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What Accounts for the Increase in the Number of Single Households?

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ABSTRACT

Between the mid seventies and the beginning of the nineties the share of single females grew dramatically in the U.S. (from 17% to 30%). So did the share of single mothers (from 12% to 17%). At the same time relative wages within and between sexes underwent huge changes. In this paper we measure the contribution that changes in relative wages had in accounting for these and other demographic facts. We construct a model where agents differ in sex, take marital status and fertility decisions and invest in their children's human capital. Our findings show that changes in relative earnings potential account for: *i)* almost ninety percent of the observed change in the share of single women, and *ii)* all the observed change in the share of single mothers, with a sharper increase among poor women. This occurs mainly through a drop in the model economy marriage rate that mimics the pattern found in the data.

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1 Introduction

Between the mid seventies and the beginning of the nineties the share of non-married females grew dramatically in the U.S. In the 30 to 44 age group it increased from 17% to 30%. This was accompanied by a steady growth in out of wedlock child-raising that increased the share of single mothers from 12% to 17% of the total female population, while total fertility remained constant over the same period.[1] During the same time period, relative wages both within and between sexes changed significantly in the U.S.[2] The wage premia within sexes widened, while the sex wage premium shrank.

The findings of a recent body of empirical literature imply that a change in household structure as large as that observed might have important consequences on human capital accumulation.[3] Moreover, children's outcomes are shown to be affected by the type of family children live in.[4] It is important therefore to understand what has determined this shift in marital status of the population.

The purpose of this paper is to measure the contribution that changes in absolute and relative wages between and within sexes had in accounting for the observed shift in the marital status of the U.S. population. To this end, we construct a model where agents differ in age and sex, search for a mate, make marital status, fertility and investment in their children's human capital decisions. We calibrate the model to the economic and demographic characteristics of the mid seventies. Then we recompute the equilibrium of the model after adjusting wages to match their values in the nineties and compare the two sets of steady state allocations. We decompose the pattern of wage variation into three components: a general increase, a rise in the skill premium and a decline in the gender wage. This decomposition allows us to assess the role of each single component in accounting for the observed shift in

[1]In the U.S. the total fertility rate was 2.01 in 1972 and 2.08 in 1990. See *Vital and Health Statistics, NCHS, Series 23 No. 17 February 1995.*

[2]See Katz and Murphy (1992), Krusell, Ohanian, Rios-Rull, and Violante (2000), Gottschalk (1997).

[3]Keane and Wolpin (1997) show that factors that take place in early stages of life are crucial determinants of children's later success. Neal and Johnson (1996) find that differences in educational achievements by the time of high-school completion account for almost all the observed black-white wage gap.

[4]McLanahan and Sandefur (1994) document differences in later achievements, in particular in terms of education, between children raised in single or in two parent families.

marital status of the population from the mid seventies to the early nineties. We find that in our model economy changes in wages account for: *i)* almost ninety percent of the observed change in the share of single women, and *ii)* all the observed change in the share of single mothers with a sharper increase among poor women. This occurs mainly through a drop in the model economy marriage rate that mimics the pattern found in the data. The drop in the male-female wage gap is the driving force. The increase in within sex wage dispersion plays a minor but still important role. Our model economy also accounts for the increase in the positive assortative matching of couples observed in the data.

The baseline model economy matches remarkably well the set of demographic and economic statistics chosen for the calibration. However, to ensure a satisfactory calibration process we also look at properties of the model that we do not impose (a form of imposing a set of overidentifying restrictions) and we confirm that the baseline model economy matches the data along many dimensions. The baseline model economy accurately predicts that high wage men were married much more often than low wage men, it matches the number of households without children and it accounts for between a fifth and a third of the intergenerational earnings correlation reported by previous empirical work.[5] The last result is particularly interesting because it is achieved without introducing any genetic or peer group effect or comparative advantage by college graduates to affect the education of their children that would reinforce the positive correlation of outcomes between parents and children. Moreover, children who grow up in single female headed households have 23% more chance of becoming a low wage type when adults.

Associated with the changes in family structure, our model finds that under the new wage structure there are other important changes such as an increase of 12% in the intergenerational earnings correlation in the model economy. This result suggests that shifts in the marital status composition of the population might have reinforced the positive intergenerational correlation of earnings that is observed in the U.S. economy. Changes in the marital status of the model population are achieved mainly through a reduction in the marriage rate as observed in the data. As an additional independent validation of the model economy,

[5]The estimates oscillate between .41 and .68. See Solon (1992), Zimmerman (1992) and Knowles (1998).

changes in the wage regime leave women's fertility unchanged which is also consistent with the data.

The model that we use has additional features that we think are interesting by themselves. First, household formation and dissolution, fertility and investment in children's education are all simultaneously determined in the context of a growth model. Second, the intra-household decision process is solved by sequencing fertility and time and resource allocation decisions in a manner that avoids the need to explicitly model strategic behavior within a couple. Third, we convexify certain decisions to guarantee continuity of the operator whose fixed point is the Bellman equation. The convexification yields two important properties that are needed to use this model: it allows us to compute the solution, and it makes the statistics of the model continuous in the parameters of the model, a necessary condition for a satisfactory calibration process.

We build on the work of Becker and Tomes (1976), Becker and Tomes (1979) and their quality-quantity trade off model of parental fertility decisions. Unlike Knowles (1998), and Aiyagari, Greenwood, and Güner (1997) our model incorporates endogenous household formation and dissolution, endogenous fertility, and inter-temporal investment in the dynasty in the form of parents' investment of time and resources in children's education.

Other attempts in the literature to answer the question of what accounts for this observed marital status shift of the U.S. population have focused on the role played by sex-ratio imbalances and by welfare programs. Wilson and Neckerman (1986) looked at the reduction in the number of marriageable men due to incarcerations, unemployment and mortality rates and do not find that this can account for the facts. Hoynes (1997) found that there is no evidence that welfare contributed to increase propensities to form female headed households for either whites or blacks and that the already weak welfare effect found in previous studies is affected by an upward bias that basically disappears when individual effects are taken into consideration in the econometric set up. The role of welfare policies is hard to assess since while the share of single female welfare recipients among low wage women increased, the share of single female non-welfare recipients actually tripled (see Table 17).

In Section 2 we report the features of the data that we are most interested in. We turn to the model in Section 3 where we also define its equilibrium. Section 4 deals with how we solve some technical problems of the model though convexification of the agents decisions. Section 5 describes how we calibrate the baseline model economy to the main features of the data in the seventies, and discusses the main margins over which agents are making decisions. In Section 6 we carry out our measurement exercises by analyzing the equilibria of economies that differ from the baseline only in terms of absolute and relative wages. Section 7 concludes. In Appendix A we describe how we constructed our measure of wages, while in Appendix B there is a brief account of the computational procedures that we use to find equilibria. Finally, Appendix C shows some additional tables of interest.

2 Main Features of the Data

Most of our data are computed from the Panel Study of Income Dynamics (PSID). We look at men's and women's marital status, number of children and wages and sort them into wage groups. It is the behavior of these wage groups that we concentrate on.

We take all PSID females aged 30-44 years and males aged 35-49 years in 1974 and in 1989. We choose these age groups in attempting to study the phenomenon of the increase in the number of women that live in single households while avoiding the overlap with the increase in the age of marriage. We also want to consider women only during their child rearing years. Finally, we want to look at typical partners.[6] To sort people between high and low wages we construct a wage measure using data on wages for the 1973-1976 and 1988-1991 periods (see Appendix A for details on how we construct this measure), and then we divide them into the top (the rich, the high wage group) and the bottom (the poor, the low wage group) halves.

Table 1 shows the key statistics that we are looking at. First, note that the fraction of women living as single increase by more than two thirds. Second, the fraction of single mothers increases by two fifths. Third, in 1974 singles were much more prevalent among rich women, but by 1991 there were almost as many singles among the rich than among the

[6]The age gap between males and females is justified because, within a couple, males tend to be older than females. In fact, almost 85% of married males aged 35-49 years are married to females aged 30-44 years.

Table 1: **Demographic and economic characteristics of rich and poor women.**

	1974	1991	Change
Wages and marital behavior of women			
Fraction of singles among all women	0.175	0.296	69%
Fraction of singles among rich women	0.211	0.305	44%
Fraction of singles among poor women	0.139	0.288	107%
Fraction of single mothers among all women	0.122	0.172	41%
Fraction of single mothers among rich women	0.132	0.135	2%
Fraction of single mothers among poor women	0.112	0.209	87%

Source: PSID.

poor. Fourth, in 1974 single motherhood was slightly more common among the rich women, while by 1991, this is much more common among the poor; in fact most of the increase in the number of single mothers comes from poor mothers.[7]

Table 2: **Wage changes between the sexes and between the rich and the poor.**

	1974	1991	Change
Men's average absolute wage increase*	19.32	20.67	7%
Wage premium between men and women	1.86	1.56	-19%
Wage premium between rich and poor women	2.13	2.52	18%
Wage premium between rich and poor men	2.27	2.58	14%

Source: PSID.
* Corrected for CPI missmeasurement.

These variations in household composition have been accompanied by large changes in absolute as well as in relative wages within and between sexes as shown in Table 2. The first column reports that women's wages have moved much closer to men's wages but are still considerably lower. Our measure of the sex premium corrects for changes in the relative

[7]Ellwood and Crane (1990) report that the decline in of married women was almost as large among whites as among blacks.

education of the two sexes but not for the changes in the relative experience (see Appendix A for details on how we constructed our measure of the sex premium). The last two rows in Table 2 show the increase in the relative wages between the top and the bottom. This is the well known increase in the skill premium.[8]

Table 3: **Marriage and divorce rates in the U.S.**

	1974	1991	Change
Marriage rate	0.11	0.09	-18%
Divorce rate	0.02	0.02	0%

Sources: Statistical Abstract and Vital Statistics of the U.S.

