Online Appendix

Appendix A. Data Construction

Appendix A.1. Raw Data Series

All raw data series retrieved from the Bureau of Economic Analysis (BEA; www.bea.gov) and the Bureau of Labor Statistics (BLS; www.bls.gov) for the period 1948:Q1–2006:Q4 were current as of April 19, 2007.

National Income and Product Accounts (NIPA-BEA)

1. Table 1.1.5: Consumption of Durable Goods (CD_t), Change in Inventories (ChInv_t)
2. Table 1.7.5: Gross National Product (GNP_t)
3. Tables 2.3.3 and 2.3.5: Quantity Index (QCONS_t^i) and Nominal (CONS_t^i) Nondurables Consumption (excluding Energy) and Services (excluding Housing)[14]
4. Table 3.9.5: Government Investment in Equipment (GovIEQ_t), Government Investment in Structures (GovIST_t)
5. Table 5.3.5: Private Fixed Investment in Equipment ($\mathrm{PrivIEQ}_t$), Private Fixed Investment in Structures ($\mathrm{PrivIST}_t$)

Fixed Asset Tables (FAT-BEA)

1. Table 5.3.4: Official Price Index for Investment in Equipment (OPIEQ_t)

Bureau of Labor Statistics (BLS)

1. Aggregate Hours Index (H_t), BLS ID PRS85006033
2. Civilian Noninstitutional Population +16 ($Pop16_t$), BLS ID LNU00000000

Cummins and Violante (2002), 1947–2000

1. Annual Quality-Adjusted Price Index for Investment in Equipment ($\mathrm{QAPIEQ}_{year}^{CV}$)
2. Annual Quality-Adjusted Depreciation Rates for Total Capital (δ_{year}^{CV})

[14] Goods i correspond to nondurables consumption in food, clothing and shoes, and others, and services in household operations, transportation, medical care, recreation, and others.

Capital Utilization Data

The data is available from the Statistics & Historical Data page for Principle Economic Indicators – Industrial Production and Capacity Utilization (G.17) at *www.federalreserve.gov/econresdata/releases/statisticsdata.htm*.

1. Electric Power Use: Manufacturing and Mining, Total industry from the survey of industrial electric power conducted by the Board of Governors of the Federal Reserve System. The voluntary survey was discontinued with the publication on December 15, 2005, of data for October 2005 since the response rate dropped significantly during the early 2000s.
2. TCU: capacity utilization: total industry provided by the Board of Governors of the Federal Reserve System. The Federal Reserve Board constructs estimates of capacity utilization for a given industry by dividing an output index by a capacity index. Capacity indexes try to capture the so-called sustainable maximum output, that is, the largest level of output that a plant can achieve given the resources available when operating the plant. Capacity indexes are constructed for 89 detailed industries (71 in manufacturing, 16 in mining, and 2 in utilities), which mostly correspond to industries at the three- and four-digit NAICS level. In the estimation exercise, we use quarterly averages of the monthly series on the percent capacity. The data are available from 1967:1.

Appendix A.2. The Relative Price of Quality-Adjusted Investment

We construct the relative price of quality-adjusted investment, P_t^I, as a Tornquist aggregate of the price index of quality-adjusted equipment investment and the price index of structures investment. We use the price index of consumption, P_t^C, as a proxy for the price of structures investment.[15] Based on P_t^I and P_t^C, we define the relative price of investment goods (using the consumption good as numeraire) as

$$P_t = \frac{P_t^I}{P_t^C}.$$

[15] As is the standard in previous literature, we use the consumption deflator as the price index for investment in structures (see Fisher (2006) and Canova et al. (2010)). This provides internal consistency in the way we compute the quality-adjusted price index for total (equipment + investment) investment —one of the elements of output is investment; hence, the alternative use of an output (instead of a consumption) deflator potentially distorts the very same measure we are trying to compute: an investment deflator.

Its inverse, $V_t = \frac{1}{P_t}$, is investment-specific technical change. We set $V_0 = \frac{1}{P_0} = 1$, that is, we assume real capital is equal to capital in efficiency units in 1947.

