all such equilibria.

The simplest way to find the equilibrium is to use the primal approach to optimal taxation, as described in Chari and Kehoe (1998). This involves stating the constraints in terms of allocations alone, with no explicit reference to prices or tax rates. As it turns out, the characterization of the equilibrium allocations under balanced budget rules is slightly different than those without such a constraint. The specific result proved by Stockman (2001) is as follows.

Proposition 1 *The primal statement of the balanced budget Ramsey problem is to* *maximize (1) subject to*

$$F\left[k\left(s^{t-1}\right),h\left(s^{t}\right),z\left(s^{t}\right)\right] = c\left(s^{t}\right)+k\left(s^{t}\right)-(1-\delta)\,k\left(s^{t-1}\right)+g\left(s^{t}\right) \quad \text{and} \tag{5}$$

$$u_c\left(s^{t}\right)k\left(s^{t-1}\right) = \beta E_t\left[u_c\left(s^{t+1}\right)\left(c\left(s^{t+1}\right)+k\left(s^{t}\right)\right)+u_h\left(s^{t+1}\right)h\left(s^{t+1}\right)\right] \tag{6}$$

for all s^t*, and given* k_0 *and* θ_0*.*

Proof. See Stockman (2001). ■

Note that equation (5) is the usual feasibility constraint that also appears in the standard Ramsey problem, (the one without the balanced budget constraint). This constraint applies at every node. Equation (6), on the other hand, is the balanced budget constraint, where prices and taxes have been substituted away using the household's first order conditions (intratemporal and intertemporal) in order to achieve primal formulation. Note that it implies the implementability constraint of the standard Ramsey problem. This constraint also has to be satisfied at every node (date-event).

The balanced budget Ramsey equilibrium in a deterministic version of the growth model has some of the same properties as the standard Ramsey equilibrium, namely that in the long run,[4] capital should not be taxed. Formally we have the following proposition.

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[4] We use the term *long run* instead of the, perhaps more familiar, *steady state* because we refer to a value of the state vector such that if it is achieved it is repeated, rather than a state vector such that if the economy starts in it, it is repeated forever. The point we refer to satisfies the former requirement but not the latter.

Proposition 2 *Under a balanced budget rule, the Ramsey optimal allocation of a deterministic version of this economy has the property that, if the economy converges to a stationary allocation, then capital accumulation is undistorted in the long run.*

Proof. This follows straightforwardly from requiring all variables to be constant in the first order condition for k_{t+1} of the Ramsey problem. ■

A recursive formulation of the problem is needed to solve for the balanced budget Ramsey equilibrium numerically. Even with the aid of Proposition 1 there are two approaches to do this. The direct approach involves having as state variables the marginal utility of consumption and the state of nature in the previous period, and the stock of capital newly available, and then to solve for consumption, labor and investment *simultaneously* in all states of nature. This approach allows for the use of standard dynamic programming. However, it is quite cumbersome as a computational problem. We follow instead the alternative approach described in Marcet and Marimon (1995). They define the state space via (s, k, η), where η' is the Lagrange multiplier associated with (6). This allows us to solve for consumption, labor input and investment in each state independently, a much simpler problem computationally. The functional equation associated to this approach is not, however, a standard maximization problem, but takes the form of a recursive Lagrangian. So stated, standard methods can be used to solve this problem. Once we obtain a solution, we back out the associated tax rates by using the first order conditions of the household's problem and the government budget constraint. We then generate time series of all relevant variables and study their properties. We turn next to the study of the economy without commitment.

4 The economy without commitment

To characterize the economy without commitment we have to be precise about who chooses what when. We repeat the one-period commitment to a capital income tax rate feature that was present in the first period of the economy with commitment, although this time it will be in place every period. More precisely, each period the government inherits a commitment to a certain capital income tax rate θ and observes the current realization of the stochastic shocks. Then it sets a vector of state contingent capital income tax rates (in the sense that it can make them depend on the realization of the shocks one period later) that the next period's government has to honor. Then the private sector moves, choosing how much to work and save. Finally, the labor income tax rate is determined so as to balance the budget.[5] We allow tax rates to be state

[5] This gives rise to a multiplicity of equilibria because of the Laffer curve; there are typically two distinct tax rates at which the budget exactly balances. We ignore the equilibrium to the right of the peak of the Laffer curve and focus on the one to the left. This ordering of moves could possible lead to the type of indeterminacy pointed out by Schmitt-Grohe and Uribe (1997). We have not found instances of this type of multiplicity in the economies that we study. This is not surprising, since our parameter values imply elasticities of substitution within the standard ranges that are measured and

contingent in order to ensure that the degree of commitment is the only difference between the two economies that we study.[6]

An economy where the government has to set the capital income tax rate for the following period in an unconditional form is a very different economy to the one we study (although that economy would have the same steady state as the current one). Note also that if the government inherited a commitment both to the tax rate on capital and to the tax rate on labor, the balanced budget constraint would not be automatically satisfied, since hours worked by households would depend also on future taxes.

As we discussed above we will focus on the properties of Markov perfect equilibria. We follow here the approach stated in a political economy context by Krusell, Quadrini, and Rios-Rull (1997), Krusell, Quadrini, and Rios-Rull (1996) and Krusell and Rios-Rull (1997) which is also related to that in Kydland and Prescott (1977). The minimal

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the type of examples where Schmitt-Grohe and Uribe (1997) report multiplicity of equilibria imply are outside this range. In particular, tax rates are too low and the elasticity of labor supply too small. An alternative sequence of decisions is to let the government set the labor tax rate before the private sector moves. This would remove the Laffer-type multiplicity, but has the disadvantage that one would have to specify the consequences of (out-of-equilibrium) private behavior leading to a non-balanced budget. We thank Larry Christiano and Marco Bassetto for enlightening discussions about this issue.

[6] Alternatively, we could have set the current period commitment to a certain labor income tax rate and have the capital income tax rate balance the budget. This possibility is considered in section 6.8.

aggregate state vector is the current realization of the shocks, $s = \{g, z\}$, the aggregate stock of capital, K, and the inherited tax rate θ. For notational simplicity we refer to the aggregate state vector as $x = \{g, z, K, \theta\}$. For individual agents there is an additional state variable, individual capital, k. Next, we characterize the behavior of an economy with an arbitrary law of motion for the capital income tax rate θ given by $\theta(s') = \psi(x)$.

This should be interpreted as giving the capital income tax rate for next period if shock s' occurs given that todays state is x. In this world an individual agent solves the following problem

$$v\left(x, k\,; \psi\right) = \max_{c,k',h}\; u\left(c, h\right) + \beta \sum_{s'} \Gamma^{ss'}\; v\left(x'\left(s'\right), k'\,; \psi\right) \tag{7}$$

subject to

$$c + k' = (1-\theta)\, r(z, K, H)\, k \,+ (1-\tau)\, w(z, K, H)\, h + k \tag{8}$$

$$K' = D_K(x; \psi) \tag{9}$$

$$H = D_H(x; \psi) \tag{10}$$

$$\tau = \phi(x; \psi) \tag{11}$$

$$\theta(s') = \psi(x) \tag{12}$$

Note that we have indexed the value function as well as some other functions by ψ; this just reminds us that function ψ is completely exogenous at this stage (although it will not be later), and therefore the objects in the above problem vary when ψ varies.

In order to solve the problem the household needs to know functions $\{D_K, D_H, \phi\}$. These functions will be determined in the economic equilibrium by the standard representative agent conditions and by the balanced budget condition. Equation (8) is the individual agent budget constraint, equation (9) is the aggregate law of motion of capital, equation (10) determines aggregate employment, equation (11) gives the labor income tax rate today and equation (12) gives the set of state contingent capital income tax rates for tomorrow. Finally, we take $r(z, K, H)$ and $w(z, K, H)$ to be expressions for the rental prices of factors.