Table 3 shows marriage and divorce rates in those two years. We see that while the divorce rate has remained basically constant throughout the period, the marriage rate has declined by 18%.[9] The main mechanism that accounts for the increase in the number of singles is, therefore, the drop in marriage rates.[10]

3 The Model

The model has overlapping generations of agents that live many periods and that differ according to various characteristics (sex, age, marital status, number of children, wage, education). Agents make decisions about the family they live in and about the investments they make in their children towards whom they have altruistic feelings. We start describing the demographics of the model in Section 3.1, then turn to the determinants of wages in Section 3.2. At this point we can describe the individual and the aggregate states of the economy and we do so in Section 3.3. We turn to agents' preferences in Section 3.4, while in Section 3.5 we discuss the special assumptions that we have made and we argue that they

[8]See Katz and Murphy (1992), Gottschalk (1997) or Krusell, Ohanian, Rios-Rull, and Violante (2000).

[9]Note that the notion of marriage that we use in the PSID is not exactly the same. In the data from the PSID we use permanent rather than legal partners, while divorce and marriage rates are for legal partners. Since most permanent couples are married we believed that the differences are small.

[10]The increase in divorce had already happened by 1974, it started in the mid sixties.

are appropriate in light of the goal of this paper. We describe the behavior of the agents in Section 3.6, which involves successive decisions on marital status, fertility and human capital investments decisions. In Section 3.7 we describe how the population evolves over time while in Section 3.8 we define steady state equilibrium.

3.1 Demographics

Agents can be either males or females, $g \in \{m, f\}$, and one of three ages: children, adults and old, $i \in \{1, 2, 3\}$, and they age exponentially. Agents care about their own consumption $c \geq 0$, the quality of the match if married $\eta \in H$, and their children's (if they have any) utility. Agents start the period either single or married $q \in \{0, 1\}$. If single they live in a household without any other adult and if married they live in a household with another adult of the opposite sex. Agents choose whether they want to be married this period, which is identified by q', but the marriage only ensues if both prospective spouses want it. Changing spouse requires to stay single for at least a period. In households where a woman is present there may be children $n \in \{0, 1, 2, \cdots\}$. The women choose every period whether or not to have one more child $n' \in \{n, n+1\}$ regardless of their marital status. Therefore, during childhood, agents are attached to an adult female, their mother, who can be married or single. We further assume that the adults in the household do not know the sex distribution of their children.[11] All members of a family age at the same time, children into adults and adults into old, and exponentially, according to a certain constant probability. Women also keep the custody of the children upon marriage dissolution. Old agents become detached from their off-spring.

3.2 Wages

Adult agents, besides being indexed by their demographic characteristics, also differ by the salary that they can command, $w \in W_g$, a finite set that differs by sex. This salary is persistent but random, with Markov matrix $\Gamma_g[w'|w]$. Its initial value is affected by the investments that the agent's parents made in the agents' education. To describe this let

[11]See below for details on why we make this assumption.

$P_g[w, \psi(y, l_f, l_m, n)]$ be the conditional probability of getting wage w for a young adult of sex g. This probability is such that the expected wage is increasing in ψ which is an aggregator of parental investment in the education. The arguments of ψ are investment in goods, y, time of the mother l_f, time of the father l_m, and number of children in the household n. Function ψ is increasing in its three first arguments and decreasing in the last one. We also assume that function ψ satisfies the Inada conditions in the first three arguments to ensure interior solutions to the maximization problems that we will define below. The time endowment of the agents while adults can then be used either to invest in their children or to work in the market at the wage w.

As we will see in the calibration stage, we will decompose W_g into two subsets that correspond to education levels, so that this variable will simultaneously provide information about both the education and the wage.

3.3 The individual and the aggregate state

At any point in time an agent of age 2, the only relevant age, is indexed by the vector $\{w, q, n, w^*, \eta\}$, that we denote compactly as z, and that tells us what the agent's wage is w, whether she is married q, how many children she has n, what the wage of the prospective husband is w^* (if $q = 1$ this person is also the current husband, while if $q = 0$ this person is a date), and the quality of the match, η. Note that we are using stars to denote prospective partner's variables, both for the wage and the gender. For an age 2 male when $q = 0$, we should understand that n is the number of children of his date, since males cannot be single fathers. Recall that the evolution of w is given by transition matrix Γ_g, and hence that of w^* by Γ_{g^*}. The two agents will only marry if both want do so. Finally, the quality of the match η from a partner that the agent just met is drawn from some γ_η distribution; if the partner is instead the spouse, then η follows a Markov process with matrix $\Gamma_\eta[\eta'|\eta]$.

The state of the economy is given by a probability measure $x = \{x_f, x_m\}$ that depicts the fraction of agents of each type. Note that the set of possible states is finite, so x is a finite dimensional vector. Not all possible states are feasible. The restriction implied by the fact that men and women are married to each other can be written as

$$x_m(w^*, 1, n, w, \eta) = x_f(w, 1, n, w^*, \eta), \qquad \text{for all} \quad w, w^*, n, \eta. \tag{1}$$

3.4 Preferences

Every period, agents get utility $u_g(c, q', n', \eta)$, where the family type matters through its size (even though there are some local externalities in consumption), and through its quality. Agents discount their own future utility at rate β. At any point in time t, the agent's assessment of her own utility is given by

$$u\left(c_t, q_{t+1}, n_{t+1}, \eta_t\right) + E_t \Bigg\{ \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \pi^{\tau-t-1} \Big\{ \pi \; u\left(c_\tau, q_{\tau+1}, n_{\tau+1}, \eta_\tau\right) + (1-\pi) \; \Omega_g\left(w_{\tau-1}, w^*_{\tau-1}, \eta_{\tau-1}\right) \Big\} \Bigg\} \tag{2}$$

where the first term inside the expectation includes all future flows of own utility conditional on not aging, and the second term the utility that ensues upon getting old, and where $(1-\pi)$ is the probability of aging. Agents care about the utility of the children in their household by weighting the expected utility of their children with concave function $b(n)$. Let's denote $E_t\{V_\tau(\bar{z}_\tau)\}$ the expected utility of n_τ children that become young adults in period τ as of period t, where $\bar{z}_\tau$ denotes the state of the children. We add a term to equation (2), where $b(n_\tau)$ weights the number of children, to obtain the total utility of an agent.

$$u\left(c_t, q_{t+1}, n_{t+1}, \eta_t\right) + E_t \Bigg\{ \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \pi^{\tau-t-1} \Big\{ \pi \; u\left(c_\tau, q_{\tau+1}, n_{\tau+1}, \eta_\tau\right) + (1-\pi) \Big[\Omega_g\left(w_{\tau-1}, w^*_{\tau-1}, \eta_{\tau-1}\right) + \beta^{-1} b\left(n_\tau\right) V_\tau\left(\bar{z}_\tau\right) \Big] \Big\} \Bigg\} \tag{3}$$

3.5 Special features of the model

We have made many potentially controversial assumptions over the structure of households. In making these assumptions we have attempted to balance the commonly held notions of what is a family with our concerns about computability and with our desire to abstract from issues of within household disagreements. Calibrating the model requires solving a system of 37 equations and unknowns and evaluating the systems requires the computation of the steady state. We think that we are pushing the boundaries of what can be computed, and that we do not have a lot of room to spare to make assumptions that more accurately describe the institution of the family. In our model agents agree over the allocation of resources between family consumption and investment in the children (there are no interpersonal transfers of utility). The members of a couple do not always agree on whether to have more children or whether to get (or stay) married. This is handled by sequencing the decisions. Marriage and fertility decisions are made earlier so that at the time of investing both parents agree. We next discuss those special assumptions of which the first two are explicitly devoted to the avoidance of bargaining issues.

• Utility is not transferable, hence there is no possibility for any of the prospective spouses to convince the other to stay.

• Every period the woman chooses unilaterally whether to have a child in addition to those she already has, and she does so before the investment decision. Note that the two parents need not agree over their desired number of children because in case of separation, it is the mother that keeps them. However, given the number of children they both agree between how much to consume and how much to invest in the children. The sequencing of decisions is what avoids the need to explicitly model bargaining.

• Children are attached to their mothers. Within the context of the model, this assumption implies that the fathers forget about their children *after* a divorce. To do otherwise would have required us to keep track of men's previous family histories, a large record keeping cost.

• A related assumption is that in married couples with children we do not distinguish men that are the biological fathers from those that are not. We make an important distinction, however, at the time of dating, when a single man dates a single mother. When deciding whether to marry or not, a man gets negative utility from the woman's children. This feature contributes dramatically to the fact that single mothers have low marriage rates.[12]

• Investment in children has returns only if and when they age. Because of this assumption we do not need to keep track of previous investment.

• Parents do not know the sex of their children. Boys and girls are not equal (in fact their rate of return of human capital investment is different). Accordingly if parents knew the sex of their children their investment decisions would be affected. To keep track of the sex distribution of the children would increase the state space substantially.[13]

• There is no genetic transmission of characteristics. In particular, a higher wage of the parents does not make it more likely for the child to have a higher wage beyond that ensured by the higher investments that the parents make.[14]

• Divorce is free of costs[15] and there is no child support or alimony.[16]

We leave for future research the relaxation of these assumptions, to which we could add the fact that we only look at steady states.

3.6 The agents decisions

The purpose of this Section is to describe the problems that the agents face in steady state and to obtain expressions for their value functions $V_g(z)$, for $z = \{w, q, n, w^*, \eta\}$. It is easier to understand the problem of the agents if we break it into three successive instances going backward that correspond to the consumption-investment decision (Section 3.6.1), the fertility decision (Section 3.6.2) and the marriage decision (Section 3.6.3).