Quarterly Quality-Adjusted Price Index for Investment in Equipment, QAPIEQ$_t$. We use the U.S. 1947-2000 annual series provided by Cummins and Violante (2002) for the price index of equipment investment, $\text{QAPIEQ}^{CV}_{year}$, and impute the quarterly movements of the official FAT-BEA price index of equipment investment, OPIEQ_t, using the Denton method. For the years after 2000, we use the official price index OPIEQ_t, rescaled such that it equates the value in Cummins and Violante (2002) in the year 2000. Thus, we assume that the hedonic methods used to compute the official price index correctly quality-adjust most types of equipment investment after 2000.

Quarterly Quality-Adjusted Price Index for Total Investment, P^I_t. We use a Tornquist price index aggregate that weights growth rates of the price index of investment in equipment and the price index of investment in structures by their nominal shares s^{IEQ}_t and s^{IST}_t. Nominal equipment investment is the sum of private equipment investment (PrivIEQ_t), government equipment investment (GovIEQ_t), changes in inventories (ChInv_t), and consumer durables (CD_t). Nominal structures investment is the sum of private structures investment (PrivIST_t) and government structures investment (GovIST_t). The growth rate of the quarterly quality-adjusted price index for total investment is

$$\lambda(P^I_t) = \left(\frac{s^{IEQ}_t + s^{IEQ}_{t-1}}{2}\right)\lambda(\text{QAPIEQ}_t) \; + \; \left(\frac{s^{IST}_t + s^{IST}_{t-1}}{2}\right)\lambda(P^C_t),$$

where $\lambda(x_t) = (x_t - x_{t-1})/x_t$ and changes in the price index for consumption goods, $\lambda(P^C_t)$, serve as proxy for inflation in the price of structures. The level of quarterly quality-adjusted price index for total investment is recovered recursively,

$$P^I_t = P^I_{t-1}[1 + \lambda(P^I_t)].$$

We use the initial value P^I_0 suggested in Cummins and Violante (2002).

Quarterly Price Index for Consumption, P^C_t. We use a Tornquist price index aggregate that weights growth rates of price indexes for nondurables consumption (food, clothing and shoes, and others) and services (household operations, transportation, medical care, recreation, and others) by their nominal shares. Let $P^{C,i}_t$ be the price index for nondurable consumption/service good i in quarter t computed as the ratio between nominal consumption of good i, CONS_t^i, and the quantity index of good i, QCONS_t^i. Let s_t^i be the corresponding nominal share of good i in period t. Then, the growth rate of the price index for consumption is

$$\lambda(P_t^C) = \sum_i \frac{s_t^i + s_{t-1}^i}{2} \lambda(P_t^{C,i}).$$

The level of the consumption price index is recovered recursively,

$$P_t^C = P_{t-1}^C \left[1 + \lambda(P_t^C)\right],$$

where we set P_0^C such that the initial relative price of investment is equal to one; see below.

Appendix A.3. Neutral Technical Change

The series of neutral technical change is computed using measures of real output Y_t, real capital K_t, and labor input H_t, together with an estimate of the input shares of production. Real output Y_t is computed as the nominal gross national product, GNP_t, deflated by P_t. We convert output, capital, and hours in per capita terms dividing by civilian noninstitutional population $Pop16_t$. We explicitly consider capital quality improvement represented by the historical fall in the real price of investment. To do so, we build quarterly series for investment in efficiency units and physical depreciation rates that we use to construct series of quality-adjusted capital stock. Quality adjustments substantially change the series of capital — real capital falls below capital in efficiency units and affects the trend of neutral technical change.

Quarterly Quality-Adjusted Investment, X_t. Total investment in efficiency units is defined as total deannualized nominal investment deflated by the quality-adjusted price of investment,

$$X_t = \frac{\text{InvEQ}_t + \text{InvST}_t}{P_t^I}.$$

Quarterly Quality-Adjusted Depreciation Rates, δ_t. We build on the time-varying annual physical depreciation rates for total capital provided in Cummins and Violante (2002) for the period 1947-2000, δ_{year}^{CV}. For the years after 2000, we assume a constant depreciation rate equal to that in year 2000. We define δ_0 as the average quarterly depreciation rate over the period 1955:Q3 to 2006:Q4: $\delta_0 = 0.013$.