Let $k' = d_k(x, k; \psi)$ and $h = d_h(x, k; \psi)$ denote the solution to (7). We are now ready to define an equilibrium for a given policy function ψ.

Definition 2 *A recursive (stationary) economic equilibrium for policy rule ψ is a set of functions $\{v\,(.;\psi)\,,\ D_K(.;\psi),\ D_H(.;\psi),\ \phi(.;\psi),\ d_k(.;\psi),\ d_h(.;\psi)\}$ such that*

(i) Functions $\{v\,(.;\psi)\,,\ d_k(.;\psi),\ d_h(.;\psi)\}$ solve the individual problem (7) given functions $\{D_K(.;\psi), D_H(.;\psi), \phi(.;\psi), \psi\}$.

(ii) The agent is representative, i.e.,

$$D_K(x;\psi) = d_k(x, K;\psi) \tag{13}$$

$$D_H(x;\psi) = d_h(x, K;\psi) \tag{14}$$

(iii) The government satisfies the budget constraint

$$(15)\ g = \theta\, r\left(z, K, D_H\left(x;\psi\right)\right) K + \phi\left(x;\psi\right)\ w\left(z, K, D_H\left(x;\psi\right)\right)\ D_H\left(x;\psi\right)$$

We will now consider what is optimal for the benevolent government to do in the current period, given that its successors will all follow policy ψ. We begin by defining the aggregate value function, i.e. the function used by the benevolent government to assess welfare, via

$$V(x;\psi) = v(x, K;\psi). \tag{16}$$

The next step is to determine the policy function ψ. For this we have to do the thought experiment that governments carry out when assessing different policy alternatives. The current government chooses the following period's state contingent capital income tax rates. It also sets the current period labor income tax rate as long as it balances the budget, which means that it has no real control over τ directly. Indirectly, however, it does. The government can affect it via its choice of the following period capital income tax rate, since this affects the return on savings, and hence the choices of households, in particular the labor choice. This means that it can affect the necessary current period tax rate needed to balance the budget.

Some expectations of future policy choices are required in order to assess current policies. We assume that the government *correctly* assumes that after next period, polices revert to be chosen according to the policy rule ψ. Let's define the problem of an agent under arbitrary, state contingent, capital income tax rates tomorrow given by $\vec{\theta'} := \{\theta(s')\}_{s' \in S}$, and after that we revert to the economic equilibrium described above. Then

$$\widehat{v}\left(x, \vec{\theta'}, k\,; \psi\right) = \max_{c,k',h} \; u\left(c,h\right) + \beta \sum_{s'} \Gamma^{ss'} \; v\left(x'\left(s'\right), k'\,; \psi\right) \tag{17}$$

subject to

$$c + k' = (1-\theta)\, r(z,K,H)\, k \, + (1-\tau)\, w\left(z,K,H\right)\, h \, + k \tag{18}$$

$$K' = \widehat{D}_K\left(x, \vec{\theta'}; \psi\right) \tag{19}$$

$$H = \widehat{D}_H\left(x, \vec{\theta'}; \psi\right) \tag{20}$$

$$\tau = \widehat{\phi}\left(x, \vec{\theta'}; \psi\right) \tag{21}$$

A few things are important to note. One is that, in the next period, the economy reverts to the economic equilibrium associated to the function ψ. Another is that we have marked a few functions with hats. This is because even though they play the same role as their no-hats counterparts above, they are indeed different as they are the result of agents' actions under the expectations that capital income taxes are set to $\vec{\theta'}$ rather than to whatever is prescribed by the function ψ. We next define intermediate equilibrium as the equilibrium that results from making the solutions to problem (17) consistent with the functions that the agents take as given. Obviously, these functions satisfy the property that

$$D_K(x;\psi) = \widehat{D}_K\left(x, \psi(x); \psi\right) \tag{22}$$

$$D_H(x;\psi) = \widehat{D}_H\left(x, \psi(x); \psi\right). \tag{23}$$

Definition 3 *An intermediate economic equilibrium given a policy rule* ψ *and a stationary economic equilibrium* $\{v\left(.;\psi\right),\ D_K(.;\psi),\ D_H(.;\psi),\ \phi(.;\psi),\ d_k(.;\psi),\ d_h(.;\psi)\}$ *is a set of functions* $\left\{\widehat{v}\left(.;\psi\right), \widehat{D}_K(.;\psi), \widehat{D}_H(.;\psi), \widehat{\phi}(.;\psi), \widehat{d}_k(.;\psi), \widehat{d}_h(.;\psi)\right\}$ *such that*

(i) Functions $\left\{\widehat{v}\left(.;\psi\right),\ \widehat{d}_k(.;\psi),\ \widehat{d}_h(.;\psi)\right\}$ *solve the individual problem (17) given functions* $\left\{\widehat{D}_K(.;\psi), \widehat{D}_H(.;\psi), \widehat{\phi}(.;\psi),\ \text{and } \psi\right\}$.

(ii) The agent is representative, i.e.,

$$\widehat{D}_K\left(x, \vec{\theta}'; \psi\right) = \widehat{d}_k\left(x, \vec{\theta}', K; \psi\right) \tag{24}$$

$$\widehat{D}_H\left(x, \vec{\theta}'; \psi\right) = \widehat{d}_h\left(x, \vec{\theta}', K; \psi\right) \tag{25}$$

(iii) The government satisfies the budget constraint

$$g = \theta r\left(z, K, \widehat{D}_H\left(x, \vec{\theta}'; \psi\right)\right) K + \widehat{\phi}\left(x, \vec{\theta}'; \psi\right) w\left(z, K, \widehat{D}_H\left(x, \vec{\theta}'; \psi\right)\right) \widehat{D}_H\left(x, \vec{\theta}'; \psi\right) \tag{26}$$

From the individual value function of the intermediate equilibrium we get an aggregate value function capable of assessing alternative policies for the next period when policies after that are given by ψ. The aggregate value function is

$$\widehat{V}\left(x, \vec{\theta}'; \psi\right) = \widehat{v}\left(x, \vec{\theta}', K; \psi\right). \tag{27}$$

Now what the benevolent government that expects other governments to use ψ does is to solve

$$\Psi(x;\psi) = \underset{\vec{\theta}'}{\operatorname{argmax}} \;\; \widehat{V}\left(x, \vec{\theta}'; \psi\right). \tag{28}$$

At this point it should be clear that the political equilibrium is characterized by the condition that when the benevolent government expects future governments to follow a certain policy, then it prefers that same policy. Clearly, this is a fixed point problem. Formally, then, we have

Definition 4 *A political equilibrium is a function ψ such that given its associated recursive and intermediate equilibria satisfies, for all x,*

$$\vec{\theta}' = \Psi(x;\psi) = \psi(x). \tag{29}$$

5 Calibration

We now describe the class of economies that we study. They are for the most part standard in the literature. Later, we discuss what are the parameters that make a difference quantitatively. Unlike in most of the real business cycle literature, we choose the time period to be a year rather than a quarter. This requires some adjustments to the representation of the process for the shocks.

The main reason for choosing a year as the period length in the baseline parameterization is that it facilitates comparison with the results in Chari, Christiano, and Kehoe (1994), who also use a yearly model. The fact that they do is of course no coincidence; tax rates are typically adjusted no more often than annually in the United States. It is of course another matter whether a year corresponds closely to the lag between legislation and implementation. But even this is not so unreasonable. In any case, we perform a detailed sensitivity analysis with respect to the period length. Note that the length of the period is not necessarily related to the issue of the balanced budget rule. The former basically restricts the length of time before capital income taxes can be levied, while the latter restricts the ability to redistribute government revenue across time.