[12] Otherwise single mothers could have children as a way to attract men.

[13] We are not interested in understanding the issue of differential investments in boys and girls. See Echevarria and Merlo (1995) and Foster and Rosenzweig (1999) for two different ways of approaching this issue.

[14] In future research we plan to use this model to measure the contribution of inherited talent versus the contribution of higher investments in time and money in accounting for the intergenerational autocorrelation of life-time earnings and education.

[15] See Cubeddu and Ríos-Rull (1996) for a model with divorce costs.

[16] It would be possible to incorporate divorce costs as either a loss of current consumption or a degradation of the people's own types. In the same fashion, child support could be thought of as a transfer from the man to the woman of a fraction of his current wage type.

3.6.1 The consumption-investment decision

For simplicity we analyze first the problem of a type $z = \{w, q, n, w^*, \eta\}$ female that becomes (or stays) single, and that has chosen to have n' children this period. We denote her value with $\widehat{G}_f(z, 0, n')$. For the male that becomes single there is no decision problem; his value is $\widehat{G}_m(z, 0, 0)$. Next, we move to decisions of the agents that marry.

A female that becomes single. Assuming the existence of a solution, the problem of a female that becomes single and has n' children can be written as

$$\widehat{G}_f(z, 0, n') = \max_{c,y,l_f>0} u_f\left(c, 0, n', 0\right) + \ \pi\,\beta\, E\Big\{V_f\left(w', 0, n', w^{*\prime}, \eta'\right)|w, x_m\Big\} + \tag{4}$$

$$(1-\pi)\Big\{\beta\,\Omega_f(w, 0, 0) + b(n^{'})E\left\{V(\bar{z}')|y, l_f, n', x\right\}\Big\}$$

Subject to the budget constraint:

$$c + y = (1 - l_f)\, w. \tag{5}$$

where the first term of the right hand side is the current period utility, the second is the expected utility conditional on not aging, which is affected by both the current wage of the woman and the distribution of available single men, the third term is the expected utility conditional on retirement and the fourth is the utility obtained through the children, which depends on the investment of time and money in the children and on the distribution of singles of the opposite sex that children will face when they grow up.[17]

[17]The explicit expressions for the conditional probabilities are,

$$E\Big\{V_f\left(w', 0, n', w^{*\prime}, \eta'\right)|w, x_m\Big\} =$$

$$\int_{W\times W^*\times H} V_f\left(w', 0, n', w^{*\prime}, \eta'\right)\ \frac{x_m(dw^{*\prime}, 0, 0, ., .)}{x_m(., 0, 0, ., .)}\ \gamma_\eta[d\eta']\ \Gamma_w[dw'|w]$$

for the value of the expected utility of the woman if she does not age where the quotient is the probability distribution of prospective draws from the single male population, and

$$E\Big\{V(\bar{z}')|y, l_f, n', x\Big\} = \frac{1}{2}\ E\Big\{\ V_f(\bar{z}')\ +\ V_m(\bar{z}')\,|\ y, l_f, x\Big\} = \frac{1}{2}\ \Big\{$$

We denote the solutions of this problem by functions $\widehat{y}_f(z,0,n')$, $\widehat{c}_f(z,0,n')$, $\widehat{l}_f(z,0,n')$.

A male that becomes single. Since a single male is not attached to any children he limits himself to consume what he earns working full time. His value is given by

$$\widehat{G}_m(z,0,0) = u_m\,(c,0,0,0) + \pi\beta\; E\left\{V_m\,(w^{*\prime},0,n',w',\eta')\,|w^*,x_f\right\} + (1-\pi)\,\beta\,\Omega_m(w^*,0,0) \quad (6)$$

where $c = w^*$, and where the conditional expectation of future values is analogous to that of single females. We now turn to discuss the investment problem of the married agents.

Investment problem of married agents. Let's write the problem of a married agent as if she were the dictator in her family

$$\widehat{G}_f(z,1,n') = \max_{c,y,l_f,l_m>0} u_f\,(c,1,n',\eta) + \;\pi\beta\; E\Big\{V_g\,(w',0,n',w^{*\prime},\eta')\,|w,\Big\} + \quad (7)$$

$$(1-\pi)\;\Big\{\beta\;\Omega_f(w,w^*,\eta) + b(n')E\left\{V(\bar{z}')|y,l_f,l_m,x\right\}\Big\}$$

Subject to the budget constraint:

$$c + y = (1-l_f)\,w + (1-l_m)\,w^*. \quad (8)$$

Note that the decision variables only enter as arguments of the first and last terms of equation (7), and those terms are independent of the gender of the decision maker. Henceforth both parties agree at this stage. Let's denote the solution to this problem by functions $\widehat{y}_g(z,1,n')$, $\widehat{c}_g(z,1,n')$, $\widehat{l}_g(z,1,n')$. We now turn to the fertility decision.

$$\int_{\bar{Z}} V_f\,(\bar{w}',0,0,\bar{w}^{*\prime},\bar{\eta}')\;\frac{x_m\,(d\bar{w}^{*\prime},0,.,.,.)}{x_m\,(.,0,.,.,.)}\;\gamma_\eta\,[d\bar{\eta}']\;\;P_f[d\bar{w}',\psi(y,l_f,0,n')] +$$

$$\int_{\bar{Z}} V_m\,(\bar{w}^{*\prime},0,0,\bar{w}',\bar{\eta}')\;\frac{x_f\,(d\bar{w}',0,d\bar{n}',.,.)}{x_f\,(.,0,.,.,.)}\;\gamma_\eta\,[d\bar{\eta}']\;\;P_m[d\bar{w}^{*\prime},\psi(y,l_f,0,n')]\Big\}$$

for the conditional probability of children's utility conditional on aging.

3.6.2 The fertility decision

As we have stated it is females who unilaterally decide whether to have another child or not by solving

$$\arg\max_{n' \in \{n, n+1\}} \quad \widehat{G}_f(w, q, n, w^*, \eta, q', n') \tag{9}$$

with solution $n^*(w, q, n, w^*, \eta, q')$. We now are in a position of defining the value for agents of each marital status.

$$\begin{aligned} G_f(w, q, n, w^*, \eta, q') &= \widehat{G}_f\left(w, q, n, w^*, \eta, q', n^*\left(w, q, n, w^*, \eta, q'\right)\right) \\ G_m(w^*, q, n, w, \eta, q') &= \widehat{G}_m\left(w^*, q, n, w, \eta, q', n^*\left(w, q, n, w^*, \eta, 1\right)\right) \end{aligned} \tag{10}$$

First, note that men perfectly forecast the fertility decision of their spouses or dates. Second, note that in equation (11) the number of children n is that of the women. Third, note that in equation (11) we are using $q' = 1$ to index the decision rule for the number of kids that pertains to the female. The reason is that if and only if a man stays or ends up married, the fertility decision of his spouse will affect the value of his marital status. We can now turn to the marriage decision stage.

3.6.3 The marriage decision

With the $G_g(z, q')$ functions we can assess whether agents prefer to be married or to be single, by simply evaluating,

$$\max \quad \{G_f(w, q, n, w^*, \eta, 0), G_{f,m}(w, q, n, w^*, \eta, 1)\}. \tag{11}$$

$$\max \quad \{G_m(w^*, q, n, w, \eta, 0), G_m(w^*, q, n, w, \eta, 1)\}. \tag{12}$$

It takes the agreement of both parties to marry or to stay married. So the value functions are given by

$$V_g(w,q,n,w^*,\eta) \equiv \begin{cases} G_g(z,1), & \text{if } \begin{cases} G_f(z,1) > G_f(z,0) \\ \text{and} \\ G_m(z,1) > G_m(z,0) \end{cases} \\ \\ G_g(z,0), & \text{otherwise.} \end{cases} \tag{13}$$

Let $q'_g(z)$ denote the marriage outcome for a type z, gender g agent. We can use fertility decisions $n'_g(z,q')$ and marriage outcomes $q'_g(z)$ to substitute in the decision rules $\{\widehat{y}_g(z,q',n')$, $\widehat{c}_g(z,q',n')$, $\widehat{l}_g(z,q',n')\}$ and to obtain decision rules that only depend on the state variables, $\{y_g(z), c_g(z), l_g(z)\}$.

3.7 Population dynamics

We now describe how the population evolves over time from agents' decisions and today's distribution of the population x, a process that we write as $x' = F(x)$.

In this model the population may (and does) change in size. Since we are interested in stationary equilibria, we renormalize function F to keep the population constant. Therefore, the first thing to compute is the growth rate of the population which is

$$\lambda = \pi + \frac{1}{2}(1-\pi) \int_Z n'(z) \; dx_f. \tag{14}$$

Those that start the period married were adults last period and can be easily calculated,

$$x'_g(w',1,n',w^{*\prime},\eta') = \frac{\pi}{\lambda} \int_Z 1_{n'(z)} \; q'_g(z) \; \Gamma_g[w'|dw] \; \Gamma_{g^*}[w^{*\prime}|dw^*] \; \Gamma_\eta[\eta'|d\eta] \; dx_g \tag{15}$$

where $1_{n'(z)}$ denotes the indicator function: one if the statement is true and zero otherwise.

