Methods versus Substance: Measuring the Effects of Technology Shocks☆

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Abstract

Calibration and modern (Bayesian) estimation methods for a neoclassical stochastic growth model are applied to make the case that the identification of key parameters, rather than quantitative methodologies per se, is responsible for empirical findings. For concreteness, the model is used to measure the contributions of technology shocks to the business cycle fluctuations of hours and output. Along the way, new insights are provided in the parameter identification associated with likelihood-based estimation, the sensitivity of likelihood-based estimation to the choice of structural shocks is assessed, and Bayesian model averaging is used to aggregate findings obtained from different DSGE model specifications.

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1. Introduction

Over the past three decades, quantitative macroeconomics has witnessed controversies about the role of formal econometric methods. While there is some broad consensus that the neoclassical stochastic growth model — potentially augmented by heterogeneity, various types of frictions, and different sources of idiosyncratic as well as aggregate uncertainty — provides a useful framework for substantive empirical work, there is less consensus on how such models should be parameterized in view of the available economic data. Following Kydland and Prescott (1982), many researchers calibrate dynamic stochastic general equilibrium (DSGE) models. Other researchers use formal econometric methods to parameterize DSGE models and study their quantitative implications. The methodological controversies between the calibration and the estimation camp are summarized, for instance, in a Journal of Economic Perspectives symposium with contributions by Hansen and Heckman (1996), Kydland and Prescott (1996), and Sims (1996).

Despite the use of similar theoretical frameworks, quantitative results for specific macroeconomic questions reported in the literature often vary dramatically. Some of this variation is seemingly associated with the choice of empirical methods. For instance, estimates of the relative importance of technology shocks for fluctuations in hours worked range from less than 10% to more than 65%. Similar figures for output can be as low as 20% or as high as 100%. Among the numerous papers published on this topic, Hansen (1985)'s calibration of a stochastic growth model with indivisible labor yielded numbers at the high end, whereas the likelihood-based estimation of a DSGE model with New Keynesian frictions by Galí and Rabanal (2005) produced estimates at the low end.

In this paper, we argue that it is not the choice of quantitative methodology (per se) that is responsible for empirical findings, but rather the identification of key model parameters associated with the methodology. To present the argument we consider a specific, illustrative empirical application throughout this paper. The neoclassical stochastic growth model with neutral and investment-specific technology shocks is used to measure the contribution of the two types of technology shocks to business cycle movements of aggregate hours worked and output. We choose this application because it allows us to focus much of the exposition on one key parameter: the aggregate labor supply elasticity. As is well known, the variability of hours and output in this model is very sensitive to the labor supply elasticity. Moreover, the magnitude of this elasticity has remained controversial with values in the literature ranging from close to zero to infinity. Generalizations to applications that involve more elaborate DSGE models, including one with variable capital utilization, are discussed toward the end of the paper.

Both calibration and estimation are concerned with mapping observations into parameter values for DSGE models. Since much of the paper is about the identification of key parameters, we begin by providing a definition of identification that can be applied to both calibration and estimation approaches. To a good approximation, parameters are determined by minimizing the discrepancy between empirical moments and their model-implied counterparts, and the choice of objective function determines the parameter identification. The starting point of our empirical illustration is a fairly standard calibration objective function as well as a widely used likelihood function to construct an econometric estimate. While calibration approaches tend to use external information to identify the labor supply elasticity, e.g., micro-level elasticities estimated from household-level data, estimation approaches, in particular ones that are based on a likelihood function, tend to extract information directly from the joint dynamics of output and hours worked.

In calibration approaches, the connection between identification of key parameters and choice of objective function tends to be transparent. However, the researcher often has to choose among a large variety of target values. In addition to the baseline calibration of the labor supply elasticity, we consider various alternatives in the empirical illustration: alternative micro-level elasticity estimates, information about the composition of hours worked fluctuations into intensive and extensive margins, and balanced growth path restrictions. In a calibration framework, the ambiguity caused by alternative identification approaches is typically communicated by reporting quantitative findings associated with the various plausible identification schemes.

In econometric approaches, in particular those that rely on autocovariance features of model and data, the identification of key parameters is less transparent because the model-implied autocovariances are complicated nonlinear functions of the underlying model parameters. However, in our application we can shed some light on the identification. The DSGE model incorporates enough restrictions to identify technology shock innovations from the observables. We show how the relative response of labor productivity and hours to technology shocks, that is, prices and quantities in the labor market, can identify the labor supply elasticity. In autocovariance-based estimation approaches, parameter identification is sensitive to the choice of observables and the assumptions about the probabilistic structure of the shocks driving the model economy. We explore this sensitivity in our empirical illustration. The relative fit of the various model specifications associated with different identification schemes can be used to weight different quantitative results.

While making the case that sources of identification, and not a controversy over the use of formal statistical methods, should be at the center of the debate in quantitative macroeconomics, we make several distinct contribution. First, we provide an economic interpretation of the way the likelihood function extracts information about important model parameters. Second, we document the sensitivity of likelihood-based estimates to the inclusion of additional shocks that complete the probabilistic structure of the DSGE model and implicitly affect the identification of key parameters. The tacit identification implications of auxiliary shocks have been a neglected topic and are likely to be very important for large-scale DSGE models that build on the work by Smets and Wouters (2003, 2007). Third, we use Bayesian model averaging to aggregate quantitative results from the estimation of DSGE models with different auxiliary shock specifications.

With the exception of the use of Bayesian model averaging, we do not discuss the problem of model evaluation in the context of calibration and estimation approaches in this paper. While model evaluation is an important problem, it is not essential to our main points about identification. Traditionally, calibrated DSGE models have been evaluated based on their ability to replicate stylized facts of business cycle dynamics, e.g., certain correlations among macroeconomic time series that have not been used to determine the parameters of the model. This evaluation is based on a weaker notion of model fit than the widely used notion in the econometrics literature of the models' ability to track and forecast macroeconomic time series. For detailed expositions of how model evaluation approaches under the calibration paradigm relate to econometric approaches, we refer the reader to Schorfheide (2000) and Geweke (2010) for a Bayesian perspective, and to Watson (1993) and Dridi et al. (2007) for a frequentist perspective. These four papers provide further references to the literature.

Starting with Canova and Sala (2009), the topic of identification has recently received particular attention in the DSGE model literature, as it has become apparent that estimation objective functions are often not very informative about important model parameters. Guerron-Quintana (2010) documents how parameter estimates of a variant of the Smets-Wouters model are affected by the choice of observables used in the Bayesian estimation. Iskrev (2010) and Komunjer and Ng (2011) develop formal conditions for the identifiability of DSGE model parameters based on first and second moments of model variables. The focus of our paper is to dissect the identification assumptions underlying typical calibration and likelihood-based estimation approaches.

The quantitative findings in the empirical application support our main methodological point. If the labor supply elasticity is set to 0.72 to match recent micro-level evidence provided by Heathcote et al. (2007), the fractions of hours and output fluctuations explained by technology shocks are 9% and 26%, respectively. If the elasticity is identified by the labor supply behavior along the intensive margin of prime-age white males, the fractions drop to 1% and 20%. If the elasticity is determined by a sufficient condition on preferences to guarantee a balanced growth path, then the fractions increase to 29% and 33%. The estimation can generate a similar range of results, depending on the choice of data set and the shock specification. If the DSGE model is estimated based on observations for labor productivity, hours worked, the relative price of investment, as well as investment, the labor supply elasticity is around 0.1, which translates into variance ratios of 1% for hours and 22% for output. If, on the other hand, the relative price of investment is excluded from the observables, and a government spending shock rather than a preference shock is used to complete the probabilistic structure of the model, then the labor supply elasticity estimate increases to 1.56, and the variance ratios rise to 22% for hours and 33% for output. Thus, within each empirical methodology, differences in the identification of key parameters can generate a wide spectrum of quantitative results. Therefore, macroeconomists should place more emphasis on searching for reliable sources of identification of key parameters and making them transparent to their audiences.

The remainder of this paper is organized as follows. The stochastic growth model is presented in Section 2. An overview of calibration and estimation methods is provided in Section 3. We begin the empirical analysis with a restricted version of the model in which capital utilization is constant. Section 4 reviews identification schemes for the labor supply elasticity that are associated with standard calibration procedures. Section 5 reports on the DSGE model estimation using state-of-the-art Bayesian techniques. In Section 6 we allow for variable capital utilization. Recommendations for practitioners that generalize to other models and applications are provided in Section 7. Section 8 concludes. An Online Appendix provides detailed information on the data set, the implementation of the empirical analysis, and parameter estimates.

2. The Model Economy

Following Greenwood et al. (2000) and Fisher (2006), we consider a stochastic growth model with a neutral technology shock, an investment-specific technology shock, and variable capital utilization. The model economy is populated with a continuum of households with the following preferences:

$$\max_{\{C_t, X_t, H_t, K_{t+1}\}} \quad I\!E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - B_t \xi \, \frac{H_t^{1+1/\nu}}{1+1/\nu} \right) \right]. \tag{1}$$

Here, C_t denotes consumption and H_t hours worked. An appealing feature of these preferences is that the parameter ν is the Frisch elasticity of substitution of labor. β is, as always, the discount rate. ξ affects the marginal rate of substitution between consumption and leisure and affects steady-state hours. B_t is an exogenous disturbance that shifts the preferences of the representative households and thereby contributes to business cycle fluctuations.

Households have access to a constant returns to scale production function given by

$$C_t + \frac{1}{V_t} X_t = A_t (u_t K_t)^{\alpha} H_t^{1-\alpha}. \tag{2}$$

The left-hand side of (2) can be interpreted as a linear transformation curve between consumption and investment goods. The slope of this curve is shifted by the investment-specific technology disturbance V_t. The right-hand side takes the standard Cobb-Douglas form, A_t is an exogenous total factor productivity (or neutral) technology process, K_t is capital, and u_t is the capital utilization. Thus, the production technology is subject to two exogenous shocks: a standard neutral technology shock A_t and a sector-specific technology shock V_t. Capital depreciates geometrically at the utilization-dependent rate $\delta(u_t)$, yielding

$$K_{t+1} = (1 - \delta(u_t)) K_t + X_t. \tag{3}$$

Let R^* denote the steady-state rental rate of capital. We use a depreciation function of the form $\delta(u_t) = \delta_0 + \delta_1 (u_t^{1+1/\zeta} - 1)$ and assume that $\delta_1 = R^*/(1 + 1/\zeta)$ such that the steady-state utilization rate equals $u^* = 1$. If $\zeta = 0$, then our model reduces to one with fixed capital utilization, the case we discuss in Sections 4 and 5. The extension to variable capital utilization, $\zeta > 0$, is subsequently provided in Section 6.

Under competitive markets, the relative price of a unit of the investment good using the consumption good as numeraire, P_t, is equal to the reciprocal of the technology shock, $1/V_t$. Define $I_t = P_t X_t$. Consumption goods $\mathcal{C}_t$ can be used for private consumption C_t or government consumption G_t. Hence, the aggregate resource constraint can be written as

$$Y_t = C_t + G_t + I_t = A_t(u_t K_t)^{\alpha} H_t^{1-\alpha}. \tag{4}$$

We assume that government expenditures are financed by lump-sum taxes and are determined exogenously as a time-varying fraction of total output, $G_t = (1 - 1/g_t)Y_t$. This specification leads to the relationship $\ln Y_t = \ln(C_t + I_t) + \ln g_t$ and implies that the government share of output is stationary. More generally, g_t can be interpreted as a demand disturbance. The static Euler equation using prices is given by $H_t = \left(\frac{1}{\xi}\frac{W_t}{C_t}\right)^{\nu}$, while the dynamic Euler equation takes the form $1 = \beta \mathbb{E}_t \left[\frac{P_{t+1}/C_{t+1}}{P_t/C_t}\Big((1-\delta(u_{t+1})) + u_{t+1}R_{t+1}\Big)\right]$. The optimal choice of the utilization rate is given by $\delta'(u_t) = R_t$. In turn, the equilibrium wage equals the marginal product of labor $W_t = (1-\alpha)A_t(u_t K_t)^{\alpha} H_t^{-\alpha}$, while the rate of return satisfies $R_t P_t = \alpha A_t (u_t K_t)^{\alpha-1} H_t^{1-\alpha}$.