5.1 Preferences and technology

The functional forms are standard. The utility function is of the CRRA form (moreover in the baseline case we use log utility), and technology is Cobb-Douglas.

$$(30) \qquad u(c,h) = \frac{\left(c^{\alpha}\,(1-h)^{1-\alpha}\right)^{1-\sigma}}{1-\sigma} \qquad \text{and} \qquad F(z,K,H) = z\,AK^{\nu}H^{1-\nu}$$

where A is a normalization parameter chosen so that output per period in the full commitment economies is 1 on average. Meanwhile, the average value of g is calibrated so that the equilibrium is such that government purchases are 20 % of output in the no commitment case as well. Of course, g remains a parameter beyond the control of the government in all cases. The rest of the parameters are calibrated in a way suited to analyze the public sector with values for the discount rate and the depreciation rate slightly below their counterparts in models without a government and with standard values for labor share and risk aversion. In addition, we use a high appreciation for leisure to take into account that the total amount of time spent working out of the working age population may be around a fourth or a fifth of total available time.

5.2 Stochastic shocks

In order to facilitate comparison with Chari, Christiano, and Kehoe (1994), we assume, as they do, that shocks to government purchases and productivity are independent (in the data, these shocks are positively correlated). We model those processes as symmetric two-state Markov chains with mean .2 of GDP and 1, respectively. We calibrate the parameters for the z process so as to match the variance and autocorrelation reported in Prescott (1986) and the g process so as to match the moments reported in Jonsson and Klein (1997).

The specific parameterization that we use is depicted in Table 1. Table 2 shows the implied main statistics for the exogenous stochastic processes for productivity and government expenditures. As we see, lengthening the period reduces the standard deviation and the autocorrelation due to time averaging. For details, see Appendix B.

Parameter values				
β	σ	α	δ	ν
0.97	1	0.20	0.08	0.36
$g \in \{.184, .216\}$			$\Gamma_g^{11} = .835$	
$z \in \{.976, 1.024\}$			$\Gamma_z^{11} = .946$	

Table 1: *Parameter values for the baseline economy with a period length of one year.* Γ^{11} *is the probability of remaining in the current state.*

STATISTICS OF THE SHOCKS

	One Year	Two years	Four Years
Technology shock			
Mean	1.0	1.0	1.0
Standard Deviation	0.024	0.023	0.022
Autocorrelation	0.88	0.83	0.74
Government purchases			
Mean	0.20	0.20	0.20
Standard Deviation	0.016	0.014	0.013
Autocorrelation	0.66	0.59	0.48
Correlation between the shocks	0.0	0.0	0.0

Table 2: *Properties of the shocks in the model economies.*

The results in the case without commitment are very sensitive to some of these parameter values, especially the length of the time period as incorporated in β, δ, Γ_g^{11} and Γ_z^{11} and the degree of intertemporal substitutability as measured by σ. We report below the sensitivity analysis with respect to these parameters. We discuss in section 6.4 the reasons why the quantitative properties are so dependent on both the length of the period and the intertemporal elasticity of substitution.

6 Quantitative results

In order to provide a background against which to compare and contrast the quantitative properties of time-consistent fiscal policy, we begin by presenting the results for the commitment economy in section 6.1. In section 6.2 we proceed to present the main contribution of this paper: a quantitative characterization of optimal fiscal policy when the government cannot commit beyond the next period's capital income tax rate. In the next section, we describe the outcome of sensitivity analysis with respect to those parameters that turn out to matter, and explain why they matter. Section 6.3 presents some cyclical properties of U.S. data in order to give some perspective on the theoretical results. Sections 6.4 and 6.5 contain some sensitivity analysis. Finally, in sections 6.6 and 6.7, we trace the sources of the results in the previous sections by considering economies hit by one type of shock (government purchases or productivity) only.

6.1 Commitment

The key findings of the economy with commitment are depicted in Table 3. We can summarize these findings as follows.

1. The capital income tax rate is close to zero on average.

2. The capital income tax rate is very volatile.

3. The expected capital income tax rates, i.e. the conditionally expected value of next period's capital income tax rate given current information, has a standard deviation that is about two thirds of that of the realized tax rate.

4. The labor income tax rate is large in the sense that on average, the entire burden of taxation is borne by labor.

5. The labor income tax rate is very smooth.

6. The tax rates are negatively correlated with each other.

7. Both tax rates are countercyclical. (This is the outcome of the interaction with both shocks. Later we look at the relation with each shock separately.)

These patterns are for the most part very similar to those described by Stockman (2001) even though the class of economies that he looked at are slightly different (he considers economies that grow and hold a constant yet positive amount of debt).

ECONOMIES WITH COMMITMENT: One-year periods		
	$\sigma = 1$	$\sigma = 5$
Capital income tax rate		
Mean	-0.0047	-0.0074
Standard Deviation	0.18	0.19
Coefficient of Variation	39.40	25.69
Autocorrelation	0.68	0.71
Correlation with expected capital income tax rate	0.99	0.99
Correlation with output	-0.26	-0.29
Correlation with technology shock	-0.41	-0.42
Correlation with government purchases	0.89	0.90
Expected capital income tax rate		
Standard Deviation	0.13	0.13
Coefficient of Variation	34.20	20.93
Autocorrelation	0.69	0.72
Correlation with output	-0.33	-0.36
Correlation with technology shock	-0.49	-0.50
Correlation with government purchases	0.82	0.84
Labor income tax rate		
Mean	0.31	0.31
Standard Deviation	0.009	0.009
Coefficient of Variation	0.028	0.029
Autocorrelation	0.88	0.93
Correlation with realized capital income tax rate	-0.40	-0.34
Correlation with expected capital income tax rate	-0.45	-0.37
Correlation with output	-0.27	-0.40
Correlation with technology shock	-0.04	-0.02
Correlation with government purchases	-0.24	-0.25
Output		
Mean	1.0	1.0
Standard Deviation	0.040	0.043
Coefficient of variation	0.040	0.043
Autocorrelation	0.93	0.95
Correlation with technology shock	0.97	0.92
Correlation with government purchases	0.11	0.05

Table 3: *Properties of taxes in the model economies with commitment.*

In general the properties that we found are also quite similar to those described in Chari, Christiano, and Kehoe (1994), with the exception, perhaps, of the fact that in their case the capital income tax rates are even more volatile which points to the role that the balanced budget constraint has in our economy. The rationale for this is clear: with positive autocorrelation the news are bigger than the value of the shocks, in the sense that the needs for financing are larger than the period increase in government expenditures. If the government could carry resources across periods, then it would tax more in the event of bad news to reduce the need for future taxation.

So to summarize, under commitment the burden of taxation is borne almost completely by labor while capital taxation accommodates all surprises.

6.2 No Commitment

The first column of Table 4 shows the key characteristics of taxes in the baseline economy without commitment. The differences with the commitment case are dramatic. A summary of those characteristics is[7]

1. The tax rate on capital income is very large.

2. The capital income tax rate is volatile. However, as measured by the standard deviation of the capital income tax rate, volatility is only about two thirds as

[7] We will look at the responses of taxes to the specific shocks in sections 6.6 and 6.7.

high in the economy without commitment.

3. Expected capital income tax rates are about two thirds as volatile as realized capital income tax rates, a similar relation that the one that holds in economies with commitment.

4. The tax rate on labor income is quite small; on average it is about a third of its level in the economy with commitment. This is not surprising, but also not obvious. For given levels of capital and hours worked, the higher capital income tax rate in the no commitment economy lowers the labor income tax rate required to balance the budget. However, general equilibrium effects could and presumably do partly offset this effect.

5. Labor income taxes are fairly volatile, more so than in the economy with commitment. Given the lower level of labor income taxes, the coefficient of variation in the economy without commitment is an order of magnitude greater than in the full commitment economy.

6. The tax rates are negatively correlated with each other as they were in the economy with commitment, only more strongly so.

7. Capital income tax rates are slightly procyclical and labor income taxes are slightly countercyclical. This is the general outcome of the interaction with both shocks. The same did not happen in the economy with commitment where both

tax rates where countercyclical.