Women that start the period as single mothers ($n' > 0$) were also all adults last period because we have assumed no children's pregnancy and they are given by

$$x'_f(w', 0, n', w^{*\prime}, \eta') = \gamma_\eta[\eta'] \; \frac{x'_m(w^{*\prime}, 0, ., ., .)}{x'_m(., 0, ., ., .)} \frac{\pi}{\lambda} \int_Z \Gamma_f[w'|dw] \; 1_{n'(z)} \left[1 - q'_f(z)\right] dx_f \qquad (16)$$

Single women with no children could have been children or adults last period

$$x'_f(w', 0, 0, w^{*\prime}, \eta') \;\; = \gamma_\eta[\eta'] \; \frac{x'_m(w^{*\prime}, 0, ., ., .)}{x'_m(., 0, ., ., .)} \left\{ \frac{\pi}{\lambda} \int_Z \Gamma_f[w'|dw] \; 1_{n'(z)=0} \left[1 - q'_f(z)\right] dx_f + \right.$$

$$\left. \frac{1-\pi}{2\lambda} \int_Z P_f[w, \widetilde{\psi}(z)] \; n'(z) \; dx_f \right\} \qquad (17)$$

Finally, we turn to the males. Newly aged males can be associated to children only via the women they associate with. We write the evolution of the distribution of single men as

$$x'_m(w^{*\prime}, 0, n', w', \eta') \;\; = \gamma_\eta[\eta'] \;\; \frac{x_f(w', 0, n', ,, ., .)}{x_f(., ., 0, ., .)} \left\{ \frac{\pi}{\lambda} \int_Z \Gamma_m[w^{*\prime}|w^*][1 - q'_m(z)] dx_m \right. \quad (18)$$

$$\left. + \quad \frac{1-\pi}{2\lambda} \int_Z P_m[w^{*\prime}, \widehat{\psi}(z)] \; n'(z) \; dx_f \right\}$$

In the last two expressions we use some elements of tomorrow's distribution to determine tomorrow's distribution. However, those elements are marginals and are only used to determine the matching ratios, and can be readily determined from today's distribution.

3.8 Competitive equilibrium

We are now ready to define a steady state, or stationary, equilibrium.

Definition. *A stationary equilibrium is a set of decision rules for consumption $c_g(z)$, resources $y_g(z)$, and time $l_g(z)$ investment in children's education, number of children chosen by women $n'_f(z)$, marriage outcomes $q,(z)$, value functions for males and females $V_g(z)$, auxiliary functions G_g and $\widehat{G}_g$, stationary distributions of agents $x_g(z)$, and a rate of population growth λ such that:*[18]

[18]Note that we are indexing the consumption and investment functions by sex. This facilitates the record

1. *Functions V_g, G_g, and $\widehat{G}_g$ satisfy equations (4-13) (i.e. agents maximize).*

2. *Consistency between male and female actions $q'_g(z) = q'_{g^*}(z^*)$ and decisions $c_g(z) = c_{g^*}(z^*)$ and $y_g(z) = y_{g^*}(z^*)$.*

3. *Individual and aggregate behavior are consistent: the population is stationary, $x = F(x)$.*

4 Convexification of the decision space to solve some technical problems

The model as it has been described has certain properties that induce severe difficulties when attempting to solve it. The root of these problems is a discontinuity at two different stages. We review in Section 4.1 how the discreteness in the marital and fertility choices induces a discontinuity in the operators that we use to solve for the value functions. Then, in Section 4.2 we describe how the same discreteness induces discontinuous mappings from the model's parameters to the model's statistics. In the calibration stage we use these mappings to search for parameter values that induce statistics in the model economy to resemble those in the data. The non–linear equation solvers that we use require those mappings to be continuous.

Henceforth in Section 4.3 we introduce a slight modification of the model economy to get around these problems that consists in convexifying the choices of the agents introducing effort functions that affect the probability of achieving the desired outcome. Because the effort functions exist to overcome a technical problem and do not play any economic role, we will ensure that the model economy has the property that there are very few agents that do not achieve their most preferred outcomes.

4.1 Discontinuity in the updating of the value functions

To solve the problem of the agents, the natural procedure is to build a Bellman type equation and to iterate on it. The problem is that the operator defined in this equation is not a

keeping and simplifies notation.

contraction, as it is not even continuous. We write the Bellman type equation as

$$\{V_m^1, V_f^1\} = T(V_m^0, V_f^0) \tag{19}$$

where $\{V_m^1, V_f^1\}$ is just a point in an Euclidean space given the finite dimensionality of the state space. In a standard problem, or in a problem with transferable utility this mapping would be continuous. However this is not the case in our economy. To see this, it suffices to show an example. Take a sequence of V's where the male's function is constant and the female's function converges to a V_f where she is indifferent between marriage and being single while for all elements of the sequence she chooses to marry. The image under operator T for the female is continuous. However, the value for the male suffers a sudden drop at that point when the female is no longer willing to marry him. The same type of reasoning can be applied at the fertility stage.

4.2 Discontinuity in the mapping from parameter values to model statistics

Another problem arises when states are discrete and there is positive mass on them. This problem consists in discontinuous behavior of the model statistics as functions of the parameter of the model.

To see this note that the parameters may be moving continuously without inducing a change of behavior in a certain state. Eventually, the change in the parameters induces a change of behavior of all agents that were in that state, which in turn induces a jump in next period's distribution, and, typically, also in the steady state distribution. Consequently, the statistics of the model, which are just integrals with respect to the stationary distribution also jump, which makes the calibration process tremendously difficult.

Theorem 12.13 in Stokey and Lucas (1989) provides some help. It gives sufficient conditions so that the moments with respect to the stationary measure are continuous with respect to some parameters. It has been used to show existence of steady states in economies with idiosyncratic shocks.[19] It states that if the state space is compact, and if the transition

[19]See Ríos-Rull (1998) for a discussion.

probabilities, that are constructed from individual decision rules and from the process for the shocks, are continuous in the parameters, and if for each set of parameters there is a unique stationary distribution, then the moments of such stationary distribution are continuous in those parameters. To take advantage of this theorem we have to make the transition probabilities continuous.

A way to do this is to make the choices of the agents continuous in the parameters which can be achieved by making the outcomes of agents' choices stochastic. In this manner a small change in a parameter changes in a small way the actions of the agents which in turn induces just a small change in the probability of next period outcomes.

4.3 Convexification of agents choices

We convexify the problem of the agents by introducing effort variables at both the fertility and the marriage stage that affect the probability of a new baby being born and of the marriage occurring. In the fertility stage we substitute the first term of equation (7), $u_f(c, 1, n', \eta)$ with an expression that includes effort as an argument. Effort generates disutility and it affects the probability of the desired outcome. The following expression already incorporates the functional forms that we choose for the probability of achieving each outcome as a function of effort and for the disutility of effort

$$\left(\frac{e^s}{e^s + \kappa e^{-s}}\right) \; u_f(c, q, n, \eta) + \left[1 - \left(\frac{e^s}{e^s + \kappa e^{-s}}\right) \;\right] u_f(c, q, n + 1, \eta) - \varsigma s^2 \tag{20}$$

where $s \in [-\infty, \infty]$ is the effort variable and κ and ς are parameters. When a woman maximizes equation (20) over s the resulting decision rule is continuous. Moreover, if a mass of agents have the same state, a fraction of them ends up with n and another fraction with $n + 1$ children which is also continuous in the parameters that determine the maximization process.

The convexification of the marriage stage is very similar except for the fact that the outcome depends on the effort of the two prospective spouses that we assume take as given the other's effort when determining their own (they play Nash).[20] The probability of getting or remaining married is given by

$$\frac{e^{s_g+s_g^*}}{e^{s_g+s_g^*}+\rho_q e^{-(s_g+s_g^*)}} \tag{21}$$

where ρ_q is a parameter indexed by the marital status. The disutility of effort is given by $-\alpha_{q,g}\, s_g^2$ which is gender and marital status specific. Of course, we rewrite current utility to accommodate the new effort terms as well as the randomization of marriage.

5 Calibration of the Baseline Economy (the Seventies)

We want to calibrate the model economy to the demographic characteristics as well as to the wage structure of the seventies. As a result of the model having a large number of parameters, we use a large set of U.S. economic and demographic statistics as calibration targets, and hence we have to solve a large system of equations and unknowns, a daunting task. Consequently, as with some modelling choices, many of our calibration choices are guided by the principle of keeping the number of parameters manageable.

The calibration of this model economy consists in solving a non-linear system with a large number of equations and unknowns. The unknowns are the parameters of the model. In this case we have 37 parameters, so we require at least 37 equations. In addition, the model has to be a steady state which requires that each time we want to evaluate the statistics of the model economy, we have to solve a very large problem. Each of the 37 non-linear equations sets to zero the squared difference between one of the chosen calibration targets computed from the data and the corresponding statistic computed from the model's steady state equilibrium allocation. Then, we proceed to minimize a weighted sum of these squared differences searching over all parameters.

We start describing the specific functional forms that we choose and the number of parameters that we have to solve for in Section 5.1. We then describe the calibration targets in Section 5.2. To make it easier to follow, the subsection titles have the relevant number

[20] In all our economies this stage produced a unique Nash equilibrium.

of parameters or targets. In Section 5.3 we describe the result of the calibration process that we call the baseline model economy. In Section 5.4 we discuss the performance of the baseline model economy by means of how well it matches other statistics of the data that are not explicitly targeted in the calibration. Finally, in Section 5.5 we discuss how the most important margins of the model work, the fertility decision and the gains from marriage.

5.1 Functional forms and parameters: 37

We divide the functional forms in five groups and describe them accordingly: demographics, wages, preferences, achievement of desired outcomes and approximation parameters.

5.1.1 Demographics: 1

There is really only one population parameter, π, the probability of remaining in the child–rearing age. When agents age we do not need to keep track of them anymore.

