Labor Share and Productivity Dynamics

Sekyu Choi
University of Bristol

José-Víctor Ríos-Rull
University of Pennsylvania,
UCL, CAERP, CEPR and NBER*

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Abstract

We present a novel way to model technological shocks, with the feature that it can be biased towards more recently installed production units. We show that at one extreme, the shock is like a neutral technological shock, while at the other end of the spectrum, it resembles investment specific technological shocks, since it affects newly created machines the most. To make these ideas operational, we embed our proposed shocks in a model with putty-clay technology (where the notion of new and old firms is clear). We estimate the process for the shocks requiring that the model replicates the volatility properties of the Solow residual and the overshooting property of the labor share of output. Our estimates point to three factors to be capable of replicating well such overshooting property: putty-clay nature of technology, a time bias in the shocks towards new plants and competitive wage setting.

Keywords: Factor Shares, Technology shocks, Putty-Clay, Business Cycles
JEL Codes: **E01, E13, E25, E32**

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1 Introduction

In this paper we present a new way of modelling aggregate productivity shocks, where we introduce a "time" bias with respect to installed capital: shocks may affect newer firms in a stronger way than older firms. On the one end of the spectrum, when shocks do not exhibit time bias, these shocks are like standard TFP shocks. If the time bias is extreme (only new machines are affected by it), the shocks point to behavior akin to that of investment specific technological shocks.[1]

To make this notion operational we embed the technology shocks into a model with putty-clay technology where new and old firms have a sharp distinction, as in Johansen (1959), Atkeson and Kehoe (1999), Gilchrist and Williams (2000), Gourio (2011) and Wei (2003). Under this technology assumption, individual productive units consisting of one worker each are indexed by their installation dates and differ in the "quality" (or size of its capital). The pre-installation size menu available to firms is Cobb-Douglas (i.e., 'putty'), but once installed, plants remain in place without the possibility of changing its quality (i.e., 'clay'). The putty-clay assumption creates a Leontief productive structure in the short-run between capital and labor: workers must be attached to machines in order for the firm to produce. This is a major departure from the traditional Cobb-Douglas setup, which implies an elasticity of substitution between inputs fixed to one for all time horizons. Since there is dispersion of productivities across plants, we explore both competitive wages and Nash bargained wages as is standard in the search and matching literature.[2]

We use this model to address the cyclical behavior of the labor share, an issue of importance for different strands of the literature: Gali and Gertler (1999) and Sbordone (2002) relate it to inflation dynamics while Santaeulalia-Llopis (2012) argues that it matters for measurement of the effects of productivity shocks. More specifically, we aim to replicate the overshooting property of the labor share as studied by Choi and Ríos-Rull (2009) and Ríos-Rull and Santaeulalia-Llopis (2010): after a positive technological shock, the share of output that corresponds to labor falls temporarily but it quickly rises, staying persistently above average for around thirty quarters. Consequently, we estimate the process for the shock together with the degree of time bias, targeting not only the standard deviation and persistence of the model implied Solow residual, but also the impulse response function of the U.S.

[1]Investment specific technology shocks have gained prominence as a source of aggregate fluctuations. The relative importance of these shocks has been studied extensively, using diverse methods. See for example Fisher (2006b), Fernandez-Villaverde and Rubio-Ramirez (2007), Justiniano, Primiceri, and Tambalotti (2010),Justiniano, Primiceri, and Tambalotti (2011), Bai, Ríos-Rull, and Storesletten (2011), Schmitt-Grohe and Uribe (2011) and the survey in by Ramey (2016).

[2]See Pissarides (2000).

labor share, using the method of simulated moments.[3]

We find that putty-clay technologies can match the overshooting property of the labor share. The best estimates are sharp and they point to a high degree of time bias towards new technologies. In our baseline sample, the exercise also leans toward the existence of competitive labor markets. In our framework, the response of the labor share is closely related with the behavior of employment, which reacts significantly to technology shocks due to its one-to-one relationship with installed capacity (the *putty-clay* effect): firms can only make use of improved conditions by opening new plants *and* hiring workers for those plants. This effect is further leveraged by the presence of time bias, since firms have added incentives to create employment because of the short-lived nature of shocks under this specification. Furthermore, the increase in employment does not have a negative effect on productivity of other workers and we show that the overshooting is maintained both under competitive wages and Nash bargaining.

The workings of our model are quite different from what happens in setups with Cobb-Douglas technology: there, factor shares move little and any departures of labor share from its steady state value are short lived (see Choi and Ríos-Rull (2009)). The main difference between putty-clay and Cobb-Douglas with putty-putty factors (when shocks are neutral) is the implied elasticity of substitution between inputs in the short run: zero in putty-clay, one in Cobb-Douglas. Hence, the way in which capital and labor can be replaced is key to understand intuitively the overshooting of the labor share in our setting. We show this through simulation of economies with different types of technology, different degrees of bias in the technology shocks different and wage setting mechanisms and compare how they manage to replicate the overshooting (it at all).

Our findings are related to two important issues in modern macroeconomics: the equilibrium unemployment puzzle and the effects of productivity shocks on the labor input. Strands of the literature following Shimer (2005) on the former and Galí (1999a) on the latter, see these findings as telltale signs of the importance of wage rigidities in the aggregate. Here we present an alternative framework which shares model features with setups used in both literatures, but does not rely in any form on wage rigidity to produce significant amounts of responses in unemployment and employment.

With respect to the unemployment puzzle, the literature studying models with labor market frictions

[3]Note that our identification relies on the structure we impose on aggregate technology. For a general discussion on the feasibility of identification of both neutral and non-neutral technology shocks, see Diamond, McFadden, and Rodriguez (1978) and Leon-Ledesma, McAdam, and Willman (2010).

find very poor responses of unemployment to realistic shocks (the so called *Shimer* puzzle).[4] The main proposition from this literature is that the terms of bargaining in real wages matter for labor market outcomes during the cycle:[5] incentives for firms to post vacancies during favorable aggregate conditions depend on how much profits they can get from matched employees (i.e., wedge between labor productivity and real wages). Hence, rigid wages give more incentives for employment creation by increasing the dynamic difference between labor productivity and real wages. Beyond the ability of our model to produce high levels of employment and unemployment volatility, our setup has two interesting implications for this literature: First, putty-clay technology is able to produce a delayed response in employment after a technology shock, in a model *without matching frictions*. This is because firms need to invest in units before production and hirings can take place. Second, the confluence of Nash bargaining and time bias mimics rigid wages: new unit wages react to changes in aggregate conditions much more than older units.

With respect to the effects of technology shocks, our results contrast with the literature following Galí (1999b), who uses long run restrictions on a vector autoregression framework and finds a negative effect of productivity shocks on aggregate hours in the U.S. economy. Our model specifications where shocks are more time biased thus are in line with Fisher (2006a), who constructs a model where only neutral and investment-specific technological shocks can have a long-run impact on labor productivity and finds that hours rise in response to technology shocks. Related to this point, Cantore, Ferroni, and Leon-Ledesma (2019) show that both a textbook and a large scale version of the New Keynesian model are unable to predict the behavior of labor share after a monetary policy shock. This inability mirrors the problems of the standard real business cycle model with Cobb-Douglas technology in explaining cyclical movements of the labor share following technology shocks. We see this as evidence that labor share is an important source of validation for modern macroeconomic models.

Recent papers have studied the cyclical properties of the labor share in the U.S. Shao and Silos (2014) pose a model with frictional labor markets and costly entry of firms and highlight the low correlation of real interest rates and output. León-Ledesma and Satchi (2019) also generate the overshooting property of the labor share in a model where there are adjustment costs to technology choices that generate an elasticity of substitution between capital and labor that is lower in the short run than in the long run.

The structure of the rest of the paper is as follows: In Section 2 we present our model and its

[4]See Shimer (2005), Hall (2005), Blanchard and Galí (2007), Costain and Reiter (2008) and Hall (2009), among others.
[5]See Mortensen (1992), Caballero and Hammour (1996) and Rogerson and Shimer (2010).

variants. Section 3 poses and discusses the estimation of the shock process for the various models. Section 4 discusses the performance of the models first in terms of the overshooting property of the labor share and then in relation to the standard business cycle second moments. Section 5 concludes. Various appendices provide additional details.

2 The Model

The model is a growth model in discrete time with infinitely lived agents. Production of the unique good of this economy takes place in independent plants that have one worker each and that may differ in the amount of capital installed. We denote this amount as k and we think of it as the "size" or "quality" of the plant. We assume that workers provide labor inelastically, thus k also represent capital/labor ratios at the plant level. A plant takes one period to become operational and once built, its capital cannot be changed and its scrap value is zero. Productive units face an exogenous break-down probability of δ.

Firms can find workers after paying a hiring cost c^v that can be thought of as due to vacancy posting and training. This cost partly replicates the hiring frictions in a standard search and matching framework (except for the increased difficulties of finding a worker in expansions). Moreover, given the required "incubation" period for plants, aggregate employment in the economy moves with some lag, as if there were matching frictions (because the capital that the worker will use has to be installed), adding a mechanism to stall the fast expansion of employment right after a positive shock in aggregate conditions. Once a worker is hired her wage is set by bilateral Nash bargaining between the firm (owner of the plant) and (the family of) the worker.

A plant of size k produces an amount $e^z k^\alpha$ of goods per period, where z is an aggregate productivity shock. Its evolution is given by

$$z = \begin{cases} \widetilde{z} & \text{if new machine,} \\ \lambda\widetilde{z} & \text{otherwise,} \end{cases} \tag{1}$$

with

$$\widetilde{z}' = \rho\widetilde{z} + \epsilon', \tag{2}$$

where $\rho \in (0,1)$ and $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. *New machine* denotes productive units which were installed one period ago and will start production in the current period. Note that when $\lambda = 1$, the shock reverts to a classic factor neutral aggregate shock, which in our case, it could be understood also as machine-cohort-neutrality. On the other extreme, if $\lambda = 0$, the shock affects only the newest productive units installed which makes it look similar to an investment specific technical shock (in Appendix A we provide a discussion of how the two shocks are related).[6]

The aggregate state of the economy S, is summarized by the aggregate shock $\widetilde{z}$ and $X(k,t)$, the measure of installed plants of capital amount k, and age or type $t \in \{0,1\}$, one if it is a new plant and zero otherwise.

2.1 Households

The economy is populated by a unit measure of households (families), each of them consisting of a mass of identical consumers/workers (normalized to one). Each household has preferences over consumption and leisure streams of their members. Every household member is part of the labor force, but employment is limited to the number of machines operating in the economy. We assume perfect capital markets, so there is no distinction in terms of consumption between employed or unemployed household members.