8. The autocorrelation of taxes is lower than in the economy with commitment and lower than that of output. This indicates that the level of the capital stock has an important role in shaping the tax rates that is not present in the economy with commitment.

There are a few other important characteristics of the two model economies that differ. Mostly, they relate to the properties of the allocations, not the tax rates themselves. They are

8. Output is much lower in the no commitment economy (about 85%) as a result of the high capital income tax rate that reduces the average amount of capital in the economy.

9. Output volatility is smaller in the no commitment economy, not only as a result of the lower output, but also in terms of coefficients of variation (about 90% as volatile in relative terms).

10. Autocorrelation of output is also smaller in the economy without commitment (about 90% of that in the full commitment case).

To summarize, the properties of capital and labor income taxes are very different under no commitment than under commitment.

ECONOMIES WITHOUT COMMITMENT				
	1 Year	2 Years	4 Years	$\sigma = 5$
Capital income tax rate				
Mean	0.65	0.47	0.36	0.81
Standard Deviation	0.11	0.12	0.11	0.44
Coefficient of Variation	0.17	0.25	0.31	0.54
Autocorrelation	0.56	0.47	0.31	0.60
Correlation with expected capital income tax rate	0.93	0.92	0.88	0.98
Correlation with output	0.11	0.08	0.00	0.58
Correlation with technology shock	-0.58	-0.54	-0.51	-0.75
Correlation with government purchases	0.46	0.66	0.73	0.46
Expected capital income tax rate				
Standard Deviation	0.069	0.060	0.041	0.26
Coefficient of Variation	0.11	0.12	0.11	0.30
Autocorrelation	0.66	0.54	0.32	0.73
Correlation with output	0.042	0.058	-0.018	0.47
Correlation with technology shock	-0.62	-0.56	-0.55	-0.83
Correlation with government purchases	0.12	0.33	0.34	0.42
Labor income tax rate				
Mean	0.12	0.20	0.23	0.01
Standard Deviation	0.031	0.023	0.016	0.180
Coefficient of Variation	0.25	0.12	0.07	18.00
Autocorrelation	0.68	0.58	0.40	0.57
Correlation with realized capital income tax rate	-0.74	-0.72	-0.69	-0.99
Correlation with expected capital income tax rate	-0.90	-0.91	-0.92	-0.97
Correlation with output	-0.26	-0.30	-0.26	-0.59
Correlation with technology shock	0.35	0.27	0.25	0.74
Correlation with government purchases	0.14	-0.086	0.17	-0.38
Output				
Mean	0.86	0.92	0.95	0.7
Standard Deviation	0.031	0.030	0.028	0.063
Coefficient of variation	0.036	0.033	0.030	0.088
Autocorrelation	0.84	0.84	0.84	0.64
Correlation with technology shock	0.74	0.78	0.84	-0.018
Correlation with government purchases	0.32	0.29	0.29	0.44

Table 4: *Properties of taxes in the model economies without commitment. Note that we define the coefficient of variation as the modulus of the ratio of the standard deviation and the mean.*

To further show how different the two cases are, Figure 1 shows the dynamic adjustment of an economy where the government loses its ability to commit in the absence of stochastic shocks. In this case, g is held constant throughout, so the new steady state will not be such that government purchases are 20% of GDP; in fact they will be more, because output falls.

In particular, Figure 1 illustrates the paths for the capital stock, output, hours worked, and tax rates, starting from the long run situation under commitment, and then at period zero, this commitment is lost, so that capital income taxes can be changed starting in period 1. The figure shows the outcomes when all shocks are set constant and equal to their unconditional means and the period length is set to one year.

What we see in Figure 1 is that an anticipated increase in the capital income tax rate encourages people to choose high leisure and consumption in period zero. Thus hours plunge and the capital stock starts to fall. Since hours fall so much, the labor income tax required to balance the budget initially increases dramatically, but eventually falls below its initial level as hours recover and capital income tax revenues rise to a higher level. In the long run, output falls as a result of the lower capital stock, but in the short run it falls even more in response to the initial fall in hours.

Conversely, one can calculate what happens if a commitment technology were suddenly and unexpectedly to be invented. As we mentioned in the introduction, the

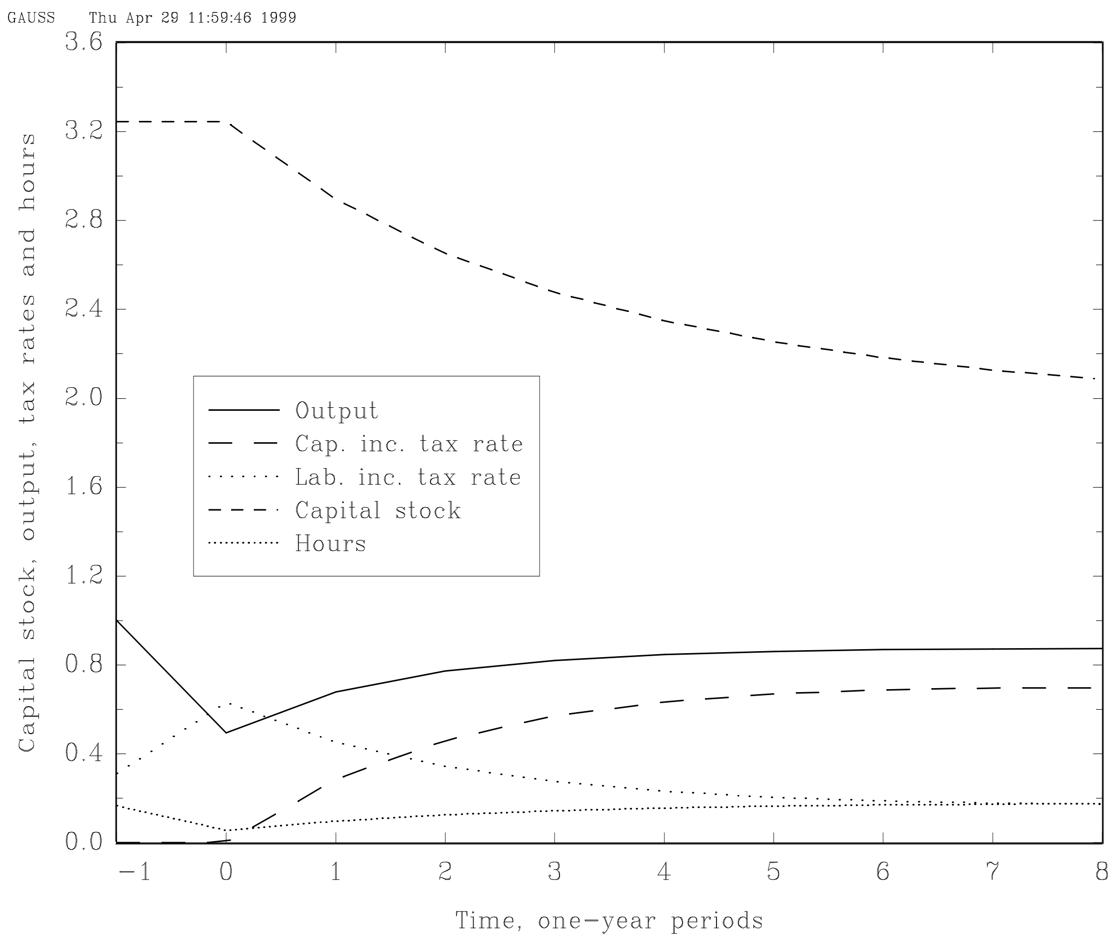

Figure 1: *Simulated capital stock, output, hours, and tax rates after a loss of commitment.*

subsequent transition involves huge welfare gains. On a period-by-period basis, the gain corresponds to an increase in consumption of 5.2%. Translating the gain into a one-time payment, it turns out that an economy in the no-commitment steady state could afford to destroy 34% of its capital stock and still be no worse off if it only gained access to a commitment technology.