We assume that the two technology processes fluctuate around linear deterministic trend paths, given by $\ln A_0 + \gamma_a t$ and $\ln V_0 + \gamma_v t$, respectively:

$$\begin{aligned}
(\ln A_t - \ln A_0 - \gamma_a t) &= \rho_{a,1}(\ln A_{t-1} - \ln A_0 - \gamma_a(t-1)) \qquad (5)\\
&\quad +\rho_{a,2}(\ln A_{t-2} - \ln A_0 - \gamma_a(t-2)) + \sigma_a \epsilon_{a,t}\\
(\ln V_t - \ln V_0 - \gamma_v t) &= \rho_{v,1}(\ln V_{t-1} - \ln V_0 - \gamma_v(t-1)) \qquad (6)\\
&\quad +\rho_{v,2}(\ln V_{t-2} - \ln V_0 - \gamma_v(t-2)) + \sigma_v \epsilon_{v,t}.
\end{aligned}$$

The most widely used specifications for the neutral technology process can be easily obtained as special cases of (5). If $0 \le \rho_{a,1} < 1$ and $\rho_{a,2} = 0$, then technology follows a stationary AR(1) process. If $\rho_{a,1} + \rho_{a,2} = 1$, then technology has a unit root and the serial correlation of its growth rates is $-\rho_{a,2}$, which is often assumed to be zero. In this case, the fluctuations are non-stationary and the technology processes can be rewritten as AR(1) processes in terms of growth rates. In order to restrict the autoregressive processes in (5) and (6) to trend stationarity, it is convenient to reparameterize them in terms of partial autocorrelations ψ_1 and ψ_2. Omitting the a and v subscripts, we let

$$\rho_1 = \psi_1(1-\psi_2), \quad \rho_2 = \psi_2. \tag{7}$$

In case of a unit root, $\psi_1 = 1$. The preference shock B_t and the government spending shock g_t are assumed to evolve according to independent AR(1) processes:

$$\ln B_t = \rho_b \ln B_{t-1} - (\sigma_b/\nu)\epsilon_{b,t} \tag{8}$$

$$\ln(g_t/g^*) = \rho_g \ln(g_{t-1}/g^*) + \sigma_g \epsilon_{g,t}. \tag{9}$$

The analysis in this paper is conducted under the assumption that the innovations $\epsilon_{a,t}$, $\epsilon_{v,t}$, $\epsilon_{b,t}$, and $\epsilon_{g,t}$ are normally distributed with zero mean and unit variance. Moreover, we assume that they are uncorrelated at all leads and lags. We will comment on this assumption later on.

Regardless of whether the technology shocks have a stochastic trend component (unit root) or are trend stationary, the following transformations generate stationary variables:

$$\frac{Y_t}{Q_t}, \quad \frac{C_t}{Q_t}, \quad \frac{I_t}{Q_t}, \quad \frac{X_t}{Q_t V_t}, \quad \frac{K_{t+1}}{Q_t V_t}, \quad \frac{W_t}{Q_t}, \quad \text{where} \quad Q_t = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}.$$

A lot of judgment is involved in the selection of the model economy to study. For instance, we restricted the production function to be Cobb-Douglas and the utility function to be logarithmic in consumption. These particular choices are based on balanced growth path considerations, that is, the fact that over the last century or so, output per capita has been growing at about 2% per year and throughout this period there has been no trend in either factor shares or interest rates, and these features are consistent with very few specifications of preferences and technology. In this sense, the choice of model is already based implicitly on some observations in a manner that is agreeable to researchers from the calibration and estimation camps.[3] While one could enrich the parameter space of the DSGE model to relax these functional form restrictions and start the calibration and estimation from a more elaborate model specification, but would not change the arguments presented in the remainder of this paper, it would only make the exposition more tedious.

[3] In fact, the choice of log utility in consumption is not irrelevant. It implies an intertemporal elasticity of substitution of consumption of 1, which is on the high end of what is typical. This means that households are relatively quite willing to have a non-smooth consumption path, which gives the model a good chance to have volatile hours worked.

3. Two Quantitative Methodologies

In the remainder of this paper, two prominent quantitative approaches in macroeconomics are compared, namely, calibration and estimation of DSGE models. At an abstract level, both approaches are concerned with mapping observations into parameter values. The comparison is presented as part of an empirical application in which the stochastic growth model of Section 2 is used to measure the importance of technology shocks for output and hours fluctuations. Before delving into the details of the application, we introduce some notation, provide a definition of identification that can be applied to calibration and estimation approaches, and provide an outline of our main argument.

Some Notation. The DSGE model described in Section 2 generates a probability distribution for a collection of observable macroeconomic variables:

$$\mathcal{Y} = \{Y, C, I, X, H, K, W, R, P\}.$$

From the definition of $\mathcal{Y}$ we omitted G, which can be obtained as $Y - C - I$, as well as the shocks A, V, and B because they are not directly observed. The probability distribution of $\mathcal{Y}$ is indexed by a $k \times 1$ vector of DSGE model parameters θ, which is given by

$$\theta \quad = \quad [\alpha, \beta, \delta_0, \xi, \ln A_0, \ln V_0, g^* \gamma_a, \psi_{a,1}, \psi_{a,2}, \sigma_a, \gamma_v, \psi_{v,1}, \psi_{v,2}, \sigma_v, \rho_b, \sigma_b, \rho_g, \sigma_g, \nu, \zeta]'.$$

We denote the density of $\mathcal{Y}$ conditional on θ by $p(\mathcal{Y}|\theta)$. If the data $\mathcal{Y}$ are held fixed and $p(\mathcal{Y}|\theta)$ is interpreted as a function of θ, then it is called the likelihood function.

Determining Parameter Values Based on Observed Data. Both calibration and estimation are concerned with determining a range of appropriate values for θ based on available data. The data may include some or all of the variables contained in $\mathcal{Y}$ as well as additional observations that are generically denoted by $\mathcal{X}$. In the context of our representative-agent model, $\mathcal{X}$ might comprise micro-level observations on hours worked and wages that are informative about the labor supply elasticity ν, or, more directly, of estimates of ν that have been computed based on micro-level observations by other researchers.

In the calibration literature, the mapping from observations into parameter values is either explicitly or implicitly defined as the solution of an extremum problem in which the researcher minimizes the discrepancy between an $l \times 1$ vector of targets $\tau(\mathcal{Y}^o, \mathcal{X}^o)$, computed from *observed* data, and their model analogues $m(\theta)$:

$$\hat{\theta}_c = \text{argmin}_{\theta \in \Theta}\, Q_c(\theta; \mathcal{Y}^o, \mathcal{X}^o), \quad \text{where} \quad Q_c(\theta; \mathcal{Y}^o, \mathcal{X}^o) = \|\tau(\mathcal{Y}^o, \mathcal{X}^o) - m(\theta)\|^2_\Omega, \tag{10}$$

where Ω in (10) is a symmetric positive-definite weight matrix and $\|x\|_\Omega$ denotes the norm $\sqrt{x'\Omega x}$. The elements of $\tau(\mathcal{Y}^o, \mathcal{X}^o)$ could include, for example, the average labor share WH/Y, the average investment-to-output ratio I/Y, and an estimate of a labor supply elasticity from micro-level data. In Section 4 we consider the case in which $l = k$ and one can recursively solve the system of equations $Q_c(\hat{\theta}_c; \mathcal{Y}^o, \mathcal{X}^o) = 0$ for the calibrated parameter values $\hat{\theta}_c$. In this case, the weight matrix Ω is irrelevant.

In many estimation approaches parameter values are also determined as an extremum of an objective function, which we denote by $Q_e(\theta; \mathcal{Y}^o, \mathcal{X}^o)$. Let $\hat{\theta}_e$ be the resulting estimator. This objective function could be identical to $Q_c(\theta; \mathcal{Y}^o, \mathcal{X}^o)$, could represent more general moment conditions in which $\tau(\mathcal{Y}^o, \mathcal{X}^o) - m(\theta)$ is replaced by a non-separable function $g(\mathcal{Y}^o, \mathcal{X}^o; \theta)$ as in Hansen (1982)'s generalized methods of moments (GMM) approach, or could be defined as the likelihood function $p(\mathcal{Y}^o_{(s)}|\theta)$ for a subset of model variables $\mathcal{Y}_{(s)}$. The omission of $\mathcal{X}^o$ from the likelihood function highlights that the likelihood function is based on variables that explicitly appear in the model.

Much of the methodological controversy between the estimation and calibration camps—see, for instance, Hansen and Heckman (1996), Kydland and Prescott (1996)—and Sims (1996), can be summarized as follows. Econometricians tend to use the probabilistic structure of the DSGE model to determine the choice of the objective function $Q_e(\theta; \mathcal{Y}^o, X^o)$ and to derive measures of uncertainty for θ. Calibrators view the probabilistic structure of the DSGE model as sufficiently misspecified as to state that the choice of objective function should be based more on judgment than on statistical considerations. Statistical measures of uncertainty are replaced by robustness exercises in which model implications are explored under different plausible parameterizations.

Identification of Parameters. We do not focus on answering whether or how to construct statistical measures of uncertainty based on the probabilistic structure of the DSGE model. Instead, our paper focuses on establishing how calibration and estimation objective functions affect the identification of key model parameters. We will make the argument that researchers, rather than resorting to default choices of $Q_c(\theta; \mathcal{Y}^o, \mathcal{X}^o)$ and $Q_e(\theta; \mathcal{Y}^o, \mathcal{X}^o)$, should construct these objective functions with parameter identification in mind. This requires a definition of identification that is applicable to both calibration and estimation approaches.

In the context of classical econometrics (see Rothenberg (1971)), two parameter values θ_1 and θ_2 are said to be observationally equivalent if $p(\mathcal{Y}|\theta_1) = p(\mathcal{Y}|\theta_2)$ for all $\mathcal{Y}$. A parameter value θ_0 is said to be identifiable if there is no other $\theta \in \Theta$ that is observationally equivalent. The classical definition is based on a pre-experimental perspective: the equality $p(\mathcal{Y}|\theta_1) = p(\mathcal{Y}|\theta_2)$ has to hold for all possible observations $\mathcal{Y}$ instead of just the actual observation $\mathcal{Y}^o$. Given that the stylized representation of the calibration objective function in (10) depends on observations $\mathcal{X}$ for which no sampling mechanism has been specified, the classical definition of identification is not practical for our purposes. Instead, we shall adopt a post-experimental notion of identification that conditions on the actual observations $\mathcal{Y}^o$ and $\mathcal{X}^o$.

To appreciate the subtle difference between the pre- and post-experimental perspectives, consider the following example. Suppose the neutral technology process in the DSGE model of Section 2 is replaced by a technology process with regime switches. Under Regime 1 the growth rate of technology is $\gamma_{a,1}$, and under Regime 2 it is $\gamma_{a,2}$. In addition, assume a researcher observes data only from Regime 1. From a pre-experimental perspective, Regime 2 could have been observed and hence $\gamma_{2,a}$ is identifiable. From a post-experimental perspective Regime 2 was not observed and the data provide no information about $\gamma_{2,a}$. We use a concept of identification— or, in other words, informativeness— that is based on the curvature of the objective function $Q_c(\theta; \mathcal{Y}^o, \mathcal{X}^o)$ with respect to θ conditional on the observations $\mathcal{Y}^o$ and $\mathcal{X}^o$. Informally, the data are not informative or the parameter is not identified if the objective function is flat in the vicinity of its extremum or, more generally, if the extremum is not unique. Accordingly, $\gamma_{a,2}$ in the regime-switching example is not identified. It should also be clear that identification is inherently tied to the choice of the objective function $Q_c(\theta; \mathcal{Y}^o, \mathcal{X}^o)$.