Quarterly Quality-Adjusted Capital Stock, K_t. We have created quarterly quality-adjusted investment series, X_t, and quarterly series for the quality-adjusted depreciation rate, δ_t. Then we can construct the series of capital in efficiency units recursively using the perpetual inventory method,

$$K_{t+1} = (1 - \delta_0)\ K_t + X_t$$

where the initial capital stock in efficiency units, K_0, is calibrated using the steady-state investment equation

$$\frac{K_0}{Y_0} = \frac{V_0\ I_0}{Y_0} \left(1 - (1 - \delta_0) \exp\left(-\lambda_K\right)\right)^{-1}.$$

We obtain the unconditional mean of the investment-output ratio is 0.284, and the quarterly capital per capita growth rate averages 1.08%. This yields an initial quarterly capital-output ratio of 11.6 (or 2.92 annually), which together with the initial value of real output pins down an initial efficient capital stock.

Neutral Technical Change, A_t. The series of neutral technical change is computed as

$$A_t = \frac{Y_t}{K_t^{\alpha}\ H_t^{1-\alpha}},$$

where $\alpha = \sum_t \frac{\alpha_t}{T}$ is the baseline capital share in Ríos-Rull and Santaeulàlia-Llopis (2010).

Appendix B. The Model

In terms of the transformed variables, the deterministic steady state of our model is characterized by the following set of equations:

$$\begin{aligned}
q^* &= e^{\frac{1}{1-\alpha}\gamma_a + \frac{\alpha}{1-\alpha}\gamma_v} && \text{(B.1)} \\
v^* &= e^{\gamma_v} \\
R^* &= \frac{q^* v^*}{\beta} - (1 - \delta_0) \\
\frac{K^*}{Y^*} &= \frac{\alpha q^* v^*}{R^*} \\
\frac{X^*}{Y^*} &= \left(1 - \frac{1 - \delta_0}{q^* v^*}\right) \frac{K^*}{Y^*} \\
I^* &= X^* \\
\frac{I^*}{K^*} &= 1 - \frac{1 - \delta_0}{q^* v^*} \\
\frac{C^*}{Y^*} &= \frac{1}{g_*} - \frac{I^*}{Y^*}
\end{aligned}$$

For the technology shock processes, let $\hat{A}_t = \ln A_t - \ln A_0 - \gamma_a t$ and $\hat{V}_t = \ln V_t - \ln V_0 - \gamma_v t$. For other variables X_t, let $\hat{x}_t = ln(X_t/X^*)$. Then the log-linearized equilibrium conditions are given by (we scale the labor supply shock $\ln B_t$ by the factor $-\nu$ such that $\hat{b}_t = -\nu \ln B_t$):

$$\begin{aligned}
\hat{r}_t &= \hat{y}_t - (\hat{k}_t + \hat{u}_t) + \frac{1}{1-\alpha}(\hat{a}_t + \hat{v}_t) \\
\hat{w}_t &= \hat{y}_t - \hat{h}_t \\
\hat{c}_t &= E_t[\hat{c}_{t+1}] - \frac{R^*}{R^* + 1 - \delta_0} E_t[\hat{r}_{t+1}] + \frac{1}{1-\alpha} E_t[\hat{a}_{t+1} + \hat{v}_{t+1}] \\
\hat{h}_t &= \nu(\hat{w}_t - \hat{c}_t) + \hat{b}_t \\
\hat{y}_t &= g_* \frac{C^*}{Y^*}\hat{c}_t + g_* \frac{I^*}{Y^*}\hat{i}_t + \hat{g}_t \\
\hat{y}_t &= (1-\alpha)\hat{h}_t + \alpha(\hat{k}_t + \hat{u}_t) - \frac{\alpha}{1-\alpha}(\hat{a}_t + \hat{v}_t) \\
\hat{k}_{t+1} &= \left(1 - \frac{I^*}{K^*}\right)(\hat{k}_t - R^*\hat{u}_t) + \frac{I^*}{K^*}\hat{i}_t - \frac{1 - I^*/K^*}{1-\alpha}(\hat{a}_t + \hat{v}_t) \\
\hat{u}_t &= \zeta \hat{r}_t.
\end{aligned} \tag{B.2}$$