Figure 2 exhibits a sample path of the full commitment and of the no-commitment economies for identical realizations of the shocks. We can see that capital income tax rates and output move in the same direction in both economies (they respond to shocks in a qualitatively similar way) but that the magnitudes of the responses are quite different. We can also see how labor income taxes are not only lower in the economy without commitment but they are also more volatile. Still the volatility of labor income taxes is lower than that of capital income taxes.

6.3 Data

Table 5 presents some cyclical properties of U.S. tax rates. Without using any formal comparison procedure, these properties look more similar to our economy without commitment.

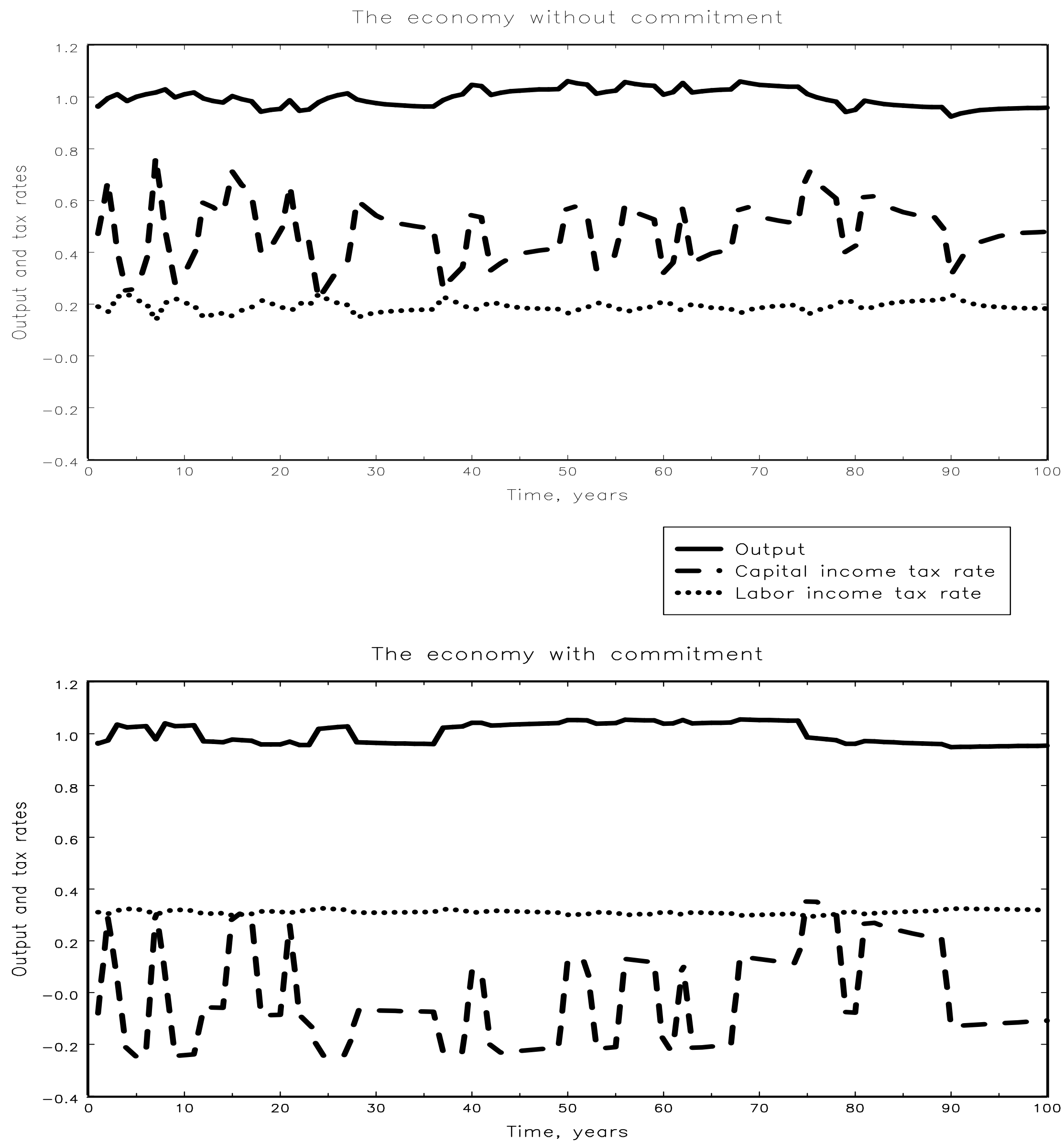

Figure 2: *Sample realizations for output and both tax rates for the economy without commitment (top panel) and with commitment (bottom panel).*

	U.S. DATA 1947-1990
	One Year Periods
Capital tax rate	
Mean	0.51
Standard Deviation	0.04
Coefficient of Variation	0.09
Autocorrelation	0.76
Correlation with output	0.17
Correlation with technology shock	0.18
Correlation with government purchases	0.42
Labor tax rate	
Mean	0.24
Standard Deviation	0.03
Coefficient of Variation	0.14
Autocorrelation	0.95
Correlation with realized cap. inc. tax rate	-0.60
Correlation with output	-0.08
Correlation with technology shock	-0.00
Correlation with government purchases	-0.42
Output	
Mean (normalized)	1.0
Standard Deviation	0.056
Autocorrelation	0.84
Correlation with technology shock	0.93
Correlation with government purchases	0.80

Table 5: *Properties of the data. Source:*

6.4 What Matters?

Obviously, changes in any of the parameters changes the quantitative answers that we obtain. For most of the parameters these changes are both predictable and small and we omit them for the sake of brevity. However there are two parameters that have dramatic implications for the quantitative findings. These are the elasticity of substitution and the length of the period. The intertemporal elasticity of substitution is, under our functional form assumption, the inverse of the coefficient of risk aversion. The value of this parameter only affects the quantitative properties of the economy without commitment (see Table 4), while it leaves essentially unchanged those of the full commitment economy (see Table 3). The intuition for this is as follows. The intertemporal elasticity of substitution determines the extent to which households will respond to a one-shot change in the expected capital income tax rate by substituting consumption and leisure across time.

If the response is large, this will act as a deterrent against levying high tax rates on capital income. Thus we should expect a higher capital income tax rate the higher is σ, since $1/\sigma$ is the intertemporal elasticity of substitution.

The role of the length of the period is, perhaps, more obvious since a longer period means (given our setup) a greater degree of commitment for the government.[8,9]

In the economy with commitment the properties of optimal taxation barely change at all when we vary both parameters. Table 4 shows the properties of the tax rates and allocations for the model economies for different values of the length of the period and for an economy with a low elasticity of substitution (the high σ economy where periods are set to mean years). The most striking properties are the following.

1. The tax rate on capital income is much higher in economies with low commitment and with low intertemporal elasticity of substitution. Increasing the length of the period results in a substantial decrease of the capital income tax rate and in an increase in the labor income tax rate. A low elasticity of substitution implies lower reductions in work effort and savings for any given capital income tax

[8] It might be thought that the length of the period is intimately related to the assumption of balanced budgets: the longer the period, the less restrictive this assumption seems to be. From the point of view of the government's ability to smooth stochastic shocks, this is true. But the really important aspect of the balanced budget assumption is not the limits it sets on the government's ability to shift the tax burden across contingencies. Rather, its main importance comes from the the limits it sets on the ability of governments to shift the tax burden across *time*, i.e., between its successors and itself.

[9] It is perhaps not obvious that the experiment we carry out is equivalent to retaining the same calibration but letting the government commit n periods in advance. However, in the environment of Krusell, Quadrini, and Rios-Rull (1997), the two are quantitatively very similar, and we have no reason to think that the present environment is relevantly different.

rate tomorrow which reduces the cost of taxing capital and hence increases the willingness of the government to use capital taxation.