When discussing estimation approaches in the remainder of this paper, we focus on Bayesian inference. Bayesian inference updates a prior distribution for θ, represented by the density $p(\theta)$, in view of the observed data $\mathcal{Y}^o$ to obtain a conditional distribution (posterior) of θ given $\mathcal{Y}^o$, represented by the density $p(\theta|\mathcal{Y}^o)$. The posterior is obtained by applying Bayes' theorem: $p(\theta|\mathcal{Y}^o) \propto p(\mathcal{Y}^o|\theta)p(\theta)$, where $\propto$ denotes proportionality and $p(\mathcal{Y}^o|\theta)$ is the DSGE model-implied likelihood function. We use Bayesian estimation techniques for several reasons. First, Bayesian methods are widely employed in the literature and by now are the preferred method of inference in applications in which the researcher uses the full model structure rather than specific equilibrium conditions for parameter estimation. Second, Bayesian inference utilizes the likelihood function, which plays a central role in optimal inference for econometric models that are indexed by a finite-dimensional parameter vector. This allows us to contrast the information content of the likelihood function with the information content of prototypical calibration objective functions. Third, under the assumption that $p(\mathcal{Y}|\mathcal{X},\theta) \approx p(\mathcal{Y}|\theta)$, it is possible to incorporate external information through the prior distribution, which can be specified conditional on $\mathcal{X}$. Fourth, Bayesian inference is post-experimental and the relevant concept of identification is related to the curvature of the posterior $p(\theta|\mathcal{Y}^o,\mathcal{X}^o) \propto p(\mathcal{Y}^o|\theta)p(\theta|\mathcal{X}^o)$. This facilitates the comparison with the calibration approach.

In the remainder of the paper, we often refer to *sources* of identification. By this we mean the following. Suppose, for instance, that the calibration objective function (10) takes the additive form

$$Q_c(\theta;\mathcal{Y}^o,X^o) = \|\tau_1(\mathcal{Y}^o,X^o) - m_1(\theta_1)\|^2_{\Omega_1} + \|\tau_2(\mathcal{Y}^o,X^o) - m_2(\theta_2)\|^2_{\Omega_2}, \tag{11}$$

where we use the convention that vectors z are partitioned into a scalar z_1 and a vector z_2. For concreteness, assume that $\theta_1 = \alpha$ and $\tau_{(1)}(\mathcal{Y}^o,X^o)$ is the labor share of income WH/Y. Since $m_2(\cdot)$ does not depend on α, the sole source of identification for the parameter α in this example is the labor share. Calibration objective functions $Q_c(\theta;\mathcal{Y}^o,X^o)$ are often explicitly constructed (or, at least, discussed) with identification in mind by linking each parameter with particular observations. The fact that the objective function can include variables that do not explicitly appear in the DSGE model, e.g., micro-level wages and hours as opposed to aggregate wages and hours, leads to additional flexibility.

If estimation and calibration are based on the same objective function, then the sources of identification for each parameter are identical. However, econometricians rely more frequently on the dynamic properties of DSGE models to identify parameters. In particular, if the estimation is based on the likelihood function $p(\mathcal{Y}^o|\theta)$, then the estimator tries to match the model-implied autocovariance sequence for $\mathcal{Y}$ with the sample autocovariances of $\mathcal{Y}^o$. In this case, the sources of identification are less transparent because it is difficult to link particular autocovariance patterns with individual parameters. Moreover, the identification of model parameters becomes sensitive to misspecification of model dynamics. In much of Section 5, we examine how the aggregate labor supply elasticity of the stochastic growth model is identified in the likelihood-based estimation of the DSGE model.

Outline of the Main Argument. In Sections 4 and 5, we present a prototypical calibration and estimation of the stochastic growth model. For each method we highlight the source of identification of the key model parameters and illustrate how the source of identification affects the substantive conclusion about the importance of technology shocks for business cycle fluctuations of hours and output. We then modify the source of identification and examine its effect on the measured importance of technology shocks. In the calibration analysis, this is done explicitly by changing the calibration of the labor supply elasticity. In the econometric analysis, it is done implicitly by changing the subset of observations $\mathcal{Y}_s$ that enter the likelihood function and by varying the set of non-technology shocks included in the DSGE model specification.

The key message of this paper is that sources of identification should be disassociated from empirical methodologies. The first step of a quantitative analysis should be to consider various plausible ways of identifying those parameters that are highly influential in shaping the quantitative findings. The second step is to incorporate these identification schemes into the quantitative analysis, be it a calibration or an econometric estimation of the DSGE model.

4. Calibration

This section examines the identifying information that is used in a typical calibration of a stochastic growth model. We begin with a benchmark calibration in Section 4.1. Throughout this section, capital utilization is restricted to be constant, leaving the labor supply elasticity ν as the parameter for which very different sources of identification are available and that affects the empirical results in an important manner. In turn, we consider alternative ways of identifying the labor elasticity in Section 4.2 and examine the sensitivity of the empirical findings. In each subsection, once the parameter values have been determined, output and hours worked data are simulated from the log-linear model economy. Since we are interested in the question of what fraction of business cycle variation in these series is due to technology shocks, we do not need to determine the parameter values of the non-technology shock processes. More generally, in contrast to the estimation in Section 5, the subsequent calibration analysis is agnostic about the sources of business cycle fluctuations other than technology shocks. Precise data definitions are provided in the Online Appendix.

4.1. Benchmark Calibration

Throughout this section we use a calibration approach in which the DSGE model parameters are determined sequentially. This makes the source of identification for each parameter very transparent. Implicit in the subsequent procedure is an objective function $Q_c(\theta; \mathcal{Y}^o, \mathcal{X}^o)$, and we focus the discussion on the vector of targets $\tau(\mathcal{Y}^o, \mathcal{X}^o)$.[4] We start by determining the steady-state-related parameters, then we identify the parameters of the technology shock processes, and at last quantify the parameters that affect the endogenous propagation of the DSGE model but not its steady state. The calibrated model is then used to determine the importance of technology shocks for business cycle fluctuations.

Steady-State-Related Parameters. Steady-state-related parameters are usually identified based on long-run averages of macroeconomic time series. We target a 66% labor share, an annual real interest rate of 4%, and a quarterly depreciation rate of 0.013. The definition of the labor share has many subtleties that we avoid here altogether (see Cooley and Prescott (1995) and Ríos-Rull and Santaeulàlia-Llopis (2010)). The depreciation rate target is obtained from the average physical depreciation rate of total capital calculated by Cummins and Violante (2002). These choices yield values of

$$\alpha = 0.34,\ \beta = 0.99, \text{ and } \delta_0 = 0.013 \tag{12}$$

in quarterly terms. The value of the parameter ξ determines the steady-state level of hours worked and leisure. In our specification, ξ does not affect the decision rules in a log-linear approximation and is irrelevant for the behavior of the model.

Parameters of the Shock Process. The identification of the parameters associated with the law of motion of the exogenous shock processes is best understood in two steps. The first step is the construction of an empirical measure of the exogenous disturbances. In the version of the DSGE model with fixed capital utilization, the construction of such measures only

[4] Strictly speaking, our $\tau(\cdot)$ function also depends on θ such that the objective function has the form $Q_c(\theta; \mathcal{Y}^o, \mathcal{X}^o) = \|\tau(\mathcal{Y}^o, \mathcal{X}^o, \theta) - m(\theta)\|^2_\Omega$.

involves the (previously identified) steady-state-related parameters, but not the endogenous propagation parameters. The second step consists of determining the parameters of the shock processes based on the measured structural disturbances.

In the model economy, the investment-specific technology shock is equivalent to the (reciprocal of the) relative price of investment in terms of consumption. This relative price is obtained by combining a price index for quality-adjusted equipment investment with a price index for investment in structures. We build on the annual series of Cummins and Violante (2002) to construct our quarterly series of quality-adjusted equipment investment. Quarterly movements are imputed based on the official index reported by the Bureau of Economic Analysis (BEA) in the Fixed Asset Tables (FAT-BEA). As a price index for investment in structures, we use the consumption deflator, P_t^C. The two indices are combined to obtain a quality-adjusted price index for total investment, P_t^I. We then define $P_t = P_t^I/P_t^C$ and $V_t = 1/P_t$, normalizing the index such that $P_0 = 1$ in 1947.

The series for the neutral technology process A_t is computed from the aggregate resource constraint (4) using measures of per capita real output Y_t, capital K_t, labor input H_t, and the previously calibrated capital share $\alpha = 0.34$. What is non-standard in our analysis is that we have to construct a quality-adjusted capital stock. To do so, we generate a quarterly series for investment in efficiency units $X_t = (I_t^E + I_t^S)/P_t^I$, where I_t^E and I_t^S are total nominal investment in equipment and structures, respectively, and P_t^I is the quality-adjusted price index mentioned above. The quality-adjusted capital stock is obtained by the forward iteration of the capital accumulation equation (3) using the calibrated depreciation rate $\delta_0 = 0.013$. The initial capital stock K_0 is chosen to match the observed level of output and the investment-to-output ratio in 1947.

Based on the empirical measures of $\ln A_t$ and $\ln V_t$, it is straightforward to determine the coefficients for the autoregressive processes (5) and (6). Typically, the calibration targets for the parameters would be obtained by a least-squares regression. For internal consistency within this paper and to facilitate a comparison with the estimation results reported in Section 5, we replace the least-squares estimates by Bayesian posterior mean estimates. Since the properties of the stochastic growth model are sensitive to the persistence of the technology processes, we estimate (5) and (6) with and without imposing unit roots on the process. The resulting parameter values are as follows:

$$\begin{aligned} \text{Deterministic trend} \ &: \ \gamma_a = -0.001,\ \psi_{1,a} = 0.97,\ \psi_{2,a} = -0.03,\ \sigma_a = 0.007, \\ &\quad\ \gamma_v = 0.008,\ \psi_{1,v} = 0.99,\ \psi_{2,v} = -0.76,\ \sigma_v = 0.003 \\ \text{Stochastic trend} \ &: \ \gamma_a = 0,\ \psi_{1,a} = 1,\ \psi_{2,a} = -0.06,\ \sigma_a = 0.007, \\ &\quad\ \gamma_v = 0.007,\ \psi_{1,v} = 1,\ \psi_{2,v} = -0.81,\ \sigma_v = 0.003. \end{aligned} \tag{13}$$

The details of the estimation are provided in the Online Appendix. Both technology processes are highly persistent and the estimates of ψ_1 exceed 0.97. The growth rates of the neutral technology process are essentially uncorrelated, that is, $\hat{\psi}_{2,a}$ is near zero, whereas the growth rates of $\ln V_t$ are strongly serially correlated with $\hat{\psi}_{2,v} \approx 0.8$. It is interesting to note that the deterministic component of technology growth is solely due to $\ln V_t$, which implies that it is embodied in the physical capital stock.

Endogenous Propagation Parameters. The only parameter value that remains to be determined is the labor supply elasticity ν. In order to identify the endogenous propagation parameters, a typical calibration does not use evidence from time series movements of endogenous variables that are centrally related to the question that is addressed. In the context of our application, which aims at measuring the fraction of cyclical variation in output and hours that is due to technology shocks, this would imply not using the aggregate dynamics of output and hours as a source of information about the labor supply elasticity. Our benchmark calibration relies on a direct estimate of the Frisch elasticity based on the analysis of micro-level data. While there are many of such estimates available in the literature (we will return to this issue below), we rely on a recent study by Heathcote et al. (2007), who estimate a value for the household elasticity of 0.72 using a definition of the household that includes both a husband and a wife (see Chetty et al. (2011b) for a recent assessment of the literature).