Notice that $\delta'(u_t) = \delta_1(1 + 1/\zeta)u_t^{1/\zeta}$ and $\delta''(u_t) = \delta_1(1 + 1/\zeta)(1/\zeta)u_t^{1/\zeta - 1}$. Since in steady state $R^* = \delta'(u^*)$, we deduce that $\delta''(u^*) = R^*/\zeta$, which delivers the last equation. The exogenous shock processes evolve according to

$$\begin{aligned}
\hat{a}_t &= \hat{A}_t - \hat{A}_{t-1} \\
\hat{v}_t &= \hat{V}_t - \hat{V}_{t-1} \\
\hat{A}_t &= \psi_{1,a}(1 - \psi_{2,a})\hat{A}_{t-1} + \psi_{2,a}\hat{A}_{t-2} + \sigma_a \epsilon_{a,t} \\
\hat{V}_t &= \psi_{1,v}(1 - \psi_{2,v})\hat{V}_{t-1} + \psi_{2,v}\hat{V}_{t-2} + \sigma_v \epsilon_{v,t} \\
\hat{b}_t &= \rho_b \hat{b}_{t-1} + \sigma_b \epsilon_{b,t} \\
\hat{g}_t &= \rho_g \hat{g}_{t-1} + \sigma_g \epsilon_{g,t}.
\end{aligned} \tag{B.3}$$

For the likelihood-based estimation of the technology shock processes and the complete DSGE models, we use the Kalman filter. Since $\hat{A}_t$ and $\hat{V}_t$ are potentially non-stationary, we initialize the filter by assuming that all hat-variables are equal to zero in period $t = -20$, where $t = 1$ corresponds to the first observation in our sample. In order to allow a marginal data density comparison between the DSGE model and the VAR, the estimation in Section 4 is based on the likelihood function that conditions on the first four sample observations ($t = 1, \ldots, 4$). The variable utilization model is estimated based on the unconditional likelihood function. Parameter estimates for our benchmark specification are tabulated in Table A-2.

Appendix C. Bayesian Estimation of Technology Shock Processes

As discussed in Section 2, the AR(2) shock processes are parameterized in terms of partial autocorrelations ψ_1 and ψ_2; see (7). These processes are trend stationary if $-1 < \psi_1, \psi_2 < 1$ and become difference stationary if $\psi_1 = 1$. We estimate the parameters subject to $0 \leq \psi_1 < 1$ (deterministic trend) and $\psi_1 = 1$ (stochastic trend). In the former case, we assume that the first-order partial autocorrelation has a Beta distribution with mean 0.95 and standard deviation of 0.02. For both the difference-stationary and trend-stationary specification, it is assumed that the second-order partial autocorrelation is uniformly distributed on the interval $(-1, 1)$. Our priors are fairly agnostic with respect to the average growth rate of the technology processes and the location parameters $\ln A_0$ and $\ln V_0$, which determine the log levels of the technology disturbances. The priors for the innovation standard deviations are centered at 1% with a large variance. A summary is provided in the first five columns of Table A-1.

We estimate the two AR(2) processes independently, based on a sample that ranges from 1955:Q3 to 2006:Q4, conditioning on observations from 1954:Q3 to 1955:Q2. Posterior means and 90% probability intervals are reported in the last four columns of Table A-1.

Appendix D. Impulse Response to a Technology Shock

We will show that the impulse response function of labor productivity and hours worked suffices to identify the labor supply elasticity. It is apparent from (B.2) that the two technology shocks enter the system in an identical manner, at least as far as detrended output, consumption, wages, hours, capital, and the rental rate of capital are concerned. Hence, without loss of generality we will focus on the response to an investment-specific technology shock. We will assume that $\psi_{1,v} = 1$ and define $\tilde{v}_t = \hat{v}_t/(1-\alpha)$ and omit the hats from all other variables. Thus, the impulse response function has to satisfy the following equilibrium conditions:

$$\begin{aligned} r_t &= y_t - (k_t + u_t) + \tilde{v}_t \\ w_t &= y_t - h_t \\ c_t &= E_t[c_{t+1}] - r^* E_t[r_{t+1}] + E_t[\tilde{v}_{t+1}] \\ h_t &= \nu(w_t - c_t) \\ y_t &= s_c c_t + s_i i_t \\ y_t &= (1-\alpha)h_t + \alpha(k_t + u_t) - \alpha\tilde{v}_t \\ k_{t+1} &= (1-\delta^*)(k_t - R^* u_t) + \delta^* i_t - (1-\delta^*)\tilde{v}_t \\ u_t &= \zeta r_t \\ \tilde{v}_t &= -\psi_{2,v}\tilde{v}_{t-1} + \frac{\sigma_v}{1-\alpha}\epsilon_{v,t}, \end{aligned} \tag{D.1}$$

where $r^* = R^*/(R^* + 1 - \delta_0)$, $s_c = g_* C^*/Y^*$, $s_i = g_* I^*/Y^*$, and $\delta^* = 1 - (1-\delta_0)/(q^* v^*)$. To construct the impulse response function, we assume that the system is in its steady state prior to $t = 1$, that $\epsilon_{v,1} = 1$, and $\epsilon_{v,t} = 0$ for $t > 1$. Thus, the time-path of the technology growth process is given by

$$\tilde{v}_t = (-\psi_{2,v})^{t-1}\frac{\sigma_v}{1-\alpha}, \quad E_t[\tilde{v}_{t+1}] = \tilde{v}_t. \tag{D.2}$$

After period 1 there is perfect foresight along the impulse response, and for any variable x_t it is the case that $E_t[x_{t+1}] = x_{t+1}$. With this in mind, we write the system for $t > 1$ as

$$\begin{aligned} w_t &= y_t - h_t \\ \Delta c_{t+1} &= r^* y_{t+1} - r^*(k_{t+1} + u_{t+1} - \tilde{v}_{t+1}) - \tilde{v}_{t+1} \\ h_t &= \nu(w_t - c_t) \\ y_t &= s_c c_t + s_i i_t \\ w_t &= \alpha(k_t + u_t - \tilde{v}_t) - \alpha h_t \\ k_{t+1} &= (1-\delta^*)(k_t - R^* \zeta r_t) + \delta^* i_t - (1-\delta^*)\tilde{v}_t \\ r_t &= \frac{1}{1+\zeta}(y_t - k_t + \tilde{v}_t). \end{aligned} \tag{D.3}$$

The Frisch elasticity can be obtained from the response function of wages, i.e., labor productivity, and hours worked, because it has to satisfy

$$\Delta h_{t+1} = \nu(\Delta w_{t+1} - \Delta c_{t+1}). \tag{D.4}$$

While we do not use direct information on consumption in our empirical analysis, we can deduce from (D.3) that

$$\begin{aligned} \Delta c_{t+1} &= r^* y_{t+1} - r^*(k_{t+1} + u_{t+1} - \tilde{v}_{t+1}) - \tilde{v}_{t+1} \\ &= r^*(w_{t+1} + h_{t+1}) - r^*(\alpha^{-1} w_{t+1} + h_{t+1}) - \tilde{v}_{t+1} \\ &= r^*(1 - \alpha^{-1}) w_{t+1} - \tilde{v}_{t+1}. \end{aligned}$$

Thus, for $t > 1$ the impulse response function of wages and hours needs to satisfy

$$\Delta h_{t+1} = \nu \left[\Delta w_{t+1} - r^*(1-\alpha^{-1}) w_{t+1} + (-\psi_{2,v})^t \frac{\sigma_v}{1-\alpha} \right]. \tag{D.5}$$

Since r^*, α, $\psi_{2,v}$, and σ_v can be identified independently from information other than that contained in the impulse response function of hours and wages to a technology shock, we deduce that ν is identifiable as long as the initial response of hours worked to a technology shock is non-zero. Moreover, ν remains identifiable in the presence of variable capital utilization $\zeta > 0$.

Appendix E. Further Results

Table A-3 reports the full set of parameter estimates for the highest posterior probability specifications based on the data sets *Y/H, H, P*, *Y/H, H, X*, and *Y/H, H, P, X*.