We write the per period utility function of the family as $u(c) + v(n) = \log(c) + (1-n)b$, where c is the common consumption level for everyone inside the family, n is the fraction of household members that work this period and b is the additional value of leisure to the non-working members of the family (we normalize the leisure value to those working at zero). In each family, its working members do so in a variety of firms or plants, each one characterized by size k and type t. Family employment then satisfies $n = \Sigma_{t\in\{0,1\}} \int X(k,t)\ dk$, where x is the size/type distribution of firms where the family members are employed at. The state of the household is comprised also of its assets, $s = \{a, x\}$. In addition, households discount the future at rate β and take the appropriate expectations whenever necessary. Their value function is

$$V(S,s) = \max_{c,a'} \quad \log(c) + (1-n)b + \beta\ E\{V(S',s')\} \qquad \text{s.t.} \tag{3}$$

$$c + a' = R(S)\ a + \sum_{t=\{0,1\}} \int w(k,t,S,s)\ X(dk,t) + \pi(S), \tag{4}$$

$$x'(k,1) = m^k(S), \quad \forall i, \tag{5}$$

$$x'(k,0) = (1-\delta) \sum_{t=\{0,1\}} X(k,t) \quad \forall i, \tag{6}$$

and the law of motion of the aggregate state S. Here $R(S)$ is the aggregate gross rate of return,

[6] We thank Joachin Hubmer, who pointed inaccuracies in our comparison to investment specific technological shocks in a previous draft of our paper.

$w(k,t,S,s)$ is the real wage a household member can command in a plant with characteristics (k,t), $\pi(S)$ represents current period profits of firms owned by the household and $m^k(S)$ represent the measure of hires of household members into $(k,1)$ firms. The last two equations in the problem of the household above, show that households need to forecast the measure of employment in different types of firms. We avoid the description of the evolution of the aggregate state because the evolution of the aggregate measure of firms X is the same than that of the employers of the workers of the representative household.

The optimization of the household involves the standard Euler equation

$$1 = \beta \; E \; \left\{ \frac{c(S,s)}{c(S',s')} \; R(S') \right\}. \tag{7}$$

2.2 Firms and Investment Decisions

The value of an active plant of size k and type t, attached to a worker from a household with characteristics in s is given by[7]

$$\Pi(k,t,S,s) = e^z k^\alpha - w(k,t,S,s) + (1-\delta) \; E\left\{Q(S') \; \Pi\left(k,0,S',s'\right)\right\}, \tag{8}$$

where $w(k,t,S,s)$ is the wage paid to the worker attached to this firm, and $Q(S')$ is the discount rate for the firm, $Q(S') = [R(S')]^{-1}$. Once a plant is installed, the firm does not make any decisions: plants get destroyed at the exogenous rate δ and their productivity is given by z (depending whether they are new or old) and the installed capacity k.

We can define the value of marginally increasing plant size k:

$$\Pi_k(k,t,S,s) = e^z \alpha k^{\alpha-1} - w_k(k,t,S,s) + (1-\delta) \; E\left\{Q(S') \; \Pi_k\left(k,0,S',s'\right)\right\}, \tag{9}$$

This expression is useful to calculate the optimal installation size, which solves the following problem

$$\max_i \quad -k + E\left\{Q(S') \; \Pi\left(k,1,S,s\right)\right\}, \tag{10}$$

[7] With this notation, we are assuming that firms and workers get matched before the investment decision. As we show in the next section, in equilibrium, this timing is irrelevant.

where the solution $k^*(S, s)$ satisfies the first order condition

$$1 = E\left\{Q(S')\ \Pi_k\left(k^*(S, s), 1, S, s\right)\right\}. \tag{11}$$

On the other hand, the number of plants $M^*(S, s)$ that are installed is determined by a zero profit condition that includes the cost c^v of hiring a worker:

$$k^*(S, s) + c^v = E\left[Q(S')\ \Pi\left(k^*(S, s), 1, S, s\right)\right] \tag{12}$$

$M^*(S, s)$ and $k^*(S, s)$ also determine overall capital investment in the current period, since total investment equals $M^* k^*$.

2.3 Wage Determination

In putty-clay economies there is no operational notion of marginal productivity of labor that is a natural candidate for the wage because the ratio of workers to machines is fixed. Yet, households do have a marginal rate of substitution that is equated to the wage and new firms have a zero profit condition that disciplines their behavior allowing for a competitive determination of the wage. The natural alternative to competition is a bargaining mechanism between workers and firms. Moreover, the competitive wage turns out to be the wage that results when the bargaining weight of workers is zero. The bargaining process between workers and firms requires two additional specifications. Whether it is done within each firm or whether it is done economy-wide,[8] and what occurs in case of breakup of the negotiations (whether the match is dissolved or whether the firm and worker are still attached the following period).[9] We explore both competitive and bargained-for wages. As we use North American data, we pose decentralized bargaining. We choose to assume that matches do not dissolve in case of the break down of the negotiations. This choice requires a discussion as the standard in the search and matching literature is to assume complete breakdown of the match. In models where all jobs are identical, the bargaining problem under match breakup that solves for the wage collapses to a simple form because both the worker and the firm could be matched in similar terms to what they are now in the future. In

[8] The latter is mostly used in Northern European contexts. See for instance Krusell and Rudanko (2012).

[9] Typically, the literature assumes that the match is dissolved (e.g. Andolfatto (1996), Choi and Ríos-Rull (2008)). Christiano, Eichenbaum, and Trabandt (2016) pose a model with alternating offers and random breakdowns at a rate that they estimate.

a putty-clay economy, this is not the case: if negotiations are not successful and the match breaks, the unmatched firm faces a different prospect than all the new firms who will be available next period to the worker (because they are of a different size) and hence the outlook of the unmatched worker and the unmatched firm is not to the same type of match. This yields a bargaining problem that depends on the distribution of existing firms and the continuation values of both workers and firms. Our assumptions avoid these complications and make the resulting wage equation similar to what one finds in the standard Mortensen-Pissarides framework a feature to which we will return later.

If we assume that the match survives then, the value of the worker for the firm is $e^z k^\alpha - w(k,t,S,s)$, while the value of the job for the household is just its wage net of the value of being unemployed, $w(k,t,S,s) - b\ c(S,s)$ (in units of consumption).[10] Firms and workers bargain over the match surplus, and if the bargaining power of the household is μ, the wage is just fraction μ of the surplus which is

$$w(k,t,S,s) = \mu\ e^z\ k^\alpha + (1-\mu) b\ c(S,s). \tag{13}$$

Given this wage function, plant specific profits each period are given by $(1-\mu)\ [e^z k^\alpha - b\ c(S,s)]$. This expression can be used to rewrite Equation (11). Note that in this setting, all firms have the same information and installation costs are the same for them. Thus, only one size k^* is installed each period.

Wages in Standard Search and Matching Models In standard search and matching models the expression for wage setting under bargaining differs for two reasons.[11] One is the aforementioned assumption of breakdown of the match in case of unsuccessful negotiations that yields an additional term that is weighted by the workers' weight, $c^v \frac{V}{1-N}$ that is the cost of posting a vacancy times the ratio of vacancies to unemployed. The other one has to do with the value of the worker for the firm. With Cobb-Douglas putty-putty technology a worker is worth its marginal productivity (while in the putty-clay technology a worker means output and not worker means no output) which yields the following

[10]In terms of "utils", the value of an additional worker for the household is given by $w(k,t,S,s)u_c(S,s) - b$ where $u_c(S,s)$ is the marginal utility of consumption. Given log preferences, transforming this expression into units of consumption gives the equation in the text.

[11]See Andolfatto (1996) and Merz (1995).

expressions for the wage (details are in Appendix B),

$$w = \mu\left[\alpha\ e^{z}\ K^{\alpha} N^{-\alpha} + c^{v}\frac{V}{1-N}\right] + (1-\mu) C\ b \qquad \text{(Match does not Survive)}, \tag{14}$$

$$w = \mu\left[\alpha\ e^{z}\ K^{\alpha} N^{-\alpha}\right] + (1-\mu) C\ b \qquad \text{(Match Survives)}. \tag{15}$$

Note that in steady state employment is almost 1, so the main difference between Cobb-Douglas putty-putty and putty-clay economies for wage setting is the existence of the parameter α in the wage formula of the earlier. The particular values for the bargaining weights depend not only on the nature of the technology but also on the calibration targets that we set.

2.4 Aggregation and a simplified problem

This model can be simplified dramatically (following Gourio (2011)) to get rid of the measure $X(k,t)$ of plants and substitute it by a few aggregates. Define aggregate installed capacity by type 0 (old) firms as

$$\overline{Y}^{0} = \int k^{\alpha}\ X(dk, 0). \tag{16}$$

In a similar same vein, capacity of new firms the following period is

$$\overline{Y}^{1'} = M^{*}\ (k^{*})^{\alpha}. \tag{17}$$

These two expressions give a recursive characterization that allows to avoid using the measure of firms:

$$\overline{Y}^{0'} = (1-\delta)\left[\overline{Y}^{0} + \overline{Y}^{1}\right]. \tag{18}$$

Given the shock process described in Equation (1), total output is given by

$$Y = e^{\lambda\tilde{z}}\ \overline{Y0} + e^{\tilde{z}}\ \overline{Y}^{1}. \tag{19}$$

Aggregate employment can also be defined recursively with the aid of the number of new plants

$$N' = (1-\delta)N + M^{*}. \tag{20}$$

Similarly, the entire wage bill for the economy is just

$$\omega = \mu \ Y + N \ (1 - \mu) \ b \ C. \tag{21}$$

Note that the aggregate feasibility constraint in the economy is now written as

$$Y = C + M^* \ \left[k^* + c^v\right]. \tag{22}$$

In a steady state, employment becomes $N = Q/\delta$, while output simplifies to $Y = (Q/\delta) \ k^\alpha$. Since the capital stock is defined as the number of operational machines (which is the same as the level of employment), this implies that in a steady state

$$\begin{aligned} Y &= \left(\frac{Q}{\delta}\right)^{1-\alpha} \left(\frac{Q}{\delta} k\right)^\alpha \\ \Rightarrow Y &= N^{1-\alpha} K^\alpha \end{aligned} \tag{23}$$

where K is total capital. Thus, the aggregate technology in the long run is Cobb-Douglas. This is important for a couple of reasons: In the long-run, the properties of the economy are guided by the usual notions applied to technology in RBC and New Keynesian models under this type of technology. Second, even if in the short run the production technology is Leontief (fixed proportions), the elasticity of substitution is higher (equal to one) in the long run.

Given these simplifications, the aggregate state of the economy reduces to $\{\widetilde{z}, \overline{Y}^0, \overline{Y}^1, N, \}$. The nature of the shock process that distinguishes between new and old installed capacity requires us to add one state variable to those posed by Gourio (2011). The model in turn simplifies to a set of equilibrium equations, which we solve using standard approximation methods. These equilibrium conditions can be summarized as the Euler equation of the household, the equations for optimal current plant size and optimal number of plant installations by firms and aggregate market consistency: Equation (7), Equations (10) and (11) and Equations (2), (13), (18), (20) and (22) respectively.