Relative Importance of Technology Shocks. We simulate the linearized DSGE model for 310 periods using only the neutral technology process (A), only the investment-specific technology process (V), and both technology disturbances (A+V). We discard the first 100 observations and calculate the variances of HP-filtered output and hours based on the remaining 210 observations. These variances are then divided by the variances of HP-filtered postwar U.S. aggregate output and hours worked. The simulation steps are repeated 1,000 times, and the entries in Table 1 correspond to the means and standard deviations of the variance ratios across the 1,000 simulations. If we combine the steady-state-related parameters in (12), the shock parameters in (13), and the Frisch elasticity $\nu = 0.72$ from Heathcote et al. (2007), then the variance ratios for hours are 0.09 (deterministic trend) and 0.07 (stochastic trend), respectively. Most of the variability in hours is generated by the investment-specific technology shock. The variance ratios for output are 0.26 (deterministic trend) and 0.23 (stochastic trend), respectively. The output fluctuations are almost exclusively driven by the neutral technology shock.

<u>Insert Table 1 here</u>

4.2. Alternative Calibrations

As is well known, e.g., King and Rebelo (1999), the variance ratios for hours worked are very sensitive to the choice of labor supply elasticity. For this reason, we now consider several alternative calibration strategies for ν that utilize different identifying information.

Other Direct Estimates of ν. There is a large literature that directly provides estimates of the Frisch elasticity based on the analysis of micro-level data, e.g., MaCurdy (1981), Browning et al. (1985), and Altonji (1986), to name a few classic papers. These estimates are typically very small. In his survey paper, Pencavel (1986) reports that most estimates for men are between 0 and 0.45. More recent surveys of the microeconomic literature that include studies along the extensive margin are Chetty et al. (2011a) and Chetty et al. (2011b). These studies pose a slightly higher value for the Frisch elasticity that takes into account the extensive margin. However, Keane and Rogerson (2011) argue for a much higher value of the elasticity to be used in macro models. Their reasoning is based on the fact that earlier estimates abstracted from several features, especially human capital accumulation, leading to estimates that are dramatically negatively biased. Their reasoning also reflects a different reading of the evidence of the extensive margin and of various structural estimates. We consider the value of $\nu = 0.2$ as a typical point estimate of the Frisch elasticity based on micro evidence and use it for the calibration of the stochastic growth model.

Information on the Composition of Hours Fluctuations. Most of the variation in hours over the business cycle occurs along the extensive margin (number of workers rather than hours per worker). To account for such variation, Rogerson (1988) developed a model where agents care about leisure but face a non-convexity in the opportunities to work. These agents can use a lottery arrangement to maximize the utility ex ante. Ex post some will work and others will not. After aggregation, this arrangement yields quasi-linear preferences for a representative consumer, which corresponds to $\nu = \infty$ in our model. Hansen (1985) used these lottery arrangements as the source of his calibration strategy in a seminal paper and not surprisingly found that hours move a lot in response to productivity shocks. Chang and Kim (2006) in multi-person households and Rogerson and Wallenius (2010) explicitly considering retirement have recently revisited the role played by non-convexities without using lotteries. Both papers argue that the macro elasticity, the elasticity that applies to an aggregate model with a stand-in household, is much larger than the elasticity that arises from studies using micro data, and advocate a value for the aggregate Frisch elasticity of around $\nu = 1$. Recently, Dyrda et al. (2012) have argued that cyclical movements in household size due to young people moving in mostly with their parents during recessions generate an aggregate, or macroeconomic, Frisch elasticity of about 1 when people living in stable households have a Frisch elasticity of 0.72.

Turning the Labor Supply Elasticity into a Steady-State-Related Parameter. Functional-form restrictions on the representative household's preferences have been used to identify the labor supply elasticity based on long-run growth patterns. A reasonable description of the last 100 years of Western experience is the statement that while there has been a massive increase of wages, by an order of magnitude if not more, interest rates and the allocation of hours per capita have not displayed any such long-run trend. Business cycles research typically uses utility functions that are consistent with this long-term behavior. The most popular among them are Cobb-Douglas utilities, $(c^\gamma \ell^{1-\gamma})^{1-\sigma}/(1-\sigma)$, where $\ell = 1 - H$ is leisure and the endowment of time is normalized to one. For this utility function, the Frisch elasticity depends only on the steady-state allocation of hours worked, H^*, via the expression $\nu = (1 - H^*)/H^*$. If the average hours worked per adult per week is set to one-third,[5] which is a standard choice, the implied elasticity is $\nu = 2$.

Quantitative Results. In addition to the benchmark results for $\nu = 0.72$, Table 1 also contains variance ratios for $\nu = 0.2$, $\nu = 1$, and $\nu = 2$ based on the alternative identification

[5] The U.S. Bureau of Labor Statistics reports (http://www.bls.gov/opub/working/page17b.htm) 67 hours per week worked by a married couple between the ages of 25 and 54. Conventionally, we think of 100 hours per week per person as the available discretionary time.

strategies discussed above. Elasticity based on males' responses along the intensive margin ($\nu = 0.2$) yields essentially no contribution of productivity shocks to movements in hours. A labor supply elasticity of $\nu = 1$ that is consistent with aggregate fluctuations generated by a heterogeneous agent economy with indivisible labor supply results in a variance ratio for hours of 0.14. The elasticity based on the Cobb-Douglas utility function and one-third of the time allocated to work ($\nu = 2$) yields a variance ratio of 0.32. Interestingly, the investment-specific technical change is the main culprit of hours variation with over 60% of the total variation in all model economies. The variance ratios for output are also increasing in ν and range from 20% to 37%. Output variation is directly affected by the neutral technology shock and indirectly through the fluctuation of hours and capital over the business cycle. We find that output variation is mostly driven by the neutral technology process. As in the benchmark analysis, the results are somewhat sensitive to the parameterization of the shock processes. The contribution of technology shocks is larger for the deterministic trend than for the unit root specification.

Aggregating the Results. We have considered various methods of identifying the aggregate labor supply elasticity. Due to the sensitivity of business cycle fluctuations to the choice of ν, the alternative calibrations generate a wide range of variance ratios. Unlike in the econometric framework considered in Section 5, there is no formal way of weighting results based on different values of ν. Thus, the practice in the literature is to report results for a variety of parameter choices, which is often called sensitivity analysis, or the method of assigning researcher-specific weights to particular parameter combinations. These weights are typically based on judgments as to how reliable and relevant particular sources of information are.

In the context of our application, we note that the choice of utility function based on sufficient conditions for a balanced growth path ends up implying a value for the Frisch elasticity based on steady-state considerations that really have nothing to do with the willingness of households to substitute hours worked across time. Moreover, the balanced growth path requirement by itself does not restrict the value of the Frisch elasticity, which in fact can be anything, provided that one chooses a per-period utility function that is rich enough.[6] This consideration leads us to down-weigh results from $\nu = 2$.

[6]Consider, for example the preferences given by $\frac{C^{1-\sigma}[1+(\sigma-1)\xi H^{1+\frac{1}{\nu}}]^{\sigma}-1}{1-\sigma}$. Here, the Frisch elasticity is ν and there is a balanced growth path. See Shimer (2009) and Trabandt and Uhlig (2009).

The value of $\nu = 0.2$ is also a concern because it strongly relies on information about the intensive margin of prime-age white males, who are full-time workers in most periods. However, it is well documented that a large fraction of hours fluctuations is accounted for by movements in and out of employment (see, for instance, Kydland and Prescott (1991), who describe the extensive margin as responsible for two-thirds of the variation), and that those that have the most hours variation along the cycle are not prime-age males (see Kydland (1984), Kydland and Prescott (1988), and Ríos-Rull (1993)). In addition, Low (2005) and Domeij and Floden (2006) argue that, due to borrowing constraints, the estimates may be biased downward (the latter by up to 50%). In sum, we regard the results based on $\nu = 0.72$ and $\nu = 1$ as the most credible ones.

5. Bayesian Estimation of the DSGE Model

We now turn to formal econometric methods to determine the importance of productivity shocks for movements in hours. Our benchmark estimation is presented in Section 5.1. Section 5.2 examines how the choice of model specification and observables affects the identification of the labor supply elasticity and the substantive findings. Bayesian model averaging is used to aggregate results from different model specifications.

5.1. Benchmark Estimation

The estimation of the DSGE model involves a number of choices, including the set of observables $\mathcal{Y}_s^o$ ($\mathcal{Y}_s$ is a subset of $\mathcal{Y}$) and $\mathcal{X}^o$, potential restrictions on the parameter vector θ, and an estimation objective function. As explained in Section 3, we use Bayesian estimation techniques, which are reviewed in detail in An and Schorfheide (2007). Thus, inference is based on the posterior density

$$p(\theta|\mathcal{Y}_s^o, \mathcal{X}^o) \propto p(\mathcal{Y}^o|\theta)p(\theta|\mathcal{X}^o). \tag{14}$$

In the notation of Section 3, the posterior density $p(\theta|\mathcal{Y}_s^o, \mathcal{X}^o)$ plays the role of the objective function $Q_e(\theta; \mathcal{Y}^o, \mathcal{X}^o)$. After describing the choice of observables and parameter restrictions, we discuss the prior distribution, present posterior estimates, explain the identification of the labor supply elasticity, and summarize the findings with respect to the importance of technology shocks.

Choice of Observables and Parameter Restrictions. For the benchmark estimation we use observations on labor productivity, hours worked, and the relative price of investment: $\mathcal{Y}_s = \{Y/H, H, P\}$. Since the goal of the analysis is to determine the contribution of productivity shocks to the variation of hours and output, these series should be ingredients for our estimation. Moreover, the model implies that wages are proportional to labor productivity. Hence, our observables span prices and quantities in the labor market. The relative price of investment, according to our theory, provides a measure of the investment-specific technology shock, which in turn exogenously shifts the labor supply equilibrium. This heuristic argument suggests that the observables contain information that can identify the labor supply elasticity ν.

In order to guarantee that the model-implied distribution of $\mathcal{Y}_s$ is non-degenerate, meaning that there does not exist a linear combination of the three series that is perfectly predictable based on past observations, the DSGE model must be driven by at least three shocks. Given the findings of, among many others, Hall (1997) and Chari et al. (2007), a natural third shock in addition to the two technology shocks is the preference shock B_t that affects the choice of hours worked. For the benchmark analysis we set the government spending shock g_t to zero ($g_* = 1$ and $\sigma_g = 0$). Based on the non-zero shock processes, we refer to the benchmark specification as $\{A, V, B\}$ specification.

Priors. The joint prior distribution for the DSGE model parameters is constructed conditional on some external information as a product of marginal distributions, which are summarized in Table 2. While priors could in principle be formed by pure introspection, in reality most priors (as well as most model specifications) are based on some empirical observations, which we denoted by $p(\theta|\mathcal{X}^o)$. The prior elicitation for the steady-state-related parameters α, β, and δ_0 uses the same sources of information as in Section 4.1. This elicitation is based on long-run averages of observables that are not included in the likelihood function, such as real interest rates, the labor share, and the size of the capital stock.[7] We use degenerate priors for two of the parameters: the discount factor and the depreciation rate are fixed at $\beta = 0.99$ and $\delta_0 = 0.013$. Based on the labor share data, we choose a prior for α that is centered at 0.34 with a standard deviation of 0.02.

[7] In slight abuse of notation, we assume that this information is part of $\mathcal{X}^o$. Del Negro and Schorfheide (2008) show how to automate this kind of prior elicitation.

