Institution Building without Commitment*

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Abstract

Time inconsistency is a pervasive problem for policymakers, and societies have developed institutions to mitigate their consequences. However, institution building does not happen overnight. This paper proposes a new framework that is tractable even in the presence of state variables and features allocations that gradually transit from that implied by a Markov perfect equilibrium towards one that weighs both short- and long-term concerns, stopping short of the Ramsey outcome. The key elements of this equilibrium concept are: (1) agents are allowed to ignore the history and restart the equilibrium; (2) agents can wait for future agents to start the equilibrium.

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1 Introduction

In this paper we pose an equilibrium concept especially suited for the study of policy settings in macroeconomics, where the time-inconsistency problem is pervasive and the environment has state variables. Our concept builds upon renegotiation proofness, but adapts it to the challenge of dealing with a dynamic, rather than repeated game. We obtain allocations that are vastly superior to those of Markov equilibria, but are not supported by trigger-strategy reversion to dominated outcomes.

We argue that equilibria should satisfy three conditions in environments with a sequence of decision makers that see themselves in a similar spot—a form of stationarity even if there are state variables. The first such condition, a no-restarting condition, is that any outcome should have the property that no decision maker would rather become an earlier member of the decision-making sequence, and it limits the use of trigger strategies as a future punishment. A second condition, a no-delay condition, prevents free riding at the start of the process: no agent can do better by sitting out the system (playing Markov) and waiting for future agents to start on a given equilibrium path and eliminates the intrinsic advantage that is often associated with the first agent. This condition prevents jumping to desirable allocations fast. We interpret the implications of this condition as the need for institutions to slowly earn good will, like earning a reputation for good behavior without need of unobserved types or triggers. Finally, the third condition is an optimality requirement within the class of allocations that satisfy the previous two requirements. An outcome path that satisfies these three conditions is an organizational equilibrium.

The class of environments with time consistency issues that we analyze have typically been modeled as a specific game that has a sequence of decision makers sometimes described as the future selves or future governments. In these games the time zero agent has a special position. We pose our equilibrium concept as a particular refinement of the set of subgame perfect Nash equilibria of this game that tries to deemphasize the preeminent role of the time zero agent. But we also pose an alternative game to model the same class of environments, where any agent has the ability to hide history and become the agent that immediately precedes the time zero agent, effectively eliminating the anointment of a particular agent as the time zero one. We argue that this alternative game eliminates the specificity of the time zero agent, conveying a recursive spirit to the passage of time. We show how for this game, the no-delay condition becomes a necessary condition for any symmetric, Pareto optimal subgame perfect equilibria.

Our notion of equilibrium is closely related to Reconsideration Proofness in Kocherlakota (1996) and to the equilibrium concept for overlapping-generations economies in Prescott and Rios-Rull (2005),
but it allows us to extend the analysis to economies with state variables.\textsuperscript{1} The extension that we introduce is defined for environments that display a weak separability property: preferences can be decomposed between a set of actions that we label “re-scaled actions” and the state of the economy.\textsuperscript{2} Separability allows us to compare actions and outcomes across periods in which state variables are different. We can then impose the three conditions above as a way of selecting among subgame perfect equilibria in the standard implementation of our environment. We show that no restarting is the natural adaptation of Kocherlakota’s symmetry requirement: factoring out the effect of the state variable, the utility that an agent receives from its actions and those of its successors is independent of the past history of play. In contrast, our no-delay condition is related to Prescott and Rios-Rull (2005), although the form it takes here is different. In their overlapping-generations economy, there is an implicit form of no-delay motivated by the presence of an initial elderly cohort which is assumed to have played their best one-shot response before time zero. Here, we argue that explicitly imposing no-delay becomes desirable because it ensures that the coordination that gives rise to the initial equilibrium is not as generous as to tempt the first player to sit it out, play its best one-shot action, and count on the same coordinating mechanism to arise in the future. This is particularly appealing in many situations where state variables are present. Without state variables all symmetric equilibria that achieve the same payoff for the initial player continue to do so for all future players as well, so that there is no reason to choose one over another, but this is no longer the case when a state is present and the way agents coordinate among symmetric outcomes matters.

Under mild conditions, we prove the existence of an organizational equilibrium. While we provide explicit strategic foundations for our equilibrium, it can be computed recursively and directly in terms of the equilibrium path, without the need to specify the complete underlying strategy that supports this outcome. This property makes it much easier to apply to specific macroeconomic problems, particularly where the blend of strategic and competitive elements would otherwise require an intricate description of the agents’ strategies.\textsuperscript{3} The equilibrium converges to a stationary allocation that we refer to as a steady state.\textsuperscript{4} From this steady state, the entire transition path then can be solved recursively. Crucially, due to the no-delay condition, agents’ actions converge only gradually.

\textsuperscript{1}Kocherlakota defines a “state” in his work, but this state only depends on the expectation about current and future actions and is thus purely forward looking. In our case, we define a state as arising from past actions (including possibly past actions of nature, if randomness is present). This is in line with the literature on optimal control and dynamic programming. The role of expectations about current and future actions arises in hybrid environments where some elements of competitive-equilibrium behavior coexist with strategic interactions; we tackle this in Section 5.

\textsuperscript{2}The same type of separability property is also explored in Halac and Yared (2014) and Halac and Yared (2017), but they focus on different SPE refinements where agents’ type is private information.

\textsuperscript{3}Bassetto (2005) describes some of the challenges that such hybrid environments entail.

\textsuperscript{4}We call the state vector of the long-run stationary allocation a steady state, but note that it does not have the property that, if the economy starts there, it will remain there for ever, as sometimes steady state is understood. This is because the equilibrium sequence of actions converges to the constant action that supports this steady state only asymptotically.
The stationary allocation to which our equilibrium converges weighs the concerns over immediate events associated to time inconsistent environments with those later. It has a larger weight into the future than the allocation of the Markov equilibrium but not so much as that implied by the Ramsey solution. It is also easy to calculate and characterize. In fact it is the best constant action from the point of view of the initial decision maker.

On the technical side, while the approach in Abreu et al. (1986, 1990) cannot be adopted to construct the set of continuation values to compute organizational equilibria or reconsideration-proof equilibria, our method exploits some similar ideas in its quest for a recursive representation, which we think may have independent value.

We solve for an organizational equilibrium in two benchmark environments. First, we analyze the well studied growth model with quasi-geometric discounting, which is one of the simplest time-inconsistency problems (just due to the nature of preferences). Less simply, but perhaps more interestingly, we solve for the choice of a government that is financed via capital income taxes (other tax instruments can be characterized in essentially similar ways), an environment subject to time inconsistency previously studied by Klein et al. (2008), among others. In both environments, the equilibrium allocation is much better (Pareto dominates) than that of the Markov perfect equilibrium. The economy slowly moves towards a high/saving or low taxation behavior, that is, the model slowly overcomes the time consistency problem. We interpret this to be a notion of slowly building reputation, without any need for unobservable types. Here what we call reputation is the result of having displayed in the past a form of patience beyond that implied by the behavior in the Markov perfect equilibrium. We think that this type of behavior helps us understand the value that modern institutions such as governments or central banks pose in showing that they have concerns over the long run, and hence do not take actions such as large capital levies or fast inflationary policies that may have been predicted by models where the present is taken to be the initial period. The limiting tax rates predicted by our equilibrium turn out to be a lot more measured than the reckless policies that arise from the standard Markovian equilibrium giving some hope to have reasonable policies as outcomes in environments without commitment and without resorting to trigger-strategies that may or may not be credible ex post.

Our paper is related to the literature that studies macroeconomic environments with time-inconsistency features typically characterized in terms of their Markov equilibria (e.g., Cohen and Michel (1988), Currie and Levine (1993), Krusell and Rios-Rull (1996), Klein and Rios-Rull (2003), Klein et al. (2005), Bassetto and Sargent (2006), Klein et al. (2008), Bassetto (2008), Krusell et al. (2010), Martin (2011), Azzimonti (2011)). It also addresses the type of environments previously studied by posing trigger strategies (Chari and Kehoe (1990), Phelan and Stacchetti (2001)). Our workhorse ex-
ample builds upon the quasi-geometric discounting growth model analyzed by Strotz (1956), Phelps and Pollak (1968), Laibson (1997), Krusell and Smith (2003), Bernheim et al. (2015), Chatterjee and Eyigungor (2016), Cao and Werning (2018), Halac and Yared (2017), among others. Finally, we build on the literature on refinements of subgame perfect equilibrium, particularly in relation to renegotiation proofness (Farrell and Maskin (1989), Kocherlakota (1996), Asheim (1997), Ales and Sleet (2014)).

Other papers that have analyzed dynamic institution building applied to macroeconomic problems include Acemoglu and Robinson (2000), Acemoglu et al. (2012), and Acemoglu et al. (2015). These papers emphasize the role of changing the distribution of power within groups in the context of Markov equilibria as the mechanism that generates slow institutional buildup.

Two other papers are of special relevance. Like us, Nozawa (2018) extends the notion of a reconsideration-proof equilibrium to economies with state variables. However, his extension imposes too strict requirements and leads to nonexistence of an equilibrium in many applications. By relying on weak separability, our approach allows us to define “state-free” notions of the economic environment and to establish existence. Brendon and Ellison (2018) analyze optimal policy in the Ramsey tradition, but they restrict the planner to choosing policies that satisfy a recursive Pareto criterion: this criterion disallows sequences that benefit policymakers in the early periods but are dominated for all policymakers from a given time onward. Like them, we also reject policies that allow early decision makers to dictate future paths that lead to early benefits purely at the expense of future decision makers. Rather than developing an optimality criterion, we propose a solution concept aimed at positive analysis, where implicit cooperation across policymakers at different times builds over time. Because of this different motivation, our “no-restarting condition” is imposed on a period-by-period basis. The presence of state variables causes problems in their environment as well, and our approach based on weak separability could be fruitfully applied there too. Interestingly the allocation that they propose coincides with the steady state to which our equilibrium converges to, the one that maintains behavior constant and is the best one among those.

We start by posing the issues with time-inconsistent preferences in the context of the well understood quasi-geometric discounting growth model with log utility and full depreciation in Section 2. We define organizational equilibrium for separable economies in Section 3, where we also describe the connections to game theory. Section 4 suggests a strategy to study non-separable economies by using suitably approximating separable economies. We study the implications of organizational equilibrium for public policy in environments with time-consistency problems in Section 5, adapting our concept to hybrid settings of competitive and strategic behavior. Section 6 concludes.

Our approach encompasses the more specific cases introduced by Brendon and Ellison in their latest version to account for state variables.
2 A Motivating Example

Our equilibrium concept can be heuristically described as the best among those requiring that:

- in equilibrium no period- \( t \) agent can do worse than any period- \( \tau \) agent for \( \tau < t \) because then it could become a \( \tau \) agent;
- in equilibrium no period- \( t \) agent can do worse than by staying out of the plan and letting the equilibrium unfold as if the economy started in the following period.

To provide the basic intuition, we revisit the canonical growth model with quasi-geometric discounting, log utility and full depreciation and compare what our equilibrium notion implies relative to other standard equilibrium concepts. This example is easy to characterize (it has some closed form solutions), and it allows us to ignore any consideration related to a competitive equilibrium emerging from the interaction with other agents, a case that we analyze in Section 5. More importantly it displays a form of separability that allows us to decompose the rewards of any feasible allocation as a separable function of the initial capital and the subsequent sequence of saving rates. We will exploit this decomposition to provide a way of comparing rewards across agents who may be endowed with different levels of capital.

Assume that the production function is

\[ f(k_t) = k_t^\alpha, \]

and the agent’s period utility function is

\[ u(c_t) = \log(c_t). \]

The relevant state of the economy is capital \( k_t \) with law of motion

\[ k_{t+1} = f(k_t) - c_t. \]

The lifetime utility for the agent at period \( t \) is

\[ u(c_t) + \delta \sum_{\tau=1}^{\infty} \beta^\tau u(c_{t+\tau}). \]

It is easy to see that the agent will disagree with itself in the next period if \( \delta \neq 1 \).

To see the separability property, it is useful to work with saving rates defined as \( s_t = k_{t+1}/k_t^\alpha \). Any sequence of saving rates \( \{s_j\}_{j=0}^{\infty} \), together with an initial capital stock \( k_0 \), implies a sequence of
capital levels \( k_t = k_0^{\alpha_t} \Pi_{j=0}^{t-1} s_j^{\alpha_j} \). The corresponding lifetime utility for the agent in period 0 is

\[
U(k_0, s_0, s_1, \ldots) = \log[(1 - s_0)k_0^\alpha] + \delta \sum_{j=1}^{\infty} \beta^j \log[(1 - s_j)k_j^\alpha] = \frac{\alpha(1 - \alpha\beta + \delta\alpha\beta)}{1 - \alpha\beta} \log k_0 + V(s_0, s_1, \ldots),
\]

where

\[
V(s_0, s_1, \ldots) \equiv \log(1 - s_0) + \frac{\delta\alpha\beta}{1 - \alpha\beta} \log(s_0) + \delta \sum_{j=1}^{\infty} \beta^j \left( \log(1 - s_j) + \frac{\alpha\beta}{1 - \alpha\beta} \log(s_j) \right). \tag{1}
\]

The same logic follows for the period-\( t \) agent, its lifetime utility is the sum of a term that depends on the period-\( t \) capital and a term that depends only on saving rates of periods \( t \) and after. We write it compactly as

\[
U(k_t, s_t, s_{t+1}, \ldots) = \frac{\alpha(1 - \alpha\beta + \delta\alpha\beta)}{1 - \alpha\beta} \log k_t + V(s_t, s_{t+1}, \ldots). \tag{2}
\]

Two relevant implications of separability are that, as of period \( t \), the relative preferences over a sequence of saving rates \( \{s_t, s_{t+1}, \ldots\} \) are independent of the initial level of capital, and also the set of feasible sequences is the same no matter what initial capital is.\(^6\)

Before we discuss the our proposed notion of equilibrium, we first characterize the allocations implied by some commonly used equilibrium concepts, including the Ramsey outcome, the (differentiable) Markov equilibrium,\(^7\) and the best allocation supported by a constant saving rate.

**Ramsey outcome** The assumption that the period-0 agent is able to commit to a particular sequence of saving rates \( \{s_t\}_{t=0}^{\infty} \) chosen at time 0, gives us a useful benchmark. The problem is

\[
\max_{\{s_t\}_{t=0}^{\infty}} u(c_0) + \delta \sum_{t=1}^{\infty} \beta^t u(c_t),
\]

subject to

\[
k_{t+1} = s_t k_t^\alpha, \quad c_t = (1 - s_t) k_t^\alpha, \quad k_0 \text{ given}.
\]

\(^6\)While this example satisfies additive separability, weak separability is sufficient for our results. When the production function is linear in capital, as is the case for an individual who takes the interest rate as given, separability holds for all CRRA utility functions.

\(^7\)More precisely, we look at the Markov equilibrium that is the limit of finite economies. See Krusell and Smith (2003) for details of the trigger-strategy equilibria that can be represented via non-differentiable Markov perfect equilibrium.
The solution to the Ramsey problem can be summarized as

\[
  s_t = \begin{cases} 
    s^R_0 = \frac{\alpha \delta \beta}{1 - \alpha \beta + \delta \alpha \beta}, & t = 0, \\
    s^R = \alpha \beta, & t > 0. 
  \end{cases}
\]

The initial agent discounts the future more heavily than its future selves, so it is willing to apply a lower saving rate than those in the future, \(s^R_0 < \alpha \beta\).

**Markov equilibrium** Another useful equilibrium concept is the Markov equilibrium. Its implied allocation satisfies a generalized Euler equation (GEE). Let \(g(k)\) denote the policy function for tomorrow’s capital \(k’\), the GEE is

\[
  u_c(f(k) - g(k)) = \beta u_c\left( f[g(k)] - g[g(k)] \right) \left[ \delta f_k[g(k)] + (1 - \delta) g_k[g(k)] \right],
\]

which yields a closed form solution for the policy function, \(g(k) = \frac{\alpha \delta \beta}{1 - \alpha \beta + \delta \alpha \beta} k^\alpha\), and a constant saving rate

\[
  s^M = \frac{\alpha \delta \beta}{1 - \alpha \beta + \delta \alpha \beta}, \tag{3}
\]

Note that the saving rate in the Markov equilibrium is the same as the first period’s saving rate in the Ramsey outcome, and \(s^M < \alpha \beta\). Note also that this saving rate is independent of the level of capital: hence, the current player cannot influence the future saving rates, and makes a choice taking those future rates as given. This is a general consequence of separability.

**Best constant savings rate** The Markov equilibrium features a particular constant saving rate, but it may not yield the best payoff compared with other constant saving rates. Suppose that agents are restricted to choose a constant saving rate for themselves and for all future agents; then, the best constant saving rate solves

\[
  \max_s u(c_0) + \delta \sum_{t=1}^{\infty} \beta^t u(c_t)
\]

subject to \(k_{t+1} = s k_t^\alpha, \quad c_t = (1 - s) k_t^\alpha, \quad k_0 \text{ given.}\)

The solution to this problem is given by

\[
  s^B = \frac{\delta \alpha \beta}{(1 - \beta + \delta \beta)(1 - \alpha \beta) + \delta \alpha \beta}, \tag{4}
\]

and it satisfies \(s^B \in (s^M, s^R)\) when \(\delta \in (0, 1)\).
Towards Organizational Equilibrium  Separability makes it easy to discuss the properties of the allocations implied by these equilibrium concepts, and in particular whether any time-$t$ agent would prefer the sequence of savings rates given to another agent. We next explore how the two criteria mentioned at the beginning of this section relate to the previous equilibrium concepts.

In the Ramsey outcome, the initial agent achieves a higher action payoff than any subsequent agent, as $V(s^M, s^R, s^R, \ldots)$ is higher than $V(s^R, s^R, s^R, \ldots)$ and it is feasible for future agents as well. If a time-$t$ agent were to be able to become the initial agent, it would always do so, violating the first criterion in our wish list. Our notion of organizational equilibrium excludes such an allocation as an equilibrium. A similar property would hold in the best subgame perfect equilibrium that can be supported under the threat of reverting to Markov after a deviation; there too the initial agent receives a more favorable treatment than subsequent agents, in the sense that its action payoff is higher.

The best constant saving rate $s^B$ does satisfy our first criterion for an organizational equilibrium, that no agent can do better by switching to an earlier allocation. However, it does not satisfy the second, since $V(s^M, s^B, s^B, \ldots) > V(s^B, s^B, \ldots)$: the initial agent would rather choose $s^M$ and let every subsequent agent choose $s^B$, which amounts to sitting out and letting the equilibrium unfold from the following period.

The Markov equilibrium avoids these issues. First, the Markov equilibrium does not favor the initial agent, as the equilibrium path features a constant saving rate and the action payoff is the same for all agents. Second, when staying out and letting equilibrium unfold the following period, the current agent will choose the Markov saving rate itself, which yields exactly the same payoff. The question is then whether anything else can be better than the Markov equilibrium and still satisfy the two criteria.

The answer is yes. Putting the two criteria together, the organizational equilibrium points to an allocation implied by a sequence of saving rates $\{s^*_0, s^*_1, s^*_2, \ldots\}$, such that

$$ V = V(s^*_t, s^*_t, s^*_t, \ldots) \text{ for all } t, \quad (5) $$

and also

$$ V(s^*_0, s^*_1, s^*_2, \ldots) \geq V(s^M, s^*_0, s^*_1, s^*_2, \ldots). \quad (6) $$

Such allocation satisfies both criteria by construction. We will show in Section 3.4 that the maximum constant value of $\bar{V}$ that can be attained is given by $V(s^B, s^B, s^B, \ldots)$, and the equilibrium path of saving rates increases gradually such that choosing $s^M$ followed by $\{s^*_t\}_{t=0}^\infty$ does not yield a higher utility than $\bar{V}$. 