Appendix F. Variable Capital Utilization

Calibration: To implement the indirect inference procedure to calibrate $\theta_{(a)} = [\psi_{1,a}, \psi_{2,a}, \sigma_a]'$, we need to construct the model implied measured TFP, which is given by

$$A_t^m = \frac{Y_t}{(K_t^m)^\alpha H_t^{1-\alpha}}, \tag{F.1}$$

where K_t^m stands for measured capital, that is, the economy's capital stock when abstracting from utilization dependent depreciation rates. Measured capital stock evolves as

$$K_{t+1}^m = (1-\delta) K_t^m + X_t. \tag{F.2}$$

We add equations (F.1)-(F.2) to the equilibrium conditions of our model.

The indirect inference procedure to calibrate $\theta_{(a)}$ can be described by the following steps:

1. Given $\theta_{(-a)} = [\alpha, \beta, \delta_0, \nu, \zeta, \psi_{1,v}, \psi_{2,v}, \sigma_v]'$, pick $\theta_{(a)}^s \in \mathcal{T}_{(a)}$, where $\mathcal{T}_{(a)}$ is a grid with 50,000 triplets defined as follows:

   $$[0.25, 0.265, \ldots, 1.00] \otimes [-0.2, -0.192, \ldots, 0.2] \otimes [0.001, 0.0015, \ldots, 0.01]$$

   Thus, $s = 1, \ldots, 50,000$.
2. Simulate the model 10,000 periods[16] setting $\ln A_0$ and γ_a to zero.
3. Fit an AR(2) model to the model implied measured TFP

   $$\ln A_t^m = \rho_{1,a}^m \ln A_{t-1}^m + \rho_{2,a}^m \ln A_{t-2}^m + \sigma_a^m \epsilon_t^m \tag{F.3}$$

   and estimate it using least squares. Given the sample size, we do not have to worry about the small sample effects of OLS.
4. Convert the least squares estimates of $\rho_{1,a}^m$, $\rho_{2,a}^m$, and σ_a^m into $\theta_{(a)}^m$.
5. Evaluate the discrepancy function $Q(\theta_{(a)}; \theta_{(-a)})$ at $\theta_{(a)}^m$. The discrepancy function is defined as

   $$Q(\theta_{(a)}; \theta_{(-a)}) = [\bar{\theta}_{(a),D}^m - \theta_{(a),S}^m(\theta_{(a)}, \theta_{(-a)})]' \bar{V}_{(a)}^{-1} [\bar{\theta}_{(a),D}^m - \theta_{(a),S}^m(\theta_{(a)}, \theta_{(-a)})].$$

   We used the additional subscripts D and S to denote estimates computed based on the actual and simulated data, respectively. In fact, $\bar{\theta}_{(a),D}^m$ corresponds to the posterior means reported in Table A-1, and $\bar{V}_{(a)}$ is the posterior covariance matrix.

[16] We simulate the model economy for 10,200 periods and discard the first 200.

6. If $s < 50,000$, go to step 1. Otherwise, compute

$$\hat{\theta}_{(a)} = \operatorname{argmin}_{\theta_{(a)} \in \mathcal{T}_{(a)}} Q(\theta_{(a)}; \theta_{(-a)}).$$

Table A-1: PRIORS AND POSTERIORS FOR TECHNOLOGY SHOCK PARAMETERS

		Prior Distribution			Deterministic Trend		Stochastic Trend	
Name	Domain	Density	Para (1)	Para (2)	Mean	90 % Intv.	Mean	90 % Intv.
γ_a	$I\!R$	Normal	0.00	0.10	-.001	[-.002, .000]	.000	[-.001, .001]
$\psi_{1,a}$	$[0, 1)$	Beta	0.95	0.02	0.97	[0.96, 0.99]	1.00	
$\psi_{2,a}$	$(-1, 1)$	Uniform	-1.0	1.00	-0.03	[-0.15, 0.09]	-0.06	[-0.19, 0.05]
σ_a	$I\!R^+$	InvGamma	0.01	4.00	.007	[.006, .008]	.007	[.006, .008]
γ_v	$I\!R$	Normal	0.00	0.10	.008	[.007, .008]	.007	[.005, .009]
$\psi_{1,v}$	$[0, 1)$	Beta	0.95	0.02	0.99	[0.99, 1.00]	1.00	
$\psi_{2,v}$	$(-1, 1)$	Uniform	-1.0	1.00	-0.76	[-0.84, -0.69]	-0.81	[-0.90, -0.73]
σ_v	$I\!R^+$	InvGamma	0.01	4.00	.003	[.003, .004]	.003	[.003, .004]
$\ln A_0$	$I\!R$	Normal	0.00	100	4.84	[4.74, 4.95]	-2.66	[-97.4, 76.5]
$\ln V_0$	$I\!R$	Normal	0.00	100	-0.14	[-0.24, -0.06]	-0.85	[-79.9, 86.1]

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; and s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. The last four columns contain posterior means and 90% credible intervals. To estimate the stochastic trend version of the model, we set $\psi_{1,a} = \psi_{1,v} = 1$.