Insert Table 2 here

The remaining steady-state-related parameters are $\ln A_0$, $\ln V_0$, ξ, as well as the technology growth rates γ_a and γ_v. In order to operationalize the prior elicitation for $\ln A_0$ and ξ, we use a re-parameterization that replaces $\ln A_0$ by $\ln Y_0$ and ξ by $\ln H^*$. Priors for $\ln Y_0$, $\ln V_0$, and $\ln H^*$ are based on pre-sample information. The priors for the technology shock parameters are identical to the ones that were used in the estimation based on the measured shock processes in Section 4.1. The priors for these parameters are fairly diffuse. The prior distribution for the autocorrelation of the preference shock is centered at 0.95 and has a standard deviation of 0.02.

Finally, a prior for the endogenous propagation parameter, that is, the Frisch labor supply elasticity, needs to be specified. Our prior for the Frisch labor supply elasticity is centered at the balanced growth path value of $\nu = 2$, but with a standard deviation of one. Hence, a 90% *a priori* credible interval encompasses values found in studies that use micro-level data for employed males, as well as the values necessary to be able to explain most of the observed volatility in hours worked in a stochastic growth model driven by technology shocks.

Posteriors. The DSGE model is estimated based on observations from 1955:Q3 to 2006:Q4, conditioning on observations from 1954:Q3 to 1955:Q2. We use the Markov Chain Monte Carlo methods reviewed in An and Schorfheide (2007) to obtain draws from the posterior distribution of the DSGE model parameters. Based on these parameter draws, we compute posterior mean estimates and 90% credible intervals. The estimates of the technology shock parameters are very similar to the ones reported in (13). In part due to the fairly tight prior on α, the posterior mean estimates fall in the range of 0.33 to 0.40. Thus, the specific statistical methods used here did not change the estimates of these parameters. The coefficient estimates for the preference shocks are such that the model is able to capture the variation in output and hours worked that is not explained by the two technology shocks. Finally, we obtain an estimate of the Frisch elasticity of 0.85, which is a credible interval that ranges from 0.34 to 1.33. This estimate is in the range of those recently obtained by Heathcote et al. (2007, 2008) using micro-level data. A table with detailed estimation results is available in the Online Appendix.

Source of Identification of Labor Supply Elasticity. The likelihood function peaks near parameter values for which the model-implied autocovariance function of the observables matches the sample autocovariance function as closely as possible in terms of a statistical metric. It does so by forcing each shock in the model to contribute particular autocovariance features, which in total have to mimic the sample autocovariances. This matching is, unfortunately, often difficult to interpret because there is no transparent link from patterns in the data to particular parameter estimates. We now shed some light on why our likelihood-based estimation yields a fairly low estimate of the labor supply elasticity.

Broadly speaking, technology shocks for which we can construct observations independently of the labor supply elasticity play the same role as exogenous demand shifters (or instrumental variables) in the analysis of traditional simultaneous equations systems: they perturb the market equilibrium and move prices and quantities. Along the response to a technology shock, the labor supply condition, written in terms of temporal differences Δ, has to be satisfied:

$$\Delta\hat{h}_t = \nu(\Delta\hat{w}_t - \Delta\hat{c}_t). \tag{15}$$

Here, $\hat{h}_t$ denotes hours in percentage deviations from its steady state, and $\hat{w}_t$ and $\hat{c}_t$ denote percentage deviations of *detrended* real wages and consumption from their respective steady states.

Since consumption is not included in the list of observables used in our likelihood-based estimation, we replace it by a function of wages, hours, and technology shocks. To ease the exposition, assume that the technology shocks follow unit root processes by restricting $\psi_{1,a} = \psi_{1,v} = 1$. We show in the Online Appendix that in response to a one standard deviation investment-specific technology shock in period $t = 1$, the wage and hours dynamics for $t > 1$ can be expressed as

$$\Delta\hat{h}_t = \nu\left[\Delta\hat{w}_t - r^*(1-\alpha^{-1})\hat{w}_t + (-\psi_{2,v})^{t-1}\frac{\sigma_v}{1-\alpha}\right], \tag{16}$$

where $r^* = R^*/(R^* + 1 - \delta_0)$, and $R^* = e^{(\gamma_a+\gamma_v)/(1-\alpha)}/\beta - (1-\delta_0)$. Recall that r^* and α are identifiable from long-run averages of the labor share, real interest rates, and the investment-capital ratio, which enter our estimation objective function implicitly through the prior distribution. Thus, information on the impulse responses of wages (which are equal to average labor productivity in our Cobb-Douglas environment with constant factor shares) and hours worked suffices to identify the labor supply elasticity. This information is contained in the $\{Y/H, H, P\}$ sample.

In order to decode the identifying information in the sample, the following experiment is conducted. Using the observed $\{Y/H, H, P\}$ data and data simulated from the DSGE model, a structural vector autoregression (VAR) for labor productivity growth, hours worked, and investment-specific technology growth is estimated. The VAR includes four lags and its innovations are interpreted as innovations in the neutral technology shock, the investment-specific technology shock, and a non-technology shock. The structural VAR is identified by the following three restrictions that are also hardwired in the DSGE model. The first two restrictions are generated by the assumption that the investment-specific technology growth (reductions in the relative price of investment) is exogenous and follows an AR(1) process. Thus, the other two shocks have no effect on investment-specific technology. The third restriction is that the non-technology shock does not shift the labor demand schedule upon impact because capital is fixed in the short run.

<u>Insert Figure 1 here</u>

The resulting VAR-based impulse responses are depicted in Figure 1. The figure shows 90% credible bands for responses estimated based on U.S. data as well as posterior mean responses based on data that have been generated from a DSGE model with Frisch elasticities $\nu = 0.2$ (solid) and $\nu = 2$ (dashed).[8] The hours responses provide conflicting information about the labor supply elasticity. To match the responses to an investment shock and the non-technology shock, the labor supply elasticity should be small; that is, a value of 0.2 is preferred over a value of 2. To reproduce the empirical response to a neutral technology shock, on the other hand, a large Frisch elasticity is needed. The overall estimate of ν is ultimately determined by the weighting of the discrepancy between sample and DSGE model-implied autocovariance functions (and hence VAR-based impulse response function estimates) encoded in the likelihood function. It turned out to be low, which suggests that the B- and V-shock responses received relatively more weight than the A-shock response.

Relative Importance of Technology Shocks. The neutral and investment-specific technology shocks combined account for about 11% of the variation in hours worked and 26% of the variation in output. Given the estimated labor supply elasticity of 0.85, the estimates of

[8]The remaining parameters used to simulate the DSGE model are obtained by reestimating the DSGE model subject to the restrictions that $\nu = 0.2$ ($\nu = 2$) and $\psi_{1,a} = \psi_{1,v} = 1$.

the relative importance of technology shocks for output and hours fluctuations are similar to those obtained from the calibration with $\nu = 0.72$ (see Table 1). In particular, fluctuations in hours are predominantly due to the investment-specific technology shock, whereas fluctuations in output are mostly driven by the neutral technology shock.

5.2. Alternative Estimations

The estimation required us to take a stand on the non-technology shocks that contribute to the business cycle fluctuations of $\mathcal{Y}_s$ because our likelihood-based estimation technique utilizes the information contained in second moments (autocovariances) of the data. Moreover, we had to select a specific set of observables to construct the likelihood function. Since the identifying information in the likelihood function depends on both the data $\mathcal{Y}_s^o$ as well as the probabilistic structure of the exogenous shocks that ultimately generate the distribution of the data, we will now consider the robustness of our estimation to changes in the exogenous shocks as well as the set of observables.

Alternative Shock Specifications. In addition to the preference shock B_t, we can utilize the government spending shock g_t, which can also be interpreted as a generic aggregate demand shock. This leads to two additional model specifications that can be estimated based on three observables: $\{A, V, G\}$ and $\{A, V, B, G\}$. In order to estimate the model specifications with the government spending shock, we set $g^* = 1.2$ based on the average post-war government spending to GDP ratio. The prior distribution for ρ_g is centered at 0.95 and has a standard deviation of 0.02. Finally, we consider model specifications with difference stationary technology shocks, that is, $\psi_{1,a} = \psi_{2,v} = 1$. The location parameters of the Inverse Gamma priors for σ_b and σ_g (see Table 2) are chosen such that the priors for the various model specifications have similar implications with respect to the importance of non-technology shocks for the business cycle fluctuations of output and hours as well as for the volatility of output growth and hours. In particular, our priors imply that the 90% *a priori* credible intervals for the combined effect of the non-technology shock range from 0% to 25% for output and 10% to 100% for hours.

Alternative Data Sets. Since the accumulation of the quality-adjusted investment series provides a measure of the capital stock, which in combination with aggregate output and hours worked identifies the neutral technology shock via the production function (see Section 4.1), we consider the quality-adjusted investment, $\ln X_t$, as a fourth observable. In addition to the benchmark sample $\{Y/H, H, P\}$, we estimate the stochastic growth models on the samples $\{Y/H, H, X\}$ and $\{Y/H, H, P, X\}$, with the qualification that only the four-shock model is estimated based on the four-variable sample.[9]

<u>Insert Table 3 here</u>

Quantitative Results. Table 3 reports posterior means and 90% posterior credible intervals for the labor supply elasticity based on the various combinations of estimated model specifications and data sets. If the observables consist of productivity, hours worked, and the relative price of investment, the posterior mean estimates range from 0.30 to 0.85 and are somewhat larger if the unit root restrictions are not imposed. If the price of investment is replaced by the quantity, the estimates of ν tend to increase, in particular if the government spending shock is included as a shock in the model. This finding is at least qualitatively consistent with the impulse responses in Figure 1. The $\{Y/H, H, P\}$ data set contains direct information on the investment-specific technology shock. The figure indicates that the responses to an innovation of the investment-specific technology shock are better matched by a low labor supply elasticity. The $\{Y/H, H, X\}$ data set, on the other hand, in conjunction with the model-implied capital accumulation and production function, tends to identify the neutral technology shock and the responses to its innovation well. According to Figure 1, these responses are better matched with a high value of the labor supply elasticity. Unfortunately, we do not have a good explanation as to why the change in the labor supply elasticity estimate is most pronounced for the $\{A, V, G\}$ model. Finally, if the four-shock model specification is estimated based on four series, the labor supply elasticity estimates drop to 0.17 (unit root restrictions not imposed) and 0.12 (unit root restrictions imposed).

<u>Insert Table 4 here</u>

So far, the econometric analysis has generated multiple sets of parameter estimates, which in turn lead to a multitude of answers for our quantitative question. For each model

[9]Guerron-Quintana (2010) examines how data choice affects parameter identifiability in the Smets-Wouters model by eliminating subsets of the seven macroeconomic time series that are typically used to estimate the Smets-Wouters model.

specification and data set combination, Table 4 reports posterior means and standard deviations of variance ratios computed from HP-filtered simulated and actual data. All entries refer to the combined effect of neutral and investment-specific technology shocks on output and hours worked. The variance ratio results mimic the labor supply elasticity estimates: high elasticities yield large effects. The largest effect of technology shocks on hours worked fluctuations is obtained from the $\{A, V, G\}$ specifications estimated based on $\{Y/H, H, X\}$ data, explaining 20% of the observed variation when the shocks are restricted to follow unit root processes and 22% otherwise. The corresponding numbers for the output fluctuations are 32% and 33%, respectively. If, on the other hand, the DSGE model is estimated based on $\{Y/H, H, P, X\}$ observations, the labor supply elasticity is around 0.1, which translates into variance ratios of 1% for hours and 22% for output. Thus, the range of answers generated by estimating the DSGE model is about as wide as the range of results obtained by the various calibrations considered in Section 4.2. We conclude that under both empirical methodologies, differences in identification of key parameters can generate a wide spectrum of quantitative results. Hence, quantitative macroeconomists from both the calibration and estimation camps should place more emphasis on searching for reliable sources of identification of key parameters and making them transparent to their audience.