8
To construct $\{s^\ast\}_{t=0}^\infty$, recall that in our example economy, the action payoff function $V(\cdot)$ is given by equation (1). Together with condition (5), it implies a recursive relationship between $s^\ast_{t+1}$ and $s^\ast_t$

$$\beta(1 - \delta) \log(1 - s^\ast_{t+1}) = \frac{\delta\alpha\beta}{1 - \alpha\beta} \log s^\ast_t + \log(1 - s^\ast_t) - (1 - \beta)V,$$

Following this relationship, there are a continuum of paths that converge to the stationary point, which is given by the best constant saving rate $s^B$. By setting the starting point $s^0$ sufficiently low, the initial agent will find it not attractive to stay out and wait for the equilibrium to unfold. An equilibrium path has the following properties: first, it displays a gradual transition from a relatively low saving rate to a high saving rate $s^B$, as if a good reputation is built over time. Second, along the transition path, the action payoff stays the same as that implied by the best constant saving rate, and is larger than that in the Markov equilibrium.

In the next section, we formalize these notions as the result of a game-theoretical refinement, and we show that the features of this motivating example are general properties of an organizational equilibrium.

### 3 Organizational Equilibrium

This section contains the core of our contribution. After we describe the economic environment (preferences and technology), we setup a standard game in which there is a specific period 0 and the entire history is recorded. In such a game, we define the organizational equilibrium as a subgame perfect equilibrium with a particular refinement. In Section 3.2, we provide an alternative representation of the same economic environment as a different game with incomplete information that eliminates the special role of period 0. In this game, we show that the no-delay condition central to the notion of organizational equilibrium is a requirement for a sequential equilibrium rather than a refinement. In Section 3.3, we prove the general properties of an organizational equilibrium and a recursive method that directly constructs the equilibrium outcome.

Consider a generic environment of sequential decision makers (typically those that have a time-consistency problem) where there is a physical state variable $k \in K$. Specifically, given the current level of $k$, the agent making a decision will choose an action $a$ from a set $A$. The state evolves according to $k_{t+1} = F(k_t, a_t)$. Preferences for an agent making decisions in period $t$ are given by $U(k_t, a_t, a_{t+1}, a_{t+2}, \ldots)$. The first assumption is that functions $U$ and $F$ are independent of calendar time, which allows meaningful welfare comparisons across decision makers.

We restrict the environments that we study to those in which the utility is weakly separable between
the state and the sequence of actions, such that the preference ordering over sequences is independent of the initial state. Formally:

**Assumption 1.**

1. At any point in time \( t \), the set of feasible actions \( A \) is independent of the state \( k_t \);

2. \( U \) is weakly separable in \( k \) and in \( \{a_s\}_{s=0}^{\infty} \), i.e., there exist functions \( v : K \times \mathbb{R} \to \mathbb{R} \) and \( V : A^{\infty} \to \mathbb{R} \) such that

\[
U(k, a_0, a_1, a_2, \ldots) \equiv v(k, V(a_0, a_1, a_2, \ldots)). \tag{8}
\]

and such that \( v \) is strictly increasing in its second argument.

In Section 4 we discuss methods for approximating non-separable economies with separable ones, so that an organizational equilibrium can be computed for the approximate economy.

Sometimes the original problem does not satisfy Assumption 1, but it is possible to re-scale actions in such a way that it does.\(^8\) As an example, the original specification of the saving problem with quasi-geometric discounting does not satisfy Assumption 1 if we define the action to be consumption: the feasible set of consumption levels depends on initial capital.\(^9\) Formally, suppose that the set of feasible actions at any capital level \( k \) is \( \tilde{A}(k) \subseteq \tilde{A} \) and that preferences are given by \( \tilde{U}(k_t, \tilde{a}_t, \tilde{a}_{t+1}, \tilde{a}_{t+2}, \ldots) \). Our construction still applies as long as it is possible to find a set of actions \( A \) and a function \( \gamma \) such that \( \tilde{a} = \gamma(a, k) \) and that Assumption 1 holds for \( A \), where

\[
U(k, a_t, a_{t+1}, a_{t+2}, \ldots) \equiv \tilde{U}(k, \tilde{a}_t, \tilde{a}_{t+1}, \tilde{a}_{t+2}, \ldots),
\]

and where for \( t \geq 0 \), \( \tilde{a}_t \) is computed recursively as

\[
\begin{align*}
\tilde{a}_t &= \gamma(a_t, k_t), \\
k_{t+1} &= F(k_t, \tilde{a}_t).
\end{align*}
\tag{9}
\]

### 3.1 The Standard Game

The general setup that we have described is typically modeled as the following game. There is an infinity of players indexed by the time at which they act, \( \{0, 1, \ldots\} \), each of whom has preferences

\(^8\)Notice also that any one-to-one transformation of \( a \) will preserve weak separability, so the action space is only defined up to such transformations. As an example, for the case analyzed in Section 2, any monotone transformation of the saving rate would be an equally valid action, yielding the same equilibrium outcomes.

\(^9\)Note that weak separability automatically fails if certain actions are only feasible for some levels of capital, since, holding actions fixed, the left-hand side of (8) would then be well defined for some values of \( k \) and not for others.
given by (8). At each time $t$, the history of play is given by $h^t := (a_0, a_1, \ldots a_{t-1})$, with $h^0 := \emptyset$. A strategy $\sigma_t$ for player $t$ is a mapping from the set of time-$t$ histories, $H^t$, to the set of actions $A$. A strategy profile is a sequence of strategies, one for each player: $\sigma := (\sigma_0, \sigma_1, \ldots)$. As usual, it is also convenient to define a continuation strategy after history $h^t$, $\sigma|h^t$, represented by the restriction of $(\sigma_1, \sigma_{t+1}, \ldots)$ to the histories following $h^t$. From any history $h^t$, a strategy profile induces a sequence of future actions, which we denote through the following short-hand notation:

$$a_{t+1}|h^t := \sigma_{t+1}(h^t, \sigma(h^t)), \quad a_{t+s}|h^t := \sigma_{t+s}(h^t, a_{t+1}|h^t, \ldots, a_{t+s-1}|h^t).$$

Starting from the set of subgame-perfect equilibria of this game, we limit the equilibria in our analysis by first imposing the following refinements:

**Requirement 1** (State Independence). We limit attention to equilibria in which the strategies followed by all players are independent of the state $k$.

**Requirement 2** (No-restarting and optimality). We limit attention to equilibria such that:

- they are symmetric, in that the action payoff

  $$V(a_{t+1}|h^t, a_{t+2}|h^t, \ldots)$$

  is the same after any history of play;

- No other symmetric state-independent equilibrium exists that attains a higher payoff.

In the absence of a state variable (when preferences are independent of $k$), Requirement R2 corresponds to Kocherlakota’s (1996) definition of reconsideration-proof equilibrium.

Our equilibrium extends the notion of reconsideration proofness to dynamic games, rather than purely repeated games. In the presence of a state variable, we assume that players coordinate on strategies that only depend on the history of play $h^t$ and not on the physical state. Weak separability plays an important role in this selection criterion: it ensures that the same actions can indeed be played independently of the current value of the state, and that each player’s preferences over current and future actions are independent of the state. This ensures that, if $\sigma|h^t$ is a subgame-perfect equilibrium for a history that attains some level of the state $\bar{k}$, it is also a subgame-perfect equilibrium for any other history, even though the initial level of capital may be different from $\bar{k}$. In this case, an organizational equilibrium imposes symmetry only in that the payoff of the subutility $V$ is independent of the history of play, but the payoff of each time-$t$ player is still different across
histories which lead to different levels of the state. Intuitively, a different state implies a different set of possible utility levels going forward, so we should expect it to affect payoffs in the subgames going forward. However, this dependence of utility from the state takes a simple form under weak separability, and there is a natural mapping across histories with different levels of capital: the same sequences of actions are possible under any level of capital, and the preferences of player $t$ over the sequences from date $t$ on are also represented by the subutility $V$, independent of $k_t$. For this reason, imposing reconsideration proofness on preferences represented by $V$ alone is appealing.

It is useful to compare our notion to previous attempts at dealing with state variables in this context. An extension of reconsideration proofness to environments with state variables was proposed by Nozawa (2018). Nozawa requires weakly reconsideration-proof equilibria to be such that the equilibria of all subgames share the same payoff function $\Psi(k)$, which depends on the state; in the absence of the state, this reduces to Kocherlakota’s (1996) symmetry requirement. A strong reconsideration-proof equilibrium is then an equilibrium in which $\Psi(k)$ is undominated by any other equilibrium point by point. This is often too strong a requirement, and hence existence may fail. Our approach avoids this problem because symmetry is defined by a single utility level $\bar{V}$, namely the action payoff attained by each agent, rather than a function. This is possible because weak separability allows us to extend this single level to the complete payoff (which remains a function of the state) by setting it equal to $v(k, \bar{V})$.

An alternative approach adopted in the past is revision proofness, which was introduced by Asheim (1997) and made explicit as a game in Ales and Sleet (2014). In their papers, a larger class of credible punishments is allowable. Specifically, under reconsideration proofness, if $\Sigma$ is the set of equilibrium strategies of the game, each player at any time $t$ is allowed to coordinate current and future play to its favorite element of $\Sigma$. Under revision proofness, player $t$’s coordination power is limited because it is required to propose deviations from the equilibrium path of play that benefit all future players. The resulting equilibrium set is much larger. For the case of quasi-geometric discounting with linear preferences, Ales and Sleet (2014) show that all subgame-perfect paths better than the Markov equilibrium are revision proof. In environments with state variables, a limitation of revision proofness is that it is unclear how a future player could “block” a revision proposal when it would inherit a different state under the revision proposal and would thus not be able to continue with the original strategy. Our notion of organizational equilibrium retains the unilateral aspect of deviations from reconsideration proofness, but it relies on weak separability to define and impose symmetry across different levels of capital.

To prove existence of an equilibrium satisfying Requirements 1 and 2, we make the following technical

\(^{10}\)As an example, no reconsideration-proof equilibrium would exist in the example of Section 2.
assumption:

**Assumption 2.** 1. A is a convex compact subset of a locally convex topological linear space with topology $\rho_x$.  
2. $V$ is quasiconcave over $A^\infty$.  
3. $V$ is continuous over $A^\infty$ with respect to the product topology $\rho^\infty_x$.  

**Proposition 1.** Under Assumptions 1 and 2, there exists a subgame-perfect equilibrium of the game that satisfies Requirements 1 and 2.

**Proof.** Assumption 2 follows Kocherlakota (1996), who uses it in Proposition 4 to prove that a reconsideration-proof equilibrium exists for the game whose period-$t$ payoff is $V(a_t, a_{t+1}, a_{t+2}, ...)$. The strategies of such a game represent a subgame-perfect equilibrium of our game with a state variable: weak separability implies that the state does not affect the preference ordering of each player over the sequence of future actions. Moreover, these strategies satisfy Requirements 1 and 2 by the definition of a reconsideration-proof equilibrium. □

There can be many such equilibria. In fact, in Section 2, Requirements 1 and 2 imply that saving rates must satisfy the difference equation (7), but they do not provide an initial condition $s_0$. We wish to push our selection further, and capture the idea that time inconsistency is not overcome all at once out of the blue, but rather that intertemporal cooperation takes time to build. If players coordinated to start from a high degree of cooperation from the beginning, there might be an incentive for a player to defect and hope that such a coordination takes place in the future. We approach this reasoning in a formal way by altering the game in Section 3.2, but for now we simply add as a third requirement that the first player has no incentive to deviate from the equilibrium path if the threat is that the same equilibrium will be played starting from the next period, as if the initial period had not taken place. Formally:

**Requirement 3** (No Delay). Let $\sigma$ be a subgame-perfect equilibrium strategy profile satisfying Requirements 1 and 2. Then, for each possible action $a \in A$,

$$V(a_0, \sigma, a_1, \sigma, a_2, \sigma, ...) \geq V(a, a_0, \sigma, a_1, \sigma, ...)$$

We are now ready to define an organizational equilibrium:

**Definition 1.** An organizational equilibrium is the outcome of any subgame-perfect equilibrium of the game described above that satisfies Requirements 1, 2, and 3.
We choose to define an organizational equilibrium as the outcome of an equilibrium in a game-theoretic sense. In Proposition 4 and Corollary 2 below, we provide a way of characterizing this outcome directly, without describing the underlying strategies that support it. This makes it easier to apply our notion in a macroeconomic context: just as a competitive equilibrium can be defined as a single path, without reference to the underlying strategies, an organizational equilibrium also involves only the description of the path. At the same time, our paper provides the full strategic foundations that support such an equilibrium path, and the interested reader can then apply our proofs to construct them, if desired.

In order to prove existence of an organizational equilibrium, we use the following additional sufficient condition:

**Assumption 3.** $V$ is weakly separable in $a_0$ and $\{a_s\}_{s=1}^{\infty}$, i.e., there exist functions $\tilde{V}: A \times \mathbb{R} \to \mathbb{R}$ and $\hat{V}: A^\infty \to \mathbb{R}$ such that, for all sequences $(a_0, a_1, a_2, \ldots) \in A^\infty$,

$$V(a_0, a_1, a_2, \ldots) = \tilde{V}(a_0, \hat{V}(a_1, a_2, \ldots)), \quad (10)$$

with $\tilde{V}$ strictly increasing in its second argument.

Assumption 3 implies that a player’s preference ordering over the actions of future players is independent of its own choice: what player 0 views as “desirable” or “undesirable” future actions does not depend on its own choice. Although this is only a sufficient and not a necessary condition for existence of an organizational equilibrium, our Requirement 3 makes more sense when it is satisfied: specifically, this assumption implies that there is a clear way of defining what “cooperation” means, because the preferences of past players over the actions of the current and future players are not tied to the choices that those past players made. In our hyperbolic discounting, past players would like future players to adopt a saving rate which is above the Markovian saving rate and closer to the long-run Ramsey outcome, and this is independent of what they themselves chose. It is in this context that the vague notion of “gradual development of cooperation” can be given formal meaning.

**Proposition 2.** Under Assumptions 1, 2 and 3, an organizational equilibrium exists.

**Proof.** Let $(a_0^E, a_1^E, \ldots)$ be the outcome of a reconsideration-proof equilibrium for the game whose period-$t$ payoff is $V(a_t, a_{t+1}, a_{t+2}, \ldots)$, and let $\tilde{V}$ be its associated value. This means that, for any period $t$ and any actions $a \in A$, there exists a continuation sequence $(a_{t+1}^E, a_{t+2}^E, \ldots)$ which is also the outcome of a reconsideration-proof equilibrium and is such that

$$V(a_t^E, a_{t+1}^E, a_{t+2}^E, \ldots) \geq V(a, a_{t+1}^E, a_{t+2}^E, \ldots). \quad (11)$$

14
We then have
\[
V(a, a_{t+1}^E, a_{t+2}^E, \ldots) = \hat{V}(a, \hat{V}(a_{t+1}^E, a_{t+2}^E, \ldots)).
\]

Acknowledging that the sequence \((a_{t+1}^E, a_{t+2}^E, \ldots)\) is potentially a function of the deviation \(a\) (as well as of time \(t\), which we can hold fixed), define
\[
\hat{V} := \inf_{a \in A} \hat{V}(a_{t+1}^E, a_{t+2}^E, \ldots). \tag{12}
\]

By the compactness of \(A\), Tychonoff’s theorem, and continuity of \(\hat{V}\), we can find a sequence of actions \(a_0^*, a_1^*, \ldots\) that attains the infimum in equation (12) above. Exploiting Assumption 3, this sequence ensures subgame perfection and satisfies the no-restarting condition (Requirement 3):
\[
V(a_0^*, a_1^*, a_2^*, \ldots) \geq V(a, a_0^*, a_1^*, \ldots).
\]

This path attains the value \(\hat{V}\), so that it continues to satisfy the optimality condition of Requirement 2. Hence, playing \((a_0^*, a_1^*, \ldots)\) followed by a restart after any deviation is an equilibrium that satisfies Requirements 1, 2, and 3, and therefore \((a_0^*, a_1^*, \ldots)\) is an organizational equilibrium.

\[\square\]

3.2 An Alternative Game where Period 0 is not Special

We wish to go one step further and formalize the notion that intertemporal cooperation is fostered by the emergence of “good institutions,” or “good norms.” To do so, we build upon the game above, but we modify it so as to make sharper predictions about the start of play. In this alternative game, every agent has the ability to erase history and become the agent in period “minus one,” effectively letting the agent in the following period become the period zero agent. This game is another representation of environments with time consistency problems, that are our objects of interest, provided that we are willing to entertain that agents can actually erase history, or at least, provide a clean separation from their past.

More generally, our game pins down what do we mean by “the first period,” an issue generally ignored in the literature. We accomplish that by giving any agent the option to either go along with whatever time index it has from the past or to become the (or better, a) time zero agent. This approach has a recursive flavor in the sense there is nothing special to the timing of birth of any particular agent and we formalize it below. We think that it has various attractive features and an unattractive one: it allows us to use the powerful tools of dynamic games, while at the same time preventing any specific agent to be the special time-zero agent; it provides a natural justification for the no-delay condition embedded in Requirement 3, which emerges naturally as a requirement for a sequential equilibrium; finally, it provides a rationale for the unorthodox name (organizational) of the equilibrium concept.
since an organization consists of an ongoing, uninterrupted set of agents that choose to go along with a plan rather than set up their own organization. The unattractive feature of our notion of the meaning of period zero is that it requires the make-believe assumption that subsequent agents can forget the previous history whenever one of them chooses to become the type-zero agent. Such assumption may not sound appealing in a literal interpretation of the problem of an agent with time-inconsistent preferences, but we think it is less so when we think of collections of governments and their possible explicit choices of breaking with the past, claiming that they do not share anything with previous governments, which we take as becoming agent 0 or, simply, restarting history.

We choose to present this alternative game in a separate section because this interpretation or design of the strategic interaction is not strictly necessary to develop our equilibrium notion, but it makes the no-delay condition not yet one more refinement of subgame perfection, but a necessary property of a sequential equilibrium.

Formally, the game of Section 3.1 is modified as follows. We now assume that the actions of past players remain unobservable to the current player until an “organization” is set up to record past play. The opportunity to set up an organization arrives at a stochastic point in time \( t \), where the probability distribution over the time of arrival is unrestricted, except that it is assumed to have full support over \( \mathbb{N} \); the precise time is unobserved by the players, who can only know whether setting up an organization is possible when they are called to play. In sum, let \( \hat{t} \) be the time at which the opportunity to set up record keeping emerges. For \( t < \hat{t} \), players do not observe past play and choose an action \( a_t \) that cannot be conditioned on \( (a_0, ..., a_{t-1}) \).\(^{11}\) In each subsequent period \( t \geq \hat{t} \), if no record-keeping organization is in place, player \( t \) can start one, so that player \( t + 1 \) will be able to condition its actions on player \( t \)'s choice \( a_t \). This choice is taken without knowing whether the opportunity was available in the past, or whether it newly arrived in period \( t \). If record-keeping has been in place since a period \( \tilde{t} \in [\hat{t}, t) \), player \( t \) can choose to continue the current organization, so that player \( t + 1 \) can condition its actions on \( (a_{\tilde{t}}, ..., a_t) \), or it can start a new organization, in which case only \( a_t \) is known to player \( t + 1 \), or it can discontinue the current organization without replacing it, in which case player \( t + 1 \) cannot condition its actions on any of the past actions \( (a_0, ..., a_{t-1}) \).\(^{12}\)

With the limitations on record-keeping described above, the game unfold otherwise as in Section 3.1, with each player at time \( t \) choosing an action \( a_t \in A \) (after making a record-keeping choice, if a choice is available). The preferences and the evolution of the state are the same as in Section 3.1. To quickly distinguish between the two games, we will from now on refer to the game of Section 3.1

\(^{11}\) We continue to only consider equilibria in which strategies are independent of the state variable, which rules out inferring past play through this indirect channel.