The state space for the representative household also collapses, since similar arguments can be used to show that average employment and wage bills suffice to solve the problem of the household. Savings and consumption decisions depend on aggregate interest rates and total labor earnings that the household receives.

3 Mapping the Model to U.S. Data

We explore two model economies that differ in the wage setting process. We look at Nash bargaining and at competitive wages (where we set the bargaining power of workers μ to zero).

We set the model period to a month, short enough to accommodate employment flows but we aggregate to quarterly frequency for a direct comparison with U.S. data moments. [12] We set the model period to a month, short enough to accommodate employment flows. We set β to 0.9966 so that the interest rate is four percent per annum. We set another group of parameters, $\{b, \delta, \alpha, c^v, \mu\}$, (value of home production, firm destruction rate, coefficient of capital in the production function, hiring cost, and worker's bargaining weight) to match a set of long-term statistics for the US economy. Although these parameters affect jointly the numerical steady state of the model, there are close links between individual parameters and model predictions, which we use to calibrate this set. The destruction rate of plants δ is linked closely to aggregate capital and savings, thus we use the aggregate consumption-output ratio (0.75). The coefficient in the production function menu α is used to match the average value of the labor share 0.6514;[13] the cost of posting a vacancy c^v is chosen to match a flow cost of a vacancy in terms of days of pay per hire of 0.446 which closely matches an aggregate vacancy expenditure relative to annual GDP of 0.50%, reported by Cheron and Langot (2004). This is close to the value of 0.433 used by Hall and Milgrom (2008) based on Silva and Toledo (2009). Finally, the unemployment rate is targets the mean value in the US during our sample (0.058). In the Nash bargaining economy we set the value of home production b to match a total value of leisure per period that amounts to 0.70 of household consumption (as calculated by Hall and Milgrom (2008)).[14]

Competitive wage setting implies that workers have no bargaining power, this is that $\mu = 0$, which implies that in this economy the value of leisure in terms of consumption becomes exactly 1.

The implied values for the parameters are reported in Table 1. While most parameters are in range of numbers used in the literature, the main departure is in parameter α in the Nash bargaining economy, which differs from the standard value of 1/3. This deviation is due to the way bargaining operates in putty-clay environments. On the other hand, the plant destruction rate is close to the value implied by

[12]We solve for each economy using perturbation methods. We use the free package *Dynare*, version 4.5.7 in Matlab R2018a. We then simulate each economy for 33,000 months, discard the first 3,000 observations and then create quarterly series by simple averaging of the monthly data.

[13]We use the value reported by Koh, Santaeulalia-Llopis, and Zheng (2020).

[14]In their discussion this value amounts to their value of z and includes not only the value of unemployment but also the direct value of leisure.

Table 1: **Steady State Parameter values**

Parameter	Description	Nash Bargaining	Competitive Setting
b	value of leisure	0.6459	0.9226
δ	plant destruction rate	0.0084	0.0084
α	production curvature	0.5383	0.3449
c^v	vacancy cost	0.3090	0.3083
μ	workers bargaining weight	0.3596	0.0000

establishment statistics.[15]

3.1 Estimating the Productivity Shock Process

To estimate the model we use a combination of moments from aggregate US time series (a detailed description of the data used and the methodology is in Appendix C). As noted above, we use empirical moments coming from quarterly frequency, which we match by aggregating our monthly model.

We estimate the parameters controlling the process for the shock, $\{\rho, \sigma_\epsilon, \lambda\}$, using simulated method of moments. Two of the moments that we use are obtained from an AR(1) representation of the Solow residual which we estimate following closely the methodology in Ríos-Rull and Santaeulalia-Llopis (2010): Using aggregate data from output, capital and employment, we construct and linearly detrend the Solow residual. The autocorrelation and estimated variance from the AR(1) are two moments which are related to parameters ρ and σ_ϵ.[16]

More importantly, we also target the dynamics of labor share using its impulse response function from a bi-variate vector autoregression of order one between linearly detrended (log) labor share and linearly detrended (log) output. Our rationale for this is as a way of dealing with the secular decline of the labor share in the last few years. Below we extend our analysis to not detrending the labor share. The main facts also hold with some nuances.

From the VAR(1) estimates we extract the orthogonalized impulse response of labor share to a shock in the output equation, which we use as moments to be matched by simulated data of our model. Appendix D presents a discussion of our estimated impulse responses (see the right panel of Figure 6)

[15]Using information from `https://www.census.gov/programs-surveys/bds/data/data-tables/legacy-firm-characteristics-tables-1977-2014.html` we compute a monthly establishment exit rate of 0.0089.

[16]In fact, they are are identical to ρ and σ_ϵ in the case of quarterly RBC models with putty-putty Cobb-Douglas technology and competitive factor pricing, but not in putty-clay models or in monthly models.

and some robustness checks. Note that, contrary to the literature using structural VARs and some form of long run or sign restriction to identify structural shocks, we are not inferring any type of causality from our VAR estimates. Rather, we use these as moments to be matched by our model. Our methodology is similar to the estimation procedure in Christiano, Eichenbaum, and Evans (2005), who estimate a set of parameters to match empirical impulse response functions. Below we also compare the predictions of the model in terms of impulse responses of other aggregate variables. The procedure we follow for those is analogous to the one we take for the labor share: we estimate bivariate vector autoregressions between detrended output and the detrended aggregate of interest and retrieve from that estimation the response of the variable to a shock in the output equation.

Our estimation algorithm is standard: for each proposed economy, we choose model parameters to solve and simulate model data, from which we apply the same empirical methodology to obtain model counterparts to the moments described above. As we want to effectively replicate the standard deviation and the autocorrelation of the Solow residual as close as possible across the different economies, we pose very large weights on those two moments relative to those of the impulse response of the labor share (a factor of 1,000), which make these moments almost exactly equal in the model and in the data.

Table 2: **Putty-Clay Economies:**
Estimated Monthly Process for the Technology Shock

	Nash Bargaining		Competitive Setting	
	Time Bias	No time bias	Time Bias	No time bias
λ	0.0051	1.	0.2181	1.
	(0.001)	–	(0.126)	–
ρ	0.9731	0.9798	0.9835	0.9835
	(0.003)	(0.007)	(0.002)	(0.007)
σ_ϵ	0.4081	0.0055	0.0245	0.0055
	(0.035)	(0.000)	(0.013)	(0.000)
Sum Square Residuals of Labor Share IRF	1.58E-05	6.88E-05	4.35E-06	5.54E-06

Standard errors in parenthesis. The last row displays the sum of square of the residuals between the impulse response function of the data and each model (given the weights that we use for the errors of the Solow residual statistics these are zero in all cases. See Appendix C).

The estimated parameters are in Table 2 where the standard errors are computed using standard methods.[17] We compare the estimates that we obtain with those that result from imposing $\lambda = 1$, this is with economies subject to standard neutral technology shocks. We see how small are the values obtained for λ: close to zero with Nash bargaining and 0.22 for competitive wages, pointing to a time biased productivity shock rather than to a neutral one. Since we require the model to match the overall volatility of the economy (in the form of implied Solow residual behavior), a consequence of the strong concentration of the productivity increases in fewer plants, the estimates for the standard deviation of the innovations σ_ϵ are quite large for both the Nash bargaining economy and the competitive wage economy when compared to their *No time bias* counterparts.

We have a natural measure of fit of the estimates, the sum of the square of the residuals for impulse response moments between US and model generated data that are also displayed in the table. From the table we see that the competitive setting yields better estimates than the Nash bargaining ones and that time bias implies smaller errors.

These findings show that in our setup, using a different identification strategy than what is usually the case, we find strong evidence that the nature of the stochastic process for productivity seems to be more related to new plants than to established plants. Secondly, our baseline estimation favours competitive wages. The earlier result is quite robust to different treatments of the data (see Appendix D), while (as we show below) the competitive wage model fits better untargeted components behind the overshooting of labor share.

3.2 Robustness

We turn to see whether our findings are robust to the length of the period (monthly versus quarterly) and to the recent downward trend in the labor share. In Appendix D we also look at overshooting property when we use long run restrictions to overidentify the technology shocks and when we use ~~of~~ Fernald's adjusted TFP series.

The Length of the Period Recall that the model is monthly and the targets (because of data collecting periodicity) are quarterly. While a shock without time bias could be easily compared via time aggregation, this is not the case in the presence of time bias that may affect the incentives of job creation in an unpredictable way. For this reason, we have estimated a quarterly version of the model

[17]We follow Duffie and Singleton (1993): the standard errors are the square roots of the diagonal elements of matrix $\{G'SG\}^{-1}$, where G is the matrix of partial derivatives and S is a diagonal matrix with the inverse of the variance of the data moments in its diagonal.

and we displays the estimates in Table 3. The findings are reassuring: the time bias is exacerbated to the extreme under Nash bargaining and sizeably under competition in the labor market; the autocorrelation shrinks and the standard deviation of the innovation goes up except in the case of Nash bargaining and time bias. These estimates are completely consistent with those of the monthly model which is more appropriate when thinking of employment flows.

Table 3: **Putty-Clay Economies:**
Estimated Quarterly Process for the Technology Shock

	Nash Bargaining		Competitive Setting	
	Time Bias	No time bias	Time Bias	No time bias
λ	0.0017	1.	0.1245	1.
	(0.003)	–	(0.031)	–
ρ	0.9348	0.9553	0.9648	0.9649
	(0.008)	(0.014)	(0.006)	(0.0123)
σ_ϵ	0.2774	0.0078	0.0531	0.0078
	(0.021)	(0.000)	(0.010)	(0.000)
Sum Square Residuals of Labor Share IRF	3.82E-06	7.11E-05	1.94E-06	1.96E-05

Standard errors in parenthesis. The last row displays the sum of square of the residuals between the impulse response function of the data and each model (given the weights that we use for the errors of the Solow residual statistics these are zero in all cases. See Appendix C).

The Trend in the Labor Share In recent years the labor share of output has experienced an important decline which is well documented by Karabarbounis and Neiman (2014), Elsby, Hobjin, and Sahin (2013), Koh, Santaeulalia-Llopis, and Zheng (2020) and De Loecker, Eeckhout, and Unger (2020) to name a few. Above we have dealt with this problem by detrending the labor share avoiding this non-cyclical pattern. To explore the extent to which this may affect our findings, we have also estimated the process for the technology shock without detrending the series and with the shorter sample period where the overshooting property was first noticed that excludes the last few years of diminishing labor share (1954:Q1 to 2004:QIV). Figure 7 in the Appendix shows the impulse responses of the labor share for all these definitions and we can see how the overshooting property is present irrespective of whether we detrend and of the sample period. If anything, the later years imply more overshooting and detrending while reducing a bit the overshooting, it makes it happen a bit faster.