Table A-2: Posterior Estimates for Benchmark Specification

Series	Y/H, H, P	
Shocks	A, V, B	
Unit Root	No	
α	0.361	[0.326, 0.395]
ν	0.852	[0.344, 1.326]
γ_a	-0.002	[-0.004, 0.000]
$\psi_{1,a}$	0.983	[0.973, 0.993]
$\psi_{2,a}$	-0.103	[-0.242, 0.038]
σ_a	0.007	[0.007, 0.008]
γ_v	0.007	[0.007, 0.008]
$\psi_{1,v}$	0.990	[0.986, 0.993]
$\psi_{2,v}$	-0.714	[-0.796, -0.632]
σ_v	0.003	[0.003, 0.004]
ρ_b	0.968	[0.952, 0.986]
σ_b	0.012	[0.010, 0.014]
$\ln H^*$	-0.037	[-0.072, -0.004]
$\ln Y_0$	9.137	[8.628, 9.692]
$\ln V_0$	-0.095	[-0.153, -0.037]

Note: The following parameters are fixed during the estimation: $\beta = 0.99$ and $\delta_0 = 0.013$.

Table A-3: Posterior Estimates for Highest Posterior Probability Specifications

Series	Y/H, H, P		Y/H, H, X		Y/H, H, P, X	
Shocks	A, V, G		A, V, B, G		A, V, B, G	
Unit Root	Yes		Yes		No	
α	0.340	[0.306, 0.374]	0.323	[0.292, 0.353]	0.391	[0.381, 0.402]
ν	0.419	[0.168, 0.643]	0.964	[0.328, 1.539]	0.170	[0.048, 0.284]
γ_a	0.000	[-0.001, 0.001]	0.000	[-0.001, 0.001]	-0.001	[-0.001, -0.001]
$\psi_{1,a}$	1.000		1.000		0.950	[0.931, 0.968]
$\psi_{2,a}$	-0.020	[-0.148, 0.120]	-0.006	[-0.089, 0.073]	-0.088	[-0.203, 0.029]
σ_a	0.007	[0.006, 0.008]	0.007	[0.007, 0.008]	0.007	[0.007, 0.008]
γ_v	0.007	[0.006, 0.008]	0.007	[0.006, 0.008]	0.008	[0.007, 0.008]
$\psi_{1,v}$	1.000		1.000		0.991	[0.988, 0.994]
$\psi_{2,v}$	-0.694	[-0.769, -0.620]	-0.059	[-0.140, 0.025]	-0.646	[-0.722, -0.570]
σ_v	0.003	[0.003, 0.004]	0.007	[0.006, 0.008]	0.003	[0.003, 0.004]
ρ_b			0.967	[0.951, 0.983]	0.953	[0.935, 0.970]
σ_b			0.011	[0.009, 0.013]	0.009	[0.008, 0.010]
ρ_g	0.962	[0.944, 0.982]	0.972	[0.952, 0.993]	0.963	[0.949, 0.978]
σ_g	0.038	[0.021, 0.056]	0.004	[0.003, 0.006]	0.010	[0.008, 0.011]
$\ln H^*$	-0.028	[-0.067, 0.009]	-0.024	[-0.064, 0.012]	-0.027	[-0.049, -0.004]
$\ln Y_0$	-32.284	[-49.319, -17.905]	8.377	[4.489, 12.753]	8.627	[8.548, 8.704]
$\ln V_0$	27.552	[17.036, 41.493]	-0.044	[-3.062, 2.724]	-0.148	[-0.229, -0.066]

Note: The following parameters are fixed during the estimation: $\beta = 0.99$, $\delta_0 = 0.013$, and $g^* = 1.2$ (in models with G-shock).