\(^{12}\) Even in this case, player \( t \) does not know if the opportunity to set up an organization first appeared in period \( \tilde{t} \), or was available in earlier periods but was not taken up, or it was taken up but discontinued by earlier players.
as the game where record-keeping starts at time 0, and the game just described here as the game where history can be hidden. The introduction of incomplete information requires switching from considering subgame-perfect equilibria to sequential equilibria.\textsuperscript{13} We relegate the notation defining histories, information sets, and strategies to Appendix A.

Since this new game involves uncertainty, in order to find equilibria where strategies are independent of the state, we need to strengthen Assumption 1:

**Assumption 4.**

1. At any point in time $t$, the set of feasible actions $A$ is independent of the state $k_t$;

2. There exist functions $\bar{v}: K \times \mathbb{R}_{++}$, $\bar{\bar{v}}: K \times \mathbb{R}$, and $V: A^\infty \rightarrow \mathbb{R}$, such that

$$U(k, a_0, a_1, a_2, \ldots) \equiv \bar{v}(k)V(a_0, a_1, a_2, \ldots) + \bar{\bar{v}}(k).$$

This assumption is satisfied throughout all of our examples.

**Proposition 3.** Consider a state-independent sequential equilibrium that satisfies Requirement 2 from period $\hat{t}$ on. Such an equilibrium exists under Assumptions 2 and 4. Let $\hat{t}$ be the realization of the (random) first time in which record keeping is possible, and let $(a_{\hat{t}}, a_{\hat{t}+1}, a_{\hat{t}+2}, \ldots)$ be the path implied by the equilibrium, conditional on $\hat{t}$. Then $(a_{\hat{t}}, a_{\hat{t}+1}, a_{\hat{t}+2}, \ldots)$ is an organizational equilibrium.

**Proof.** See Appendix A. \qed

Notice that we do not redefine an organizational equilibrium in this section; rather, we prove that the same definition that we used in Section 3.1 also describes the equilibrium path of this new game from the point at which record keeping becomes possible.

### 3.3 Equilibrium Properties

We are interested in macroeconomic applications of the notion of organizational equilibrium. For these applications, keeping track of strategies is cumbersome. It would be desirable to characterize an organizational equilibrium directly in terms of the equilibrium sequence, without fully specifying

\textsuperscript{13}Note however that we do not need to keep track of beliefs within an information set. This is for two reasons: first, no future player will be able to distinguish between histories that are in the same information set at time $t$, which means that their actions will be the same independently of the specific node within the current information set; second, player $t$'s payoff conditional on current and future actions and on the state is also independent of the specific node within the information set.
the supporting strategies; this is similar to the way in which competitive equilibria are defined in macroeconomic models, which is also purely in terms of sequences of actions (and prices). In this section, we develop conditions that allow us to do this, and that provide further properties of the equilibrium allocation.

The following proposition proved in Appendix B provides a first step:

**Proposition 4.** Let Assumption 1 hold. A sequence \( \{\bar{a}_t\}_{t=0}^{\infty} \) that satisfies the following properties is an organizational equilibrium:

1. **No-restarting:**
   \[
   V(\bar{a}_t, \bar{a}_{t+1}, \bar{a}_{t+2}, ...) = \bar{V} \quad \forall t \geq 0;
   \]

2. **Optimality:** No other sequence satisfying no-restarting achieves a higher constant value;

3. **No-delay:**
   \[
   V(\bar{a}_0, \bar{a}_1, \bar{a}_2, ...) \geq \max_a V(a, \bar{a}_0, \bar{a}_1, ...).
   \]

Furthermore, if a sequence satisfying the three properties above exists, then all the organizational equilibria satisfy the same conditions.

When a sequence that satisfies the three properties of Proposition 4 can be found, we have a way of characterizing organizational equilibria directly in terms of sequences. We are also interested in establishing the converse: specifically, that any organizational equilibrium is a sequence that satisfies the three properties of Proposition 4 (or, equivalently, that a sequence satisfying the 3 properties exists). To do so, we require further assumptions. The same assumptions are also instrumental in establishing that some of the properties that we observed in the simple example of Section 2 are true more generally.

In Section 3.1, we introduced some structure on the action payoff function \( V \) through Assumption 3 as a sufficient condition for the existence of an organizational equilibrium. Assumption 3 implies that the current action payoff is determined by the current action and a scalar sufficient statistic for the sequence of future actions. Our next assumption imposes a recursive structure on this sufficient statistic.\(^{14}\)

---

\(^{14}\)Compared to Assumptions 1 and 3, Assumption 5 features the same function \( \hat{V} \) both on the left and right-hand side: this is what allows us to use recursive methods. Note: I am not sure this footnote is needed, but Victor complained about lack of intuition, so please advise if this makes things better or worse.
Assumption 5. Let \( \hat{V} \) be defined as in Assumption 3. There exists a function \( W: A \times \mathbb{R} \to \mathbb{R} \), increasing in the second argument, such that, given any sequence \( \{a_t\}_{t=0}^{\infty} \in A^\infty \),
\[
\hat{V}(a_0, a_1, a_2, \ldots) \equiv W \left( a_0, \hat{V}(a_1, a_2, \ldots) \right).
\] (14)

This assumption of course holds in the example of Section 2 and in many other applications of economic interest. As is often the case, a recursive structure is instrumental in constructing equilibria in infinite-horizon economies in which backward induction cannot be applied. Note however that the recursion applies only on preferences about future actions: \( a_t \) enjoys a special role in the preferences of player \( t \), which is the essence of the time-consistency problem.

We then obtain (again proved in Appendix B):

**Proposition 5.** Under Assumptions 2, 3, and 5, there exists an organizational equilibrium \( \{a_t\}_{t=0}^{\infty} \) which is recursive in the value \( \hat{V}(a_t, a_{t+1}, a_{t+2}, \ldots) \): that is, there exists a function \( g: \mathbb{R} \to A \times \mathbb{R} \) such that \( (a_t, v_{t+1}) = g(v_t) \), and \( v_t = \hat{V}(a_t, a_{t+1}, a_{t+2}, \ldots) \) for all \( t = 0, 1, \ldots \).

Proposition 5 uses values as a state variable in ways similar to Abreu et al. (1986, 1990) (APS). However, as the proof shows, constructing the set of possible values is considerably more involved than in the case of APS. In APS, the set of equilibrium values can be obtained by starting from a large (convex) set of possible continuation values, and iterating backward through a monotonically shrinking sequence of sets until convergence. In our case, the symmetry property of organizational equilibria imposes that continuation equilibria must share the same value as the equilibrium of the game from time 0. This equality constraint breaks the APS procedure and requires a more complicated argument.

Proposition 5 shows that organizational equilibria have a recursive structure, but does not provide a direct way of computing them. This is achieved by the following results proved in Appendix B:

**Proposition 6.** Under Assumptions 2, 3, and 5, the action payoff of an organizational equilibrium coincides with the maximum value that \( V(a, a, a, \ldots) \) can attain under a constant action. \( \hat{V}(a_t, a_{t+1}, a_{t+2}, \ldots) \) is increasing over time for any organizational equilibrium, and it converges to the value associated with the constant action profile that maximizes \( V(a, a, a, \ldots) \). Furthermore, if \( \hat{V} \) is a strictly quasiconcave function and the steady state that maximizes \( V(a, a, a, \ldots) \) is not a Markov equilibrium, then the initial value of \( \hat{V}(a_0, a_1, a_2, \ldots) \) is strictly below \( \hat{V}(a, a, a, \ldots) \): convergence is not immediate.

**Corollary 1.** Under Assumptions 2, 3, and 5, the payoff of an organizational equilibrium is below that of the Ramsey outcome, which is the allocation that maximizes \( V(a_0, a_1, \ldots) \). The inequality is
strict, except in the case in which the Ramsey outcome is attained by a constant allocation (in which case the constant allocation also represents a Markov equilibrium). The payoff of an organizational equilibrium is weakly better than that of the best (state-independent) Markov equilibrium, and strictly better whenever no Markov equilibrium attains the payoff of the best constant allocation.

Corollary 1 shows that an organizational equilibrium always has an intermediate payoff between the Ramsey outcome and the best Markov equilibrium. The three notions yield the same payoff only when the Ramsey outcome can be attained by a Markov equilibrium: in this case, there is no time consistency problem, because player \( t \) has an incentive to choose the Ramsey allocation if it believes that future players will also do the same. Whenever time consistency has bite, an organizational equilibrium falls short of the Ramsey outcome.

Armed with these results, we conclude this section by completing the proof that we can characterize organizational equilibria purely by looking at sequences of actions, without a full specification of strategies (again proved in Appendix B):

**Corollary 2.** Under Assumptions 2, 3, and 5, there exists a sequence \( \{\bar{a}_t\}_{t=0}^{\infty} \) that satisfies Properties 1, 2, and 3 of Proposition 4.

Proposition 6 and Corollary 2 show that the procedure we used to compute organizational equilibria in Section 2 applies more generally. Specifically, we first compute a constant action profile that maximizes \( V(a, a, a, ...) \): this is what would be chosen by the decision maker at time 0 if future players were committed to take the same action. This maximization yields a value \( V^* \), which must remain constant along the path, i.e., \( V(a_t, a_{t+1}, a_{t+2}, ...) = V^* \). The proof of Proposition 6 shows how to exploit the recursive structure implied by Assumption 5 to construct other sequences that attain the value \( V^* \); these sequences converge to the constant action profile \( (a, a, a, ...) \). Corollary 2 shows that (at least) one of these sequences satisfies the no-delay condition (which, in sequence form, is Property 3 of Proposition 4). This sequence is also characterized as that which minimizes \( \hat{V}(a_0, a_1, ...) \) among all sequences that have value \( V(a_t, a_{t+1}, a_{t+2}, ...) = V^* \). Finally, Proposition 6 proves that the convergence to the constant action profile is not immediate, unless we are in a special case in which the steady state can be supported in a Markov equilibrium with no intertemporal incentives.

### 3.4 Application to the Growth Model

We now apply the results that we derived for a generic case to the specific application of Section 2. By inspecting Equations 1 and 2, we can verify that our preferences satisfy Assumptions 1, 3, 4, and 5. Assumption 2 is satisfied if we pick an arbitrarily small \( \epsilon > 0 \) and we impose a minimum
saving rate $\epsilon$ and a maximum saving rate $1 - \epsilon$.\(^\text{15}\) Following Proposition 4 and Corollary 2, we compute an organizational equilibrium by directly looking at sequences that satisfy the properties of no-restarting, optimality, and no-delay of Proposition 4.

First, in order to attain a constant action value, as implied by the no-delay condition, the sequence of saving rates must satisfy the difference equation (7). From Proposition 5, we know that the solution has a recursive structure. In this case, instead of writing the recursion in terms of continuation values, it is more convenient to write it directly in terms of the saving rates that will be undertaken. Accordingly, using (7), we define

$$q(s; V) := 1 - \exp \left\{ \frac{-(1 - \beta)\bar{V} + \frac{\delta \alpha \beta}{1 - \alpha \beta} \log s + \log(1 - s)}{\beta (1 - \delta)} \right\}, \quad (15)$$

so that equation (7) can be rewritten as $s_{t+1} = q(s_t; \bar{V})$. The blue lines and the red line in Figure 1 represent this difference equation under different values for $\bar{V}$.

**Figure 1:** Evolution of the Saving Rate for Sequences that Attain a Constant Value

The optimality property is satisfied by the solution that achieves the highest possible action payoff; by Proposition 6 this is the payoff attained by the action $s$ that maximizes $V(s, s, s, ...)$, which is

\(^{15}\)We could prove that these bounds do not affect our results when $\epsilon$ is sufficiently close to zero. Intuitively, while household preferences are not time consistent, the degree of time inconsistency is limited, so that the player moving at time $t$ would not want to consume all of its endowment and at the same time it would not want future players to save all of theirs. We omit a formal analysis of these bounds for brevity.
given by (4). This is represented by the red line in Figure 1 and is associated with a value that we define \( V^* \).\(^{16}\)

Even after we pin down \( \bar{V} = V^* \), the difference equation (7) admits a continuum of solutions, indexed by the initial condition. From the perspective of the agent who makes the proposal, all of these sequences of saving rates yield the same payoff; however, they yield different payoffs from the perspective of future agents. As an example, consider the following two proposals: the first one is \( s_t = s^B \) for all \( t \), and the second one starts from \( s_0 = s^M \) and subsequently follows a sequence dictated by the difference equation (7). Both of these sequences imply the same action payoff \( V(s_0, s_1, \ldots) = V(s^B, s^B, \ldots) = V^* \) for the agent who makes the proposal, as a higher saving rate today is rewarded with higher saving rates in the future. The no-delay condition restricts the range of initial conditions that correspond to an organizational equilibrium; \( s_0 \) must be such that \( V(s^M, s_0, s_1, \ldots) \leq V^* \), so that the initial agent would not gain from waiting for the sequence to start in the next period.\(^{17}\) To guarantee the no-delay condition is satisfied, the sequence of saving rates has to start with a low enough point to prevent the initial agent from sitting out; in Figure 1, this is no higher than the point \( s_0 \), which corresponds to \( q(s^M; V^*) \). This condition excludes the possibility of jumping to the steady-state saving rate immediately, and a gradual transition has to take place.

While there are many organizational equilibria, all of which give the same utility to the first generation, the equilibrium in which

\[
 s_0 = q(s^M; V^*)
\]

yields the highest total utility for the subsequent generation by delivering the most capital. We select this one because it is the natural outcome if there is an arbitrarily small amount of altruism involved.\(^{18}\)

In the organizational equilibrium, time inconsistency is gradually overcome through time: at least from period 1 on, the saving rate exceeds that of the Markov equilibrium, and a virtuous cycle is started, with a monotonic increase which converges to \( s^B \). Initial saving is limited by the temptation to let the next generation start the virtuous cycle. This temptation diminishes in subsequent periods,

\(^{16}\)For points to the right of the steady-state saving rate, the difference equation implies a sequence of saving rates converging to 1. This would violate the upper bound \( 1 - \epsilon \), so eventually no solution would be possible. Even if we ignore the bound, this solution violates the transversality condition and yields infinitely negative utility rather than \( V^* \). When a higher value than \( V^* \) is used in the difference equation, there is no fixed point and all the solutions of the difference equation violate the transversality condition.

\(^{17}\)In general, the no-delay condition requires \( \max_s V(s, s_0, s_1, \ldots) \leq V(s_0, s_1, \ldots) \). The fact that this maximum is always attained by \( s^M \) independent of the sequence is due to the specific functional-form assumptions of our example.

\(^{18}\)For the same reason, a fortiori we neglect saving rates below the Markov saving rate. Technically, such saving rates would also represent organizational equilibria, up to a lower bound which is the second solution to the equation \( V(s^M, s_0, s_1, \ldots) = V^* \), with the sequence \((s_0, s_1, \ldots)\) satisfying (7).
since restarting the virtuous cycle from scratch implies giving up on the accumulated effect of previous increases in $s_t$. Note that $s^R$ is below the long-term savings rate of the Ramsey outcome, no matter how close to 1 $\delta$ is (as long as it is strictly less than 1): while the equilibrium path converges to the Ramsey outcome as $\delta \to 1$, it never coincides with it, and the folk theorem does not apply.

**Comparison with other equilibria**  We now compare the properties of the sequence of capital stocks and the lifetime utilities in the organizational equilibrium with those in the Ramsey outcome and the Markov equilibrium.

We first turn to the transition paths of different equilibria. We assume that the initial capital stock is the steady state capital stock in the Markov equilibrium, i.e., $k_0 = k^M$. The first row of Figure 2 displays the transition paths for the saving rate $s_t$ and capital $k_t$. In the Markov equilibrium, the capital stock remains unchanged at the steady-state level that we assumed as a starting point. The Ramsey outcome features the same saving rate as the Markov equilibrium in the first period, so that the capital stock remains the same at the beginning of the second period. From the second period onwards, the saving rate increases to $s^R$ permanently. The sequence of saving rates in the organizational equilibrium is induced by the transition function $s_{t+1} = q(s_t; V^*)$. Particularly, the saving rate in the first period is $s_0 = q(s^M; V^*) > s^M$, and the capital is initially higher than in the Ramsey allocation. Over time, the saving rates increase gradually and converge to $s^* < s^R$. Asymptotically, capital in the organizational equilibrium settles between the Ramsey outcome and the Markov equilibrium.

Now we turn to the welfare comparison. Given a particular sequence of saving rates $\{s_\tau\}_{\tau=0}^\infty$, based on the analysis in the last section, the lifetime utility for generation $t$ can be written as

$$U_t(k_t, \{s_\tau\}_{\tau=0}^\infty) = \frac{\alpha(1 - \alpha\beta + \delta\alpha\beta)}{1 - \alpha\beta} \log k_t + V_t.$$  

The total payoff $U_t$ and the action payoff $V_t$ are depicted in the second row of Figure 2. The total payoff in the Markov equilibrium is the lowest during the entire transition, which is the result of both the lowest capital stock and action payoff. The comparison between the Ramsey outcome and the organizational equilibrium is more subtle. In the first period, the total payoff in the Ramsey outcome is higher than that in the organizational equilibrium: this has to happen by definition, since the Ramsey outcome maximizes the total payoff from the perspective of period 0. In the following period, the comparison reverses, and the total payoff in the organizational equilibrium is actually higher than the Ramsey outcome. This happens both because the initial generation accumulates additional capital, and because the organizational equilibrium does not impose as high a saving rate,

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19 For the numerical computation, we use the following parameter values: $\alpha = 0.4$, $\beta = 0.96$, and $\delta = 0.9$.
allowing for some indulgence for the short-run impatience that arises in the second period. Our notion of organizational equilibrium treats initial capital as a bygone, factoring it out of the payoff that is relevant in computing the equilibrium itself; however, it captures the notion that the initial agent is not privileged compared to future decision makers and cannot impose on them sacrifices that it has not undertaken. For this reason, when we focus on $V_t$, an organizational equilibrium redistributes from the initial agent to all future decision makers. When comparing the total payoff, after period 0, early decision makers benefit both from a higher capital level and a higher action payoff, while eventually capital falls below the Ramsey outcome and late generations lose from this.
In terms of the steady-state capital level, Figure 3 shows how it changes with $\delta$. In the Ramsey outcome, the stationary allocation is independent of $\delta$, and we normalize it to 1. As can be seen from the Figure, as $\delta$ decreases, the capital stock in the Markov equilibrium decreases faster than that in the organizational equilibrium.

4 Equilibrium of Approximating Economies

The assumption of weakly separable utility is restrictive. In this section, we propose a strategy to study organizational equilibrium for economies where this assumption is not satisfied. Our approach is to look at an economy that is weakly separable and similar in a particular metric to the original one and then study organizational equilibrium in this alternative economy. This strategy has a strong tradition in macroeconomics where little (if anything) is known about recursive equilibrium in distorted economies that do not have a particular functional form. Consequently, the equilibrium is computed for a similar economy in a certain sense (see Kubler and Schmedders (2005) and Kubler (2007) for related discussions).

For brevity, we limit our analysis to a more specific class of economies, in which the action and the state are univariate and a recursive structure is present from the outset. Specifically, we assume the

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$20$We retain $\alpha = 0.4$ and $\beta = 0.96$. 

preferences for agent in period $t$ have the following recursive formulation

$$U(k_t, a_t, a_{t+1}, \ldots) = P(k_t, a_t) + Q(k_{t+1}, a_{t+1}) + \beta U(k_{t+1}, a_{t+1}, a_{t+2}, \ldots),$$

where $U$ is strictly concave, and, following our previous notation, $k_t$ is the period-$t$ state, $a_t$ is the period-$t$ action, and $k_{t+1} = F(k_t, a_t)$. We also assume that $F$ is weakly concave, and that, for each action $a$, $F(a, k) - k > (\leq) 0$ in a neighborhood of $k = 0$ ($k = \infty$). Appendix C provides an example, generalizing the quasi-hyperbolic discounting case to partial depreciation. The time inconsistency issue arises as the period utility function depends on future agents’ action $a_{t+1}$. In general, the life-time utility $U$ may not be separable between $k_t$ and the sequence of actions $\{a_{t+\tau}\}_{\tau=0}^{\infty}$.