2. The volatility of capital income taxes in terms of standard deviations is not affected much by the length of the period. It is very much affected, however, by the elasticity of substitution. The low elasticity of substitution economy has up to four times the standard deviation of capital income tax rates.

3. The volatility of labor income taxes in terms of standard deviations is decreasing in the length of the period. In the low elasticity of substitution economy it is huge.

4. In terms of the properties of output, low elasticity of substitution economies have a huge volatility, about double that of the baseline case, and it also has a lower autocorrelation.

To summarize, longer periods make the no commitment economy look more like the full commitment economy, while a lower intertemporal elasticity of substitution exacerbates the problems induced by lack of commitment as it makes work effort and savings inelastic with respect to future income tax rates.

6.5 Consumption-leisure substitutability

So far, we have assumed that the elasticity of substitution between leisure and consumption is one. In order to investigate what happens when we deviate from this assumption, we consider the following specification of the utility function.

$$u(c,h) = \frac{c^{1-\sigma_1}-1}{1-\sigma_1} + \gamma\frac{(1-h)^{1-\sigma_2}-1}{1-\sigma_2}.$$

These preferences are not CRRA and they are unit dependent. To start our analysis, we calibrate the model so that when $\sigma_1 = \sigma_2 = 1$, we get the same labor allocation and the same g/y as in the two–year–period baseline economy. In this case we obtain similar results as those implied by standard preferences. The response of the equilibrium taxes to changes in preferences are quite subtle and they depend on how the comparisons are specified.

The first panel of Table 6 shows the steady state tax rates as well as the relative size of the public sector and the size of the labor input across economies that only differ on the curvature parameters. Across all economies the amount needed to be financed is the same but the willingness to work is not, implying that the size of the public sector relative to output is not the same across economies. Thus it isn't necessarily the case that if one goes up the other goes down. An increase in σ_1 allows for a reduction of both steady state tax rates while an increase of σ_2 implies an increase of both tax rates which is partly due to an increase in households' willingness to work. Alternatively, Without recalibrating the economy

σ_1	σ_2	g/y	h	θ	τ
1.00	1.00	0.20	0.18	0.47	0.18
1.25	1.00	0.18	0.19	0.45	0.18
1.00	1.25	0.21	0.17	0.49	0.20
1.25	1.25	0.19	0.19	0.46	0.18

Recalibrating the economy to keep $g/y = .2$.

σ_1	σ_2	g/y	h	θ	τ
1.00	1.00	0.20	0.18	0.47	0.18
1.25	1.00	0.20	0.19	0.45	0.20
1.00	1.25	0.20	0.17	0.49	0.19
1.25	1.25	0.20	0.19	0.47	0.20

Recalibrating the economy to keep h at its $\sigma_1 = \sigma_2 = 1$ level

σ_1	σ_2	g/y	h	θ	τ
1.00	1.00	0.20	0.18	0.47	0.18
1.25	1.00	0.20	0.18	0.44	0.20
1.00	1.25	0.20	0.18	0.49	0.19
1.25	1.25	0.20	0.18	0.46	0.20

Table 6: *Changes in curvature parameters when preferences are not CRRA*

an increase in the curvature of leisure reduces the willingness to work and generates an increase of both types of taxes. The willingness to work is also responsible for the change in the relative size of the public sector.

These findings indicate that we should not interpret increases in the curvature parameter as a reduction of the intertemporal elasticity of substitution of the good to which it refers. Its interpretation is more complex than that: changes in the curvature parameters mainly affect the *intra*temporal allocation.

To partly account for the effects that changes in the willingness to work induce in the steady state allocation, the second panel of Table 6 shows the same type of parameter changes as panel 1 but the size of the public sector is calibrated so that it the same as a fraction of GDP across all steady states. The implications are no longer the same as in the previous case. An increase in the curvature of consumption reduces capital income taxes and increases labor income taxes while the opposite is true with an increase in the curvature of leisure.

Another way of accounting for the changes that the curvature parameters inflict on the willingness to work is to recalibrate the economy (by means of adjusting γ) so that the economy with constant taxes at $\theta = .47$ and $\tau = .20$) implies the same allocation of hours worked than the economy with $\sigma_1 = \sigma_2 = 1$. The third panel of Table 6 shows the economies' statistics in just this case. What we see is that an increase in the curvature of consumption induces a (somewhat surprising) reduction of capital taxation, while an increase in the curvature of leisure induces an increase in the capital tax rate. The labor taxes are unchanged.

6.6 Government expenditure shocks only

Next, we turn to the properties of the tax rates when there is only one source of fluctuations. The isolation of both shocks increases our understanding of the properties of the model economies.

Table 7 shows the main statistics when productivity shocks are set to their unconditional mean both for the case of full commitment and for the case where the government cannot commit beyond the next period.

The upper panel of Figure 3 illustrates the dynamics of the no commitment economy in response to shocks to government purchases. We may confirm that output, capital income tax rates and government purchases move together. The lower panel of Figure 3 shows the dynamics of the full commitment economy in response to shocks to government purchases. As in the no commitment economy, output, capital income taxes and government purchases move together, but the labor income tax is much flatter.

Again, we observe the main properties that we saw for the case with both shocks simultaneously. The key properties that we see are once more the high capital and low

	ECONOMIES WITH SHOCKS TO g ONLY	
Tax Rates	Full commitment	No commitment
Labor		
Mean	0.31	0.12
Standard Deviation	0.006	0.020
Coefficient of Variation	0.015	0.16
Autocorrelation	0.77	0.68
Correlation with g	-0.43	0.11
Capital		
Mean	-0.008	0.65
Standard Deviation	0.16	0.075
Coefficient of Variation	N/A	0.12
Autocorrelation	0.66	0.38
Correlation with g	0.99	0.80
Correlation between tax rates	-0.53	-0.52