Aggregating the Results. Not all estimated model specifications track the time series data equally well. In a Bayesian econometric framework, it is natural to assign more weight to parameter estimates and predictions obtained from model specifications that attain a better time series fit. Formally, one can use log marginal likelihoods to update prior model probabilities.[10] For each of the three data sets, Table 5 reports log marginal likelihood differentials (or log Bayes factors), using the specification with the highest marginal likelihood as a benchmark. The log Bayes factors are converted into posterior model probabilities under the assumption that all DSGE model specifications have equal prior probability. For the $\{Y/H, H, P\}$ data set, it appears to be slightly preferable to impose unit roots in the two technology processes and to augment the technology-driven DSGE model with a government expenditure shock instead of a preference shock. Based on the $\{Y/H, H, X\}$ observations,

[10] Consider a collection of models $\mathcal{M}_m$, $m = 1, \ldots, M$. The marginal likelihood is defined as $p(Y|\mathcal{M}_m) = \int p(Y|\theta, \mathcal{M}_m)p(\theta|\mathcal{M}_m)d\theta$. The posterior probability of $\mathcal{M}_m$ is $\pi(\mathcal{M}_m|Y) = \pi(\mathcal{M}_m)p(Y|\mathcal{M}_m)/p(Y)$, where $p(Y) = \sum_{m=1}^{M} \pi(\mathcal{M}_m)p(Y|\mathcal{M}_m)$ and $\pi(\mathcal{M}_m)$ is the prior probability of $\mathcal{M}_m$. From a non-Bayesian perspective, the marginal likelihood provides a measure of in-sample fit that is adjusted by a penalty for model complexity.

the trend-stationary specifications are preferred in the three-shock models. However, the best fit is obtained by the four-shock version with unit root technology processes. Overall, the log marginal likelihood differentials are fairly small, indicating that the data can only imperfectly discriminate among the various specifications.

Insert Table 5 here

The boldfaced entries in Table 4 indicate specifications with posterior model probabilities higher than 3%. Weighted by posterior model probabilities, we conclude from the $\{Y/H, H, X\}$ data that 5% of hours fluctuations and 25% of output fluctuations can be explained by technology shocks. The corresponding numbers for the $\{Y/H, H, P\}$ data set are 2% and 21%, respectively. Due to the small value of the estimated labor supply elasticity based on the $\{Y/H, H, P, X\}$ data set, hours essentially do not move in response to technology shocks, and they explain 22% of output fluctuations.

6. Variable Capital Utilization

This section extends the previous analysis to the case of variable capital utilization ($\zeta > 0$), a feature that mainly affects the propagation of structural shocks. The amount of capital input is no longer pre-determined upon impact of the shock because firms adjust capital services along the utilization margin. Since the marginal product of labor rises as the effective capital input increases, hours tend to respond more strongly to technology shocks. Variable capital utilization is a mechanism that is present in many business cycle models to enhance the response of hours to productivity shocks (see the comprehensive discussions in Greenwood et al. (2000) and Christiano et al. (2005)). From a methodological perspective, the variable utilization model includes an additional endogenous propagation parameter that needs to be identified. Moreover, it is not possible anymore to determine the parameters of the neutral technology shock independently of the endogenous propagation parameters, which is the case in many DSGE models. Section 6.1 reexamines the calibration analysis, and the variable capital utilization model is estimated in Section 6.2. It remains the case that within each empirical methodology, differences among the identification of key parameters can generate a wide spectrum of quantitative results. Finally, Section 6.3 provides some discussion of results obtained from various other variants of the stochastic growth model that have been considered in the literature.

6.1. Calibration

Neither the calibration of the steady-state-related parameters, nor the calibration of the parameters associated with the investment-specific technology shock and the labor supply elasticity, is affected by allowing for variable capital utilization. We only have to reconsider the calibration of the neutral technology shock parameters as well as the choice of the elasticity of capital utilization with respect to interest rates, ζ. We consider several of the strategies to calibrate ζ that have been proposed in the literature and use an indirect-inference step to determine the technology shock parameters.

Neutral Technology Shock Parameters. In the case of constant capital utilization, we constructed a sequence for capital, and in turn the neutral technology shock, based on investment data by iterating the capital accumulation equation (3) forward with $\delta(u_t) = .013$. In the case of variable capital utilization, the capital depreciation rate is time-varying and the use of the perpetual inventory method requires a utilization series as well as knowledge of the function $\delta(u_t)$. Since capital utilization is difficult to measure without error, we proceed with an indirect inference approach. We will use actual data to construct the same series of measured total factor productivity (TFP) that we used in Section 4.1. However, for $\zeta > 0$ this measured TFP cannot be equated with the neutral technology shock because it ignores the effect of time-varying utilization and, hence, it contains an incorrect measure of capital that does not account for utilization-dependent depreciation rates.

Suppose a parameter value for ζ has been selected. Let $\theta_{(-a)} = [\alpha, \beta, \delta_0, \nu, \zeta, \psi_{1,\nu}, \psi_{2,\nu}, \sigma_\nu]'$ denote the DSGE model parameter not associated with the neutral technology process and define $\theta_{(a)} = [\psi_{1,a}, \psi_{2,a}, \sigma_a]'$.[11] We proceed by simulating data from the DSGE model for various choices of $\theta_{(a)}$, compute measured TFP from the simulated data, and estimate the autoregressive coefficients $\psi^m_{1,a}$, $\psi^m_{2,a}$, and σ^m_a for the simulation-based measured TFP series. We then choose the $\theta_{(a)}$ that minimizes a discrepancy measure between the estimated parameters of measured TFP based on the actual and simulated data. As actual parameter estimates (target) we use the values reported in (13). Throughout this process, the parameters $\theta_{(-a)}$ are held fixed. Further details are provided in the Online Appendix.

[11]Since we are interested only in the business cycle properties of the model, the parameters that determine the level of the series, $\ln A_0$ and $\ln V_0$, and the growth rate parameters γ_a and γ_ν are omitted.

Direct Estimates of ζ. Basu and Kimball (1997) provide direct estimates of $1/\zeta$ using observable proxies for the latent utilization of factor inputs. These proxies are relative factor prices as well as material and energy inputs, and their relationship to utilization has been derived from the firms' cost minimization problem under some assumptions on the form of the production function. Their benchmark point estimates imply a value of ζ of about 1. However, their confidence intervals, without imposing a non-negativity constraint, range from about -0.2 to 2 for $1/\zeta$, which in turn is consistent with large values of ζ. Baxter and Farr (2001) and Mandelman et al. (2011) set the elasticity to $\zeta = 1$, essentially using Basu and Kimball (1997)'s point estimate. Christiano et al. (2005) use a direct, albeit potentially noisy, measure of capital utilization as one of the observables in a structural VAR that is used to generate a set of impulse responses, including that of utilization, to a monetary policy shock. The parameter ζ (among others) is then estimated by matching their model's impulse responses to the ones obtained from the structural VAR. Their estimation procedures insisted on a huge value of the elasticity that attempted to break the feasibility of solving the model. As a compromise, they set $\zeta = 100$.

Turning ζ into a Steady-State-Related Parameter. Greenwood et al. (1988) specify a power functional form for depreciation with only one parameter as a function of capacity utilization, and then they set the elasticity to get a steady-state depreciation rate of 10% conditional on a discount rate of 0.96. The implied value of the elasticity is $\zeta = 2.38$. Similarly, in a model with productivity shocks, endogenous capital utilization, and labor hoarding, Burnside and Eichenbaum (1996) choose $\zeta = 1.85$ to match a steady-state depreciation target given steady-state capital-output ratio, discount rate, and output growth targets. Greenwood et al. (2000) have a more complicated model with equipment and structures, and its technological structure is such that to calibrate it, seven parameters are controlled by seven targets. All of these targets are based on long-run U.S. features that the balanced growth path of the economy is set to replicate. Their value is $\zeta = 1.69$.

Relative Importance of Technology Shocks. To examine the relevance of the sources of identification of ζ, we consider the following set of values: $\{0, 0.2, 1, 2, 5\}$. The value of 0 corresponds to the benchmark case of constant utilization, and the remaining values are chosen to cover the estimates reviewed above. Our approach resembles that of King and Rebelo (1999), who recognized how little is known about the utilization elasticity and conducted a large sensitivity analysis for values of ζ between 0 to 10. The results for a Frisch labor supply elasticity[12] of $\nu = 0.72$ are summarized in Table 6. The top rows of the table document that the presence of variable capital utilization does not really affect the estimated persistence of the neutral technology shock, captured by $\psi_{1,a}$ and $\psi_{2,a}$, for the range of ζ's considered. However, the estimate of the innovation standard deviation σ_a monotonically declines with ζ. The resulting variation in the contribution of neutral technology shocks to the variance of hours is not very large, ranging from 2% to 4%, and is non-monotonic in ζ.

Insert Table 6 here

The measurement of the investment-specific technology shock is not affected by the variable capital utilization. Since the mechanism of capacity utilization tends to amplify hours movements, the variance ratio for hours fluctuations caused by the V shock increases from 6% ($\zeta = 0$) to 21% ($\zeta = 5$) depending on the choice of the parameter, a choice made at best on thin grounds. Thus, as in the benchmark model with constant capital utilization, the choice of identification scheme for a key parameter yields a lot of variation in the quantitative finding.

6.2. Bayesian Estimation

In the likelihood-based DSGE model estimation, the utilization elasticity ζ is implicitly identified from the autocovariances of the observables. Variable capital utilization either could be treated as a latent variable, meaning that information about ζ is extracted, say, from the joint dynamics of productivity, hours, and the relative price of investment, or a measure of utilization could be added to the set of observations. Including a utilization series as an observable has the advantage that its cross-correlation with the other time series can provide valuable identifying information for ζ. The disadvantage is that this information is potentially contaminated by measurement errors. We subsequently document the range of estimates that can arise from different ways of treating capital utilization.

Latent Variable Approach. When treating utilization as a latent variable, we only need to include a prior distribution for ζ, which we choose as $\zeta \sim \mathcal{G}(2, 1)$. A 90% credible interval covers all the values considered in the calibration except $\zeta = 5$.

[12] For each value of ζ, the analysis of the importance of the identification of the Frisch elasticity of labor can be easily replicated. Clearly, Section 4.2 already showed the answers for $\zeta = 0$. For instance, for $\zeta = 1$ we obtain that the variance ratio for hours ranges between 0.01 for $\nu = 0.2$ to 0.54 for $\nu = 2$.

Treating Utilization as Observable. Following the existing literature, two proxies for variable capital utilization are considered: capacity utilization data from industrial production, henceforth *TCU*, and a survey of electric power use, denoted by *E*. The *TCU* series starts in 1967:Q1, whereas the *E* series is available from 1972:Q1 to 2005:Q4. Our estimation sample is adjusted accordingly. *TCU* is divided by its sample average to ensure that the average utilization is normalized to one, and then we take the logarithm. *E* has an upward trend that reflects an increase in electricity intensity of the production process. Consequently, we remove a deterministic trend from the log of electrical power use.

To use the utilization data in the estimation, a measurement equation needs to be specified:

$$u_t^m = \hat{u}_t + \omega_t, \quad \omega_t = \rho_u \omega_{t-1} + \sigma_u \eta_t. \tag{17}$$

Here, u_t^m is the observed utilization and $\hat{u}_t$ is the utilization in the DSGE model in terms of log deviations from steady state. The AR(1) process ω_t captures a measurement error. We consider the following prior distribution for the parameters of the measurement error process:

$$\rho_u \sim B(0.5, 0.2), \quad \sigma_u \sim IG(0.01, 4).$$

At the prior mean, the measurement error variance is less than 1% of the variance of the observed utilization series.