**First-order approximation** Consider any stationary point of the mapping $F$, that is, any point $(\bar{k}, \bar{\alpha})$ such that $\bar{k} = F(\bar{k}, \bar{\alpha})$. Consider a first-order approximation of our original economy around any such stationary point: this is represented by a utility function

$$\tilde{U}(k_t, a_t, a_{t+1}, \ldots) = \bar{U} + \bar{U}_k(k_t - \bar{k}) + \sum_{j=0}^{\infty} \bar{U}_{a_j}(a_{t+j} - \bar{\alpha}),$$

(16)

where $\bar{U} = U(\bar{k}, \bar{\alpha}, \bar{\alpha}, \ldots)$ as well as

$$\bar{U}_k = \frac{P_k(\bar{k}, \bar{\alpha}) + Q_k(\bar{k}, \bar{\alpha}) F_k(\bar{k}, \bar{\alpha})}{1 - \beta F_k(\bar{k}, \bar{\alpha})},$$

$$\bar{U}_{a_0} = P_a(\bar{k}, \bar{\alpha}) + \frac{F_a(\bar{k}, \bar{\alpha})(P_k(\bar{k}, \bar{\alpha}) + Q_k(\bar{k}, \bar{\alpha}))}{(1 - \beta F_k(\bar{k}, \bar{\alpha}))},$$

$$\bar{U}_{a_1} = Q_a(\bar{k}, \bar{\alpha}) + \beta \bar{U}_{a_0},$$

$$\bar{U}_{a_j} = \beta \bar{U}_{a_{j-1}}, \quad j > 1.$$

Clearly, this approximated economy is weakly separable between $k$ and the sequence of actions, and our notion of organizational equilibrium can be applied. The remaining question is how to determine the point around which the approximated economy is constructed. The approximated economy satisfies Assumptions 2, 3, and 5, so that Proposition 6 applies. A natural requirement for the choice of $\bar{\alpha}$ is therefore that it coincides with the point that maximizes $V(a, a, \ldots)$. In our case,

$$V(a, a, a, \ldots) = \frac{1}{1 - \beta} \left[ \bar{P}_a + \bar{Q}_a + F_a \left( \frac{Q_k + \beta \bar{F}_k}{1 - \beta F_k} \right) \right] (a - \bar{\alpha}).$$

\footnote{Our assumptions about $F$ imply that, for each action $a$, there exists a unique point $k(a)$ such that $F(k(a), a) = k(a).
An interior solution is only possible if we choose \( \bar{a} \) as the point that satisfies\(^{22}\)

\[
\mathcal{P}_a + \mathcal{Q}_a + \mathcal{F}_a \left( \frac{\mathcal{Q}_k + \beta \mathcal{F}_k}{1 - \beta \mathcal{F}_k} \right) = 0. \tag{17}
\]

**Second-order approximation** Since this first-order approximation is linear in the actions by construction, it is not useful in determining the transition path of the economy towards a steady state.\(^{23}\) To this end, in Appendix C we derive a second-order approximation. The challenge is that the second-order approximation is not weakly separable; to restore separability, it is necessary to suitably transform the action set, and work with the appropriate transformation.

5 Organizational Equilibrium and Public Policy

In this Section, we put the notion of organizational equilibrium to work for the cases that we find most interesting, those of the determination of government policies when the Ramsey solution is time inconsistent.

To look at these environments, we extend the framework in Section 3 to accommodate a government (a large player) that behaves strategically and representative households who behave competitively.\(^{24}\) Given the current level of \( k \in K \), the government chooses an action \( a \) from a set \( A \), and the consumers choose an action \( s \) from the set \( s(k) \subseteq S \). The state evolves according to \( k' = F(k, a, s) \). Let the preferences for the government in period \( t \) be given by \( \Psi(k_t, a_t, s_t, a_{t+1}, s_{t+1}, a_{t+2}, s_{t+2}, \ldots) \).\(^{25}\)

**Assumption 6.** Given a sequence of government actions \( \mathbf{a} := \{a_t\}_{t=0}^{\infty} \), there exists a unique competitive equilibrium \( \mathbf{s}(\mathbf{a}) := \{s_t(\mathbf{a})\}_{t=0}^{\infty} \), where the sequence \( \mathbf{s}(\mathbf{a}) \) is independent of the state \( k_0 \).

Assumption 6 plays two roles. First, the uniqueness allows us to define government preferences directly over the sequence of government actions, taking as given that households will play the associated competitive equilibrium. Second, the fact that \( s \) is independent of the initial state extends the weak separability requirement that is at the heart of our method. We can then define the

\(^{22}\)Such a point is unique because our original utility function is assumed to be strictly concave. Since we need a compact action set, it is in principle possible that no solution to (17) exists if the \( \bar{a} \) is at the lower or upper bound, where the appropriate inequality would need to be imposed. We only consider interior equilibria here for brevity.

\(^{23}\)Proposition 6 proves that convergence is not immediate for utility functions that are strictly quasiconcave. One may also apply a first-order approximation that maintains concavity, such as the log-linear approximation, a strategy we implement in Section 5.2.

\(^{24}\)The setup here uses ideas from Stokey (1991).

\(^{25}\)As in Section 3, sometimes it may be necessary to transform the original government action so that it is feasible independently of the choices of the private sector and the current level of the physical state, and so that the desired separability property of preferences emerges. A similar re-scaling may be needed for the household choices.
government’s preferences over sequences of actions as

\[ U(k, a_t, a_{t+1}, a_{t+2}, \ldots) := \Psi(k, a_t, s_t(a), a_{t+1}, s_{t+1}(a), a_{t+2}, s_{t+2}(a), \ldots), \]  

(18)

where for \( t \geq 0 \), \( k_t \) is computed recursively as

\[ k_{t+1}(k) = F(k_t(k), a_t, s_t(a)). \]  

(19)

These preferences now take the same form as in the one-agent case, so once again we impose Assumptions 1 and 2, and we define an organizational equilibrium as in Definition 1. While the definition of an organizational equilibrium is the same in terms of sequences of actions, its connection to symmetric subgame-perfect equilibria of an underlying game is slightly different, due to the presence of competitive households that act in anticipation of the government’s future actions. In Appendix D, we adapt the analysis of Section 3 to this new environment and we provide an explicit game that describes the strategic interaction between the government and the households over time.\footnote{In the application that we will describe shortly, we assume that the government is a first mover within each period, so that households react contemporaneously to a government deviation. The definition could be adapted to environments where the opposite timing prevails, as in Ortigueira (2006).}

The interaction of private and government decisions is such that Assumptions 3 and 5 fail in our applications. Fortunately, Proposition 4 provides an alternative way of proving existence, that we will adopt in what follows.

In Section 3, an organizational equilibrium represents a refinement of a subgame perfect equilibrium based on specific beliefs that the single player at each stage entertains about future play. In the richer environment considered here, coordination of beliefs involves both the government and a continuum of private players. It is natural for this coordination to take the form of institutions and laws, which is why we call ours an “organizational equilibrium.” Nonetheless, it is important to contrast this role of institutions as purely coordinating expectations from an alternative, in which they represent forms of commitment. As in Prescott and Rios-Rull (2005), we take the view here that laws can be freely changed ex post and that government agencies can be reformed, so that they do not represent effective forms of commitment, and show how cooperation across different players over time can still be sustained, even when the self-interest of future players rules out the usual grim-trigger strategies.

### 5.1 A Taxation Example

To illustrate the general definition of an organizational equilibrium in a hybrid competitive-strategic environment, we revisit Klein et al. (2008), replacing their Markov equilibrium with our notion of
organizational equilibrium. In this problem, the government sets a tax instrument, which, depending on the case, is a flat tax on capital income, labor income, or total income. The proceeds are used to produce a public good, and the government is constrained to a balanced budget. In this subsection, we first consider a special case with inelastic labor supply and full depreciation, where a closed-form solution is possible. In the next section, we use our approximation strategy to explore a quantitative version, as in Klein et al. (2008).

The production function is given by

$$y_t = f(k_t, \ell_t) = k_t^\alpha \ell_t^{1-\alpha},$$

where labor is inelastically supplied ($\ell_t = 1$) and capital is subject to full depreciation, so that the resource constraint is

$$c_t + g_t + k_{t+1} = f(k_t, \ell_t),$$

(20)

where $g_t$ is the government provision of the public good. The preference of a representative consumer is given by

$$\sum_{t=0}^{\infty} \beta^t [\log c_t + \gamma \log g_t].$$

We derive the analytical expressions for the case in which the government instrument is a tax on capital income. The same method can be applied when the tax is levied on labor income, or on total income. The consumers’ budget constraint is

$$c_t + k_{t+1} = (1 - \tau_t) r_t k_t + w_t.$$

We take the tax rate to be the government’s action. Its domain is $(0, 1/\alpha)$ and is thus independent of initial capital.\(^{27}\) Given a sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$, we first characterize a competitive equilibrium in terms of sequences of consumption, capital, and factor prices, and then summarize it by a sequence of saving rates $s_t \in (0, 1 - \alpha \tau_t)$, which is our notion of the private sector’s actions.

Given a sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$ and an initial level of capital $k_0$, a sequence $\{c_t, g_t, k_{t+1}, w_t, r_t\}_{t=0}^{\infty}$ is a competitive equilibrium if and only if the following conditions are satisfied:

- Factor prices are equal to their marginal productivity, i.e., $r_t = f_k(k_t)$, and $w_t = f(k_t) - r_t k_t$.

\(^{27}\)Note that $\tau_t = 1/\alpha$ implies a tax rate that is greater than 100%, and it is such that $\tau_t r_t k_t = k_t^\alpha$: at that rate, the government seizes the entire output. Except possibly for the initial period, a competitive equilibrium is only possible if $\tau_t < 1$, since otherwise households would not save. Rather than imposing this ex ante, it is more convenient to let $\tau_t < 1$ emerge endogenously from the equilibrium conditions. Finally, note that we rule out the extrema of the interval to avoid dealing with infinitely negative utility.
The consumers’ intertemporal decision is optimal, which requires the Euler condition to hold
\[ u'(c_t) = \beta u'(c_{t+1})(1 - \tau_{t+1})r_{t+1}, \]
along with the transversality condition: \( \lim_{t \to \infty} \beta^t u'(c_t)k_{t+1} = 0. \)

- The government budget is balanced, i.e., \( g_t = \tau_t r_t k_t. \)
- The resource constraint (20) holds.

Substituting factor prices, the resource constraint, and the budget constraint into the Euler equation and summarizing private-sector actions by the saving rate \( s_t := k_{t+1}/f(k_t, \ell_t) \) (which also is in \( [0, 1] \) independently of initial capital), a competitive equilibrium is described by the difference equation
\[ s_t = \alpha \beta (1 - \tau_{t+1}) \]
along with the transversality condition
\[ \lim_{t \to \infty} \beta^t \frac{s_t}{1 - \alpha \tau_t - s_t} = 0. \]

The following lemma then establishes existence and uniqueness of the competitive equilibrium for an arbitrary tax sequence (proved in Appendix E).

**Lemma 1.** Assumption 6 is satisfied for this economy. Specifically, given a sequence \( \{\tau_t\}_{t=0}^{\infty} \in [\epsilon, 1 - \epsilon]\), there exists a unique competitive equilibrium.

Suppose the sequence of tax rates is \( \{\tau_j\}_{j=0}^{\infty} \), the sequence of saving rates is \( \{s_j\}_{j=0}^{\infty} \), and the initial capital is \( k_0 \). Then capital evolves according to the following sequence: \( k_t = k_0^\alpha \Pi_{j=0}^{t-1} s_j^{\alpha^{t-1-j}} \), and the current government’s total payoff is
\[ U(k_0, s_0, \tau_0, s_1, \tau_1, \ldots) = \frac{\gamma}{1 - \beta} \log \alpha + \frac{\alpha(1 + \gamma)}{1 - \alpha \beta} \log k_0 + V(s_0, \tau_0, s_1, \tau_1, \ldots), \]
where the action payoff is
\[ V(s_0, \tau_0, s_1, \tau_1, \ldots) = \sum_{j=0}^{\infty} \beta^j \left\{ \log(1 - \alpha \tau_j - s_j) + \gamma \log \tau_j + \frac{\alpha \beta(1 + \gamma)}{1 - \alpha \beta} \log s_j \right\}. \]

Clearly, the separability condition is satisfied in this environment. To construct an organizational equilibrium (and thereby prove its existence), we rely on Proposition 4 and look directly for sequences
of government actions (tax rates) that satisfy the no-restarting, optimality, and no-delay conditions of that proposition.

The no-restarting condition states that $V(s_t, \tau_t, s_{t+1}, \tau_{t+1}, \ldots)$ should equal to a constant for different $t$. This payoff is not recursive if expressed in terms of the sequence of tax rates alone, since the saving rate in period $t$ depends on past as well as present and future taxes. Nonetheless, a recursive structure emerges if we study the joint sequence of tax and saving rates, while keeping track of the households’ Euler equation (21) to ensure the private optimizing behavior required for a competitive equilibrium. This leads to the following proposition which characterizes the equilibrium outcomes. Similar to Section 2, the organizational equilibrium can be characterized by a recursive function that governs the transition and a starting point that implements the no-delay condition (proved in Appendix E).

**Proposition 7.** An organizational equilibrium exists. In any such equilibrium, the value attained by the policymaker is

$$ (1 - \beta) V^* = \max_{\tau} \log(1 - \alpha \tau - \alpha \beta (1 - \tau)) + \gamma \log \tau + \frac{\alpha \beta (1 + \gamma)}{1 - \alpha \beta} \log(\alpha \beta (1 - \tau)). \quad (24) $$

There exists a recursive equilibrium characterized by a transition function $q$ such that $\tau_{t+1} = q(\tau; V^*)$ for all $t \geq 0$, and an initial tax rate $\tau_0$ such that the no-delay condition is satisfied.

Before we visualize the transition paths of the equilibrium outcomes, it is useful to compare the steady-state tax rates in organizational equilibrium, Markov equilibrium, and Ramsey outcome. To facilitate the analysis, define the function $H(s, \tau)$ as

$$ H(s, \tau) \equiv \log(1 - \alpha \tau - s) + \gamma \log \tau + \frac{\alpha \beta (1 + \gamma)}{1 - \alpha \beta} \log s, $$

which is part of the action payoff (23) related to the current tax rate and the saving rate.

In the Markov equilibrium, the problem of the current government can be written as

$$ \max_{\tau} H(s, \tau), \quad \text{s.t.} \quad \frac{s}{1 - s - \alpha \tau} = \frac{\alpha \beta (1 - \tau')}{1 - s' - \alpha \tau'}. $$

The current government takes future $\tau'$ and $s'$ as given when choosing $\tau$.\(^{28}\) It follows that the tax rate in the Markov equilibrium is a constant $\tau^M = \gamma(1 - \alpha \beta) / \alpha(1 + \gamma)$.\(^{29}\) A high tax rate is chosen because the

\(^{28}\)In general, in a Markov equilibrium the government would consider $\tau'$ and $s'$ as given functions of $k'$. For the specific application at hand, weak separability implies that the Markov equilibrium features constant tax rate and saving rate, so the government is taking as given the future levels of $\tau'$ and $s'$ directly.

\(^{29}\)The same result is obtained in Klein et al. (2008), where the Markov equilibrium is obtained by taking the limit of the solution to a finite-horizon problem.
government fails to take into account the effects of the current tax rate on past saving choices.

In the steady state of the Ramsey outcome, the allocation is characterized by

\[ H_s = \frac{1}{\alpha} H_\tau, \quad \text{and} \quad s = \alpha \beta (1 - \tau). \]

This allocation does not maximize \( H(s, \tau) \), but it is optimal from the initial government’s perspective. The condition \( s = \alpha \beta (1 - \tau) \) is what the saving rate and the tax rate need to satisfy in a steady state. Therefore, the effect of a high tax rate on past saving decisions is internalized and a low tax rate is set in the long run.

In the organizational equilibrium, the tax rate is chosen such that it maximizes the action payoff \( H(s, \tau) \) subject to households’ Euler equation when all the variables stay constant.\(^{30}\)

\[ \max_{\tau} H(s, \tau) \quad \text{s.t.} \quad s = \alpha \beta (1 - \tau). \]

Unlike the Markov equilibrium, the intertemporal distortion is internalized. This is why the outcome in the organizational equilibrium is better than that in the Markov equilibrium. However, the allocation does not necessarily favor the initial agent and has to strike a balance across governments in different periods, which prevents it being the same as the Ramsey outcome.

Now we turn to the transition paths. As an example, we set \( \beta = 0.9, \alpha = 0.36, \) and \( \gamma = 0.5, \) and the transition function \( \tau_{t+1} = q(\tau_t; V^*) \) is plotted in Figure 4. Let \( \tau^M \) denotes the tax rate in the Markov equilibrium and \( \tau^R \) denotes the steady-state tax rate in the Ramsey outcome. We choose \( \tau_0 = q(\tau^M; V^*) \), which is the lowest tax rate that satisfied the no-delay condition. As expected, the initial tax rate \( \tau_0 \) is lower than \( \tau^M \), but it is higher than \( \tau^R \). The function \( q \) implies a gradual transition of the tax rate from \( \tau_0 \) to the steady state \( \tau^* \), which is close to that in the Ramsey outcome.

Figure 5 displays the corresponding transition paths for the tax rates and allocation in the three economies. The initial condition is the steady state capital in the Markov economy. In the Ramsey outcome, the government initially sets the tax rate as high as in the Markov equilibrium and rapidly adjusts it to the steady state value \( \tau^R \). As a result, private consumption drops initially, since households anticipate a lower tax rate in the future. At the same time, the capital stock accumulates

\(^{30}\)The action payoff excludes the utility of initial capital, which can be done thanks to weak separability, but it does take into account how actions affect utility through the evolution of capital in the future. For this reason, it contains a proper accounting of the effects of the transition and does not coincide with maximizing total utility in the steady state.
to its steady-state high level. The path of government spending is non-monotonic, since the output and tax rate move in opposite directions. In the organizational equilibrium, the tax rate starts lower than the Markov tax rate and converges to $\tau^*$. The transition of the tax rate is slower than in the Ramsey outcome, to ensure that the government’s action payoff is equalized across periods. Because of the lower tax rate, the capital stock is higher than that in the Markov equilibrium.

5.2 A Quantitative Taxation Model

In this section, we revisit the quantitative taxation problem in Klein et al. (2008). We extend the previous section to allow elastic labor supply, partial depreciation, and three types of taxation. The properties of the transition path are very similar to what we have shown in the previous section, so we just focus on the steady-state properties.

Preferences are

$$\sum_{t=0}^{\infty} \beta^t \left[ \gamma_c \log c_t + \gamma_\ell \log(1 - \ell_t) + \gamma_g \log g_t \right],$$

where $\ell_t$ stands for labor. The budget constraint for the household is

$$c_t + i_t = w_t \ell_t + r_t k_t - (\tau^T_{\ell} + \tau_t) w_t \ell_t - \left( \tau^k_{\ell} + \tau_t - \frac{\delta(\tau^k_{\ell} + \tau_t)}{r_t} \right) r_t k_t,$$

where $i_t$ is investment, $\tau^T_{\ell}$ is labor income tax, $\tau^k_{\ell}$ is capital income tax, and $\tau_t$ is total income tax. The
last term on the right-hand of the budget constraint allows for the possibility of capital depreciation
deduction. In Klein et al. (2008), the capital evolves according to
\[ k_{t+1} = (1 - \delta) k_t + i_t. \]
However, this specification does not allow the government’s lifetime utility to be separable between
capital and the sequence of tax rates. Instead, we specify the law of motion of capital to be\(^{31}\)
\[ k_{t+1} = \bar{r} k_t^{1 - \delta} i_t^\delta. \]
This can be viewed as a log-linear approximation of the original law of motion of capital, and this

\(^{31}\)For the same reason, we approximate the depreciation allowance to \(\delta r_t k_t / \bar{r}\) rather than \(\delta k_t\). See Appendix E.
Table 1: Steady State Comparison

<table>
<thead>
<tr>
<th>Aggregate statistics</th>
<th>Labor income tax</th>
<th>Capital income tax</th>
<th>Total income tax</th>
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<td>Markov</td>
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<td>0.180</td>
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<td>$\tau$</td>
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<td>0.256</td>
<td>0.269</td>
</tr>
</tbody>
</table>

Modification makes the economy weakly separable, and therefore we can apply the definition of organizational equilibrium to this approximated economy.