5.1.2 Wages: 13

We want wages to reflect educational attainment as well as life-cycle characteristics. There are 4 possible wage levels for each sex $w \in \{w_1, w_2, w_3, w_4\}$. We identify $\{w_1, w_2\}$ with the outcomes of agents that do not go to college and $\{w_3, w_4\}$ with the outcomes of those that do. Consequently, the transition between the two groups is zero. We identify w_1 and w_3 with entry level wages for each education level, that we assume equal, and w_2 and w_4 with the peak wages of each education group that we assume occur at the same ages for both education groups. Consequently, there are 3 female wage levels and 3 male wages left to determine. With respect to the transition probabilities, Γ, our assumptions require that only $\Gamma_f(w_1|w_1)$ has to be determined, since $\Gamma_m(w_1|w_1) = \Gamma_f(w_1|w_1)$ (because the two sexes' aging process is the same), $\Gamma_g(w_2|w_2) = \Gamma_g(w_4|w_4) = 1$ (because of irreversibility of age and education). These assumptions reduce the number of parameters from 32 (2 sexes times 4 levels plus 12 transitions) to 7.

In addition, we have to parameterize function the probabilities of children becoming of a certain wage type conditional on their parents' investment of time and money. Note that given our choices for the process for wages, all we need to specify is the probability of going to college that we assume sex independent and equal to

$$\left[\exp\left(\gamma_1 l_f^{\mu_1} + \gamma_2 \left(\frac{y}{n'}\right)^{\mu_2} + \gamma_3 l_m^{\mu_3}\right)\right]^{-1} \tag{22}$$

This expression has six parameters. Note that we assume that for each child what matters is the total investment in time and the per capita investment in money.

5.1.3 Preferences: 15

Utility Function: 10. We assume a current utility function that is constant relative risk aversion and separable in consumption and in the quality of the match. The utility function is indexed by sex because the contribution of the quality of the match is sex dependent.

$$u_g(c, q', n', \eta_g) = \frac{\phi_1(c, q', n')^{1-\sigma}}{1-\sigma} + \eta_g \tag{23}$$

Function ϕ_1 transforms consumption expenditures and family composition into effective consumption. We use OECD measurements that say it takes \$1.70 to provide \$1 of consumption to each member of a married couple [21] and we calibrate the parameter $\phi_{1,c}$ which determines how many additional cents per child are needed to provide the child with the same \$1 of consumption. There are four possible values for η_g. We assume that they are symmetric and centered around the mean $\bar{\eta}_g$, $\eta_{4,g} = -\eta_{1,g} + 2\bar{\eta}_g$ and $\eta_{2,g} = -\eta_{3,g} + 2\bar{\eta}_g$, which leaves 3 values to be determined, $\eta_{1,g}$, $\eta_{2,g}$ and $\bar{\eta}_g$.

Markov matrix Γ requires 12 independent parameters. We assume that the quality of the match can only move one step at a time, i.e. the $\Gamma_\eta(j|i) = 0$ if $|i-j| > 1$, which reduces the number of parameters to 6. We further reduce the number of parameters to 2 by assuming certain forms of symmetry, $\Gamma_\eta(1|1) = \Gamma_\eta(4|4)$, $\Gamma_\eta(2|2) = \Gamma_\eta(3|3)$, $\Gamma_\eta(2|1) = \Gamma_\eta(2|3)$ and $\Gamma_\eta(3|2) = \Gamma_\eta(3|4)$. We are left with $\Gamma_\eta(1|1)$, and $\Gamma_\eta(2|2)$. We assume that the unconditional

[21] This value is taken from the OECD Adult Equivalent Family Size tables.

distribution, γ_η, from which single prospective partners draw their quality of the match indicator is uniform.

Discounting 3. People discount their own future well-being at a rate β and their children's future welfare at a rate $b(n') = \beta_c \, n'^{1-\delta}$ that is increasing with n' but at a decreasing rate.

Retirement 2. The functions $\Omega_{g,s}(w)$ $\Omega_{g,m}(w, w^*, \eta)$ that represent the expected value of being retired and single, and retired and married, are another set of preference parameters. These parameters ensure that children are attractive even when the risk aversion parameter $\sigma > 1$. We restrict function Ω_g to depend on marital status but not on wages, sex or quality of the match variable, which leaves 2 additional parameters.

5.1.4 Approximation parameters: 8

As we showed in Section 4, the convexification of the model required us to specify effort functions that require a total of 8 parameters. These parameters do not carry economic meaning. Accordingly, some of the equations that we solve for have the role of ensuring that our specifications keep the convexified problem similar to the original one (see Section 5.2.4).

5.2 Calibration targets

We decompose the description of the calibration targets into demographic, wages, marriage sorting and accuracy of the approximation.

5.2.1 Demographic Targets: 23

The demographic targets pertain to properties of household composition and its flows in the U.S. during the seventies. These include the distribution of women by marital status in the different earnings groups, the distribution of women by parental status in the different earnings groups, and the average size of single and two-parent households. We choose the following 23 targets (their values are in Table 4).

1-3. The fraction of singles among women of age 30-44 in the lowest 30%, the 30%-70% and the highest 70%-100% of the earnings distribution.

4-6. The fraction of mothers among women aged 30-44 in the lowest 30%, the 30%-70% and the highest 70%-100% of the earnings distribution.

7-9. The fraction of single mothers among women aged 30-44 in the lowest 30%, the 30%-70% and the highest 70%-100% of the earnings distribution.

10-12. The average number of children per mother among women aged 30-44 in the lowest 30%, the 30%-70% and the highest 70%-100% of the earnings distribution.

13-15. The average number of children per household with a female member in the lowest 30%, the 30%-70% and the highest 70%-100% of the households' earnings distribution.

16-18. The birth rate among women of age 30-44 in the lowest 30%, the 30%-70% and the highest 70%-100% of the earnings distribution.

19-20. The average number of children per married mother, and the average number of children per single mother.

21-23. The yearly marriage rate for women older than thirty, the yearly marriage rate for women younger than thirty, and the yearly divorce rate.

5.2.2 Wages: 8

We want the baseline model economy to match the wage premia among several groups. In particular, we want to match the sex premia, the college premia and the wage dispersion within gender. Consequently we choose the following 8 targets with values reported in Table 5.

24. The fraction of the male population with college education.

25-26. The wage premia between the college and non-college men and women.[22]

27. The wage premium between men and women.

[22] Since in our model economy the distribution of women and men over educational achievements is the same we compute the college premia from the data assuming no gender differences in educational achievements. Specifically we assume that women's partition into four educational categories (non high-school degree, high-school degree but no college experience, college experience but no college degree, college degree) was the same as men's in the mid 70s. The college premia is defined as the ratio between the average wage of the population with at least some college experience and the average wage of the population with at most high-school diploma.

Table 4: **Calibration targets: Demographics**

Fraction of singles among women, age 30-44	$0-30\%$	0.142
	$30-70\%$	0.149
	$70-100\%$	0.242
Fraction of mothers among women, age 30-44	$0-30\%$	0.902
	$30-70\%$	0.885
	$70-100\%$	0.799
Fraction of single mothers, age 30-44	$0-30\%$	0.117
	$30-70\%$	0.111
	$70-100\%$	0.142
Average number of children per mother age 30-44	$0-30\%$	3.048
	$30-70\%$	2.614
	$70-100\%$	2.282
Average # of children per household with female member	$0-30\%$	2.278
	$30-70\%$	2.413
	$70-100\%$	2.172
Birth rate among women, age 30-44	$0-30\%$	0.030
	$30-70\%$	0.029
	$70-100\%$	0.023
Average number of children per married mother, age 30-44		2.686
Average number of children per single mother, age 30-44		2.501
Marriage rate of women older than 30		0.110
Marriage rate of women younger than 30		0.150
Divorce rate		0.021

28-31. The wage distribution within sexes. We look at the wage ratio between the highest 70%-100% wage group and each of the 0-30% and 30%-70% wage groups for both men and women.

5.2.3 Marriage sorting: 2

We want the baseline model economy to match the positive assortative sorting of couples that we observe in the data. Again we restrict the sample to women age 30-44. Consequently among the set of calibration targets we choose:

32. The fraction of married women of the 0-50% earning group (poor) who are married to men of the same earnings group.

Table 5: **Calibration targets: Wages**

Fraction of males who went college, age 35-49	0.347
Wage premia between college and non-college, men age 35-49	1.664
Wage premia between college and non-college, women age 30-44	1.947
Wage premia between men and women	1.860
Men's wage premia between 70%-100% wage group and 0-30%	3.279
Men's wage premia between 70%-100% wage group and 30%-70%	1.835
Women's wage premia between 70%-100% wage group and 0-30%	2.915
Women's wage premia between 70%-100% wage group and 30%-70%	2.003

33. The fraction of married women of the 50-100% earning group (rich) who are married to men of the same earnings group.

Table 6: **Calibration targets: Sorting**

Fraction of rich married women who are married to rich men	0.583
Fraction of poor married women who are married to poor men	0.599

5.2.4 Accuracy of the approximation: 4

As part of the convexification of the problem of the agents we let them choose effort that affects the probability of achieving their most desired outcome. Accordingly, we want failures to achieve the desired outcome to be minimal. Consequently we include in the set of targets the following four statistics that we want to set to zero.[23]

34. Share of married individuals who prefer to divorce but do not.
35. Share of married individuals who prefer to stay married but divorce.
36. Share of singles who either marry preferring to be single or stay single preferring to be married.

[23]As discussed above, these are not really calibration targets: they are criteria that the equation solver wants to satisfy.

37. Share of women whose fertility outcome is not the desired one.

We place more emphasis in setting to zero the first three statistics than the last one. We do not have a clear sense of the fraction of women that do not achieve their desired fertility, but it is not zero.