<u>Insert Table 7 here</u>

Posteriors and the Relative Importance of Technology Shocks. The estimation results are presented in Table 7. We estimate the $\{A, V, B\}$ specification of the DSGE model on the $\{Y/H, H, P\}$ sample, and the $\{A, V, B, \omega\}$ specification on the $\{Y/H, H, P, TCU\}$ and $\{Y/H, H, P, E\}$ samples. The latter specification includes the capital utilization measurement error process ω_t. The estimates of ζ are very sensitive to the identifying information contained in the three samples and vary considerably. The estimated utilization elasticity is 0.24 if utilization is treated as a latent variable. Since ζ affects the shape of the impulse responses and the autocovariance functions, it is identifiable in the absence of utilization data. If utilization data are included in the set of observables, the estimate of ζ increases to 0.75 (E) and 1.70 (TCU), respectively. However, the estimated autocorrelation of the measurement error process is around 0.95, and the estimate of σ_u is around 0.014. This implies that more than 90% of the variability in the utilization series is attributed to measurement errors, suggesting a mismatch between model-implied and observed utilization dynamics.

The estimated first-order partial autocorrelation of the neutral technology process drops from 0.98 to 0.96 if utilization data are included, and the standard deviation of the innovation drops from .0068 to .0057. Inference with respect to the labor supply elasticity ν is very similar across samples. The estimates are slightly lower than those obtained from the $\{Y/H, H, P\}$ sample reported in Table 3. The discussion in Section 5, in particular equation (16), implies that under variable capital utilization, the labor supply elasticity remains identifiable based on the relative response of labor productivity and hours to a technology shock. The identifying information is contained in the relative response of wages and hours to technology shocks and thus encoded in the likelihood function even if utilization is excluded from the set of observables.

The estimated contribution of technology shocks to fluctuations in hours worked and output is less sensitive to the choice of data set than the estimation of the utilization elasticity. Overall, the variance ratios remain in the range of 6% to 11% for hours worked and 24% to 28% for output. Unlike in the calibration, identification of the labor supply elasticity is linked to identification of the utilization elasticity, and the amplification effect of variable capital utilization is essentially offset by a lower estimate of the labor supply elasticity.

6.3. Further Generalizations Considered in the Literature

A recent literature uses an expanded version of the neoclassical growth model, includes various nominal and real frictions as well as several additional shocks, and provides an answer to the same question that we pursue. Justiniano et al. (2010b) estimate one of these expanded models with the same Bayesian techniques used in this paper. They assess the contribution of technology shocks (especially investment-specific shocks) to be up to 60% of hours variation at business cycle frequencies. The reasons for this discrepancy with our findings include the fact that in their environment with fixed wages, a fraction of agents is unable to reoptimize their price and forced to supply whatever number of hours is demanded at the posted price; thus, even with small Frisch elasticity, hours tend to move a lot in response to technology shocks. Another source of discrepancy, at least compared with our estimation with the $\{Y/H, H, P\}$ and $\{Y/H, H, P, X\}$ data sets, is the fact that they treat the investment shock as a latent process, which turns out to be much more volatile than the relative price of investment. Justiniano et al. (2010a) include the relative price of investment as an observable but allow for an additional (unobserved) shock to the marginal efficiency of installed investment. They find that this shock plays a big role in accounting for hours variation relative to the observed shocks to the relative price of investment. Liu et al. (2009) impute most of the role in shaping fluctuations (especially when focusing on the Great Moderation) to the role of neutral technological shocks, capital depreciation shocks, and wage mark-up shocks, while they argue that investment-specific technology shocks played a small role.

7. Lessons for Practitioners

While our methodological points were presented in the context of a specific application, there are several lessons learned for other models and applications.

Misspecification and Parameter Inference. In a perfect world, different sources of identifying information would yield mutually consistent conclusions about key model parameters after the precision of the information has been properly accounted for. Unfortunately, that is not the case if one works with fairly stylized and to some extent misspecified models, as we do in quantitative macroeconomics.[13] The reality is that different identification approaches often yield conflicting quantitative findings. Much of the literature on estimated DSGE models concentrates on characterizing uncertainty conditional on a particular identification scheme but loses track of the often more important sensitivity of results to the initial choice of identification source.

Identification First, Methods Second. The first step of the analysis should be to consider various plausible ways of identifying those parameters that are highly influential in shaping the quantitative findings. The second step is to incorporate these identification schemes into the quantitative analysis, be it a calibration or an econometric estimation of the DSGE model—constructing an objective function $Q_c(\theta; \mathcal{Y}^o, \mathcal{X}^o)$ or $Q_e(\theta; \mathcal{Y}^o, \mathcal{X}^o)$. Unfortunately, much of the existing literature has the order reversed: researchers make a choice as to whether to calibrate or estimate the model, and then rely on the "standard" identification approaches

[13] A comparison of log marginal likelihood values between the DSGE models estimated in Section 5.2 and a more flexible VAR(4) with Minnesota prior reveals that some of the theory-implied cross-equation restrictions of the DSGE models are at odds with the data.

that are typically associated with these methodologies. We deliberately followed this reverse order in our presentation to highlight three identification schemes for an aggregate labor supply elasticity that are typically associated with a calibration and to provide some new insights into the identification schemes that are hardwired into different likelihood functions.

As a recommendation for practitioners, we suggest disassociating identification schemes from quantitative methodologies. The various identification sources that we highlighted in the context of our application and that arise in other applications can be incorporated in either a calibration or an estimation approach. For instance, a calibration could use the impulse response information from Section 5.1 to parameterize the labor supply elasticity. The identification of the neutral technology shock parameters in Section 6.1 mimics the indirect inference approach in econometrics developed by Gourieroux et al. (1993) and Smith (1993). Estimation can use information from micro-level data, impose functional form restrictions that turn endogenous propagation parameters into steady-state-related parameters, or extract information from long-run averages of interest rates, capital-investment ratios, or consumption-investment ratios. This can be implemented in part by using prior distribution in a Bayesian framework or through setting up moment conditions in a frequentist GMM or minimum-distance framework.

Aggregating Results. Econometric approaches, in particular Bayesian approaches, tend to weight quantitative predictions from different model specifications by the relative fit that these specifications attain. In our application, we used Bayesian model averaging to combine results from different DSGE model specifications estimated based on the same data set. This proved to be a convenient tool but did not allow us to aggregate results obtained from different data sets. Across data sets, the variation in hours explained by technology shocks ranges from 1% to 5% and the variation in output from 21% to 25%.

For the calibration analysis, we tabulated our findings conditional on various choices of the labor supply elasticity, enabling the reader to apply her or his own weighting scheme for the plausibility of the various identification approaches. As made clear in Section 4.2, our own weighting scheme takes the following form: we have doubts about the two extreme values of the labor supply elasticity, which are based on middle-aged full-time working white men, the most irrelevant group from a business cycle point of view, or on unnecessary, and probably misguided, restrictions from balanced growth paths. The use of micro-based estimates that take into account both the work of men and women and the intensive and extensive margins as in Heathcote et al. (2007, 2008) deliver arguably the most plausible estimates of ν and imply that between 3% and 9% of hours fluctuations are explained by technology shocks. These estimates happen to be consistent with those from the likelihood-based analysis.

In the environment with capital utilization, the calibration approach for a unit value of ζ (the elasticity of utilization with respect to the interest rate) and for our favorite labor elasticity yields an estimate of the variance of hours accounted for by technology shocks of 0.15. The estimation approach yields estimates ranging from 0.06 to 0.11. Only when the utilization elasticity is set to 5 does the calibration yield very different answers.

8. Conclusion

The main contribution of our paper is to compare how key model parameters are identified in a standard calibration versus a likelihood-based estimation. We make the case that quantitative macroeconomists will benefit from thinking about particular identification sources independently of the quantitative approaches that are used to exploit them. In the context of a specific application, we shed some light on how the likelihood function extracts information about important model parameters and thus contributes to the growing econometric literature on DSGE model identification. Moreover, we carefully examine the sensitivity of likelihood-based estimates to the inclusion of additional shocks that complete the probabilistic structure of the DSGE model, and we use Bayesian model averaging to aggregate the quantitative implications of the various model specifications. The analysis conducted in this paper has convinced us that, regardless of our preferences for quantitative methodologies, we should place more emphasis on searching for reliable sources of identification of key parameters and making these sources transparent to our audience.

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Table 1: CALIBRATION

		(Old) Micro Studies of Males, Intensive Margin $\nu = 0.2$		(New) Micro Studies of All Household Margins $\nu = 0.72$		Nonconvexities Movements in Household Size $\nu = 1.0$		Balanced Growth Cobb-Douglas Utility $\nu = 2.0$	
Deterministic Trend $\alpha = 0.34$, $\beta = 0.99$, $\delta_0 = 0.013$, $\gamma_a = -0.001$, $\psi_{1,a} = 0.97$, $\psi_{2,a} = -0.03$, $\sigma_a = 0.007$, $\gamma_v = 0.008$, $\psi_{1,v} = 0.99$, $\psi_{2,v} = -0.76$, $\sigma_v = 0.003$									
Series	Shock	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
Hours	A	.004	(.001)	0.03	(.005)	0.05	(.008)	0.011	(.017)
	V	.006	(.002)	0.06	(.014)	0.09	(.021)	0.21	(.050)
	A+V	**0.01**	(.002)	**0.09**	(.018)	**0.14**	(.028)	**0.32**	(.067)
Output	A	0.20	(.017)	0.26	(.021)	0.28	(.023)	0.35	(.026)
	V	.001	(.000)	.004	(.001)	.007	(.001)	0.02	(.002
	A+V	**0.20**	(.017)	**0.26**	(.022)	**0.29**	(.024)	**0.37**	(.030)
Stochastic Trend $\alpha = 0.34$, $\beta = 0.99$, $\delta_0 = 0.013$, $\gamma_a = 0$, $\psi_{1,a} = 1$, $\psi_{2,a} = -0.06$, $\sigma_a = 0.007$, $\gamma_v = 0.007$, $\psi_{1,v} = 1$, $\psi_{2,v} = -0.81$, $\sigma_v = 0.003$									
Series	Shock	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
Hours	A	.001	(.000)	0.01	(.002)	0.02	(.003)	0.04	(.006)
	V	.006	(.001)	0.06	(.009)	0.10	(.015)	0.25	(.036)
	A+V	**.008**	(.015)	**0.07**	(.011)	**0.11**	(.018)	**0.29**	(.042)
Output	A	0.18	(.015)	0.21	(.018)	0.22	(.019)	0.25	(.021)
	V	.002	(.000)	.018	(.001)	0.03	(.003)	0.08	(.007)
	A+V	**0.19**	(.015)	**0.23**	(.019)	**0.25**	(.022)	**0.33**	(.027)

Note: We report variance ratios (simulated/actual) for HP-filtered series.

Table 2: PRIOR DISTRIBUTION FOR DSGE MODEL PARAMETERS

Name	Domain	Density	Para (1)	Para (2)
α	$[0,1)$	Beta	0.34	0.02
β		fixed	0.99	
δ_0		fixed	.013	
ν	$\mathbb{R}^+$	Gamma	2.00	1.00
γ_a	$\mathbb{R}$	Normal	0.00	0.10
$\psi_{1,a}$	$[0,1)$	Beta	0.95	0.02
$\psi_{2,a}$	$(-1,1)$	Uniform	-1.0	1.00
σ_a	$\mathbb{R}^+$	InvGamma	0.01	4.00
γ_v	$\mathbb{R}$	Normal	0.00	0.10
$\psi_{1,v}$	$[0,1)$	Beta	0.95	0.02
$\psi_{2,v}$	$(-1,1)$	Uniform	-1.0	1.00
σ_v	$\mathbb{R}^+$	InvGamma	0.01	4.00
ρ_b	$[0,1)$	Beta	0.95	0.02
σ_b	$\mathbb{R}^+$	InvGamma	.017 or .012	4.00
g^*		fixed	1.00 or 1.20	
ρ_g	$[0,1)$	Beta	0.95	0.02
σ_g	$\mathbb{R}^+$	InvGamma	.010 or .007	4.00
$\ln H_*$	$\mathbb{R}$	Normal	0.00	10.0
$\ln Y_0$	$\mathbb{R}$	Normal	0.00	100
$\ln V_0$	$\mathbb{R}$	Normal	0.00	100

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; and s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu,s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. To estimate the stochastic growth version of the model, we set $\psi_{1,a} = \psi_{1,v} = 1$. The $\{A,V,B\}$ (benchmark) specification is estimated with $Para(1) = .017$ σ_b prior. The $\{A,V,G\}$ specification is estimated with $Para(1) = .010$ σ_g prior. The $\{A,V,B,G\}$ specification is estimated with $Para(1) = .012$ σ_b prior and $Para(1) = .007$ σ_g prior.