We set $\alpha = 0.36$, $\beta = 0.96$, $\delta = 0.08$, $\gamma_g = 0.09$, $\gamma_c = 0.27$, and $\gamma_\ell = 0.64$. We choose $\gamma_g$ and $\gamma_\ell$ such that the steady state government spending to GDP ratio is 18% in the Pareto efficient allocation and the household works 32% of its available time.\(^{32}\) The rest of the parameters are standard.

Table 1 shows the steady-state comparison among the Pareto allocation, the Ramsey outcome, the Markov equilibrium, and the organizational equilibrium. The results regarding the Ramsey outcome and the Markov equilibrium are very similar to those obtained in Klein et al. (2008) despite the fact that they are obtained in an approximated economy. Across the three different tax instruments, a common feature is that the tax rates and the allocation in the organizational equilibrium always stay between the Ramsey outcome and the Markov equilibrium. This feature reinforces the analysis in Section 2 and Section 5.1: an organizational equilibrium tempers the temptation from time inconsistency that makes Markov equilibria particularly undesirable, but at the same time it does not support the same level of long-run cooperation as in a Ramsey outcome. Among the three tax instruments, capital income taxation is the most distortionary. In the Markov equilibrium with capital-income taxes, the steady-state output level is only 57% of its Pareto efficient level, while an organizational equilibrium attains 66% of the Pareto optimal steady-state allocation and achieves a value only slightly below the Ramsey outcome. We interpret this result as a large improvement over the Markov equilibrium. For the cases of a labor income tax or a total income tax, the difference between the Ramsey outcome and the Markov equilibrium is much smaller. Since the organizational equilibrium stays in between the two benchmarks, we only conclude that it brings the allocation closer to the Ramsey outcome.

\(^{32}\)By “Pareto efficient allocation,” we mean the allocation that could be attained if the government had access to lump-sum taxation. The Ramsey outcome is the Pareto efficient allocation conditional on the tax instruments available to the government.
6 Conclusion

Rome was not built in a day. Likewise, many institutions that underpin solid policy evolved slowly. Sargent (2017), and Hall and Sargent (2014; 2015; 2018) describe the way the United States acquired a reputation for honoring its debt over time. The current environment of low inflation emerged after the tumultuous '70s, during which governments gradually learned how to manage monetary policy without resorting to the anchor of a commodity standard. Social security started as a narrow and limited program and only later grew in size and scope.

In this paper, we provided an equilibrium concept that is well suited to analyzing such situations. The equilibrium eschews abrupt transitions and is not (or at least not necessarily) supported by grim triggers, but rather cooperation for the common long-term good evolves slowly and would potentially also erode slowly. While the constraints that we impose on equilibrium strategies appear very restrictive, what is most interesting to us is that in our computed examples, they still permit very good outcomes, bringing substantial improvement to the dismal predictions of Markov equilibria. At the same time, the notion of organizational equilibrium allows for clear-cut comparative statics exercises and does not suffer from the pervasive multiplicity of subgame-perfect equilibria, which is implied by the folk theorem. Because of this, it is more readily amenable to empirical analysis.
References


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Appendix for Online Publication

A Complete Formal Description of the Game of Section 3.2 in which History Can Be Hidden, and Proofs of the Theorems of that Section.

The players in the game are nature, plus an infinity of players 0,1,... indexed by the time at which they act. Nature moves first, choosing a time $\hat{t}$, from which record keeping is possible. We assume that this distribution has full support on $\mathbb{N}$.\footnote{It would be equivalent to assume that nature moves in each period up to $\hat{t}$, as long as the conditional hazard rate of the start of record keeping is the same. This is because nature’s choice is not fully observed by the agents anyway.}

Players take two actions:

- Player $t$ chooses $a_t \in A$.
- In addition, a player may choose a record-keeping action $\rho_t \in \{S,C,H\}$, where $S$ stands for starting record keeping, $C$ stands for continuing record keeping, and $H$ for hiding past records. Whether these actions are available at time $t$ depends on the past in a way that we will soon make explicit.

We are now ready to define histories and information. The first history is $\emptyset$, at which stage nature moves. In all periods $t < \hat{t}$, players only choose the action $a_t$. While the history of play is $(\hat{t}, a_0, ..., a_{t-1})$, their only information is that $t < \hat{t}$, and the current level of the state $k_t$: they do not observe any of the past players’ actions, and they only know that record keeping is not yet possible. In period $t = \hat{t}$, the history of play is also $(\hat{t}, a_0, ..., a_{\hat{t}-1})$. Player $t = \hat{t}$ observes $k_t$, does not observe any of the actions taken by the past players, but it knows that $t \geq \hat{t}$ and that either $t = \hat{t}$ or $\rho_{t-1} = H$: that is, it knows that it is either the first player with the opportunity to set up record keeping, or the opportunity was available in the past, but player $t - 1$ chose not to adopt it and to hide the previous history. Player $t = \hat{t}$ is called to choose an action $\rho_t \in \{S,H\}$ as well as $a_t$. In period $\hat{t} + 1$ and all subsequent periods, the history of play is $(\hat{t}, a_0, ..., a_{\hat{t}-1}, \rho_{\hat{t}}, a_{\hat{t}}, \rho_{\hat{t}+1}, a_{\hat{t}+1}, ..., \rho_{t-1}, a_{t-1})$. In each of these periods, if $\rho_{t-1} = H$, then player $t$ only knows that record keeping is possible and the level of $k_t$: it does not know whether $t = \hat{t}$ or $\rho_{t-1} = H$. In this case, player $t$ has the same options as player $\hat{t}$. Otherwise, let $\tilde{t}$ be the last time action $S$ was taken; player $t$ then knows that $\tilde{t} \leq \hat{t}$ and it knows $(\rho_{\tilde{t}}, a_{\tilde{t}}, \rho_{\tilde{t}+1}, a_{\tilde{t}+1}, ..., \rho_{t-1}, a_{t-1})$ (in addition to $k_t$). Player $t$ has 3 options for $\rho_t$: first, it can choose $\rho_t = H$, in which the next player will start again with no record of the past; second, it can choose $\rho_t = C$, that is, to continue record keeping; in this case, player $t + 1$ will know $(\rho_{\tilde{t}}, a_{\tilde{t}}, \rho_{\tilde{t}+1}, a_{\tilde{t}+1}, ..., \rho_{t-1}, a_{t-1})$. Finally, it can restart the history ($\rho_t = S$), disavowing the past, but recording its own actions, in which case player $t + 1$ will only observe $(\rho_t, a_t)$. In all cases player $t + 1$ will observe $k_{t+1}$.

A strategy $\sigma_t$ for player $t$ is a mapping from the set of time-$t$ histories, $H^t$, to the set of actions $A$ and (when available) record-keeping choices $\rho_t \in \{H,S,C\}$, that is measurable with respect to the information available at time $t$. As before, a strategy profile $\sigma$ is a sequence of strategies, one for each player. It is useful to distinguish between the two choices made by agents: accordingly, let $\sigma_{a,t}$ be the component of $\sigma_t(h^t)$ that
contains the prescribed action \( a \in A \) after history \( h^t \), and \( \sigma_{\rho,t}(h^t) \) be the prescribed choice of record keeping. Analogously, we define \( \sigma_a := \{\sigma_{a,t}\}_{t=0}^{\infty} \) and \( \sigma_\rho := \{\sigma_{\rho,t}\}_{t=0}^{\infty} \).

We restrict attention to equilibria that satisfy Requirement 1: that is, they involve strategies that are independent of \( k_t \).

We define a full-disclosure equilibrium to be an equilibrium in which \( \sigma_{\rho,t}(h^t) = S \) for all histories for which \( t \geq \hat{t} \) and no previous record of play is known, and \( \sigma_{\rho,t}(h^t) = C \) for all histories for which player \( t \) observes a record of past actions. In a full-disclosure equilibrium, players introduce record keeping as soon as possible and never erase any of the record available, independent of the actions of past players.

The proof of Proposition 3 relies on a sequence of lemmata:

**Lemma 2.** Let \( \sigma \) be a sequential equilibrium that satisfy Requirement 1 in the game defined above. Then:

1. There exists a full-disclosure sequential equilibrium \( \tilde{\sigma} \) that also satisfies that satisfy Requirement 1 and such that the same actions \( \{a_t\}_{t=0}^{\infty} \) are taken on the equilibrium path under \( \sigma \) and \( \tilde{\sigma} \).
2. If \( \sigma \) satisfies Requirement 2 from period \( \hat{t} \) (whatever \( \hat{t} \) turns out to be), then \( \tilde{\sigma} \) can be chosen to also satisfy the same requirement.

**Proof.** Our proof only looks at pure-strategy equilibria. It could be extended to mixed-strategy equilibria, in which players randomize over their choice of record keeping, using the same logic presented here, as long as a public randomization device is present that allows coordination across players. We omit the case of mixed-strategy equilibria for brevity.

1. Assumption 1 implies that, if future players do not condition their choices on the state \( k \) (but potentially condition their choices on all their remaining information in any arbitrary way), the optimal choice for a current player is independent of the current state. In looking at equilibria that satisfy Requirement 1, we can therefore leave the state \( k \) in the background and focus only on the history of actions, disclosures, and the time at which record keeping becomes available.

Let \( \sigma = \{\sigma_t\}_{t=0}^{\infty} \) be the strategy profile of the sequential equilibrium that contains the equilibrium action path \( \{a_t\}_{t=0}^{\infty} \).

We need to construct an alternative strategy profile \( \tilde{\sigma} \) that contains the same equilibrium action path, but involves full disclosure. We will do so by creating a suitable mapping from the set of histories to itself, and setting \( \tilde{\sigma}_{a,t}(h^t) = \sigma_{a,t}(\eta(h^t)) \). \( \eta \) is constructed recursively as follows:

- For \( t \leq \hat{t} \), \( \eta(h^t) = h^t \).
- For \( t > \hat{t} \) and histories in which \( \rho_{t-1} = H \), \( \eta(h^t) = h^t \).
- For \( t > \hat{t} + 1 \) and histories in which \( \rho_{t-1} = S \) and \( \sigma_{\rho,t}(h^{t-1}) = S \) or \( \sigma_{\rho,t}(h^{t-1}) = C \), \( \eta(h^t) = h^t \).
- For \( t > \hat{t} + 1 \) and histories in which \( \rho_{t-1} = S \) and \( \sigma_{\rho,t}(h^{t-1}) = H \), \( \eta(h^t) = (h^{t-1}, H, a_{t-1,h^t}) \), where \( a_{t-1,h^t} \) is the action taken in period \( t - 1 \) according to the history \( h^t \).
For $t > \hat{t}$ and histories in which $\rho_{t-1} = C$, we define $\eta$ recursively as $\eta(h_t) = (\eta(h_{t-1}^t), \sigma_{\rho,t}(h_{t-1}^{t-1}), a_{t-1, h_t})$. Furthermore, whenever $t \geq \hat{t}$, $\tilde{\sigma}_{\rho,t} = S$ if no record keeping is currently in place, and $\tilde{\sigma}_{\rho,t} = C$ otherwise, in line with the definition of a full-disclosure equilibrium.

In words, $\tilde{\sigma}$ is constructed from $\sigma$ by assuming that agents take the same actions under the two strategy profiles whenever they do not observe the past. When past actions are observed from $\tilde{s}$ on, the strategy profile $\tilde{\sigma}$ prescribes that the agents take the same actions they would have taken under $\sigma$ when faced with a history that has same choices for $(a_0, ..., a_{t-1})$, but in which past players from $\tilde{s}$ on chose to hide, start, or continue record-keeping according to the equilibrium profile $\sigma$. At the same time, $\tilde{\sigma}$ always prescribe full disclosure. Next, we verify that $\tilde{\sigma}$ is a measurable strategy with respect to the information sets available to the players at each point $t$. The choice of $\rho$ only depends on whether record keeping is possible and whether it is inherited from the past, which is observable to an agent at the time it makes its choice. Furthermore, by construction, the mapping $\eta$ is such that the prescribed action $\tilde{\sigma}_{a_t}(h_t)$ is the same for all histories that share the same observable record.\footnote{This assumes that the property is true for $\sigma$, which must be the case for $\sigma$ to be a valid strategy profile and therefore a valid equilibrium, provided that $\sigma$ does not condition on $k_t$, which is guaranteed by Requirement 1.}

Next, we verify that $\tilde{\sigma}$ represents a sequential equilibrium. A player’s payoff only depends on the current and future actions $a_t \in A$, and only indirectly on record keeping choices.

- In any period $t < \hat{t}$, the current choice of $a_t$ by player $t$ is not known to future players and therefore it has no impact on any future action. Furthermore, the two strategies $\sigma$ and $\tilde{\sigma}$ imply the same sequence of future actions $(a_{t+1}, a_{t+2}, ...)$ along the equilibrium path.\footnote{Notice that future actions are in general uncertain and depend on the realization of $\hat{t}$, but their stochastic process is identical in the two equilibria.} The optimality of $\tilde{\sigma}_t$ then follows directly from that of $\sigma_t$.
- Consider next periods $t \geq \hat{t}$ and histories $h_t$ such that no record is available to player $t$. For such histories, $\eta(h_t) = h_t$. There are two possibilities. First, suppose that $\sigma_{\rho,t}(h_t) = S$. Then, no matter what choice of $(\rho_t, a_t)$ player $t$ takes, the equilibrium implies that future players will take the same actions $(a_{s,t})_{s=t+1}^{\infty}$. Hence, $\tilde{\sigma}_t(h_t) = \sigma_t(h_t)$ is an optimal choice. Suppose instead that $\sigma_{\rho,t}(h_t) = H$, that is, according to the equilibrium profile $\sigma$, player $t$ should hide its action. In this case, $\eta$ is such that player $t$ gets the same payoff whether it chooses $\rho_t = S$ or $\rho_t = H$, since $\eta(h_t, H, a_t) = \eta(h_t, S, a_t)$: player $t$ is indifferent between starting record keeping or not, because in either case future players will ignore its play and behave as if no record had been taken in $t$. Starting record keeping is thus weakly optimal, and taking the same action that would have been taken under the profile $\sigma$ is optimal as well.\footnote{Since future players will ignore the action $a_t$, player $t$ will maximize its payoff assuming that its action does not affect the future, as if no record were taken, just as it would under the strategy $\sigma$, which prescribes hiding the record.}
- Consider histories $h_t$ in which a record is present. The reasoning is similar. If $\sigma_{\rho,t}(\eta(h_t)) = C$, then, no matter what choice of $(\rho_t, a_t)$ player $t$ takes, the equilibrium implies that future players will take the same actions $(a_{s,t})_{s=t+1}^{\infty}$ under profiles $\tilde{\sigma}$ and history $h_t$ as they would under $\sigma$ and history $\eta(h_t)$. Hence, if $\sigma_t(\eta(h_t))$ is optimal (taking as given that $\sigma$ will be followed in the future), then $\tilde{\sigma}_t(h_t)$ is also optimal, if future players play according to $\tilde{\sigma}$. If $\sigma_{\rho,t}(\eta(h_t)) = H$, then under $\tilde{\sigma}$ future players will ignore past actions whether player $t$ chooses $\rho_t = H$ or $\rho_t = C$, and their
Lemma 3. Let $\hat{\sigma}$ be a full-disclosure state-independent sequential equilibrium for the game in which history can be hidden. Then:

$$\hat{\sigma}$$

is such that histories with $t \geq \hat{t}$ are mapped into histories with $t \geq \hat{t}$. If $V$ is symmetric, then it achieves the same action payoff $V$ following any history that has $t \geq \hat{t}$; as a consequence, the same property is inherited by $\hat{\sigma}$. This implies that the set of values attainable by sequential equilibria satisfying Requirement 1 from period $\hat{t}$ is the same as the set of values attainable by full-disclosure sequential equilibria satisfying Requirement 1 from the same period; the maxima of the two sets will thus coincide, completing the proof.\textsuperscript{37}

\textsuperscript{37}The inability to keep records for periods before $\hat{t}$ will in general imply that the payoff in previous periods is lower.
1. $\hat{\sigma}_a|_{h^t} \equiv \sigma$ is a subgame-perfect equilibrium for the game where record-keeping starts at time 0, and it also satisfies state independence (Requirement 1);\(^{38}\)

2. If $\hat{\sigma}$ is symmetric from period $\hat{t}$ on, then $\hat{\sigma}_a|_{h^t}$ is also symmetric.

Proof. 1. In the game in which history can be hidden, in period $\hat{t}$, player $\hat{t}$ starts with no information about the past, just as in period 0 of the game where record-keeping starts at time 0. Furthermore, $\hat{\sigma}$ is such that records will be kept from $\hat{t}$ on. Take as given the choice of $\rho_t$ dictated by $\hat{\sigma}$, and focus on the choice of $a_t$. In order for $\hat{\sigma}$ to represent a sequential equilibrium, at any time $t \geq \hat{t}$ and after any history $h^t$ it must be the case that $\sigma_a(h^t)$ (along with starting record keeping if no record is present or continuing it otherwise) is optimal, conditional on the fact that future players will continue to play $\hat{\sigma}$. Let $h^t_{a,\hat{t}}$ represent the subcomponent of history $h^t$ that captures the history of actions $(a_t, a_{t+1}, \ldots, a_t)$. Since $\hat{\sigma}$ implies that future players will behave in such a way that the entire history of play from $\hat{t}$ is known, it then follows that $\sigma_a(h^t)$ must be optimal in the game where record keeping starts in period 0 after history $h^t_{a,\hat{t}}$, assuming that future players will play according to the strategy profile $\hat{\sigma}_a|_{h^t}$.

2. Symmetry implies that the action payoff $V$ on the equilibrium path conditional on attaining any history $h^t$ with $t \geq \hat{t}$ is the same. This property is inherited by $\hat{\sigma}_a|_{h^t}$ in any subgame following a history $h^t_{a,\hat{t}}$, since the action paths coincide going forward.

\[ \square \]

**Lemma 4.** Let $\sigma$ be a symmetric state-independent subgame-perfect equilibrium of the game where record-keeping starts in period 0. Then, if and only if $\sigma$ satisfies Requirement 3 as well, there exists a state-independent full-disclosure sequential equilibrium $\hat{\sigma}$ of the game where history can be hidden, which is symmetric from period $\hat{t}$ and is such that $\hat{\sigma}_a|_{h^t} \equiv \sigma$.