5.3 The Baseline Model Economy

We take that the child rearing age is 18-49 years of age. In order to compute the statistics in the model economy we have to map a period into calendar time. The calibrated value of the probability of not aging is $\pi = .898$; this implies that average adult life in the baseline is 9.81 periods, or that each period corresponds to 13 quarters. Therefore, the 30 to 44 years of age group corresponds roughly to the group of that is 4 to 8 periods of age in the model.

The task of replicating the 37 targets is daunting. We think that the baseline model economy is overall extremely successful. In any case, the reader can arrive to her own conclusions by looking at Tables 7 through 10 that describe the values of the calibration targets in the baseline model economy next to the value in the data. The parameter values that we use are in Table 16. This configuration of parameters is the best that we have been able to find in terms of matching all the targets, although, obviously, we do not match all targets exactly.[24]

Going through some of its statistics we see that the baseline model economy matches the demographic statistics remarkably well (Table 7). For instance, the model captures very well the negative relation between marriage and wages for women aged 30 to 44, as well as the association between motherhood and wages for the same age group. Perhaps its only shortcoming is to provide too steep a negative profile for the negative relation between average number of children and wages. Also, the average number of children in the baseline is lower than in the data, while baseline birth rates are higher than in the data. There is a reason for this result. Calibration targets are computed from mid 70s PSID data, i.e. when the transition towards women's lower total fertility was almost complete after the US baby-

[24] In fact some of them may be mutually inconsistent within a steady state structure.

Table 7: **Performance of the baseline model economy: Demographics**

		Data	Model
Fraction of singles among women, age 30-44	$0-30\%$	0.142	0.131
	$30-70\%$	0.149	0.143
	$70-100\%$	0.242	0.228
Fraction of mothers among women, age 30-44	$0-30\%$	0.902	0.909
	$30-70\%$	0.885	0.894
	$70-100\%$	0.799	0.791
Fraction of single mothers, age 30-44	$0-30\%$	0.117	0.104
	$30-70\%$	0.111	0.109
	$70-100\%$	0.142	0.139
Average number of children per mother age 30-44	$0-30\%$	2.498	3.048
	$30-70\%$	2.469	2.614
	$70-100\%$	2.256	2.282
Average # of children per household with female	$0-30\%$	2.278	2.254
	$30-70\%$	2.413	2.293
	$70-100\%$	2.172	1.693
Birth rate among women, age 30-44	$0-30\%$	0.030	0.049
	$30-70\%$	0.029	0.048
	$70-100\%$	0.023	0.043
Average number of children per married mother, age 30-44		2.686	2.476
Average number of children per single mother, age 30-44		2.501	2.071
Marriage rate of women older than 30		0.110	0.141
Marriage rate of women younger than 30		0.150	0.170
Divorce rate		0.021	0.017

boom period. The high number of children among PSID families in the mid 70s was the result of the baby-boom. Birth rates were instead already affected by the transition towards lower fertility. The baseline steady state allocation is trying to match features of an economy which indeed was not in a demographic steady state in the mid 70s.[25] The baseline is also very successful in reproducing the asymmetry in the flows in and out of marriage observed in the data.

Table 8 shows that the baseline model economy closely mimics the wage gender gap

[25]Ideally one would like to use women's total fertility rates per wage group as calibration targets, instead of the average number of children per mother. However this information is not available.

Table 8: **Performance of the baseline model economy: Wages**

	Data	Model
Fraction of males who went college, age 35-49	0.347	0.284
Wage premia between college and non-college, men age 35-49	1.664	3.242
Wage premia between college and non-college, women age 30-44	1.947	3.020
Wage premia between men and women	1.860	1.854
Men's wage premia between 70%-100% wage group and 0-30%	3.279	2.809
Men's wage premia between 70%-100% wage group and 30%-70%	1.835	2.275
Women's wage premia between 70%-100% wage group and 0-30%	2.915	2.630
Women's wage premia between 70%-100% wage group and 30%-70%	2.003	2.171

found in the data. For both men and women, the ratio between average wages of the top and bottom 30% is lower than in the data. At the same time the ratio between average wages of the top 30% and the 30%-70% wage group is higher than in the data. As a result the ratio between average wages of rich (top half) and poor people (bottom half) within sex in the baseline closely matches the data. This ratio is 2.146 in the model and 2.131 in the data, for women, and 2.272 in the model and 2.270 in the data, for men. The baseline model economy is largely over-estimating the wage premia between people who went to college and those who did not. This is the result of the model having all wage variation being accounted for by education. We have chosen to set the wage premium accurately.

Table 9: **Performance of the baseline Model Economy: Sorting**

Fraction of rich married women who are married to rich men	0.583	0.533
Fraction of poor married women who are married to poor men	0.599	0.530

The assortative matching (Table 9) is a little below that in the data (not surprisingly given that in the model all matching is random without any mechanism to increase the odds of matching within the same education group).

Finally, Table 10 shows the performance of the baseline model economy with respect to

Table 10: **Performance of the baseline model economy: Approximation**

Share of married individuals who prefer to divorce but do not	0.0001
Share of married individuals who prefer to stay married but divorce	0.0017
Share of singles who either marry preferring to be single or stay single preferring to be married	0.0028
Share of women whose fertility outcome is not the desired one	0.0590

our approximation targets (recall that we wanted very few agents not to achieve their desired marital status). This is exactly the case: at most 0.3% of the agents do not get the marital status that they want each year. Also, each year, just above 5.0% of women do not attain their desired fertility outcome as a result of the way we model how choices become outcomes in the model.

5.4 Overidentifying parameters

That the baseline model economy matches the 37 moments that we have reviewed so far is not surprising since the parameter values are designed to do so. We want to make sure that the baseline model economy is a satisfactory representation of the relation between wages and marital status during the seventies. To do so, we look at how it performs along a variety of other dimensions that were not targeted as calibration statistics.

There are four types of such statistics that we look at, and in all of them the baseline model economy does very well.

1. The fraction of households with a female member that have children out of all households with a female member. It is 0.864 in the data and almost the same in the model, 0.867.

2. The joint distribution of males across wages and marital status is very different from that of women. It is the low wages men that are more likely to be single both in the model and in the data. In particular the fraction of single men in the bottom half of the wage distribution is 0.187 in the data and 0.191 in the model, while that of single

men among the high wage group is 0.116 in the data and 0.097 in the model.

3. The autocorrelation of earnings across generations. This is a very important statistic that has generated a lot of attention. Its value in the data is between 0.41 as reported by Solon (1992) using the PSID, and a value of 0.68 estimated by Zimmerman (1992). In the baseline model economy this value is 0.137. We find this extremely reassuring. The model ignores all genetic transmission of ability and all comparative advantage of educated parents in improving the education of their children. In the baseline model economy the intergenerational correlation of earnings is a product only of explicit investments of time and money from the parents. Given that we abstract from the other channels that contribute to a positive autocorrelation, a value of 0.137 seems to us what it would be appropriate.

4. Moreover children who grow up in single female headed households have 23% more chance of becoming a low wage type when adults. This feature matches the pattern documented by McLanahan and Sandefur (1994) who find that, other things being equal, teenagers who spent part of their childhood apart from their biological father are twice as likely to drop out of high school.

5.5 The model's margins

In this section we discuss how the model's most important margins actually work. In particular, we look at fertility and marital status decisions.

5.5.1 Women's fertility decision

There are two factors that have a great influence over the fertility decision: *i)* the curvature of the utility function and *ii)* child-raising costs. The greater the curvature of the utility function, the larger the family size because parents like to smooth consumption over more children. If child-raising costs were independent from parents' earnings type, poorer parents would have fewer children, which is counterfactual. To get around this, the literature makes two assumptions, both of which are necessary: that the cost of raising a child is increasing with the parents' earnings type (typically achieved by assuming that children impose a fixed time cost to parents), and that the risk aversion parameter is below one.[26]

In our model, we also have child-raising costs increasing with parents' earnings. In fact, children consume a fixed fraction of total family consumption, which is bigger for wealthier families. Moreover, the calibration process determines a risk aversion parameter below one which is required to achieve a negative relationship between mothers' earnings and number of children, as it is empirically observed.

Not only poor mothers have more children in our model but, given relative returns of investment of parental time and of resources in children's education, high earning parents prefer to invest more resources and less time than low earning parents because the opportunity cost of time is higher for them.

5.5.2 The gains from marriage

In our model, marriage delivers five types of potential gains three of which are for both partners: *i)* prospective partners may like each other, i.e. $\eta > 0$, *ii)* there are increasing returns to consumption while living together, *iii)* there are returns to specialization, (either because the wages or the return of time investments are not equated between the spouses); one of the potential gains of marriage is solely for men, *iv)* children are non-exclusive goods, therefore a marriage gives utility to husbands; and another potential gain of marriage is only for women, *v)* marriage alleviates the decreasing returns of parents' time investment in childrearing.

Therefore, all agents want to marry somebody they like and who has a high wage, and they want to do it earlier rather than later. So the marriage decision involves assessing whether the current match is good enough or whether it is better to wait for a better one. The calibration process determines some specific features of this decision: it makes women less eager to accept a match by providing them often with a negative perception of their mates and it also reduces the returns to marriage by providing a negative view of it upon

[26]See Becker and Tamura (1990) and Alvarez (1999).

aging.

6 Absolute and relative wage changes

In this Section we ask to what extent the changes in wages occurred since the seventies can account for the dramatic changes that we have observed in family composition in the last few years. To answer this question, we take the baseline model economy and substitute the values for wages in the seventies for their nineties counterpart and we look at what are the implied changes in the steady state allocations. In this period, wages have changed in many dimensions, and we look at all of them both separately and together. We look at changes: in the level of all wages in Section 6.1, in the gender wage premium in Section 6.2, in the dispersion of wages both for men and for women in Section 6.3. Finally, we look at all the wage changes simultaneously in Section 6.4.