When we reestimate the technology shock with this alternative impulse responses (Tables 12 and 13) we see consistent evidence of a large time bias, with the values of λ for the Nash bargaining wage setting economies ranging ranging from .01 to .07 and with those of competitive wage setting between .2 and .5 (actually all except one are below .3). The better performance of the competitive wage setting economy relative to the Nash bargaining one is no longer the case in terms of the sum of the square of the residuals: for two of the four impulse responses the residuals are larger in the competitive economies and for the other two, they are larger in the Nash bargaining economies. However, the joint behavior of consumption and output in the competitive economies is more similar to the data than that of the Nash bargaining economies. In particular, all competitive wage economies present cyclical correlations (with respect to output) which are much closer to the data than Nash bargaining economies, making us think that the former are a better description of US economy.[18]

3.3 Economies with Cobb-Douglas (Putty-Putty) Technology

To understand the role of the different elements, we also explore standard search and matching economies with Cobb-Douglas putty-putty technology (where capital and labor are fully transferable across plants). To get further insight in the role of wage setting we pose three versions differing in the wage setting. One with dynamic wage bargaining, the standard in search and bargaining environments,[19] where unsuccessful negotiations result in the breaking up of the match. Another economy where the match survives the break up of negotiations like in our putty-clay economy (*Static Nash Bargaining*). Finally, one where the wage is determined competitively. Details of this model and its functional forms are in Appendix B.

The steady states of the putty-putty economies are the same as those of the putty-clay economies. We target average labor share (0.6514), the consumption output ratio (0.75), an unemployment rate of (0.058), the value of leisure (0.7) and the flow cost of vacancies (0.5% of GDP). Because these economies also have search and matching frictions, some additional targets are required: market labor tightness is set to 1, the elasticity of the matching function with respect to vacancies to 0.6 and the separation rate to 0.03 (See Shimer (2005) for a discussion of these targets). We estimate the shock process of these economies exactly like the putty-clay ones.

The results are in Table 4. Interestingly, we see that the static Nash bargaining worker's weight is

[18] See Appendix D for details.

[19] See Merz (1995), Andolfatto (1996).

Table 4: **Cobb-Douglas putty-putty Economies** (standard technology)

	Dynamic Nash Bargaining	Static Nash Bargaining	Competitive Setting
Steady State Parameters			
b	0.343	0.362	0.343
α	0.008	0.005	0.008
ω	0.981	0.980	0.981
χ	0.005	0.005	0.005
ψ	0.489	0.468	0.487
c^v	0.087	0.085	0.086
μ	0.644	0.612	0.922
δ	0.691	0.971	0.000
Estimated Process for the TFP Shock Standard errors in parenthesis			
ρ	0.981	0.980	0.981
	(0.007)	(0.007)	(0.007)
σ_ϵ	0.0055	0.0051	0.0055
	(0.000)	(0.000)	(0.000)
Sum Square Residuals of Labor Share IRF	2.21E-04	2.24E-04	2.13E-04

The last row displays the sum of square of the residuals between the impulse response function of the data and each model (given the weights that we use for the errors of the Solow residual statistics these are zero in all cases. See Appendix C).

larger than the dynamic one while the competitive value is (by definition) zero. This indicates that the dynamic Nash bargaining process is somewhere in between the static and the competitive process.

One thing to note is that the estimates of the shock process are essentially identical for all economies: the details of the wage setting do not affect in any relevant way how well the overshooting of the labor share is replicated. In fact, these economies do quite badly with errors that are over one order of magnitude larger than those of the putty-clay economies. Why this is the case is the question that we address in the next Section.

4 Properties of the Estimated Model Economies

We start discussing the logic of why the model economies with putty-clay technology are able to deliver the overshooting property of labor share in Section 4.1. We then move to estimate the shock directly on employment and the consumption to output ratio in Section 4.2. We finish the section describing the standard business cycle properties of these economies in Section 4.3.

4.1 The Overshooting Property of Labor Share

How do the model economies manage to match the behavior of the labor share? Recall that after an initial positive technology shock, labor share drops but subsequently exhibits a positive deviation from its long run trend, which is larger than the initial drop.

To see what are the determinants of labor share we take Equation (13), integrate over plants and divide by output to get an expression for the aggregate labor share:

$$LSH = \mu + (1 - \mu)\ b\ N\ \frac{C}{Y}. \tag{24}$$

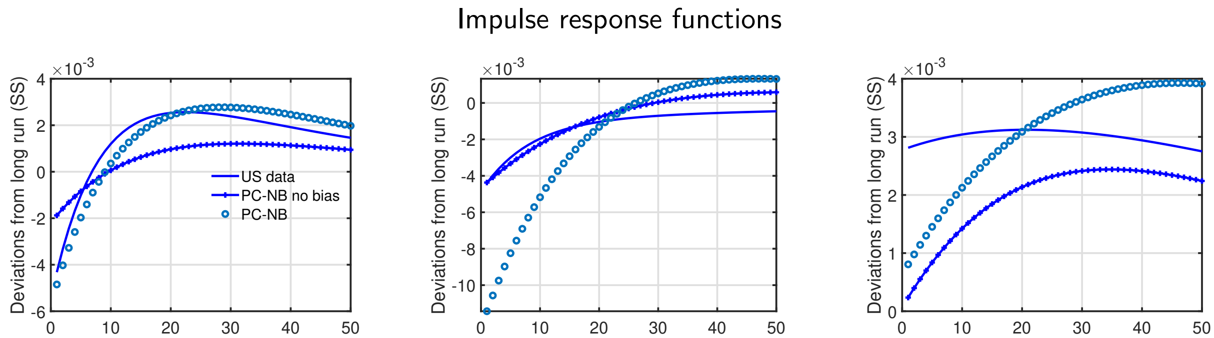
Note first that because we calibrate all the economies to the same values for the labor share, employment rate and consumption to output ratio in steady state, the value of b varies with the value of μ to satisfy this equation. In particular, we have $b \sim .65$ and $\mu \sim .36$ implying a slope of .42 for the Nash bargaining wage setting economy and $b \sim .92$ and $\mu = 0$ implying a slope of .92 for the competitive economy. Now note that any given movements in employment and the consumption to output ratio have much *stronger* implications for the labor share in the competitive economy than in the Nash bargaining one which helps explain why the former has an easier time (in terms of the size of the time bias parameter) in achieving the overshooting property.

Note also that this affine relation between the labor share and the product of the consumption to output ratio and employment is *not* an accounting identity, it is an equilibrium relation and hence it does not necessarily hold as such in the data.

Putty-Clay Nash Bargaining Economies, $\mu > 0$: Figure 1 shows the impulse response function of labor share, the consumption to output ratio and employment of the putty-clay economies with Nash bargaining wage setting with and without time biased technical change. Recall that the time biased technical change implied a time bias of 99.5% relative to the neutral one (in the sense that the estimated λ is 0.51% while the neutral is 1). Recall also that the sum of the square of the residuals of the neutral technology shock is more than four times larger than in the economy where the shock has time bias.

The left panel of the figure shows that the economy with a time bias technology shock does a

Figure 1: Putty-Clay Economies with Nash Bargaining Wage Setting with and without Technology Shock Time Bias



(a) Labor Share (b) Consumption-Output ratio (c) Employment

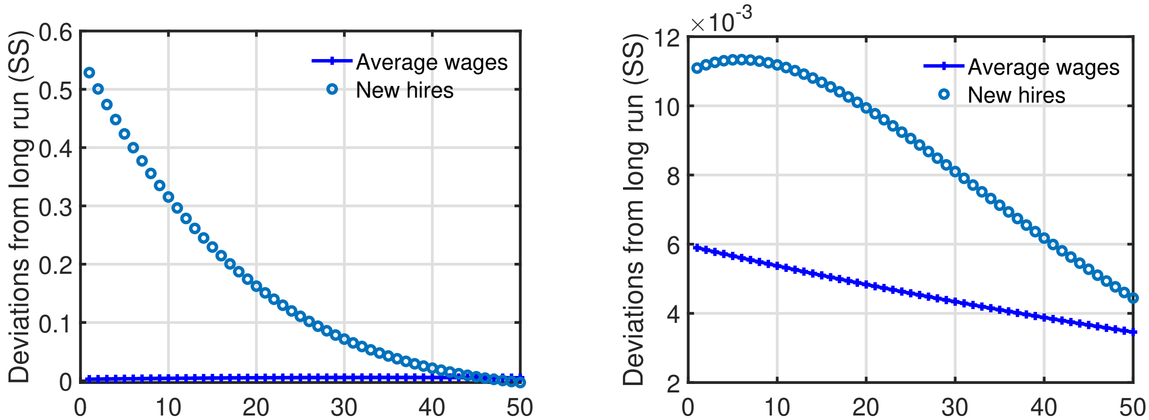
reasonably good job of generating the overshooting property of the labor share. It has the right amplitude even though it is slightly delayed. The no time bias shock manages to get only a bit of the overshooting property but much less than that in the data. When we look at how the model economies behave in terms of the components of the labor share the results are not well aligned with the data. The low slope of the relation between the labor share and the product of employment and the consumption to output ratio imply that these have to move a lot to replicate the labor share. First negatively and then positively which results in an exaggerated tremendous fall in the consumption to output ratio that slowly vanishes to the point of changing side (in fact it generates a countercyclical consumption behavior, as seen in Section 4.3). To complete the picture the response of employment is much weaker at the beginning, but accumulates over time yielding a higher response than the data at the five year mark. All of this is achieved with an extreme form of time bias. The economy with no time bias cannot generate enough of an overshoot: the consumption to output ratio does not fall that much and employment responds quite slowly

To better understand the response of employment in these two economies, Figure 2 shows the response of wages of new plants and average established plants in both Nash bargaining economies.[20] The results are striking: the extreme time bias of the technical shock together with the decentralized nature of bargaining makes for an enormous wage increase in new plants. While we do not have data on this variable, casual observation indicates that this is unlikely to be the case (we would expect to see

[20] Because of the decentralized Nash bargaining, these are the only economies that display wage dispersion.

Figure 2: Wage Responses in Putty-Clay Economies with Nash Bargaining with and without Time Bias

Impulse response functions



(a) Time Bias

(b) No Time Bias

a lot of workers switching from old to new plants in expansions).