Proof. Assume first that $\sigma$ satisfies Requirement 3. The condition $\hat{\sigma}_a|_{h^t} \equiv \sigma$ fully characterizes $\hat{\sigma}_a$ from period $\hat{t}$ on. To see this, let $h^t$ be an arbitrary history in which $t > \hat{t} + s$ and player $t$ observes $(a_{t-s}, a_{t+1-s}, \ldots, a_{t-1})$; this implies that either player $t - s - 1$ chose to hide records, or player $t - s$ chose to restart them, while all subsequent players up to $t$ chose to continue record keeping. This history is in the same information set as a history with the same sequence of actions $(a_{t-s}, a_{t+1-s}, \ldots, a_{t-1})$ in which $\hat{t} = t - s$ and players adopted full disclosure; actions for such history are determined by $\hat{\sigma}_a|_{h^t} \equiv \sigma$. This observation also implies that, following any history, the sequence of actions $a$ that are predicted to happen along a continuation equilibrium according to $\hat{\sigma}$ is the same as those in a corresponding history in the game where record-keeping starts in period 0 under $\sigma$. If all histories under $\sigma$ are followed by the same equilibrium action payoff $\bar{V}$, then the same value carries over to $\hat{\sigma}$. To verify that $\hat{\sigma}$ is indeed optimal after any history $h^t, t \geq \hat{t}$, we denote $h^t_{a,s} = (a_s, \ldots, a_t)$ to be the record available to player $t$ after history $h^t$ and proceed as follows:\(^{39}\)

\[ ^{38} \text{Note that, without further assumptions, } \hat{\sigma}_a|_{h^t} \text{ may depend on the precise realization of } \hat{t}. \text{ The property still holds: in this case, each possible continuation strategy } \hat{\sigma}_a|_{h^t} \text{ is a subgame-perfect equilibrium of the game where record-keeping starts at time } 0. \]

\[ ^{39} \text{Along the equilibrium path, the record available should start from period } \hat{t}, \text{ but we need to verify optimality even for histories that are not on the equilibrium path.} \]
• Player $t$ does not have an incentive to choose $\rho_t = C$ and any action $a \neq \sigma_t(h^t_{a,i})$. Assuming that future players will follow $\tilde{\sigma}$, the consequences of such a choice would be the same as those of choosing $a \neq \sigma_t(h^t_{a,i})$ after history $h^t_{a,i}$ in the game where record-keeping starts in period 0 when future players follow $\sigma$; since $\sigma$ represents an equilibrium, choosing $a \neq \sigma_t(h^t_{a,i})$ is weakly worse.

• Player $t$ does not have an incentive to choose $\rho_t = S$ and any action $a \in A$. Following such a choice, player $t + 1$ will behave as if $\hat{t} = t$, and future actions will unfold according to the strategy profile $\sigma$. Requirement 3 implies that, whatever action player $t$ chooses, it would be (weakly) better off playing $\rho_t = S$ and $a = \sigma(\emptyset)$, that is, choosing to restart record keeping and playing the first action of the strategy profile of the game where record-keeping starts in period 0. This latter choice gives an action payoff of $\check{V}$, which is the same as that obtained by continuing record keeping and following $\hat{\sigma}$.

• Player $t$ does not have an incentive to choose $\rho_t = H$ and any action $a \in A$. Following such a choice, player $t + 1$ and subsequent players will follow the strategy $\sigma$ as if the game in which record-keeping starts in period 0 took place from that point on. Requirement 3 implies that, faced with this prospect, player $t$ does not have any action that can guarantee a payoff higher than $\check{V}$ for herself.

To finish establishing the “if” part of the Lemma, the last step is to construct the strategy profile $\hat{\sigma}$ in periods $t < \hat{t}$. In these periods, the actions taken by player $t$ will not be observed by future players; as long as $\tilde{\sigma}$ is independent of the state, the actions of the current player will thus have no consequences on the actions taken by future players. We thus need to prove existence of a sequence of actions $(\hat{a}_0, \hat{a}_1, \ldots)$ that will be taken by players in period $t$ if $t < \hat{t}$, and that are optimal given that the same sequence will be continued up to the unknown time $\hat{t}$ and given that starting in period $\hat{t}$ actions will unfold according to the equilibrium path dictated by $\sigma$. Given $\sigma$, consider a correspondence $M : A^\infty \rightrightarrows A^\infty$ that associates to a sequence $(a_0, a_1, \ldots)$ all the sequences such that player $t$ is choosing optimally given that $(a_0, a_1, \ldots)$ will be followed up to period $\hat{t}$ and $\sigma$ will be followed from period $\hat{t}$ on. By Assumptions 2 and 4 and the theorem of the maximum, $M$ is nonempty, compact- and convex-valued, upper hemicontinuous, and independent of the state. By Kakutani’s fixed-point theorem, $M$ has a fixed point, which can be used as our desired sequence $(\hat{a}_0, \hat{a}_1, \ldots)$.

Conversely, suppose that $\sigma$ does not satisfy Requirement 3. We know from the previous part of the proof that player $t \geq \hat{t}$ can attain the action payoff $\check{V}$ by continuing record keeping and following the strategy $\hat{\sigma}$, but also by playing $\rho_t = S$ and $a = \sigma(\emptyset)$, effectively starting the sequence $(a_0, a_1, \ldots)$ of Requirement 3. However, if player $t$ hides the record and chooses $\rho_t = H$, then the strategy profile $\hat{\sigma}$ implies that record keeping will start in period $t + 1$ and the actions $(a_0, a_1, \ldots)$ will unfold from period $t + 1$ instead. If Requirement 3 fails, there exists an action $\hat{a}$ such that $V(\hat{a}, a_0, a_1, \ldots) > V(a_0, a_1, \ldots) = \check{V}$, which yields a higher payoff than following $\hat{\sigma}$; this would imply that $\hat{\sigma}$ is not an equilibrium strategy profile.

We are now ready to prove Proposition 3.

**Proof of Proposition 3.** In the game in which record-keeping starts in period 0, let $\sigma$ be a strategy profile whose equilibrium path is an organizational equilibrium. By Lemma 4 we can find a state-independent strategy profile $\tilde{\sigma}$ for the game in which history can be hidden that attains the same equilibrium path from $\hat{t}$ on, whatever
the realization of \( \hat{t} \); this equilibrium is also symmetric. To complete the proof, we need to show that there is no other state-independent equilibrium which is symmetric from period \( \hat{t} \) on and attains a higher payoff from that point onwards. By contradiction, suppose that such an equilibrium existed, let it be \( \tilde{\sigma} \). From Lemma 2, we can assume without loss of generality that \( \tilde{\sigma} \) involves full revelation. Lemma 3 implies that \( \tilde{\sigma} \vert_{\hat{t}} \) is a symmetric state-independent equilibrium of the game in which history can be hidden, which would then achieve a higher payoff than \( \sigma \); however, this would imply that \( \sigma \) does not satisfy Requirement 2 and therefore that its equilibrium path is not an organizational equilibrium, establishing a contradiction.

\(\square\)

B Proofs of Section 3.3.

B.1 Proof of Proposition 4

Proof. Suppose first that \( \{\tilde{a}_t\}_{t=0}^\infty \) is a sequence satisfying the three properties in the proposition. We construct a subgame-perfect equilibrium strategy profile as follows.\(^{40}\) We start with \( \sigma_0(\emptyset) = \tilde{a}_0 \). Let \( \tilde{h}^t, t \geq 1 \) be an arbitrary history whose predecessors are \( (\emptyset, \tilde{h}^0, \tilde{h}^1, ... \tilde{h}^{t-1}) \). If \( a_s = \sigma_s(\tilde{h}^{s-1}) \), for \( s = 0, ..., t-1 \), set \( \sigma_t(\tilde{h}^t) = \tilde{a}_0 \). Otherwise, let \( \hat{t} := \max\{s : a_s \neq \sigma_s(\tilde{h}^{s-1})\} \) and set \( \sigma_t(\tilde{h}^t) = \tilde{a}_{t-1-\hat{t}} \). In words, this strategy punishes any deviation by restarting the continuation equilibrium from the same equilibrium path that is supposed to prevail in period 0. Properties 1 and 3 ensure that such a punishment is sufficient to deter deviations, both in the initial period and in any subsequent period and history. This equilibrium is state independent (Requirement 1) and symmetric, since the equilibrium path of play attains an action value \( \tilde{V} \) independent of the past history. No equilibrium can attain a higher constant value. Suppose such an equilibrium existed, and let \( \{a_t^B\}_{t=0}^\infty \) be its equilibrium path, which attains a constant \( V^B > \tilde{V} \). Then we would have \( V(a_t^B, a_{t+1}^B, ...) = V^B > \tilde{V} \), \( \forall t \geq 0 \), which would contradict property 2 of our initial sequence. Therefore, the newly constructed subgame-perfect equilibrium satisfies Requirement 2. Finally, Requirement 3 is a direct analog of the third property that we imposed on the sequence.

Suppose now that a sequence satisfying the 3 properties of the proposition exists and its value is \( \tilde{V} \). Requirement 2 implies that all organizational equilibria feature a path of constant value \( \tilde{V} \) as well, which implies that they satisfy the first two properties; the third property follows directly from Requirement 3. \(\square\)

B.2 Proof of Proposition 5.

To prove this we rely on a useful lemma, which introduces a convenient way of representing equilibria through their values, similarly to Abreu, Pierce, and Stacchetti’s (1986; 1990) method.\(^{41}\)

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40 We defined an organizational equilibrium within the context of the game of Section 3.1, so the proposition is proven in the context of this game, although of course the results apply to the game of Section 3.2 when Assumption 4 is satisfied.

41 Note, however, that we cannot adopt their method to recursively compute the desired sets. Given \( V^* \), \( \tilde{V} \) can be computed recursively as in Abreu, Pierce, and Stacchetti. However, without further assumptions the set of values of \( V^* \) for which \( \tilde{V} \) is defined need not be convex, which makes finding its maximum difficult.
Lemma 5. Let $V^* \in \mathbb{R}$ and $\hat{V} \subset \mathbb{R}$ be a value and a set of continuation values that satisfy the following properties:

1. \[ \forall a \in A \; \exists \hat{v} \in \hat{V} : \hat{V}(a, \hat{v}) \leq V^*; \]

2. \[ \forall v \in \hat{V} \; \exists (a, \hat{v}) \in A \times \hat{V} : \hat{V}(a, \hat{v}) = V^* \land W(a, \hat{v}) = v. \]

3. There exists no value $V^{**} > V^*$ and set $\hat{\hat{V}}$ that satisfies properties 1 and 2; furthermore, there is no set $\hat{\hat{V}} \supset \hat{V}$ that satisfies properties 1 and 2 together with $V^*$.

Then:

- Construct an arbitrary sequence of actions $\{a^*_t\}_{t=0}^\infty$ recursively as follows. In period 0, pick $\hat{v}^*_0 \in \hat{V}$ and $(a^*_0, \hat{v}^*_1) \in A \times \hat{V}$ such that $\hat{V}(a^*_0, \hat{v}^*_1) = V^*$ and $W(a^*_0, \hat{v}^*_1) = \hat{v}^*_0$. In each subsequent period, pick $(a^*_t, \hat{v}^*_t+1) \in A \times \hat{V}$ such that $\hat{V}(a^*_t, \hat{v}^*_t+1) = V^*$ and $W(a^*_t, \hat{v}^*_t+1) = \hat{v}^*_t$. Constructing such a sequence is possible by the definition of $V^*$ and $\hat{V}$. The sequence so constructed is the outcome of a reconsideration-proof equilibrium;

- If $\{a^*_t\}_{t=0}^\infty$ is the equilibrium path of a reconsideration-proof equilibrium, $\hat{V}(a^*_0, a^*_1, ...) = V^*$ and $\hat{V}(a^*_t, a^*_{t+1}, ...) \in \hat{V}$ for any $t > 0$.

Proof.

First, we prove that the recursively-constructed sequence $\{a^*_t\}_{t=0}^\infty$ satisfies

\[ \hat{V}(a^*_t, \hat{V}(a^*_{t+1}, a^*_{t+2}, ...)) = V^* \; \forall t \geq 0 \] (25)

and

\[ \hat{V}(a^*_t, a^*_{t+1}, a^*_{t+2}, ...) \in \hat{V} \; \forall t \geq 0. \] (26)

Note that, if $\hat{v}^*_T = \hat{V}(a^*_T, a^*_{T+1}, a^*_{T+2}, ...) \in \hat{V}$ for some period $T$, iterating backwards we find that $\hat{v}^*_t = \hat{V}(a^*_t, a^*_{t+1}, a^*_{t+2}, ...)$ for all $t < T$, so that equations (25) and (26) hold.

Define

\[ \{a_t\}_{t=0}^\infty \in \arg\min_{\{a_t\}_{t=0}^\infty} \hat{V}(a_0, a_1, ...) \]

and similarly let $\{\tilde{a}_t\}_{t=0}^\infty$ be a sequence that attains the maximum. Both exist by the compactness of $A$ and the continuity of $\hat{V}$ (in the product topology).
Next, truncate the sequence \( \{a_t^s\}_{t=0}^{\infty} \) at time \( S > T \) and replace the continuation with \( \{a_t\}_{t=0}^{\infty} \) or \( \{\bar{a}_t\}_{t=0}^{\infty} \). By Assumption 5 and the monotonicity of \( W \), we have

\[
\hat{V}(a_T^s, a_{T+1}^s, \ldots, a_S^s, a_0, a_1, \ldots) \leq \hat{V}(a_T^s, a_{T+1}^s, \ldots, a_S^s, a_{S+1}^s, \ldots, a_{S+2}^s, \ldots) \leq \hat{V}(a_T^s, a_{T+1}^s, \ldots, a_S^s, a_0, \bar{a}_1, \ldots) \tag{27}
\]

and

\[
\hat{V}(a_T^s, a_{T+1}^s, \ldots, a_S^s, a_0, a_1, \ldots) = W(a_T^s, W(a_{T+1}^s, \ldots, W(a_S^s, W(a_0, W(a_1, \ldots)))...) \leq W(a_T^s, W(a_{T+1}^s, \ldots, W(a_S^s, \bar{v}_S^s)...) = \bar{v}_T^s \leq \\
W(a_T^s, W(a_{T+1}^s, \ldots, W(a_S^s, W(\bar{a}_0, W(\bar{a}_1, \ldots)))...) = \hat{V}(a_T^s, a_{T+1}^s, \ldots, a_S^s, \bar{a}_0, \bar{a}_1, \ldots). \tag{28}
\]

Taking limits as \( S \to \infty \) in equations (27) and (28) and exploiting the continuity of \( \hat{V} \) according to the product topology, the left-most and right-most expressions in the inequalities converge to the same value, which then implies that indeed \( \bar{v}_T^s = \hat{V}(a_{T+1}^s, a_{T+2}^s, a_{T+3}^s, \ldots) \) and (25) and (26) hold.

To complete the proof of the first point, we need to show that there exists no symmetric subgame-perfect equilibrium whose payoff is strictly greater than \( V^* \). By contradiction, suppose that there is such an equilibrium with value \( V^{**} > V^* \). Let \( \sigma^{**} \) be the strategy profile representing one such equilibrium. Define

\[
\hat{V}_b := \{ v : v = \hat{V}(a_{t+1}^{s_t|h_t}, a_{t+2}^{s_t|h_t}, a_{t+3}^{s_t|h_t}, \ldots), h^t \in A^t \},
\]

where \( \{a_s^{s_t|h_t}\}_{s=t+1}^{\infty} \) is the equilibrium path implied by the strategy profile \( \sigma^{**} \) following a history \( h^t \). The pair \( (V^{**}, \hat{V}_b) \) satisfies property 1 in the lemma, since otherwise \( \sigma_0^{**} \) would not be optimal at time 0. It also satisfies property 2 since \( \sigma^{**} \) is symmetric and by the definition of \( \hat{V}_b \). But then this implies that property 3 in the lemma does not hold for \( V^* \), establishing a contradiction.

In the previous point we proved that, given \( V^* \) and \( \hat{V} \), we can construct a reconsideration-proof equilibrium of value \( V^* \). Since all reconsideration-proof equilibria must have the same value, it must be the case that \( \hat{V}(a_0^s, a_1^s, \ldots) = V^* \). Furthermore, repeating the steps of the previous point, we can prove that the value \( V^* \) and the set

\[
\hat{V}_a := \{ v : v = \hat{V}(a_{t+1}^{s_t|h_t}, a_{t+2}^{s_t|h_t}, a_{t+3}^{s_t|h_t}, \ldots), h^t \in A^t \}
\]

satisfy properties 1 and 2. By the definition of \( \hat{V} \), it follows that \( \hat{V}_a \subseteq \hat{V} \).

While not essential for the proof of Proposition 5, the following lemma is useful for computations:

**Lemma 6.** The set \( \hat{V} \) defined in Lemma 5 is convex.\(^{42}\)

**Proof.** We first define the set \( \hat{V}_c \) by relaxing property 2 in Lemma 5 to be the following:

\[
\forall v \in \hat{V}_c \quad \exists (a, \hat{v}) \in A \times \hat{V} : V^* \land W(a, \hat{v}) = v.
\]

\(^{42}\)Lemma 5 defines a unique set, since the union of all sets satisfying properties 1 and 2 satisfies properties 1 and 2 as well.
We will later prove that \( \hat{\mathcal{Y}}_c = \mathcal{Y} \).

**Simple case.** First, if \( \hat{\mathcal{Y}}_c \) is a singleton, then it is necessarily convex and \( \hat{\mathcal{Y}}_c = \mathcal{Y} \): by property 3 of Lemma 5, \( V^* \) should be raised until \( \hat{V}(a, \hat{v}) = V^* \) at the single element \( \hat{v} \in \hat{\mathcal{Y}}_c \), with no effect on property 2 and relaxing the constraint in property 1.

From now on, we study the case in which \( \hat{\mathcal{Y}}_c \) contains at least two values.

**Step 1.** To prove that \( \hat{\mathcal{Y}}_c \) is convex, we prove that its convex hull, \( \operatorname{Co}(\hat{\mathcal{Y}}_c) \), satisfies properties 1 and 2 as well (and of course \( \operatorname{Co}(\hat{\mathcal{Y}}_c) \supseteq \hat{\mathcal{Y}}_c \) unless \( \hat{\mathcal{Y}}_c \) is convex as well). Property 1 is immediate from the monotonicity of \( V \).

Let \( v_1, v_2 \in \hat{\mathcal{Y}}_c \), and let \((a_1, \hat{v}_1), (a_2, \hat{v}_2)\) elements of \( A \times \hat{\mathcal{Y}}_c \) be two pairs of actions and continuation values that satisfy property 2 of Lemma 5. Consider their convex combination \((\alpha v_1 + (1 - \alpha)v_2, \alpha \hat{v}_1 + (1 - \alpha)\hat{v}_2), \alpha \in [0, 1]\). Since \( \hat{V} \) is continuous and quasiconcave and \( W \) is continuous, \( \hat{V}(\alpha v_1 + (1 - \alpha)v_2, \alpha \hat{v}_1 + (1 - \alpha)\hat{v}_2) \geq V^* \), and \( W(\alpha v_1 + (1 - \alpha)v_2, \alpha \hat{v}_1 + (1 - \alpha)\hat{v}_2) \) takes all values in \([v_1, v_2]\) as \( \alpha \) varies between 0 and 1. Hence, all intermediate values satisfy property 2 as well, which completes the proof that \( \operatorname{Co}(\hat{\mathcal{Y}}_c) \) satisfies property 2.

**Step 2.** To prove that \( \hat{\mathcal{Y}}_c = \mathcal{Y} \), proceed as follows. Define \( \underline{v} := \min\{\hat{\mathcal{Y}}_c\} \) and \( \overline{v} := \max\{\hat{\mathcal{Y}}_c\}. \) By definition, we can find \((\underline{a}, \underline{v})\) and \((\overline{a}, \overline{v})\) such that

\[
\hat{V}(\underline{a}, \underline{v}) \geq V^* \land W(\underline{a}, \underline{v}) = \underline{v},
\]

and

\[
\hat{V}(\overline{a}, \overline{v}) \geq V^* \land W(\overline{a}, \overline{v}) = \overline{v}.
\]

Since \( A \) is convex, we can construct within it a line from \( \underline{a} \) to \( \overline{a} \) by defining \( a(\alpha) := \alpha \underline{a} + (1 - \alpha)\overline{a}, \alpha \in [0, 1] \). By the quasiconcavity of \( \hat{V} \), we know

\[
\hat{V}(a(\alpha), \alpha \int_{\underline{v}} + (1 - \alpha)\overline{v}) \geq V^*.
\]

By property 1 of Lemma 5, for each action \( a(\alpha) \) and the monotonicity and continuity of \( \hat{V} \) we have

\[
\hat{V}(a(\alpha), \underline{v}) \leq V^*.
\]

Since \( \hat{\mathcal{Y}}_c \) is convex, we can find a (unique) value \( \hat{v}(\alpha) \) such that

\[
\hat{V}(a(\alpha), \hat{v}(\alpha)) = V^*.
\]

Monotonicity and continuity of \( \hat{V} \) imply that \( \hat{v}(\alpha) \) is a continuous function. It then follows that \( \hat{V}(a(\alpha), \hat{v}(\alpha)) \) is a continuous function of \( \alpha \). As \( \alpha \in [0, 1] \), this function must take all values between \( \underline{v} \) and \( \overline{v} \), proving that the property 2 of Lemma 5 is satisfied by \( \hat{\mathcal{Y}}_c \) and thus \( \hat{\mathcal{Y}}_c = \mathcal{Y} \). \( \blacksquare \)

We are now ready to prove Proposition 5.