We change the wages in the model economy by changing the support of male and female wage distributions exogenously while leaving unchanged all the other parameters calibrated for the baseline economy. For example, the reduction in the wage-gender gap is achieved in the model by shifting the support of women's wage distribution upwards, while keeping men's average wage and within sex wage dispersion unchanged. As a result of these changes the model changes its steady allocations which results not only in changes in the demographics but also in changes in the wage distribution.

We care about real wages. One option would be to normalize wages using the CPI. However, because it is widely recognized that the CPI tends to overstate inflation,[27] we make a correction upwards of 0.8 percentage points in the CPI to overcome the bias.

In order to assess the individual contribution of each wage change to the shift in the marital status composition of the population we first look at the effect of *i)* increasing men's

[27]The CPI fails to fully capture improvement in goods quality or the ability of consumers to substitute away from goods which experience a sudden increase in their prices (Gottschalk (1997)). Shapiro and Wilcox (1996) review the available evidence and place the median of their subjective probability distribution for the overall bias in the CPI around 1.0 percentage point per year. They also estimate that about 80% of the mass distribution lies between 0.6 and 1.5 percentage points. The CPI Advisory Commission calculated a point estimate of 1.5 percentage points for the total bias in the CPI for the last decade, with a range extending from 1.0 to 2.7 percentage points per year.

and women's average absolute wages on the steady state equilibrium allocation, keeping constant wage premia between and within sexes. Second, starting from the baseline equilibria allocation, we reduce *ii)* the sex wage premium, keeping relative wages between sexes and men's absolute average wages constant. Next, to assess the impact of the increase in within sexes wage dispersion, we change *iii)* males' and *iv)* females' wage premia keeping absolute average wages and therefore sex wage premium constant. Finally *v)* we change average absolute wages and sex, male and female wage premia together.[28]

6.1 Change in absolute average wages

We start by looking at changes in the level of all wages, while keeping relative wages between and within sexes constant. Given the way we compute wage premia between and within sexes, this means that everybody's wages increase by the same proportional amount. The main implications of the change in the level of wages are in Table 11.

Ex-ante, the effects of these wage changes on the number of singles is hard to predict: an increase in the prospective spouse's wage raises women's gains from marriage, while an increase in women's wages reduces these gains, two opposing effects. It turns out that the first effect prevails over the second, but by a small margin, so that the share of single women drops by 7% and that of single mothers by 3%.[29] The mechanism through which this works more is by making low wage men more attractive partners than before, increasing their likelihood of marriage, in particular with high wage women. This explains why the share of singles decreases more among rich than among poor women and also why a reduction in the positive assortative mating ensues, as is shown in Table 11.

6.2 Change in the gender wage premium

The implications of a reduction in the wage gender gap while men's wages as well as the wage dispersion within sexes are kept constant are in Table 12.

[28] For ease of exposition we present statistics of the population partitioned into top and bottom according to earnings. Also, we define within sex wage dispersion as the ratio between average wages of top and bottom halves of the population. The sex wage premium is the ratio between men's and women's average wages.

[29] Both effects are greater for poor women, given the curvature of the current utility of consumption.

Table 11: **Increase in average absolute wages**

	Baseline	New Al–location	Model Change	74-91 Data
Males' average wage	9.396	10.018	7%	7%
Fraction of singles among all women	0.165	0.154	-7%	69%
Fraction of singles among rich women	0.199	0.181	-9%	44%
Fraction of singles among poor women	0.131	0.128	-2%	107%
Fraction of single mothers among all women	0.116	0.112	-3%	41%
Fraction of single mothers among rich women	0.128	0.123	-4%	2%
Fraction of single mothers among poor women	0.104	0.101	-3%	87%
Marriage rate, women older than 30	0.141	0.145	3%	-18%
Divorce rate	0.017	0.016	-6%	0%
Assortative mating				
Fraction of rich married women married to rich men	0.533	0.527	-1%	7%
Fraction of poor married women married to poor men	0.530	0.526	-1%	3%

These changes induce two effects in opposite directions: men are more willing to marry because women's overall quality improved (their market productivity grew while their fertility was largely unaffected), while for women being single, and waiting for a better match is now more attractive. The latter effect overcomes the former. Moreover, it does in a very important manner. The drop in the gender wage gap accounts for 85% of the shift in the marital status of the population observed in the data as shown in Table 12.

Given the concavity of the current utility of consumption and the fact that rich and poor females' average wages increase by the same proportional amount to keep women's wage dispersion constant, the drop in the wage gender gap reduces the gains from marriage more for poor than for rich women. Only if these gains were on average still positive for all women would we observe a greater increase in the share of singles among poor females. Instead, rich women's gains from marrying poor men become negative. Poor women's average gains from

Table 12: **Reduction in the wage gender gap**

	Baseline	New	Model Change	74-91 Data
Sex wage premium	1.854	1.563	-19%	-19%
Fraction of singles among all women	0.165	0.263	59%	69%
Fraction of singles among rich women	0.199	0.323	62%	44%
Fraction of singles among poor women	0.131	0.204	58%	107%
Fraction of single mothers among all women	0.116	0.170	47%	41%
Fraction of single mothers among rich women	0.128	0.171	33%	2%
Fraction of single mothers among poor women	0.104	0.170	63%	87%
Marriage rate, women older than 30	0.141	0.090	-36%	-18%
Divorce rate	0.017	0.020	18%	0%
Assortative mating				
Fraction of rich married women married to rich men	0.533	0.549	3%	7%
Fraction of poor married women married to poor men	0.530	0.541	2%	3%

marriage with poor men are lower than before but still positive. Additionally poor women's chances of meeting poor men increase because many more poor men remain single.

As a result the share of singles increases more among rich than poor women and positive assortative mating is strengthened. The drop in the wage gender gap accounts for more than half of the increase in the share of single females and for 72% of the increase in the share of single mothers among poor women. The share of single females among rich women increases in the model more than evidence found in the data.

6.3 The widening wage dispersion

We separate the effects of wage dispersion by gender. This is a potentially important feature because in the period under study there was a major increase in the college premium for men and women.

6.3.1 Raising males' wage dispersion

An increase in males wage dispersion induces an increase in the share of single low and high wage women, the former by 58% and the latter by 46%. Low-wage men become even poorer than before, therefore they tend to be less picky when searching for a partner. Despite this, their lower earnings make them worse prospective mates and poor women are less willing to marry them.

Table 13: **Increase in men's wage dispersion**

	Baseline	New	Model Change	74-91 Data
Males' wage premium	2.272	2.584	14%	14%
Fraction of singles among all women	0.165	0.248	51%	69%
Fraction of singles among rich women	0.199	0.290	46%	44%
Fraction of singles among poor women	0.131	0.207	58%	107%
Fraction of single mothers among all women	0.116	0.176	52%	41%
Fraction of single mothers among rich women	0.128	0.172	34%	2%
Fraction of single mothers among poor women	0.104	0.179	72%	87%
Marriage rate, women older than 30	0.141	0.098	-30%	-18%
Divorce rate	0.017	0.020	18%	0%
Assortative mating				
Fraction of rich married women married to rich men	0.533	0.557	4%	7%
Fraction of poor married women married to poor men	0.530	0.551	4%	3%

Low-wage women have a hard time to substitute poor men with rich men because the latter are now pickier than before since their wages have risen. Therefore: *i)* the share of single females increases more among poor than among rich women as shown in Table 13 and *ii)* the pattern of positive assortative mating gets reinforced.

6.3.2 Raising females' wage dispersion

An increase in females' wage dispersion increases the share of single females among high wage women by 27%, and reduces the share of low wage single women by 11%.

Table 14: **Increase in women's wage dispersion**

	Baseline	New	Model Change	74-91 Data
Females' wage premium	2.147	2.527	18%	18%
Fraction of singles among all women	0.165	0.184	12%	69%
Fraction of singles among rich women	0.199	0.253	27%	44%
Fraction of singles among poor women	0.131	0.116	-11%	107%
Fraction of single mothers among all women	0.116	0.117	1%	41%
Fraction of single mothers among rich women	0.128	0.135	5%	2%
Fraction of single mothers among poor women	0.104	0.100	4%	87%
Marriage rate, women older than 30	0.141	0.119	-16%	-18%
Divorce rate	0.017	0.016	-6%	0%
Assortative mating				
Fraction of rich married women married to rich men	0.533	0.576	8%	7%
Fraction of poor married women married to poor men	0.530	0.565	7%	3%

Low-wage women become less desirable partners than before because their income falls while their fertility remains largely unaffected. At the same time low-wage women's gains from marriage increase, everything else equal. As a result they become less picky when choosing a partner. The latter effect outweighs the former.

On the other end rich females become even richer and pickier in the search for a partner. While their earnings improve, their fertility pattern does not change significantly. [30] Therefore men are even more willing to marry them. The first effect overcomes the second and

[30]The average number of children of rich women goes from 2.329 in the baseline to 2.316 in the new steady state allocation; the same figure for poor women increases from 2.498 to 2.528 as their per child costs drop.

rich women are more likely to stay single and wait for high wage partners. Consequently, the positive assortative mating of couples gets reinforced.

6.4 Overall effect of wage changes (the nineties)

We now turn to the effects of wage changes when they take place simultaneously. Table 15 reports the results.