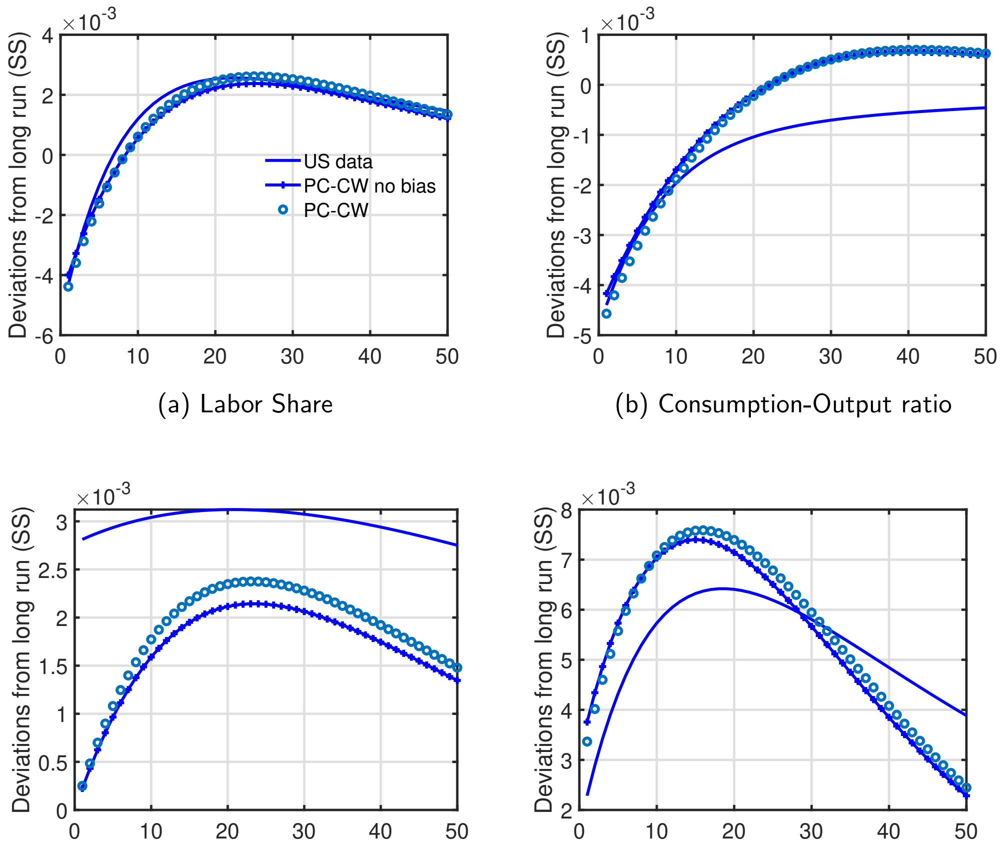
Putty-Clay Economies with Competitive Wage Setting, $\mu = 0$: Figure 3 shows the impulse response functions for putty-clay economies with competitive wage setting with and without technical time bias. They both match the impulse response of the labor share better than the economies with Nash bargaining. In fact, even the competitive economy without time bias does better than the Nash bargaining economy with time bias. Comparing the two competitive economies we see that the one without time bias economy is a bit worse (it has 27% higher sum of square residuals).

In terms of the determinants of labor share we see (panel (b)) that the consumption to output ratio of both economies are similar and their responses exhibit more amplitude than the US data, while matching well the first ten quarters after the initial shock. Both economies display a similar pattern in terms of employment creation (panel (c)): lower than that in the data, with time bias being able to produce a more pronounced response between periods 10 and 30 after the initial shock. Recall that the equation that determines the behavior of labor share in terms of employment and the consumption to output ratio is an equilibrium equation that holds in the model but not necessarily in the data, where other concerns, such as hours per worker and the participation margin (to name a few) may be quite relevant.

Both models also have a reasonable impulse response of wages, perhaps 20% more than that in the data at its peak. Overall, we can see that competitive wage setting with a productivity shock biased towards new technologies accounts well for the overshooting property of labor share even if the response of employment is not strong enough.

Figure 3: Putty-Clay Economies with Competitive Wage Setting with and without Technical Time Bias

Impulse response functions



(a) Labor Share

(b) Consumption-Output ratio

(c) Employment

(d) Wages

On the role of time bias. In putty-clay economies, all employment creation requires new investment. Time bias (λ) allows for a concentration of the effects of the productivity shocks in new plants where they can attract employment creation as opposed to what occurs in older plants where the shock acts as a rent to be shared by workers and plant owners under Nash bargaining or just by plant owners under competition. Time bias then amplifies the responses of employment and labor share to shocks, as can be seen in Figure 1 and Figure 3. When technological shocks are extremely time biased towards new machines, firms have added incentives to create employment to seize better aggregate conditions. Also, this is aided by the fact that the specifications with low values of λ go hand in hand with high levels of aggregate shock volatility: if only the newest machines can make use of TFP shocks, and the model has to be consistent with moments of the Solow residual in US data, then the estimation will lead to higher values for σ_ϵ which in turn generate higher levels of employment after positive shocks.

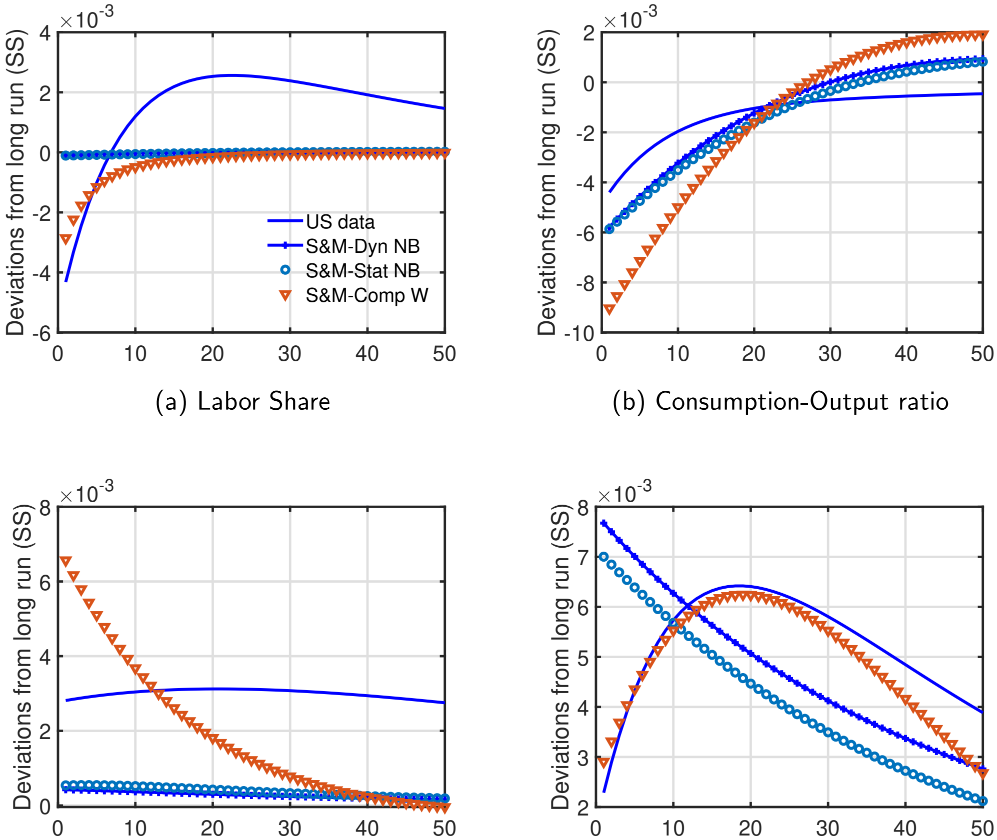
In economies with Nash bargaining wage setting, time bias has also a differential effect on wages, as seen in Figure 2. When only the newest machines are affected by technological shocks, average wages become sluggish to innovations in technology, since most productive units only see their wages react to aggregate conditions through movements in consumption (outside option of workers). This effect is interesting in that mimics "fixed" average wages in an endogenous way which also helps the model in producing more volatility of employment by altering the benefits of hiring workers: wages of new workers are relatively high only one period, while their productivity is long lasting, tied to their machine size k. However, in the estimated economy the differences are extreme given our simplifying assumption of the time bias operating only one period.

Putty-clay versus Cobb-Douglas putty-putty Technologies In Choi and Ríos-Rull (2009) is well documented the ill-performance of search and matching Cobb-Douglas putty-putty economies in replicating the overshooting property of labor share. Comparing those economies with the putty-clay economies at the centre of this paper gives us further insights in the mechanisms at work. Figure 4 shows how all putty-putty economies fail to generate any overshooting, in fact, only the economy with competitive wages manages to move the labor share at all (and only immediately after the shock). Those that have wage bargaining are subject to the puzzle noted by Shimer (2005): most of the productivity increases go to wages of existing workers not to new workers. Only the economy with competitive wage setting displays movements in employment of any significance, although employment peaks when the shock hits and then it returns to the steady-state level pretty rapidly unlike both the data and the putty-clay economies. The competitive wage economy is an extreme form of what was proposed by Hagedorn and Manovskii (2008), in that the gap between productivity and wages (which is zero in the competitive wage case) is negatively related to the ability of the model to generate movements in employment.

A valuable lesson of looking at the Cobb-Douglas putty-putty economies is the extreme similarity in the business cycle performances of the dynamic and the static Nash bargaining economies (only the steady-state value of leisure changes to maintain average labor share). This gives peace of mind with respect to assumption of static bargaining in the putty-clay economy. We do not think that it is that

Figure 4: Cobb-Douglas putty-putty Economies with Search and Matching with different wage settings

Impulse response functions



(a) Labor Share

(b) Consumption-Output ratio

(c) Employment

(d) Wages

S&M-Dyn NB refers to a model with standard Nash Bargaining. S&M-Stat NB refers to a model with static Nash Bargaining. S&M-Comp W refers to a model with competitive wage setting.

important to characterize the behavior of non competitive wage setting environments.

Intuitively, putty-clay technology implies that, in the short-run, increasing employment and investment is the only way in which temporary high productivity can be taken advantage of. This is in stark contrast to environments with Cobb-Douglas technology, where a unitary elasticity of substitution between capital and labor means that the expansion path after technology shocks tends to equalize shares across factors of production: firms can substitute away from relatively more expensive factors when increasing output. Additionally, under putty-clay each additional hire does not decrease the productivity of all other workers in the economy as is the case with Cobb-Douglas technology, making the incentives to hire in our model last longer than in the search and matching framework.

Overall, the evidence points to competitive wages and a high degree of time bias as the model economy that best matches the data. It does so through its ability to generate a strong and relatively early response of employment without needing to generate extreme movements in the consumption output ratio. It also benefits from the fact that the labor share responds more strongly to the product of employment and the consumption ratio. Another feature that goes in its favor is that the need for an extreme time biased shock is not as strong as in the Nash bargaining case. The reason is that competition breaks the link between productivity in the plant and the salary of its worker which makes wages in new plants go up much less than in the Nash bargaining economies. Lower wages in new plants imply that a lower productivity is needed to make job creation attractive.

4.2 Estimating the Shock on Employment and the Consumption to Output Ratio

We want to know whether our putty-clay structures is capable of improving their performance in terms of the responses of both employment and the consumption to output ratio, instead of focusing on the labor share alone. To this end, below we present an alternative estimation strategy where target jointly the impulse responses of both the consumption-output ratio (C/Y) and employment (N) instead of the one of the labor share.

Table 5: Targetting Employment and the Consumption Output Ratio: Estimated Processes

	Nash Bargaining	Competitive Setting
λ	0.0471	0.0504
	(0.015)	(0.011)
ρ	0.979	0.983
	(0.001)	(0.001)
σ_ϵ	0.100	0.094
	(0.024)	(0.015)
Sum Square Residuals of Labor Share IRF	2.24E-04	2.13E-04

Estimates when impulse responses of C/Y and N are targeted. Standard errors in parenthesis The last row displays the sum of square of the residuals between the impulse response function of labor share of the data and each model. Monthly process.