\(^{43}\)It is straightforward to prove that \( \hat{\mathcal{Y}}_c \) is closed, by the continuity of the functions defining it.
Proof. The second property of the value $V^*$ and the set $\hat{V}$ in Lemma 5 implies that we can construct a function $g : \hat{V} \rightarrow \mathbb{R} \times \hat{V}$ with the property that $\hat{V}(g(v)) = V^*$ and $W(g(v)) = v$. Starting from any value $v_0 \in \hat{V}$, we can construct recursively a path $(a_t, v_{t+1}) = g(v_t)$. By Lemma 5, this is the equilibrium path of a reconsideration-proof equilibrium. It will thus be an organizational equilibrium provided that $V(a_t, v_{t+1}) \geq \max_a \hat{V}(a, v_0)$ for all $t$.

By the definition of $V$, this property is satisfied by its least element, $v^*$, hence, it will be satisfied provided that the initial value $v_0$ is sufficiently low.

B.3 Proof of Proposition 6

Proof. Define a correspondence $\zeta : \mathbb{R} \times \mathbb{R} \Rightarrow \mathbb{R}$ as follows:

$$v \in \zeta(v', v^*) \iff \exists a \in A : \begin{cases} \hat{V}(a, v') = v^* \\ W(a, v') = v. \end{cases}$$

(30)

In words, given $(v^*, v')$, $v$ belongs to the correspondence if there is an action $a$ which, together with a continuation value $v'$, yields utility $v^*$ when evaluated according to the decision maker’s preferences ($\hat{V}$) and utility $v$ when evaluated with its continuation utility function $W$.

We prove that there exists a value $v^*$ for which $\zeta$ is nonempty and admits a fixed point in continuation utilities ($v = v'$). We do so by proving that a Markov equilibrium $(a^M, v^M)$ exists, such that

$$v^* = \hat{V}(a^M, v^M) = \max_a \hat{V}(a, v^M)$$

(31)

and

$$v^M = W(a^M, v^M).$$

(32)

To prove the existence of a Markov equilibrium, we construct a correspondence $\hat{a}(.)$ from $A$ into itself by setting

$$\hat{a}(a) = \max_{a_0 \in A} \hat{V}(a_0, a, a, a, ...).$$

By the usual compactness and continuity properties, this correspondence is nonempty, compact-valued, and upper hemicontinuous. Quasiconcavity of $\hat{V}$ ensures that it is also convex-valued. Hence, the correspondence has a fixed point by Kakutani’s theorem; let $a^M$ be one such fixed point. Given Assumption 5, letting $v^M := \hat{V}(a^M, a^M, a^M, ...)$, equations (31) and (32) are satisfied.

We thus know $v^M \in \zeta(v^M, \hat{V}(a^M, v^M))$. Once again, our assumptions about compactness and continuity imply that the correspondence $\zeta$ is upper hemicontinuous. Let $V^*$ be the maximal value for which $\zeta$ admits

\[44\] By the monotonicity of $\hat{V}$ in its second argument and the property 1 of $V$, $\hat{V}(a, v) \leq V^*$ for all $a \in A$. 

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a fixed point in continuation utilities. In the proofs below, it is useful to establish that

\[ v \in \zeta(v', V^*) \implies v \leq v'. \]  

(33)

Suppose (33) is not satisfied. Let \( (a, v') \) be such that \( V(a, v') = V^* \) and \( W(a, v') > v' \). Holding the action \( a \) fixed, continuity and monotonicity imply that higher values of \( v' \) lead to higher values of \( V(a, v') \) and \( W(a, v') \). As long as \( W(a, v') > v' \), we know that \( v' < \max_{(a, v) \in \zeta} \tilde{V}(a_0, a_1, \ldots) \) and can thus be raised further. Eventually, we will attain a value \( v^h > v' \) for which \( W(a, v^h) = v^h \) (this has to happen, since \( W(a, v') \) is bounded by the maximum above). Let \( V^h := V(a, v^h) > V^* \). We just established that a fixed point of \( \zeta(\cdot, V^h) \) exists, which contradicts the assumption that \( V^* \) is the highest value for which a fixed point can be found.

In our next step, we prove that there are no symmetric equilibria with value \( V^{**} > V^* \). By the definition of \( V^* \), given any combination of an action and a continuation utility \( (a, v') \), if \( \tilde{V}(a, v') = V^{**} \) then \( W(a, v') < v' \). This implies that any equilibrium path with value \( V^{**} \) would feature a strictly increasing sequence of continuation values; convergence is ruled out, because continuity and compactness would imply that the limiting point would be a fixed point of \( \zeta \), which is inconsistent with \( V^{**} > V^* \). Since the set of possible continuation values is bounded by

\[ \max_{(a, v) \in \zeta} \tilde{V}(a_0, a_1, \ldots), \]

no such equilibrium path can exist.

We now prove that there exist symmetric equilibria with value \( V^* \), which then implies that any such equilibrium is reconsideration proof. Let \( v^{SS} \) be the maximal fixed point of \( \zeta(\cdot, V^*) \). For any continuation value \( v > v^{SS} \), a repetition of the arguments described above for \( V^{**} \) imply that no equilibrium path would be possible.\(^45\)

We prove instead that there exists a convex set \( \mathcal{V} = [v_\ell, v^{SS}] \) which, together with \( V^* \), satisfies the properties of Lemma 5, where

\[ v_\ell := \min_{v' \leq v^{SS}} \min (v', V^*). \]  

(34)

To do so, prove first that, for any action \( a \in A \), \( \tilde{V}(a, \min_{(a, v) \in \zeta} \tilde{V}(a_0, a_1, \ldots)) \leq V^* \). By contradiction, suppose that an action \( a_L \) such that \( \tilde{V}(a_L, \min_{(a, v) \in \zeta} \tilde{V}(a_0, a_1, \ldots)) > V^* \) existed. We could then repeat the same steps used to prove (33) and construct a steady state with value higher than \( V^* \).

Since \( \tilde{V}(a, \min_{(a, v) \in \zeta} \tilde{V}(a_0, a_1, \ldots)) \leq V^* \ \forall a \in A \), we can define

\[ v'_\min := \min_{(a, v')} := \tilde{V}(a, v') = V^*. \]

Since there exists an action \( a^{SS} \) such that \( V(a^{SS}, v^{SS}) = V^* \), \( v'_\min \leq v^{SS} \). Also, by equations (33) and (34), \( v_\ell \leq v'_\min \). Hence, \( \tilde{V}(a, v_\ell) \leq V^* \ \forall a \in A \): Property 1 of Lemma 5 is satisfied by the value \( V^* \) and the continuation set \( [v_\ell, v^{SS}] \). To prove Property 2, let \( a_\ell \) and \( v'_\ell \) be such that \( W(a_\ell, v'_\ell) = v_\ell \) and \( \tilde{V}(a_\ell, v'_\ell) = V^* \),

\(^{45}\)If along the equilibrium path, for some \( T \geq 0, v_T > v^{SS} \), then \( v_t > v^{SS} \) for all \( t > T \). Since \( \{v_t\} \) is bounded and monotonically increasing, the limiting point will be a fixed point of \( \zeta \), which is incompatible with \( v^{SS} \) being the largest fixed point.
and \( \lambda \in [0,1] \).\(^{46}\) As we just established, \( \tilde{V}(\lambda a_\ell + (1 - \lambda)a^{SS}, v_\ell) \leq V^* \). By quasiconcavity, \( \tilde{V}(\lambda a_\ell + (1 - \lambda)a^{SS}, \nu_\ell' + (1 - \lambda)v^{SS}) \geq V^* \). Strict monotonicity implies that there exists a unique value \( v_\lambda \) such that \( \tilde{V}(\lambda a^{SS} + (1 - \lambda)a_\ell, v_\lambda) = V^* \), which must vary continuously with \( \lambda \) by the continuity of \( \tilde{V} \). It follows that \( W(\lambda a^{SS} + (1 - \lambda)a_\ell, v_\lambda) \) is a continuous function of \( \lambda \) and it takes all values between \( v_\ell \) and \( v^{SS} \), proving that Property 2 of Lemma 5 holds. Finally, from equations (33) and (34), we know that any value \( v \notin [v_\ell, v^{SS}] \) could only be attained by some action \( a \) with a continuation value \( v' > v^{SS} \), which would lead to nonexistence in subsequent periods. Hence, \( [v_\ell, v^{SS}] \) is the largest set that satisfies Properties 1 and 2 of Lemma 5 together with the value \( V^* \), completing the proof that a reconsideration-proof equilibrium has value \( V^* \), and thus that in turn the organizational equilibrium with the state variable is also associated with an action value \( V^* \). Our construction also proved that \( V^* \) is the maximal action payoff that can be attained by a constant action. Finally, suppose that \( \hat{V} \) is strictly quasiconcave. Let \( a^{SS} \) be the unique action that attains \( \max_a V(a, a, a, ..., a) \).

If this steady state is not a Markov equilibrium, then \( a^{SS} < \max_a \tilde{V}(a, v^{SS}) \). In this case, a sequence that starts at \( a^{SS} \) and stays constant violates the no-delay condition.

\[ \hat{V}(\lambda a_\ell + (1 - \lambda)a^{SS}, v_\ell) \leq V^* \]  

\[ \lambda a^{SS} + (1 - \lambda)a_\ell \in \arg\max_a V(a, a^{SS}, a^{SS}, ...)\]

this implies that \((a^{SS}, a^{SS}, a^{SS}, ...)\) is also a Markov equilibrium, and that \( a^{SS} \) achieves the highest payoff among constant allocations, which (by Proposition 6) is also the payoff of an organizational equilibrium. In particular, \((a^{SS}, a^{SS}, ...)\) is an organizational equilibrium.

A state-independent Markov equilibrium cannot depend on the past nor on calendar time, and so it is a constant sequence \((a, a, ...)\). An organizational equilibrium attains the same payoff as the best constant allocation; hence, it can be no worse than the best Markov equilibrium, and is strictly better whenever the best constant allocations do not correspond to a Markov equilibrium.\(\Box\)

\textbf{B.4 Proof of Corollary 1}

\textit{Proof.} The Ramsey outcome is the allocation that attains the highest payoff, and so by definition an organizational equilibrium cannot do better. If there is no constant allocation that attains the Ramsey outcome, then it means that the best constant allocation attains a payoff strictly smaller than Ramsey; Proposition 6 proves that the payoff of an organizational equilibrium coincides with that of the best constant allocation, and is thus strictly worse than Ramsey as well. When a constant allocation \( a^{SS} \) attains the Ramsey outcome, it must be the case that

\[ a^{SS} \in \arg\max_a V(a, a^{SS}, a^{SS}, ...)\]

this implies that \((a^{SS}, a^{SS}, a^{SS}, ...)\) is also a Markov equilibrium, and that \( a^{SS} \) achieves the highest payoff among constant allocations, which (by Proposition 6) is also the payoff of an organizational equilibrium. In particular, \((a^{SS}, a^{SS}, ...)\) is an organizational equilibrium.

A state-independent Markov equilibrium cannot depend on the past nor on calendar time, and so it is a constant sequence \((a, a, ...)\). An organizational equilibrium attains the same payoff as the best constant allocation; hence, it can be no worse than the best Markov equilibrium, and is strictly better whenever the best constant allocations do not correspond to a Markov equilibrium.\(\Box\)

\textbf{B.5 Proof of Corollary 2}

\textit{Proof.} This proof follows closely that of Proposition 6. Let \( \zeta, V^*, v_\ell \), and \( v^{SS} \) be defined as in that proof. The proof of Proposition 6 rules out symmetric equilibria with values higher than \( V^* \) by showing that there does not exist a sequence of actions that has a constant value higher than \( V^* \). It also shows how to construct

\[ v_\ell \leq v'_\ell \leq v^{SS} \text{ by (33) and (34)}.\]

\[ 46\text{We have } v_\ell \leq v'_\ell \leq v^{SS} \text{ by (33) and (34).} \]
a sequence such that \( \tilde{V}(a_0, \tilde{V}(a_1, a_2, ...)) = V^* \) and \( \hat{V}(a_0, a_1, ...) = v \) for any value in \( v \in [v_\ell, v_{SS}] \); any such sequence satisfies properties 1 and 2 of Proposition 4. Let \( \{\hat{a}_t\}_{t=0}^\infty \) be such that \( \hat{V}(\tilde{a}_0, \hat{V}(\tilde{a}_1, \hat{a}_2, ...)) = V^* \) and \( \hat{V}(\hat{a}_0, \hat{a}_1, ...) = v_\ell \). The proof of Proposition 6 establishes that \( \hat{V}(a, v_\ell) \leq V^* \) \( \forall a \in A \). Hence, \( \{\hat{a}_t\}_{t=0}^\infty \) satisfies Property 3 of Proposition 4 as well.

C  Second-Order Approximation to Non-Separable Economies

To capture the transition dynamics, we apply a second-order approximation to the original economy. The difficulty is that a second-order approximation of the economy is not separable under its original representation. To proceed, we consider a transformation of the action \( a \) to \( r \), where \( r = R(k, a) \) with the normalization \( R(k, a) = a \).\(^{47}\) For later use, we denote \( S \) as the inverse of \( R \), i.e., \( a = S(k, r) \). Our strategy is to choose this transformed action \( r \) such that the separability property is satisfied under a second-order approximation. The normalization implies that \( S_r = 1 \) and \( S_{rr} = 0 \). In a second-order approximation, we need to determine \( S_k, S_{kk} \) and \( S_{kr} \) to make sure the approximated economy is separable between \( k \) and \( r \).

With this transformation, the economy can be alternatively represented by

\[
\Psi(k_t, r_t, r_{t+1}, ...) = G(k_t, r_t) + H(k_{t+1}, r_{t+1}) + \beta \Psi(k_{t+1}, r_{t+1}, r_{t+2}, ...)
\]

with

\[
k_{t+1} = T(k_t, r_t).
\]

The transformed functions \( G, H \), and \( T \) are defined in a straightforward way as

\[
T(k, r) = F(k, S(k, r)), \quad G(k, r) = P(k, S(k, r)), \quad H(k, r) = Q(k, S(k, r)). \tag{35}
\]

In a first-order approximation, we have\(^{48}\)

\[
\begin{align*}
\overline{\Psi}_k &= \overline{G}_k + \overline{H}_k T_k + \beta \overline{\Psi}_k T_k \\
\overline{\Psi}_0 &= \overline{G}_r + \overline{H}_r T_r + \beta \overline{\Psi}_r T_r \\
\overline{\Psi}_1 &= \overline{H}_r + \beta \overline{\Psi}_0 \\
\overline{\Psi}_i &= \beta \overline{\Psi}_{i-1}, \quad i > 1
\end{align*}
\]

The lifetime utility under the first-order approximation is

\[
\Psi^{(1)}(k_0, r_0, r_1, ...) = \overline{\Psi} + \overline{\Psi}_k (k - \overline{G}) + \overline{\Psi}_0 (r_0 - \overline{H}) + \sum_{i=1}^{\infty} \beta^{i-1} \overline{\Psi}_i (r_i - \overline{H})
\]

Similar to the first-order approximation in Section 4, the steady state of the approximated economy requires

\(^{47}\)This is a normalization because our equilibrium notion is invariant to monotone transformations of \( a \) alone.

\(^{48}\)We denote \( \overline{\Psi}_k \) as the derivative with respect to the state variable \( k \), and \( \overline{\Psi}_i \) as the derivative with respect to the action in \( i \) period.
\[ \Psi_0 + \Psi_{1} = 0, \] which leads to
\[ (1 - \beta P_k)(P_a + Q_a) + Q_k P_a + \beta P_k F_a + (1 - \beta)Q_a F_a S_k = 0. \] (36)

In order for our transformation to share the same limiting steady state, we require \( S_k = 0 \).

Now we proceed to the second-order approximation, and the following elements are necessary for the approximation.

\[ \Psi_{kk} = G_{kk} + H_{kk} T_k + H_k T_k + \beta \Psi_{kk} T_k + \beta \Psi_{kk} T_k \]
\[ \Psi_{k0} = G_{kr} + H_{kk} T_r + H_k T_r + \beta \Psi_{kk} T_r + \beta \Psi_{kk} T_r \]
\[ \Psi_{k1} = H_{kr} T_k + \beta \Psi_{k0} T_k \]
\[ \Psi_{i} = \beta \Psi_{i-1} T_r, \quad i > 1 \]
\[ \Psi_{0i} = \beta \Psi_{0,i-1} T_r \]
\[ \Psi_{ii} = \beta \Psi_{i-1,i-1} T_r \]
\[ \Psi_{ij} = \beta \Psi_{i-1,j-1} T_r \]

The lifetime utility in this approximated economy is given by
\[ \Psi^{(2)}(k, \{r_r\}_{r=0}^{\infty}) = \Psi + \Psi_k (k - \overline{k}) + \frac{1}{2} \Psi_{kk} (k - \overline{k})^2 + \sum_{i=0}^{\infty} \Psi_{ki} (k - \overline{k}) (r_i - \overline{r}) + \sum_{i=0}^{\infty} \Psi_{ri} (r_i - \overline{r}) + \frac{1}{2} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \Psi_{ij} (r_i - \overline{r}) (r_j - \overline{r}) \]

Weak separability of the second-order approximation requires
\[ \frac{\partial^2 \Psi^{(2)}}{\partial \overline{k} \partial r_i} = \frac{\partial^2 \Psi^{(2)}}{\partial \overline{k} \partial r_j} \]
\[ \forall i, j, k. \]

Generically, this is only possible if \( \Psi_{ki} = 0 \) for \( i \geq 0 \). Note that once \( \Psi_{k1} = 0 \), it follows that \( \Psi_{ki} = 0 \) for \( i > 1 \). As a result, it is sufficient to impose that \( \Psi_{k0} = \Psi_{k1} = 0 \). Using the definition in conditions (35), we can express the derivatives of \( T, G \) and \( H \) as functions of \( S_{kk}, S_{kr}, \) and the primitives of the economy. For
The lifetime utility in this approximated economy is thus given by the simpler expression

\[ T_k = F_k + F_a S_k = F_k \]
\[ T_r = F_a S_r = F_a \]
\[ T_{kk} = F_{kk} + F_{ka} S_k + F_a S_{kk} + (F_{ka} + F_{aa} S_k) S_k = F_{kk} + F_a S_{kk} \]
\[ T_{kr} = F_{ka} S_r + F_{aa} S_r S_k + F_{a} S_{kr} = F_{ka} + F_a S_{kr} \]
\[ T_{rr} = F_{aa} S_r + F_{a} S_{rr} = F_{aa}. \]

Because \( \Psi_{k0} \) and \( \Psi_{k1} \) are affine functions of \( S_{kk} \) and \( S_{kr} \), we can generically set \( S_{kk} \) and \( S_{kr} \) such that \( \Psi_{k0} = \Psi_{k1} = 0 \). To simplify the expressions further, note that \( \Psi_{ki} = 0 \) directly implies also that \( \Psi_{ij} = 0 \) for all \( i \) and \( j \) except for \( i = 0 \) and \( j = 1 \). Generically, we have \( T_k = F_k \neq 0 \). We then also have \( \Psi_{01} = (T_r/T_k)\Psi_{k1} = 0 \).