Table 3: Labor Supply Elasticity Estimates

Unit Roots Imposed	Shocks	Data Set Y/H, H, P		Y/H, H, X		Y/H, H, P, X	
		Mean	90% Intv.	Mean	90% Intv.	Mean	90% Intv.
No	A, V, B	0.85	[0.34, 1.33]	0.82	[0.44, 1.17]		
	A, V, G	0.63	[0.35, 0.89]	1.56	[0.93, 2.17]		
	A, V, B, G	0.70	[0.33, 1.05]	0.77	[0.38, 1.14]	0.17	[0.05, 0.28]
Yes	A, V, B	0.30	[0.06, 0.53]	0.60	[0.30, 0.88]		
	A, V, G	0.42	[0.17, 0.64]	1.83	[1.05, 2.60]		
	A, V, B, G	0.35	[0.07, 0.63]	0.96	[0.33, 1.54]	0.12	[0.03, 0.22]

Table 4: Importance of Technology Shocks

Unit Roots Imposed	Shocks	Series	Y/H, H, P Mean	Y/H, H, P (StdD)	Y/H, H, X Mean	Y/H, H, X (StdD)	Y/H, H, P, X Mean	Y/H, H, P, X (StdD)
No	A, V, B	Hours	0.11	(.066)	**0.05**	(.025)		
		Output	0.26	(.040)	**0.26**	(.038)		
	A, V, G	Hours	**0.08**	(.046)	0.22	(.084)		
		Output	**0.25**	(.037)	0.33	(.047)		
	A, V, B, G	Hours	0.10	(.060)	0.07	(.038)	**0.01**	(.008)
		Output	0.26	(.040)	0.27	(.043)	**0.22**	(.031)
Yes	A, V, B	Hours	0.01	(.009)	0.03	(.015)		
		Output	0.20	(.028)	0.23	(.033)		
	A, V, G	Hours	**0.02**	(.010)	0.20	(.063)		
		Output	**0.21**	(.029)	0.32	(.044)		
	A, V, B, G	Hours	**0.02**	(.014)	**0.05**	(.030)	.002	(.002)
		Output	**0.21**	(.030)	**0.25**	(.040)	0.19	(.026)
Weighted		Hours	**0.02**		**0.05**		**0.01**	
		Output	**0.21**		**0.25**		**0.22**	

Notes: Variance ratios are in bold for model specifications that attain a posterior model probability of 3% or more. The last two rows (Weighted) contain weighted averages based on the marginal likelihoods in Table 5.

Table 5: Log Marginal Likelihood Differentials and Posterior Probabilities

Unit Roots Imposed	Shocks	Data Set Y/H, H, P		Y/H, H, X		Y/H, H, P, X	
		$\ln p(Y)$	(Prob.)	$\ln p(Y)$	(Prob.)	$\ln p(Y)$	(Prob.)
No	A, V, B	-11.7	(.000)	-3.32	(.035)		
	A, V, G	-3.38	(.032)	-12.6	(.000)		
	A, V, B, G	-6.01	(.002)	-5.63	(.003)	0.00	(.992)
Yes	A, V, B	-8.67	(.000)	-7.33	(.001)		
	A, V, G	0.00	(.932)	-15.01	(.000)		
	A, V, B, G	-3.32	(.034)	0.00	(.961)	-4.82	(.008)

Notes: For each data set, the log marginal likelihood differences are computed relative to the DSGE model specification with the highest marginal likelihood. The log marginal likelihoods for these specifications are 2278.12, 1951.07, and 2820.11, respectively.

Table 6: CALIBRATION OF VARIABLE CAPITAL UTILIZATION MODEL ($\nu = 0.72$)

	$\zeta = 0$		$\zeta = 0.2$		$\zeta = 1$		$\zeta = 2$		$\zeta = 5$	
Neutral Technology Shock Parameters										
$\psi_{a,1}$	0.974		0.974		0.985		0.9846		0.985	
$\psi_{a,2}$	-0.027		-0.027		-0.004		0.0367		0.094	
σ_a	.0070		.0067		.0053		0.0048		.0043	
Decomposition of Hours										
	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
A	0.03	(.005)	0.04	(.006)	0.02	(.004)	0.02	(.003)	0.02	(.003)
V	0.06	(.014)	0.08	(.020)	0.12	(.030)	0.16	(.039)	0.21	(.055)
A,V	**0.09**	(.018)	**0.12**	(.025)	**0.15**	(.033)	**0.18**	(.042)	**0.23**	(.057)
Decomposition of Output										
	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
A	0.26	(.021)	0.28	(.023)	0.23	(.019)	0.23	(.020)	0.24	(.021)
V	.004	(.001)	.007	(.001)	0.02	(.003)	0.03	(.005)	0.05	(.009)
A,V	**0.26**	(.022)	**0.29**	(.024)	**0.25**	(.021)	**0.27**	(.022)	**0.29**	(.025)

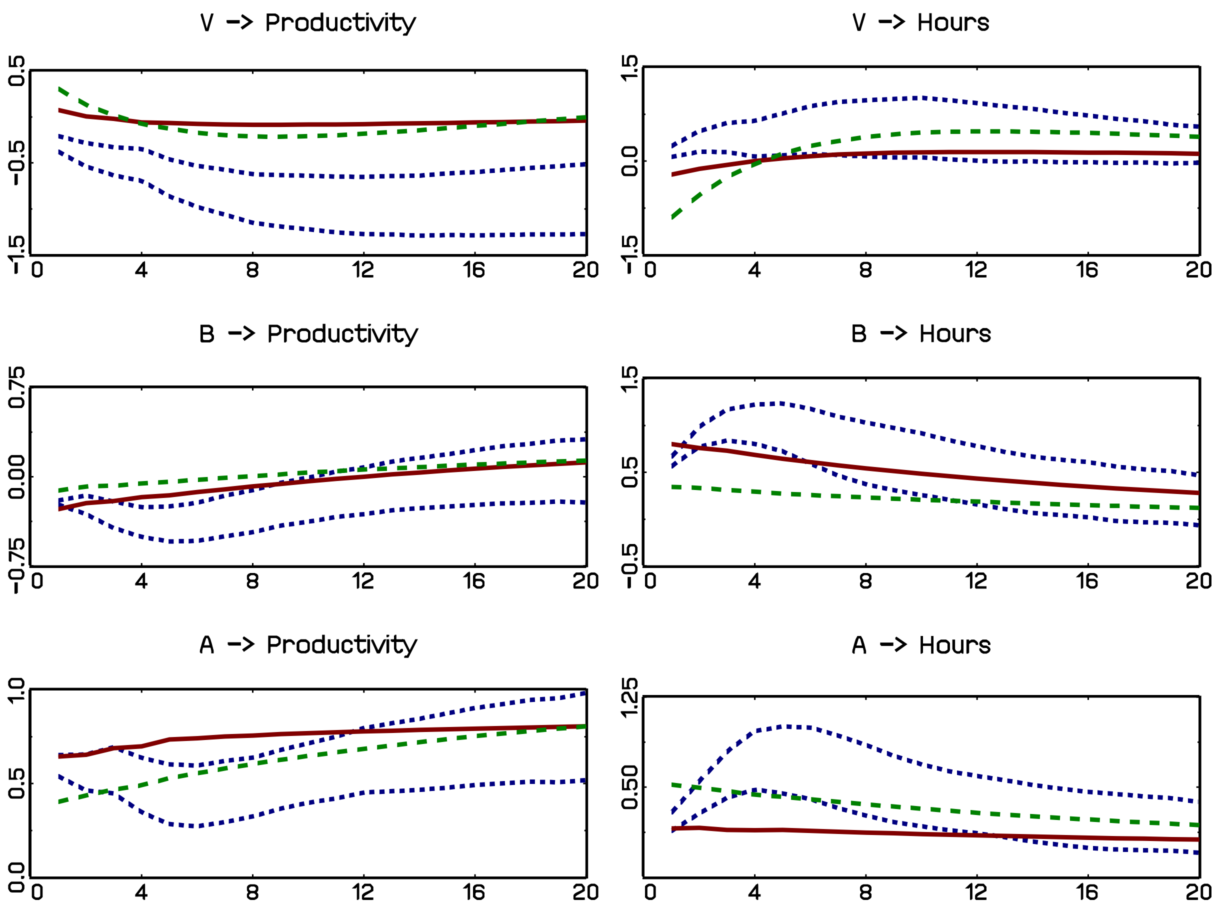
Notes: ζ is the elasticity of capital utilization with respect to the rental rate. The parameter values for $\theta_{(-a)}$ are the same as in Table 1: $\alpha = 0.34$, $\beta = 0.99$, $\delta_0 = 0.013$, $\psi_{1,v} = 0.99$, $\psi_{2,v} = -0.76$, $\sigma_v = 0.003$. The first column ($\zeta = 0$) corresponds to the case of constant utilization and reproduces the variance ratios from Table 1. Since for $\zeta = 0$ measured TFP corresponds to the neutral technology shock, the values for $\theta_{(a)}$ correspond to the values in (13).

Table 7: Estimation of Variable Capital Utilization Model

	Data Set					
	Y/H, H, P		Y/H, H, P, TCU		Y/H, H, P, E	
	Selected Parameter Estimates					
	Mean	90% Intv.	Mean	90% Intv.	Mean	90% Intv.
ν	0.457	[0.158, 0.737]	0.346	[0.105, 0.570]	0.459	[0.160, 0.753]
ζ	0.238	[0.047, 0.426]	1.703	[0.969, 2.431]	0.746	[0.310, 1.168]
$\psi_{a,1}$	0.984	[0.975, 0.994]	0.981	[0.965, 0.996]	0.964	[0.941, 0.989]
$\psi_{a,2}$	-0.110	[-0.247, 0.016]	-0.130	[-0.274, 0.011]	-0.168	[-0.322, 0.012]
σ_a	.0068	[.0062, .0074]	.0057	[.0052, .0062]	.0057	[.0051, .0062]
	Decomposition of Hours					
	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
A	0.02	(.015)	0.01	(.094)	0.02	(.022)
V	0.04	(.032)	0.07	(.076)	0.10	(.099)
A,V	0.06	(.042)	0.09	(.082)	0.12	(.110)
	Decomposition of Output					
	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
A	0.24	(.038)	0.26	(.045)	0.24	(.046)
V	.005	(.003)	0.02	(.012)	0.02	(.011)
A,V	0.24	(.039)	0.28	(.051)	0.25	(.052)

Notes: The estimation based on the $\{Y/H, H, P\}$ data set treats capital utilization as a latent variable. In the other two estimations, utilization is treated as observable (with measurement error). The data set includes either capacity utilization data from industrial production (TCU) or electricity usage survey data (E).

Figure 1: VAR Responses, Actual versus Simulated Data



Note: The figure depicts 90% credible bands for a VAR(4) (dotted, blue) estimated based on actual data and posterior mean responses for VAR(4)'s estimated on long samples of DSGE model generated observations with $\nu = 0.2$ (red, solid) and $\nu = 2.0$ (green, dashed).