Table 7: *Properties of taxes with shocks to government consumption only.*

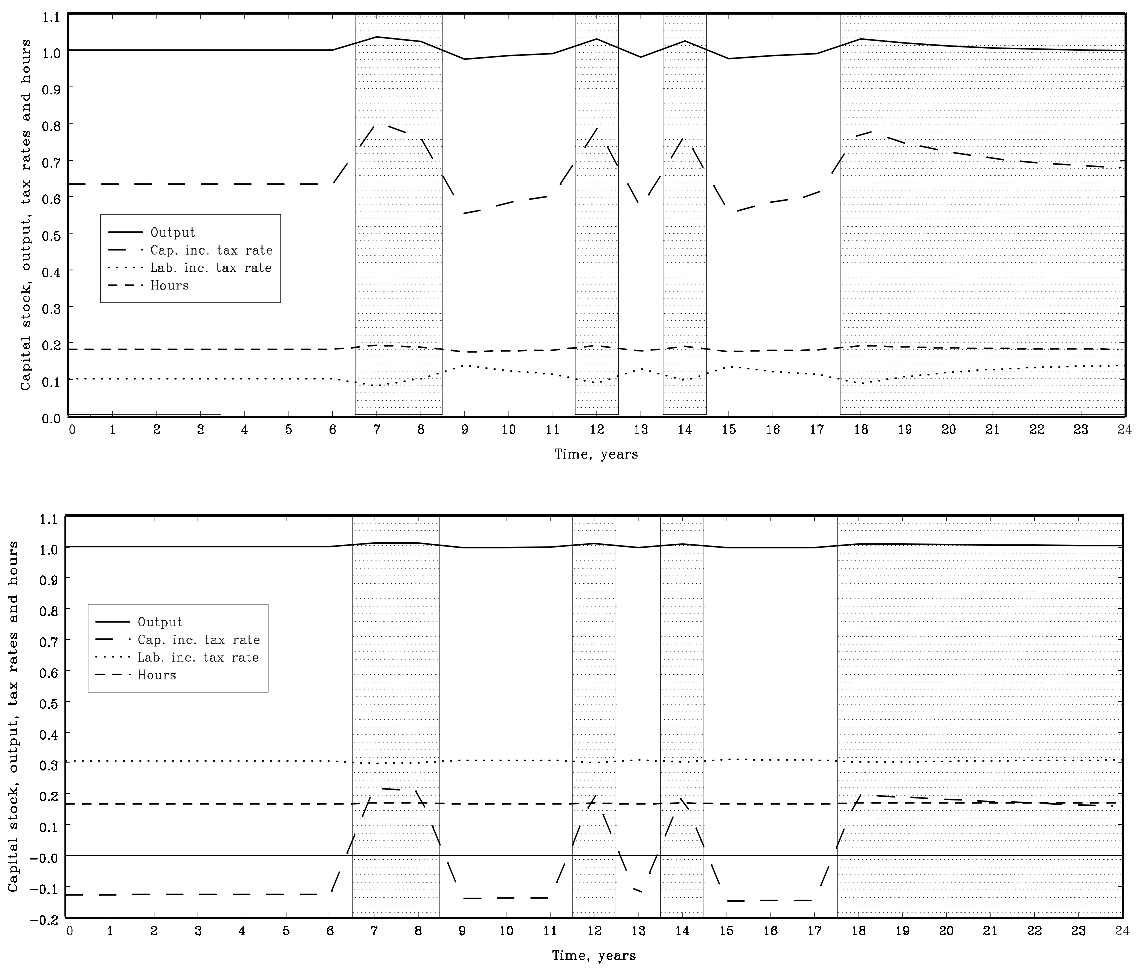

Figure 3: *Simulated output, tax rates and hours for the baseline parameterization with shocks to government purchases only. The shaded areas are "wars", i.e. periods with high government purchases. The upper panel exhibits the no commitment economy. The lower panel exhibits the full commitment economy.*

labor taxes without commitment, the lower standard deviation of capital income taxation and the higher standard deviation of labor income taxes under no commitment, even though overall capital income tax rates are more volatile than labor income tax rates are. Under commitment labor income tax rates are negatively correlated with the shock while with no commitment this correlation is small and positive.

This seems to indicate that in the full commitment economy a government expenditure shock that requires extra financing is purely dealt with an increase in capital income taxes. This is not the case in the economy without commitment where an increase in public expenditures is associated first (see period 7) with a decrease in labor taxation and, as times passes and the capital is depleted, with an in increase in labor taxation. The correlations of the tax rates with the shocks are smaller in absolute value in the economy without commitment. This type of behavior, points to the stock of capital playing a prominent role in shaping the tax system and it is in our view what is mostly responsible for the overall higher volatility of labor taxation in the economy without commitment.

The differential response to government expenditure shocks result in a higher response of output in the economy without commitment. The differences that commitment generates in the correlations between taxes and the shock do not translate into changes in the correlation between the tax rates themselves which is clearly negative in both economies.

6.7 Productivity shocks only

Table 8 shows the main statistics when government expenditures are set to their unconditional average both for the case of full commitment and for the case where the government cannot commit beyond the next period.

	ECONOMIES WITH SHOCKS TO z ONLY	
Tax Rates	Full commitment	One-period commitment
Labor		
Mean	0.31	0.12
Standard Deviation	0.0075	0.023
Coefficient of Variation	0.024	0.19
Autocorrelation	0.93	0.62
Correlation with z	-0.16	0.54
Capital		
Mean	0.0089	0.65
Standard Deviation	0.078	0.091
Coefficient of Variation	N/A	0.14
Autocorrelation	0.83	0.69
Correlation with z	-0.90	-0.80
Correlation between tax rates	-0.39	-0.99

Table 8: *Properties of taxes in the model economies with shocks to productivity only.*

The upper panel of Figure 4 illustrates the dynamics of the no commitment economy in response to technology shocks, and the lower panel of Figure 4 illustrates the dynamics of the full commitment economy in response to productivity shocks.

In the economy with commitment the tax rates responses only depend on the state

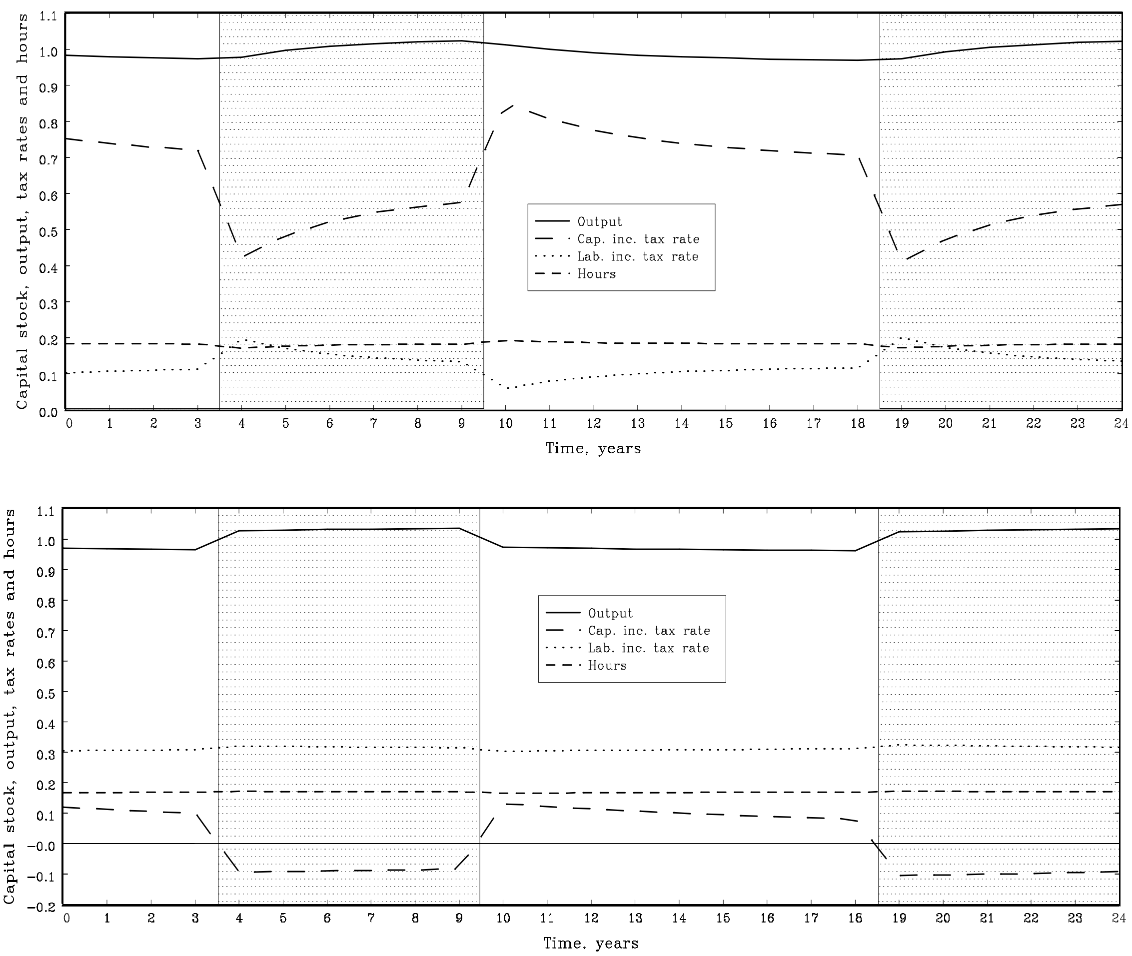

Figure 4: *Simulated output, tax rates and hours for the baseline parameterization with shocks to productivity only. The shaded areas are the "booms", i.e. the periods with a high value of the productivity variable z. The upper panel exhibits the no commitment economy, and the lower panel exhibits the full commitment economy.*

of nature and they are relatively unchanged for different values of the stock of capital. This is not the case for the economy without commitment where the response to shocks changes dramatically with the level of capital. In this regard, a positive productivity shock in the economy with commitment induces a cut in capital income taxes and a very small change in labor taxes for as long as the boom lasts. In the economy without commitment good news are followed by a cut in capital income tax rates and an increase in the labor income tax rate. As the expansion continues the labor tax rate goes down and the capital income tax rate goes up. Therefore, the specific stochastic response to the shocks in the economy without commitment seems to depend in the persistence of the shocks. What this implies for the correlations structure is that under commitment labor income tax rates are mildly negatively correlated with productivity while without commitment this correlation is sizable and positive. Also note that without commitment the two tax rates are perfectly negatively correlated while they are only mildly negatively correlated under commitment.