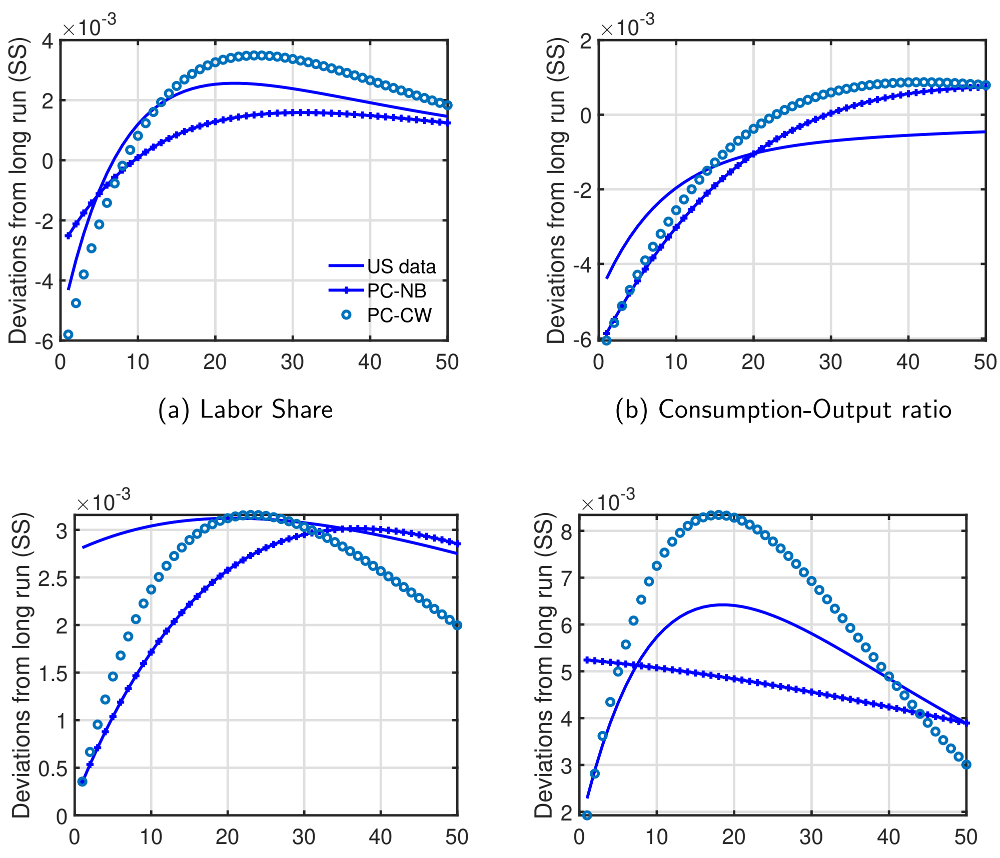
The estimated parameters are in Table 5 for both time biased economies while Figure 5 displays the associated impulse responses. The table shows that parameter estimates are quite similar for both economies and that their ability to match the responses of labor share is worse than the previous models, as shown by the sum of squared residuals.

The main difference with the results from the previous models (Figure 1 and Figure 3) is that this estimation strategy results in bigger responses of both employment and the consumption to output ratio for both economies, with the Nash bargaining economy producing less overshooting, and the competitive wage economy producing more.

As noted earlier, our model does not have a participation margin for workers: there are only employed and unemployed individuals. Thus, although putty-clay can yield a strong response from employment, the model is not yet capable of replicating the strong and speedy response observed in US data.

Figure 5: Putty-Clay Economies with Technical Time Bias (Alternative Estimation)

Impulse response functions



(a) Labor Share (b) Consumption-Output ratio

(c) Employment (d) Wages

4.3 Other Business Cycle Statistics in Data and Model Economies

How to these model economies behave in terms of standard aggregate variables? Table 6 reports relative standard deviations (columns σ_x/σ_y), correlations with output (columns $\rho_{x,y}$) and autocorrelations (columns $\rho_{x,x'}$). For model simulated data, we aggregate monthly series to quarterly data by simple averaging. For all series (data and simulated), we take logs and detrend using the Hodrick-Prescott filter, with smoothing parameter set to 1600, as is standard in the literature.

Table 6: Cyclical volatility of U.S. and Estimated Economies

	US data			Nash Bargaining						Competitive					
				Time Bias			No time bias			Time Bias			No time bias		
x	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$
Emp	0.64	0.80	0.90	0.44	0.49	0.96	0.20	0.33	0.96	0.20	0.43	0.96	0.18	0.42	0.96
Une	8.50	-0.87	0.89	10.27	-0.38	0.95	4.29	-0.28	0.96	3.86	-0.38	0.96	3.39	-0.38	0.96
LSh	0.66	-0.22	0.63	0.61	-0.64	0.78	0.26	-0.79	0.79	0.58	-0.73	0.78	0.53	-0.74	0.78
Wag	0.62	0.55	0.74	0.37	0.98	0.85	0.73	1.00	0.79	0.54	0.91	0.88	0.56	0.94	0.86
Con	0.74	0.78	0.81	0.66	-0.19	0.87	0.51	0.95	0.84	0.54	0.91	0.88	0.56	0.94	0.86
Inv	4.49	0.85	0.81	4.88	0.91	0.77	2.62	0.98	0.78	2.63	0.97	0.78	2.49	0.97	0.78

Note: US data from the Bureau of Labor Statistics (see Appendix C for details) for the period 1947:QI-2018:QIV. σ_x/σ_y is the relative standard deviation of variable x with respect to output; while $\rho_{x,y}$ is the correlation of variable x also with respect to output. All series are logged and HP-filtered.

The volatility of employment is more subdued than its data counterpart, pointing to the main shortcoming of these economies. The behavior of other variables is much more in line with the data. Especially, that of unemployment, an artifact of the lack of a labor force participation margin. As hinted before the time biased Nash bargaining economy has a very counterfactual property in terms of the correlation between consumption and output: it is negative in that economy, while this correlation is large and positive in the data, across time periods and countries making it a very counterfactual feature for this economy. We see that the small excessive response of wages to a technology innovation in the competitive wage setting economies does not translate into excessive wage moments. The persistence of all the variables are in line with the data.

Table 7: Cyclical volatility of Alternate Estimations with Time Bias

	US data			Putty-clay - NB			Putty-clay - comp. w		
x	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$
Employ	0.638	0.803	0.902	0.257	0.378	0.962	0.255	0.468	0.961
Unempl	8.497	-0.872	0.893	6.339	-0.302	0.958	5.756	-0.388	0.960
Labor Sh	0.658	-0.216	0.633	0.339	-0.763	0.785	0.753	-0.699	0.781
Wages	0.625	0.553	0.742	0.642	0.999	0.791	0.472	0.748	0.938
Cons	0.743	0.780	0.808	0.409	0.795	0.913	0.472	0.748	0.938
Invest	4.483	0.848	0.806	3.139	0.970	0.777	3.107	0.952	0.775

Note: US data is taken from the Bureau of Labor Statistics (see Appendix C for details) for the period 1947:QI-2018:QIV. y represents output; σ_x/σ_y is the relative standard deviation of variable x with respect to output; $\rho_{x,y}$ is the correlation of variable x with respect to y. All series are logged and HP-filtered.

Table 8: Cyclical volatility of U.S., No time bias and Search and Matching models

	US data			S&M dyn-NB			S&M stat-NB			S&M comp. w		
x	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$	σ_x/σ_y	$\rho_{x,y}$	$\rho_{xx'}$
Employ	0.64	0.80	0.90	0.08	0.94	0.83	0.11	0.94	0.84	0.62	0.97	0.82
Unempl	8.50	-0.87	0.89	1.30	-0.94	0.83	1.75	-0.94	0.84	12.25	-0.89	0.81
Labor Sh	0.66	-0.22	0.63	0.01	-0.92	0.69	0.01	-0.98	0.77	0.19	-0.77	0.52
Wages	0.62	0.55	0.74	0.92	1.00	0.79	0.88	1.00	0.78	0.28	0.86	0.87
Cons	0.74	0.78	0.81	0.33	0.90	0.85	0.26	0.89	0.85	0.28	0.86	0.87
Invest	4.48	0.85	0.81	3.19	0.99	0.79	3.69	0.99	0.80	3.27	0.99	0.83

Note: US data is taken from the Bureau of Labor Statistics (see Appendix C for details) for the period 1947:QI-2018:QIV. y represents output; σ_x/σ_y is the relative standard deviation of variable x with respect to output; $\rho_{x,y}$ is the correlation of variable x with respect to y. All series are logged and HP-filtered.

The standard business cycle statistics for the economies with search frictions and Cobb-Douglas putty-putty technology are in Table 8. As highlighted by Shimer (2005), this technology coupled with Nash bargaining with a high value of the workers bargaining weight have a hard time matching the aggregate volatility of employment and unemployment. The competitive wage setting can do a lot better, (as pointed to by Hagedorn and Manovskii (2008)), but, as noted by Choi and Ríos-Rull (2009), it does not not generate the overshooting of the labor share. The main reason for this is the fact that employment peaks immediately after the shock hits and it does not have a sufficiently long-lived response to generate any overshooting.

Our main takeaway from this subsection is the ability of the putty-clay model with competitive wages to replicate volatility of labor market aggregates when faced with shocks estimated from data.

5 Conclusions

In this paper we have proposed a flexible way to introduce productivity shocks into a framework with putty-clay technology with an additional parameter whose value tells us whether we should think of technology shocks as being neutral or as being akin to investment specific shocks. The main insight from our analysis, is the fact that the latter can be thought of as neutral shocks which have a *time* bias, which favors recently created productive units. Our results show that, when targeting volatility of the Solow residual and the impulse response of the labor share in the US economy, a model with competitive wage setting and a strong time bias of the aggregate shock does best (quite well actually). In particular, it does better than a model with Nash bargaining and extreme time bias. Still, matching both the response of the consumption to output ratio and of employment is not yet achieved.

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Appendix

A Time Biased Shocks and Investment Specific Technological Innovations

To show the distinction between our time biased shock and standard investment specific technological (IST) shocks, in this section we depict the dynamics of both shocks. In Table 9 we show the time biased shocks, while the dynamics of standard investment specific shocks are in Table 10. For the latter, we imagine an economy where capital accumulation is given by $K_{t+1} = (1 - \delta)K_t + e^{\widetilde{z}_t} Q_t$, where $\widetilde{z}$ evolves according to Equation (2) and Q_t is the amount of capital installed in period t.

		machine age (in periods)				
time	shock	1	2	3	4	...
0	1	1	λ	λ	λ	...
1	ρ	ρ	$\lambda\rho$	$\lambda\rho$	$\lambda\rho$	...
2	ρ^2	ρ^2	$\lambda\rho^2$	$\lambda\rho^2$	$\lambda\rho^2$	...
3	ρ^2	ρ^3	$\lambda\rho^3$	$\lambda\rho^3$	$\lambda\rho^3$	...
4	ρ^3	ρ^4	$\lambda\rho^4$	$\lambda\rho^4$	$\lambda\rho^4$	...
...	...	...	...	...	...	...