Table 15: **Change in all wage premia and increase in absolute wages**

	Baseline	New	Model Change	74-91 Data
Females' wage premium	2.147	2.527	18%	18%
Males' wage premium	2.272	2.584	14%	14%
Sex wage premium	1.854	1.563	-19%	-19%
Men's absolute average wage	9.396	10.018	7%	7%
Fraction of singles among all women	0.165	0.265	60%	69%
Fraction of singles among rich women	0.199	0.331	66%	44%
Fraction of singles among poor women	0.131	0.200	53%	107%
Fraction of single mothers among all women	0.116	0.164	41%	41%
Fraction of single mothers among rich women	0.128	0.161	26%	2%
Fraction of single mothers among poor women	0.104	0.169	62%	87%
Marriage rate, women older than 30	0.141	0.088	-37%	-18%
Divorce rate	0.017	0.019	12%	0%
Assortative mating				
Fraction of rich married women married to rich men	0.533	0.560	5%	7%
Fraction of poor married women married to poor men	0.530	0.550	4%	3%

The overall effect of all wage changes depends on the relative strength of the different effects at work. The results show a dramatic increase in the number of single women and of single mothers, essentially of the same enormous size that we observe in the data.

The model over predicts the effects of the wage changes among high earning women and underpredicts the effects among low wage women. Moreover, the changes in the marital status of the population are achieved in the model mainly through a reduction in the marriage rate, the same mechanism that does the job in the data, although its drop in the model economy is twice as big as in the data. The model economy is also successful in accounting for the rising trend in the positive assortative mating of couples which is observed in the data.

Together with the changes in family structure, our model finds that under the new wage structure there are other important changes such as an increase in the inter-generational earnings correlation by 12% in the model economy. Additionally, being born in a single female headed household increases the likelihood of becoming a low-earning type adult by a third. This result suggests that shifts in the marital status composition of the population might have reinforced the positive inter-generational correlation of earnings that is observed in the U.S. economy. After the change in the wage regime the inter-generational earnings correlation is 0.153. The model economy of the nineties accounts for 25%-40% of the inter-generational earnings correlation found in empirical studies without considering any peer group or genetic effect which might strengthen the correlation between parents' and children's outcomes.

As an additional independent validation of the model economy, changes in the wage regime leave women's fertility virtually unchanged.[31] This is consistent with the stability in US women's total fertility rates observed since the mid 70s.

7 Conclusion

In this paper we have constructed a general equilibrium model where agents choose to form families, have children and invest in their human capital. We calibrated the model to the distribution of families and wages of the seventies. We used the model to measure the role played by changes in wages in accounting for the enormous changes observed in family

[31] The average number of children per mother goes from 2.497 to 2.498 among poor women and from 2.329 to 2.282 among rich women.

structure in the last few years. According to our model, the observed changes in wages (they have gone up, the college premium has increased and the gender premium has decreased) have played a fundamental role in generating the increase in the number of people living as singles that is observed in the data.

In doing this project, we have, not only created a novel model susceptible of calibration, with agents differing in age, gender, family status and education levels, but we also have made a few technical developments to get around certain technical problems that arise in models with discrete choice and lack of transferable utility.

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Appendices

A The wage indicator

The wage indicator is a measure of earnings potential computed both for people in and out of the labor force. We take all PSID females aged 30-44 years and males aged 35-49 years in 1974 and in 1989 and follow their wage history for years 1973 to 1976 and 1988 to 1991 respectively. We divide the sample of observations of PSID individuals into twenty four groups according to gender, age, work experience, completed education and race. If individuals work for at least two of the four years taken into consideration we average individual yearly hourly wages over those years to get a measure of wage potential unaffected by short term fluctuations. When wages are not observed they are imputed, i.e. each individual is assigned a wage that is a function of the average wage of the education, gender, race, work experience group she/he belongs to.

Agents whose wages are imputed are more likely to be associated with unobservable characteristics that select them out of the labor market. Because of this selection bias our measure of potential wages is most likely an upper bound of agents' actual earnings potential. The wage indicator turns out to be age independent. When we sort women according to their wages, poor (bottom half of the wage distribution) and rich (top half) females on average have the same age 36.9 and 37.1 years respectively in 1974, 36.4 and 36.8 years in 1989.

A.1 The wage gender gap

When we compare our measure of wage gender gap with that previously found in the gender pay gap literature, it might at first seem too high both at the beginning and at the end of the period. O'Neill and Polacheck (1993) use annual earnings of full-time year-round workers to compute the male-female pay gap. They report a value of 1.67 in 1979 and 1.51 in 1988. Since full-time male employees work about 8%-10% hours more than full-time female employees the wage gender gap that they report is likely to overestimate that obtainable using yearly hourly earnings. In fact Blau and Kahn (1997) using yearly hourly earnings of full-time non agricultural, non self-employed workers aged 18-65 years report a male-female pay gap of 1.60 in 1979 and 1.38 in 1988. Like us they use the PSID data. For this reason we compare our statistics with what they obtain. The male-female pay gap that we compute from the PSID is 1.90 in 1974 and 1.59 in 1991. The difference in the starting year between the periods that we and Blau and Kahn consider should not be relevant because changes in the wage gender gap started towards the end of the seventies. Our measure of the wage gender gap differs from theirs for the following two reasons.

First, we use both observed and imputed wages. It is more likely that we impute wages for women than for men because women are notoriously less attached to the labor force. Secondly, women with long spells out of the labour force are more likely to be low productivity women, with less education and less accumulated work experience. Including these observations in our sample drags down women's average yearly hourly earnings across occupations. In fact if we used only observed wages, our measure of the wage gender gap would drop to

1.72 in 1974 and 1.54 in 1991. Notice that the drop is relatively higher for the gender pay gap computed at the beginning of the period of interest. The share of women over the total female sample who are out of the labour force and for whom we impute earnings is indeed higher in the mid seventies than at the beginning of the nineties.

Secondly, our age cohorts (30-44 for women and 35-49 for men) differ from those usually considered in the literature. Blau and Kahn (1997) include male and female workers aged 18-65 years. If we use the same age brackets and only observable wages our measure of the male-female pay gap would be 1.60 in 1974 and 1.46 in 1991. These figures are comparable with those computed by Blau and Kahn (1997)). An approximate 6% discrepancy between our figures and theirs for the end of the eighties might be due to the fact that they exclude part-time workers and self-employed. It is nevertheless impossible to verify this conjecture because they do not specify the criteria used to distinguish part-time from full-time workers.

Since our model does not account for the gender difference in educational achievements found in the data, when we compute relative wages within and between sexes we do control for this difference. This means that we compute women's average wage and women's wage dispersion assuming that the educational composition of the female population is equal to that of the male population in the two periods taken into consideration. Under this hypothesis the wage gender gap obtained is 1.86 in 1974 and 1.56 in 1991.

B Computational Procedures

There are various parts to the computational procedure. The calibration process is like solving a large system of equations and unknowns. We proceed by minimizing the weighted sum of the square of the residuals of the 37 equations described in Section 5 by choosing the 37 parameters described there using standard of the shelf routines.

To evaluate the sum of weighted residuals for a given parameterization we have to solve the steady state of the model economy for that parameterization, compute the relevant statistics and compare them with data.

Computing the steady state involves finding the stationary measure and the decision rules associated to it. To do so given that the state space is finite, we proceed as follows:

(i) Start with initial guesses for the measure of males x_m^0 and x_f^0 females and for the value functions V_m^0 and V_f^0

(ii) Given x_m^0 and x_f^0 solve the households' problem by iterating on the Bellman type equation defined in Section 4. At each step of the iteration we proceed backward solving all stages by means of first order conditions

  (ii.1) Solve the single female and the married females' investment problem.

  (ii.2) Solve females' effort decision to determine the number of children.

  (ii.3) Solve females' and males' effort decision to determine marital status.

(iii) Using the resulting decision rules, update the initial guesses of the measures x_m^0 and x_f^0; if at the first iteration on the the $x's$ the difference between their old and new is less than some tolerance value then we reached the stationary distribution. Otherwise, iterate on the $x's$ until convergence is reached, and then update the initial guesses x_m^0 and x_f^0, and go back to step (ii).

The mapping T is not monotonic which implies that iterations need not converge. We get around this problem by slow updating.

Computing the relevant statistics amounts to evaluate simple integrals that is done with standard methods. Comparing the model statistics with their data counterparts is trivial.

C Other Tables of Interest

Table 16 shows the parameter values of the baseline model economy while Table 17 shows some statistics regarding welfare coverage in the period under study.

Table 16: **Calibrated parameter values for the baseline model economy**

Demographics		Wages		Preferences		Approximation	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
π	0.898	w_1	1.416	σ	0.561	κ	0.62792
		w_2	3.222	β	0.729	ρ_0	0.56784
		w_4	9.737	β_c	0.156	ρ_1	0.80962
		w_1^*	3.512	$\phi_{1,c}$	0.513	ς	0.12891
		w_2^*	5.743	$\eta_{1,m}$	6.191	$\alpha_{1,m}$	6.1964
		w_4^*	18.630	$\eta_{2,m}$	6.582	$\alpha_{0,m}$	0.59733
		$\Gamma_w(w_1\|w_1)$	0.073	$\bar{\eta}_m$	7.500	$\alpha_{1,f}$	0.00005
		μ_1	0.402	$\eta_{1,f}$	-2.291	$\alpha_{0,f}$	0.00050
		μ_2	0.489	$\eta_{2,f}$	-1.070		
		μ_3	0.900	$\bar{\eta}_f$	-.704		
		γ_1	0.488	$\Gamma_\eta(\eta_1\|\eta_1)$	0.929		
		γ_2	1.435	$\Gamma_\eta(\eta_2\|\eta_2)$	0.189		
		γ_3	3.661	Ω_s	7.891		
				Ω_m	7.094		

Table 17: **Fraction of welfare and non-welfare recipients among poor single women**

	1974 PSID	1991 PSID
Welfare recipient	0.064	0.076
Non welfare recipient	0.075	0.212
Total	0.139	0.288