When we look at the implications for output we have the reverse result of the economy that is subject to shocks to government expenditures only. With shocks only to productivity, output is less volatile in the no commitment economy; the coefficient of variation is just 0.033 as compared to 0.040 in the full commitment economy. Recall that the overall effect of the lack of commitment is to make output less volatile, about 10% less volatile.

6.8 Constitutional Differences

We now turn to describe how small changes in the constitutional design can and do have large implications for the policies chosen. Consider an alternative environment which is like the one we have considered so far in that the government is subject to a balanced budget rule, but that instead of inheriting a commitment from the previous government to a capital income tax rate, suppose it inherits a commitment to a labor income tax rate. The capital income tax is then whatever it has to be to balance the budget, and the government's current period choice variable is next period's labor income tax rate.

The issue is whether this arrangement is the same as the one we have studied so far. In a static model, the answer is yes. The government has to balance the budget and has one instrument to do so, so it has no degrees of freedom.

In a dynamic context, things are very different. By choosing one of tomorrow's tax rates, the government affects today's incentives to save and work. But this means that it can indirectly affect the current tax rate (the one that it does not inherit), because current labor input and prices determine what tax rate is required today in order to balance the budget. These indirect effects are an essential part of the incentives that the current government faces and the opportunities it has for manipulating its successor.

Moreover, the mechanics of these indirect effects are very different depending on what tax is inherited. If the government inherits a capital income tax rate, then its choice of tomorrow's capital income tax rate will affect the rate of return on postponing consumption and leisure. For the sake of argument, say it is considering an increase in the capital income tax rate. Such an increase will reduce incentives to work today, thus increasing the labor tax rate that is required to balance the budget. Since the labor tax rate is distortionary, this is a fairly powerful deterrent against prohibitively high capital income tax rates.

With labor income tax rates being inherited from the past (and chosen for the future), the corresponding deterrent is absent. When contemplating a lower labor income tax rate tomorrow (and hence a higher capital income tax rate tomorrow), the government still has to take into account that this reduces incentives to work today. But in this setting, that means that the current capital income tax has to increase in order to balance the budget, and the current capital income tax is not distortive, since it is levied on a factor that is inelastically supplied from today's point of view.

These considerations suggest that capital income tax rates should be higher in the economy where labor income taxes are inherited (and chosen for the next period) than in the economy where the capital income tax plays these roles. This is indeed the case. In the baseline economy, the steady state capital income tax is 77% (as compared to 65% in the economy where the capital income tax is the state variable).

This shows the large quantitative importance of seemingly minor constitutional details.

7 Conclusions

We have found that the properties of optimal fiscal policy depend crucially on the extent to which there is commitment on the part of the government. With the limited (one-period) commitment that we study in this paper, we observe huge capital income tax rates and fairly volatile labor income tax rates. These properties are nothing like the ones we and others have found for the case of full commitment.

We also find that, when the government cannot commit itself fully, the average level of the capital income tax rate varies significantly with the degree of intertemporal substitutability and the parameters associated with the period length. By contrast, these parameters are essentially irrelevant in the full commitment case.

We leave for future research the issue of what optimal fiscal policy looks like under imperfect commitment but when the government can issue debt.

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Appendix

A Computation

The computations were done using linear-quadratic approximations of the period return function for the no-commitment economy and linearization of the first order conditions for the full commitment economy. In neither case do we approximate around the deterministic steady state; rather, we choose a different point around which to approximate for each realization of the stochastic shock. The details of the procedures are in the subsections that follow.

A.1 Computation of the model with commitment

From an abstract point of view, we can write the first-order conditions as

$$f^j\left(x^j, (x')^j, \overrightarrow{d}'\right) = \sum_{l=1}^{n} \Gamma^{j,l} f^{j,l}\left(x, d^j, (x')^j, (d')^l\right) = 0 \tag{31}$$

for each $j = 1, ..., n$ where $x = (k, \eta)$ and $d = (c, h, \mu)$ and $\overrightarrow{d'} = (c^1, ..., c^n, h^1, ..., h^n, \mu^1, ..., \mu^n)$ is a vector containing the full contingency plan for d' with one possible value for each of the possible realizations of next period's stochastic shock. We seek decision rules of the form $d = D^j(x)$ and $x' = G^j(x)$. We iterate on linear policy functions, where updating is done by linearizing and solving the first order conditions. Although this is a linearization-based approach, we achieve more precision than is usually associated with linearization-based methods by carefully choosing the points around which to linearize.

The approach proceeds as follows. The points around which we linearize (a separate one for each possible realization of the current state j) are chosen by making use of the previous guess of the policy function. More precisely, for each j, the point around which to approximate, call it $\left(\widetilde{x}^j, \widetilde{d}^j, (\widetilde{x}')^j, \left(\overrightarrow{\widetilde{d}'}\right)^j\right)$ is chosen as follows, where we will denote the most recent guesses of the decision rules by D_0^j and G_0^j. $\widetilde{x}^j$ is chosen as the simulated sample average of x conditional on the stochastic shock being equal to j. $\widetilde{d}^j$ is set via $\widetilde{d}^j = D_0^j\left(\widetilde{x}^j\right)$. Similarly, $\left(\widetilde{x}'\right)^j = G_0^j\left(\widetilde{x}^j\right)$. Finally, $\left(\overrightarrow{\widetilde{d}'}\right)^j$ is set so that, for each l, $\left(\left(d'\right)^l\right)^j = D_0^l\left(\left(\widetilde{x}'\right)^j\right)$. Notice that we need the decision rules in order to choose the points around which we approximate, and that we need points in order to update the decision rules. This means that we have a fixed point problem for the choice of points, and hence an extra level of iterations on top of the iterations on the decision rule itself.

A.2 Computation of the equilibrium of the economy without commitment

Given a policy function, the computation of the stationary economic equilibrium is done by value function iteration, once a current return has been generated by linear quadratic approximation. The points around which we approximate are chosen using an analogous method to the one we described for the full commitment economy. The intermediate economic equilibrium is then calculated in a single step.

Finally, we update the policy function by solving the government's maximization problem. We then solve for the economic equilibrium (stationary and intermediate) again, using the updated policy function. This procedure is repeated until we have found a fixed point. In this context, it is worth noting that if the initial guess of the policy function is linear, and the value function is quadratic, then the updated policy function is also linear. See Krusell, Quadrini, and Rios-Rull (1996), and Krusell and Rios-Rull (1997) for details.

B Varying the period length

Varying the period length consists in choosing values for the parameters β, δ, Γ_g^{11}, Γ_z^{11} and the high and low values for g and z.

Given the yearly β and δ, we can k-double the period length by setting the new values according to $\beta = \beta^k$ and $\delta = 1 - (1 - \delta)^k$.

Setting new values for Γ_g^{11}, Γ_z^{11} and the high and low values for g and z is a little bit more tricky. What we do is to consider a corresponding AR(1) process with the same unconditional variance and autocorrelation as the original two-state Markov process. Call this AR(1) process $< x_t >$. Now to double the length of the time period, we construct a new process $< y_t >$ via $y_t = (1/2)(x_t + x_{t-1})$ and calculate the variance and autocorrelation of $< y_t >$. We then define a new two-state Markov process with the same variance and autocorrelation as $< y_t >$.