Table 9: Representation of shock dynamics.

		machine age (in periods)				
time	shock	1	2	3	4	...
0	1	0	0	0	0	...
1	ρ	1	0	0	0	...
2	ρ^2	ρ	1	0	0	...
3	ρ^2	ρ^2	ρ	1	0	...
4	ρ^3	ρ^3	ρ^2	ρ	1	...
...	...	...	...	...	...	...

Table 10: Representation of investment specific shocks.

For both tables, we imagine an economy that is in steady state, and introduce a shock to productivity of size one (just for exposition). Given the AR(1) representation of the shocks, in both economies the value of the shock declines according to ρ.

In our time-biased representation of the shock (see Equation (1) and Equation (2)), machines that are 1 period old (installed just last period), enjoys productivity e^1 the first period that is operational (time 0 in the table), and enjoys the declining productivity sequence $\{\lambda\rho, \lambda\rho^2, \lambda\rho^3, ...\}$. Furthermore, all older machines enjoy the productivity shock, in amount λ, and as time passes, they also are exposed to a declining value of the shock.

On the other hand, in Table 10 we see that capital/machines (Q) which is 1 period of age, enjoys IST for all of its existence (sequence of 1's in the table). In a model with IST, installed machines do not enjoy these shocks (thus the zeros in the table).

		machine age (in periods)				
time	shock	1	2	3	4	...
0	1	1	0	0	0	...
1	ρ	ρ	0	0	0	...
2	ρ^2	ρ^2	0	0	0	...
3	ρ^2	ρ^3	0	0	0	...
4	ρ^3	ρ^4	0	0	0	...
...	...	...	...	...	...	...

Table 11: Representation of shock dynamics with $\lambda = 0$

The extreme case of full time biased shocks, i.e., $\lambda = 0$, is shown in Table 11. There, we can see the similarities and differences between this type of shock and standard IST. On the one hand, both shocks give the same value of the shock to the newest machines. However, the $\lambda = 0$ case do not affect machines of any cohort beyond those 1 period old, which is the main difference with how IST affects dynamically investment. However, although our formulation removes the effect of the shock on older cohorts of machines/capital, these are depreciating at exponential rate δ, giving more weight to the newest cohorts.

Thus, although there are strong similarities between IST and our shock when $\lambda = 0$, they are not the same.

B Standard Search and Matching Model in General Equilibrium

We use the model put forward by Andolfatto (1996), which is a general equilibrium version of the standard search and matching framework. The model is described by perfect insurance inside the household (so employed and unemployed workers consume the same) and search and matching frictions in the labor market. Households derive utility from consumption and the leisure enjoyed by its members, with per period utility given by $\log(C) + (1 - N)b$, where C is consumption, N is the measure of household members who are unemployed while b is a parameter related to the extra amount of leisure enjoyed by the unemployed. Households decide how much to consume and save every period. Firms produce using a standard Cobb-Douglas production function $Y = e^z K^\alpha N^{1-\alpha}$, rent capital K and search for workers by posting vacancies V, at flow cost c^v. Matches in the economy are given by the matching function $m(V, 1-N) = \omega V^\psi (1-N)^{1-\psi}$ and there is a lag of one period between match and production. Real wages are the solution to bilateral Nash bargaining between the firm and the household, with the bargaining weight of the worker being μ. We consider two alternative versions of the model: (i) standard Nash bargaining and (ii) an ad-hoc wage to mimic our baseline economy.

The following equations characterize the equilibrium of this model (where primes denote future values):

Aggregate consistency and evolution of states:

$$
\begin{aligned}
Y &= e^z K^\alpha N^{1-\alpha} \\
Y &= I + C + c^v V \\
N' &= (1-\chi)N + \omega V^\psi (1-N)^{1-\psi} \\
K' &= (1-\delta)K + I \\
z' &= \rho z + \epsilon, \quad \epsilon \sim N(0, \sigma_\epsilon^2)
\end{aligned}
$$

Job Filling Rate:

$$
\Phi = \omega V^{\psi-1}(1-N)^{1-\psi}
$$

Euler equation:

$$
1 = \beta E\left[\frac{C}{C'}\left(1 - \delta + (1-\alpha)\frac{Y'}{K'}\right)\right]
$$

Optimal vacancy posting decision:

$$
\frac{c^v}{\Phi} = \beta E\left[\frac{C}{C'}\left(\alpha\frac{Y'}{N'} - w' + (1-\chi)\frac{c^v}{\Phi'}\right)\right]
$$

Wages:

$$
\begin{aligned}
w &= \mu\left[\alpha\frac{Y}{N} + c^v\frac{V}{1-N}\right] + (1-\mu)Cb \qquad &\text{(Dynamic bargaining)}, \\
w &= \mu\left[\alpha\frac{Y}{N}\right] + (1-\mu)Cb \qquad &\text{(Static bargaining)}.
\end{aligned}
$$

C Data and Estimation

Data is taken from the Bureau of Labor Statistics (BLS)'s website, the FRED site of the Saint Louis Fed and the Bureau of Economic Analysis (BEA) tables. The series we use are Output (Table 1.1.6., Real Gross Domestic Product in Chained Dollars from BEA), Labor Share Index (BLS series PRS85006173), Employment (BLS series LNS12000000Q), Unemployment (BLS series LNS13000000), Real Hourly Compensation (BLS series PRS85006153), Personal consumption expenditures (FRED series PCECC96), Real Gross Private Domestic Investment (FRED series GPDIC96) and the chain-type quantity Indexes for Net Stock of Fixed Assets and Consumer Durable Goods, table 1.2 from the Fixed Asset tables from the BEA (as capital). All variables refer to non-farm business sector and are seasonally adjusted. We further normalize all variables by the non-institutionalized population in the US (BLS series LNU00000000Q). For all series, we consider information between the first quarter of 1948 and the fourth quarter of 2018 (last available data point for fixed assets).

As for the Solow residual, we follow closely the procedure in Ríos-Rull and Santaeulalia-Llopis (2009). It is defined as

$$\widehat{z}_t = y_t - \overline{LSH}\ n_t + (1 - \overline{LSH})\ k_t,$$

where $\overline{LSH}$ is the average labor share, y_t is detrended output, n_t is detrended employment and k_t is detrended capital, where the detrending procedure is linear: For variable X, it's residual is

$$x_t = \log(X_t) - \alpha_t^x t$$

where t is a linear trend and α_t^x is the least square estimator of a linear trend on an equation on $\log(X_t)$.

From here, a regression of the form

$$\widehat{z}_t = \gamma\ \widehat{z}_{t-1} + \eta_t$$

produces a linear least square estimate for the persistence, $\hat{\gamma}$ and the volatility of the Solow residual, $\hat{\sigma}_\eta$.

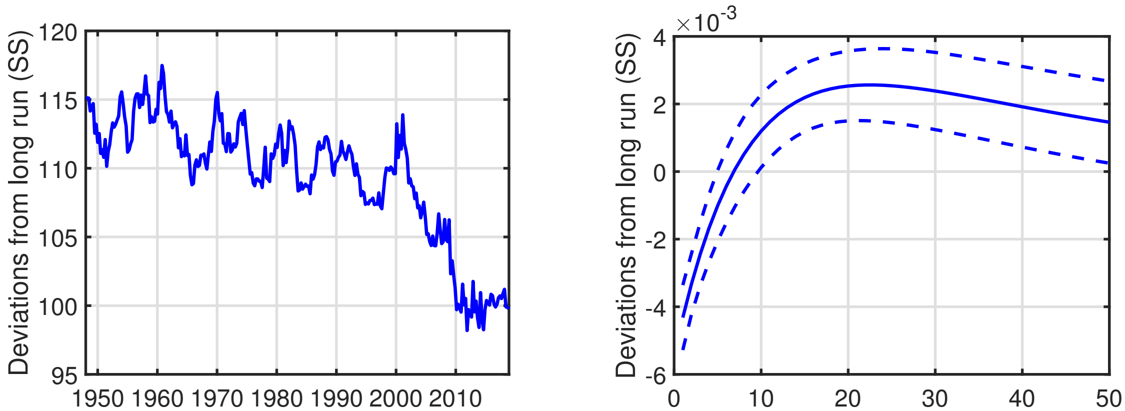
In our estimation exercise (and across all models we present) we estimate the same equation using simulated data and match the estimates from our data sources:

$$\begin{aligned} \hat{\gamma} &= 0.9611774 \quad (0.0137514) \\ \hat{\sigma}_\eta &= 0.0077764 \quad (0.00032759) \end{aligned}$$

where number in parentheses are standard deviations from the estimates. To maintain comparability across models in terms of the overall amount of volatility that they are able to generate, we force each model to match $\hat{\gamma}$ and $\hat{\sigma}_\eta$ almost perfectly. We accomplish this by way of increasing the weight of those targets in the estimation routine, when minimizing the sum of squared residuals between data and model generated moments. In total, we target 32 moments: the two moments related to the Solow process plus 30 elements in the impulse response function of labor share. We give the first two targets a weight of one thousand while the rest receive a weight of one. In all our estimations, the models match the estimates of $\hat{\gamma}$ and $\hat{\sigma}_\eta$ very accurately.

D Labor Share During the Business Cycle

Here we look in detail at the behavior of the labor share. On the left panel of Figure 6 we depict the labor share series (described in the previous section). We can see clearly its downward trend, which becomes even more pronounced around the mid 2000s. On the right panel, we present the estimated orthogonal impulse response from a vector autoregression of order 1 between linearly detrended log labor share and linearly detrended log GDP with its corresponding 95% confidence intervals.[21]



(a) Labor Share (b) Impulse Response of LSH

Figure 6: Left panel is a labor share Index for non-farm business US, 1948:Q1 to 2018:QIV. Right panel is an impulse response function of labor share to a shock in output, from VAR(1) model. Source: Bureau of Labor Statistics.

The figure summarizes various facts with respect to the labor share which have been previously discussed in Choi and Ríos-Rull (2009) and Ríos-Rull and Santaeulalia-Llopis (2010): Upon impact, labor share falls giving rise to the well known counter cyclical behavior of the series. In time, labor share starts to rise, and peaks around 15 to 20 quarters after the initial shock. This positive response is persistent and long lasting, making the aggregate gains of labor due to an expansion positive in the medium to long run, more than compensating the initial drop in this share (hence the overshooting label). The estimates of the bivariate VAR show no effects whatsoever of lagged labor share on productivity. See Ríos-Rull and Santaeulalia-Llopis (2010) for details. Also, given that we are using a different time period and different variable definitions, this is also a sign of the robustness of the overshooting fact. As seen in the figures, this response is statistically significant at the 95% level.

When we refer to empirical impulse responses to other aggregates (consumption to output ratio, employment, average wages for example) in the main text, the procedure we follow to compute them, both in the model and US data, is analogous to what we have described above for the response of the labor share.

[21]See Hamilton (1994).

Labor Share Trends

We turn to explore whether our findings are an artifact of the detrending. As seen in the left panel of Figure 6, the labor share exhibits a clear trend downwards, perhaps becoming steeper after the turn of the century. So we perform our analysis also on a shorter sample that avoids the recent years of falling labor share and by comparing the extent to which detrending affects the impulse response.

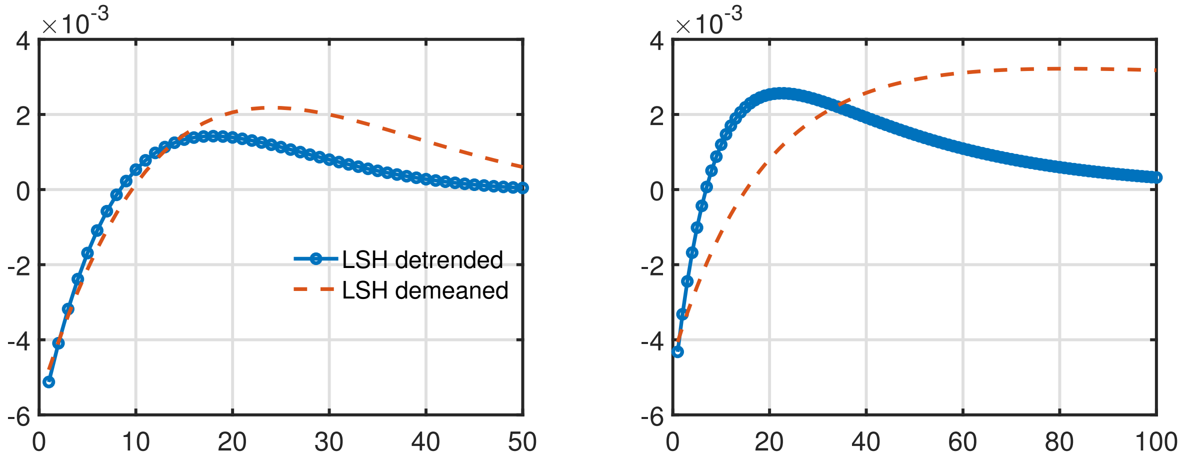


Figure 7: Impulse response functions of labor share in US data. Left panel: sample used in Ríos-Rull and Santaeulalia-Llopis (2010) (1954:Q1 to 2004:Q4). Right panel: full sample (1948:Q1 to 2018:Q4).

Figure 7 depicts the impulse response functions of the labor share as described above, both log detrended and not (just demeaned). In the left panel we use the same sample used when the overshooting property was first noted by Ríos-Rull and Santaeulalia-Llopis (2010) which spans 1954:QI to 2004:QIV, while the right panel contains our full sample. As seen from the figure, overshooting is present irrespective of whether we detrend or not. Moreover, the procedure of linearly detrending the labor share we use throughout the exercise in the main text, produces a lower overshooting versus when we do not. Further, we see that detrending pulls the overshoooting a bit earlier.

We proceeded to estimate the shock process with these alternative impulse responses and the results are depicted in Tables 12 and 13. As seen from both tables, the main message of the main exercise is maintained, in that in both subsamples and detrending procedures, the estimation finds low values of λ, implying high levels of time bias. On the other hand, the sum of squared residuals of the labor share IRF is not as clear in prefering competitive wage setting as in the main text. However, the competitive wage setting still behaves better in terms of the relative performance of the consumption aggregate. As noted in the main exercise in the text, the economy with Nash bargaining and time bias has a counterfactual behavior in terms of consumption: it is counter-cyclical with respect to output. In Table 14 and Table 15 we see that, even though this countercyclical feature does not repeat in all cases, all competitive wage economies are closer to the data in terms of the correlation of consumption relative to output ($\rho_{x,y}$) and, in all but the estimation on the "full sample, detrended labor share", the competitive wage setting

Table 12: Model Estimated Using 1954-2004 Sample

	Nash Bargaining		Competitive Setting	
	Detrended	Not	Detrended	Not
λ	0.075	0.015	0.491	0.289
	(0.028)	(0.004)	(0.488)	(0.233)
ρ	0.982	0.982	0.987	0.986
	(0.002)	(0.003)	(0.001)	(0.002)
σ_ϵ	0.066	0.237	0.011	0.018
	(0.020)	(0.033)	(0.010)	(0.014)
Sum Square Residuals of Labor Share IRF	4.65E-05	2.06E-05	6.97E-05	1.46E-05

Table 13: Model Estimated Using Full (1948-2018) Sample

	Nash Bargaining		Competitive Setting	
	Detrended	Not	Detrended	Not
λ	0.005	0.015	0.218	0.2252
	(0.001)	(0.005)	(0.126)	(0.164)
ρ	0.973	0.977	0.984	0.984
	(0.003)	(0.004)	(0.002)	(0.003)
σ_ϵ	0.408	0.234	0.024	0.024
	(0.035)	(0.042)	(0.013)	(0.016)
Sum Square Residuals of Labor Share IRF	1.58E05	4.11E-05	4.35E06	7.44E-05

produces relative volatilities of consumption more in line with the data. However, this latter case is where the Nash bargaining economy with time bias produces counter-cyclical consumption.

Table 14: Cyclical volatility of U.S. and Estimated Economies (1954-2004 sample)

	US data		Detrended				Not detrended			
			Nash bargaining		Competitive		Nash bargaining		Competitive	
x	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$
Employ	0.62	0.80	0.23	0.36	0.18	0.42	0.33	0.42	0.18	0.42
Unempl	7.31	-0.87	5.53	-0.29	3.38	-0.38	8.06	-0.31	3.56	-0.38
Labor Sh	0.69	-0.30	0.29	-0.78	0.52	-0.74	0.44	-0.72	0.53	-0.74
Wages	0.60	0.49	0.69	1.00	0.58	0.95	0.53	1.00	0.56	0.94
Cons	0.81	0.88	0.46	0.90	0.58	0.95	0.41	0.36	0.56	0.94
Invest	4.19	0.91	2.85	0.98	2.44	0.97	3.78	0.95	2.49	0.97

Note: σ_x/σ_y is the relative standard deviation of variable x with respect to output; while $\rho_{x,y}$ is the correlation of variable x also with respect to output. All series are logged and HP-filtered.

Table 15: Cyclical volatility of U.S. and Estimated Economies (full 1948-2018 sample)

	US data		Detrended				Not detrended			
			Nash bargaining		Competitive		Nash bargaining		Competitive	
x	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$	σ_x/σ_y	$\rho_{x,y}$
Employ	0.64	0.80	0.44	0.49	0.20	0.43	0.34	0.43	0.20	0.43
Unempl	8.50	-0.87	10.27	-0.38	3.86	-0.38	8.25	-0.33	3.84	-0.38
Labor Sh	0.66	-0.22	0.61	-0.65	0.58	-0.73	0.46	-0.71	0.58	-0.73
Wages	0.62	0.55	0.37	0.97	0.54	0.91	0.52	1.00	0.54	0.91
Cons	0.74	0.78	0.66	-0.19	0.54	0.91	0.42	0.28	0.54	0.91
Invest	4.48	0.85	4.88	0.91	2.63	0.97	3.89	0.95	2.62	0.97

Note: σ_x/σ_y is the relative standard deviation of variable x with respect to output; while $\rho_{x,y}$ is the correlation of variable x also with respect to output. All series are logged and HP-filtered.

Robustness of overshooting: Long run restriction

On the left panel of Figure 8, we show impulse response functions of labor share: the solid line is ours, while the dotted line represents the impulse response function from a SVAR with long run restrictions. We follow the baseline estimation in Santaeulalia-Llopis (2012), who uses a standard procedure to identify technology shocks: that the effect of technology on labor inputs is zero in the long run, as is done in Galí (1999b) and Fisher (2006b). We can see that the overshooting fact is maintained under the standard SVAR specification, even if slightly smaller.

Robustness of overshooting: Fernald's adjusted TFP

We also look at whether the overshooting property of the labor share occurs when using Fernald's utilization adjusted TFP series. [22] The right panel of Figure 8 shows the impulse response of labor share. The overshooting shape is there, but the size is not. We do not see this as evidence against our analysis. The Fernald TFP series is an attempt to get at some true technology series that takes

[22] Downloaded from
`https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp`. See Fernald (2012).

into account some variability in capital utilization. As such it is not that different than what our own model could produce: a technology series conditional on a particular theory. Our simple model does not contain a mechanism to vary factor utilization so it cannot speak directly to what produces this lower reaction of labor share to this alternative TFP measure.

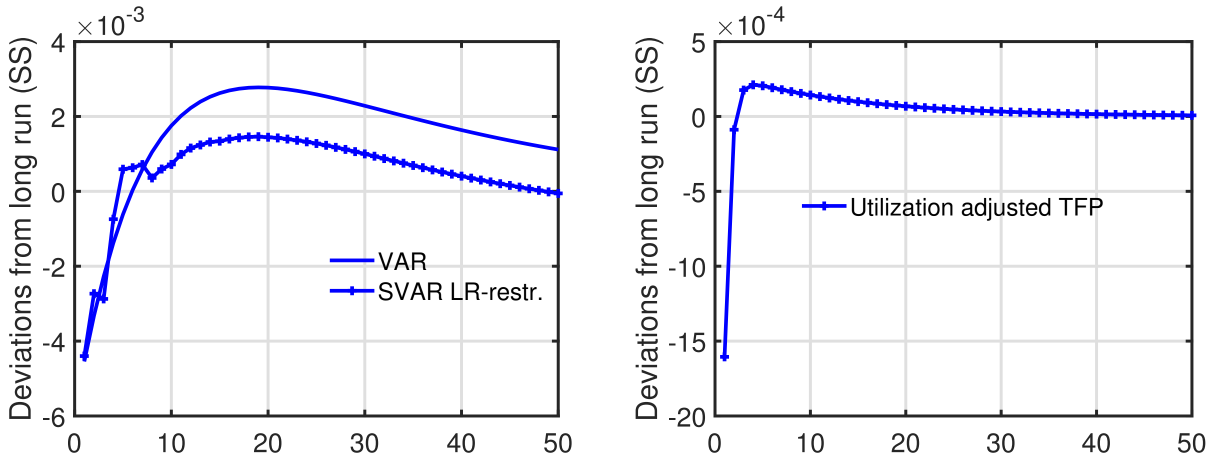


Figure 8: Impulse response functions of labor share in US data: structural vector autoregression with long-run restrictions (left) and VAR using Fernald's utilization TFP series. (right).