The lifetime utility in this approximated economy is thus given by the simpler expression

\[ \Psi^{(2)}(k, \{r_{r}\}_{r=0}^\infty) = \Psi + \Psi_{k}(k - \bar{k}) + \frac{1}{2} \Psi_{kk}(k - \bar{k})^2 + \sum_{i=0}^\infty \Psi_{ri}(r_i - \bar{r}) + \frac{1}{2} \sum_{i=0}^\infty \Psi_{ii}(r_i - \bar{r})^2 \]
\[ = \Psi + \Psi_{k}(k - \bar{k}) + \frac{1}{2} \Psi_{kk}(k - \bar{k})^2 + \Psi_{00}(r_0 - \bar{r}) + \sum_{i=1}^\infty \beta^{-1} \Psi_{1i}(r_i - \bar{r}) + \frac{1}{2} \Psi_{00}(r_0 - \bar{r})^2 + \frac{1}{2} \sum_{i=1}^\infty \beta^{-1} \Psi_{11}(r_i - \bar{r})^2. \]

By construction, this economy is separable between the state variable \( k \) and the action \( r \), and our organizational equilibrium definition can be applied. The action payoff \( V(r_0, r_1, \ldots) \) is simply the part that is only related to \( r \)

\[ V(r_0, r_1, \ldots) = \Psi_{00}(r_0 - \bar{r}) + \sum_{i=1}^\infty \beta^{-1} \Psi_{1i}(r_i - \bar{r}) + \frac{1}{2} \Psi_{00}(r_0 - \bar{r})^2 + \frac{1}{2} \sum_{i=1}^\infty \beta^{-1} \Psi_{11}(r_i - \bar{r})^2. \] (37)

With a constant sequence of \( r \), i.e., \( r_t = r \), we have

\[ V(r, r, \ldots) = \left( \Psi_{00} + \frac{\Psi_{11}}{1 - \beta} \right) (r - \bar{r}) + \frac{1}{2} \left( \Psi_{00} + \frac{\Psi_{11}}{1 - \beta} \right) (r - \bar{r})^2 = \frac{1}{2} \left( \Psi_{00} + \frac{\Psi_{11}}{1 - \beta} \right) (r - \bar{r})^2, \]

where the second equality arises from the fact that we already established that \( \frac{\Psi_{00} + \Psi_{11}}{1 - \beta} = 0 \) from the first-order approximation. \( V(r, r, \ldots) \) is thus maximized at \( \bar{r} \), where it takes a value of zero.

Let \( \{r_0, r_1, \ldots\} \) denote the equilibrium path. The no-restarting condition leads to

\[ V(\bar{r}, \bar{r}, \ldots) = V(r_t, r_{t+1}, \ldots), \quad \text{for any } t \geq 0. \]

Using the particular form of \( V(r_0, r_1, \ldots) \) in (37), the no-restarting condition implies a first-order difference
functions are therefore
\[ \Psi_0(r_t - \tau) + \frac{1}{2} \Psi_{00}(r_t - \tau)^2 + (\Psi_1 - \beta \Psi_0)(r_{t+1} - \tau) + \frac{1}{2}(\Psi_{11} - \beta \Psi_{00})(r_{t+1} - \tau)^2 = 0, \]
which leads to the transition function \( q^* \).
\[ r_{t+1} - \tau = q^*(r_t - \tau) = \frac{\Psi_0 + \sqrt{\Psi_0^2 - 2(\Psi_{11} - \beta \Psi_{00})(\Psi_0(r_t - \tau) + \frac{1}{2} \Psi_{00}(r_t - \tau)^2))}}{\Psi_{11} - \beta \Psi_{00}}, \]
where
\[ \Psi_0 = \overline{P}_a + \overline{P}_a \overline{Q}_k + \frac{\beta \overline{P}_a (\overline{P}_k + \overline{P}_k \overline{Q}_k)}{1 - \beta \overline{P}_k}, \]
\[ \Psi_{00} = \frac{\overline{P}_a \overline{P}_a \overline{Q}_k + \frac{\beta \overline{P}_a (\overline{P}_k + \overline{P}_k \overline{Q}_k)}{1 - \beta \overline{P}_k}}{\overline{P}_a \overline{Q}_k (1 - \beta \overline{P}_k)}, \]
\[ \Psi_{01} = \overline{Q}_a + \beta \Psi_{00}. \]

Now consider the no-delay condition. If the initial agent chooses to wait for the next one to start the equilibrium, its best choice is the Markov action \( r_M \) that solves
\[ r_M = \arg \max_r \Psi_0(r - \tau) + \frac{1}{2} \Psi_{00}(r - \tau)^2 = \tau - \frac{\Psi_0}{\Psi_{00}}. \]
Note that the transition function \( q^*(r - \tau) \) is minimized at \( r_t = r_M \). An organizational equilibrium is \( \{r_0, r_1, \ldots\} \) with \( r_0 = q^*(r_M - \tau) \) and \( r_{t+1} = q^*(r_t - \tau) \).

**Example** We apply the second-order approximation to the hyperbolic discounting economy considered in Section 3.4. With full depreciation and log utility, the saving rate and the capital is separable in the lifetime utility, and we can compare the solution to the organizational equilibrium in the original economy and that in the approximated economy. For the parameter values, we set \( \beta = \delta = 0.9 \), and \( \alpha = 0.4 \). The corresponding functions are therefore
\[ P(k, s) = \log((1 - s)k^\alpha), \quad Q(k, s) = -\beta(1 - \delta) \log((1 - s)k^\alpha), \quad F(k, s) = sk^\alpha. \]

The left panel shows the transition path of the saving rates. Notice that the saving rates in the approximated economy and the original economy are almost on top of each other.

With partial depreciation, the economy is no longer separable between saving rates and capital. The corre-

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\footnote{Condition (38) represents a hyperbola. As \( \Psi_0 = \overline{Q}_a < 0 \) and \( \Psi_{11} + \frac{\Psi_0}{\beta} = 0 \), it follows that \( \Psi_0 < 0 \). Therefore, the selected solution (39) has the property that \( q^*(0) = 0 \).}
responding functions become

\[ P(k, s) = \log((1-s)(k^\alpha + (1-d)k)) \quad Q(k, s) = -\beta(1-\delta)\log((1-s)(k^\alpha + (1-d)k)), \quad F(k, s) = s(k^\alpha + (1-d)k). \]

The right panel shows the transition path of the saving rates with the depreciation rate \( d = 0.7 \) and the initial capital 5% below the steady state. Similar to the full depreciation case, the saving rates also gradually converge to a level that is between the steady-state saving rates in the Ramsey outcome and the Markov equilibrium.

\section*{D Organizational Equilibrium in Policy Problems}

In Section 3, there is one player for each period. Here, the policymaker is still represented by one player for each period, but we also include a continuum of identical households that face a dynamic problem.\footnote{The notion of an equilibrium can be readily extended to environments with finite types of households or to economies with overlapping generations. Extending organizational equilibrium to economies with a continuum of types could be done by interacting the analysis here with distributional notions of equilibrium as in Jovanovic and Rosenthal (1988).} In this appendix, we describe explicitly the strategic interaction between the government and the households at different points in time. The game unfolds as follows. In each period, the government in power takes an action \( a \in A \) first. Then, the households move simultaneously. Each household takes an action \( s \in S \). The aggregate state for next period evolves according to \( k' = F(k, a, s) \). A full description would require us to specify what happens when households take different actions, so that, while they are identical ex ante, they may end up being different ex post. However, in most of the applications that are of interest, the household optimization problem has a unique solution. Hence, there can be no equilibrium in which identical households take different actions. Moreover, a deviation by a single household has no effect on aggregates. We exploit these properties and specify the evolution of the economy and preferences only after histories in which (almost) all households
have taken the same action. Starting from an arbitrary period $t$ and state $k_t$, household preferences are given by a function

$$Z(k_t, \{a_v, s_v, s_v^+\}_{v=t}^{\infty}),$$

(40)

where $s_v$ represents the action taken by the individual household, and $s_v^-$ is the action taken by (almost) all other households. We assume that $S$ is a convex compact subset of a locally convex topological linear space and that $Z$ is jointly continuous in all of its arguments (in the product topology), strictly quasiconcave in the own action sequence $\{s_v\}_{v=t}^{\infty}$, and weakly separable between the state and the remaining arguments. We also assume that household preferences are time consistent. More precisely, we assume that, given an initial level of the state $k_t$ and a sequence of other households’ actions $\{a_v, s_v\}_{v=t}^{\infty}$,

$$Z(k_t, \{a_v, s_v, s_v^+\}_{v=t}^{\infty}) = \max_{\{s_v\}_{v=t}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v^+\}_{v=1}^{\infty}) \Rightarrow Z(F(k_t, a_t, s_t), \{a_v, s_v, s_v^+\}_{v=t+1}^{\infty}) = \max_{\{\tilde{s}_v\}_{v=t+1}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v^+\}_{v=t+1}^{\infty}).$$

(41)

Equation (41) states that, if it is optimal from period $t$ to follow the same sequence of actions that all other households are taking, then it is also optimal to follow that sequence in subsequent periods, as long as other households also continue to do the same. Notice that we exploit the fact that each household has no effect on the aggregates to leave the continuation preferences over several histories unspecified; this is convenient, because it prevents us from having to explicitly introduce individual state variables. To be concrete, consider the taxation game to which we apply this general definition; in that game, $s_t$ is the individual saving rate. Equation (41) is written from the perspective of a household that starts with the same level of $k_t$ as the aggregate, which allows us not to draw a distinction between the two. If that household finds it optimal to follow the same saving rate as all other households, then it will optimally choose to have the same level of $k_{t+1}$, and equation (41) ensures that the continuation plan will remain optimal from period $t+1$ onwards. If instead the household chooses a different saving rate from others, then it would potentially enter period $t+1$ with a different level of the state from the aggregate; however, whenever this choice does not maximize (40), we know this would not be an optimal individual choice without need to specify the entire continuation path; moreover, the individual deviation does not affect aggregate incentives; hence, we do not need to keep track of it for the purpose of computing other households’ best response either.

We define a competitive equilibrium from period $t$ and a state $k_t$ as a sequence $\{a_v, s_v\}_{v=t}^{\infty}$, such that

$$Z(k_t, \{a_v, s_v, s_v^+\}_{v=t}^{\infty}) = \max_{\{s_v\}_{v=t}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v^+\}_{v=1}^{\infty}).$$

Proposition 8. Given any sequence of policy actions $\{a_v\}_{v=t}^{\infty}$, a competitive equilibrium exists.

Proof. Fix $k_t$ and $\{a_v\}_{v=t}^{\infty}$. Given our assumptions on $S$ and $Z$, the best-response function

$$br(\{s_v\}_{v=0}^{\infty}) := \arg \max_{\{s_v\}_{v=t}^{\infty}} Z(k_t, \{a_v, \tilde{s}_v, s_v^+\}_{v=1}^{\infty})$$

is well defined and continuous. By Brouwer’s theorem, it admits a fixed point, which is a competitive equilibrium. ∎
Equation (41) ensures that the continuation of a competitive equilibrium is a competitive equilibrium itself. Also, the separability assumption about $Z$ implies that, if $\{a_v, s_v\}_{v=t}^{\infty}$ is a competitive equilibrium from a state $k_t$, then it is also a competitive equilibrium from any other state $k'_t$.

In what follows, we proceed by assuming that the competitive equilibrium is unique given a sequence of policy actions, which can be verified in each specific application.\textsuperscript{51}

At time $t$, government preferences are given by a function $\Psi^g(k_t, a_t, s_t, a_{t+1}, s_{t+1}, a_{t+2}, s_{t+2}, \ldots)$. We assume that this function is also weakly separable in $k_t$ and its other arguments. For each given sequence of government actions $\{a_s\}_{s=t}^{\infty}$, a unique competitive equilibrium exists. The resulting sequence of private sector actions is given by a sequence $\{s_s\}_{s=t}^{\infty}$, which is independent of $k_t$, since household preferences are also separable in $k_t$. We thus specify the government utility from its sequence of actions as that experienced in the competitive equilibrium associated with those actions. With this specification, government preferences can be represented as in equation (8), and an organizational equilibrium can be defined in the same way as in Section 2. Existence of an organizational equilibrium is guaranteed by Proposition 2 when Assumptions 2 and 3 hold. However, these assumptions are significantly more restrictive in tax applications. As is well known, optimal tax problems frequently feature nonconvexities, in which case existence may have to be established in the specific application, as we do in our examples. Moreover, anticipation effects from the competitive equilibrium imply that Assumption 3 often does not hold either. It is worth noting that this assumption can be weakened. Its central role in our proof of Proposition 2 is to establish that the continuation sequence $(a^E_t, a^E_{t+2}, \ldots)$ in equation (11) can be made independent of the current deviation $a$. In our tax example, we prove this result by showing instead that the static best-response $\arg \max_a V(a, a_0, a_1, a_2, \ldots)$ is independent of the sequence $\{a_t\}_{t=0}^{\infty}$; hence, any continuation which deters deviation to this action will also be sufficient to deter deviation to any other choice.

As we did for the simpler case of Section 3, we relate an organizational equilibrium to a strategic notion of equilibrium. To do so, we need to keep track of histories of play. A symmetric history of play is a record of all actions taken in the past; we distinguish between histories at which the government is called to play, which are given by $h^0 := \emptyset$ and
\[
h^t := (a_0, s_0, a_1, s_1, \ldots, a_{t-1}, s_{t-1}), \quad t > 0,
\]
and histories at which households are called to play, that take the form of $h^{p,0} := a_0$ and
\[
h^{p,t} := (a_0, s_0, a_1, s_1, \ldots, a_{t-1}, s_{t-1}, a_t), \quad t > 0.
\]
Let $H$ be the set of histories at which the government is called to play, and $H^p$ the set of histories at which households are called to play. For the reasons discussed above, we only keep track of histories in which almost all households have taken the same action.

A strategy for the households is a mapping $\sigma^p : H^p \to S$; likewise, a government strategy is a mapping

\textsuperscript{51}Non-uniqueness can be accommodated by assuming a selection rule on how households coordinate when multiple equilibria are possible, as long as this rule has the properties that the continuation of a selected competitive equilibrium is selected itself as a continuation competitive equilibrium and that the selection is continuous.
\( \sigma : H \rightarrow A \). A symmetric strategy profile is a pair \( (\sigma^p, \sigma) \), representing how all households and the government will act following any symmetric history; it recursively induces a path of play \( \{a_t, s_t\}_{t=0}^\infty \).

A symmetric strategy profile \( (\sigma^p, \sigma) \) is a sequential equilibrium if the following is true:

- Given that the government will follow \( \sigma \) and other households will follow \( \sigma^p \), the actions dictated by \( \sigma^p \) are optimal for each household. After any history \( h^t \), each household takes as given the government policy action \( a_t \) and the initial state \( k_t \), which is recursively determined by the history of past play. Moreover, the strategy \( \sigma^p \) followed by other households and the government strategy \( \sigma \) determine the future path of aggregate play, \( \{s_v, a_{v+1}\}_{v=t}^\infty \). Household optimality requires that the sequence of actions prescribed by \( \sigma^p \) is optimal along this path: equivalently stated, it requires the actions prescribed by \( \sigma^p \) to be a competitive equilibrium from period \( t \) on, following any arbitrary (symmetric) history.
- Given that households will follow the strategy \( \sigma^p \) and that future governments will follow the strategy \( \sigma \), and given any past history \( h^t \), the current government choice \( \sigma(h^t) \) is optimal.

**Proposition 9.** Given any organizational equilibrium, there exists a sequential equilibrium whose outcome coincides with the organizational equilibrium.

*Proof.* Let \( (a_0^*, a_1^*, a_2^*, ...) \) be an organizational equilibrium, and let \( (s_0^*, s_1^*, ...) \) be the competitive-equilibrium associated with it. We construct a strategy profile recursively as follows:

- \( \sigma(\emptyset) = a_0^* \);
- For any \( t > 0 \) and any history \( h^t = (a_0, ..., a_{t-1}) \) such that \( a_s = a_s^* \) \( \forall s = 0, ..., t-1 \), \( \sigma(h^t) = a_t^* \);
- For any \( t > 0 \) and any history \( h^t = (a_0, ..., a_{t-1}) \) such that \( \exists s : a_s \neq a_s^* \), define \( T := \max\{s < t : a_s \neq \sigma(a_0, ..., a_{s-1})\} \) and set \( \sigma(h^t) = a_{T-1}^* \).
- For any history \( h^t = (a_0, s_0, a_1, s_1, ..., a_{t-1}, s_{t-1}, a_t) \) at which households are called to play, let \( \{a_s^*\}_{s=t+1}^\infty \) be the sequence of government actions that follow from period \( t+1 \) if the government plays the continuation of the strategy \( \sigma \) defined above following \( (a_0, ..., a_t) \). Set \( \sigma^p(h^t) \) to be the competitive equilibrium that is associated with \( (a_t, a_{t+1}^*, a_{t+2}^*, ...) \), which exists and is unique by assumption.

By construction, the household strategy satisfies the second condition for a sequential equilibrium for any history of play. For the government, following any history, the strategy prescribes to play the organizational equilibrium sequence, either from its beginning or from some element \( a_t^* \), \( t > 0 \). Should the government deviate from its strategy, the continuation strategy restarts the organizational equilibrium sequence from \( a_0^* \).

By the definition of an organizational equilibrium, continuing along the sequence is always weakly preferred to playing the best one-shot deviation followed by a restart; hence, the government optimality condition is satisfied and the strategy above describes a sequential equilibrium. \( \square \)
E Proofs and Computational Details for Section 5

E.1 Proof of Lemma 1

First consider the following social planner’s problem

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t,$$

subject to the resource constraint

$$c_t + k_{t+1} = k_t^{\alpha_t}.$$

Note that $\alpha_t$ in the production function can be time-varying in a deterministic fashion. The Euler condition is

$$\frac{1}{c_t} = \alpha_{t+1} \beta k_{t+1}^{\alpha_{t+1}-1} \frac{1}{c_{t+1}}.$$

Let $\mu_t$ denote the saving rate, i.e., $k_{t+1} = \mu_t k_t^{\alpha_t}$, then the Euler condition above can be rewritten as

$$\frac{1}{(1 - \mu_t)k_t^{\alpha_t}} = \alpha_{t+1} \beta k_{t+1}^{\alpha_{t+1}-1} \frac{1}{(1 - \mu_{t+1})k_{t+1}^{\alpha_{t+1}},$$

which can be further simplified to

$$\frac{\mu_t}{(1 - \mu_t)} = \alpha_{t+1} \beta \frac{1}{(1 - \mu_{t+1})}.$$

The associated transversality condition is

$$\lim_{t \to \infty} \beta^t \frac{k_{t+1}}{c_t} = \lim_{t \to \infty} \beta^t \frac{\mu_t}{(1 - \mu_t)} = 0.$$

By the standard concavity arguments, the planning problem has a unique solution and the Euler condition and the transversality condition are necessary and sufficient for optimality. Hence, there must be a unique sequence of saving rates that satisfies them.

Now consider the tax-distorted competitive equilibrium in Section 5. In the tax-distorted competitive equilibrium, define $\varphi_t$ as the after-tax saving rate, i.e., $k_{t+1} = \varphi_t (1 - \alpha_t \tau_t) k_t^{\alpha_t}$, the Euler condition for households is

$$\frac{1}{c_t} = \alpha_{t+1} \beta k_{t+1}^{\alpha_{t+1}-1} \frac{1 - \tau_{t+1}}{c_{t+1}}$$

or

$$\frac{1}{(1 - \varphi_t)(1 - \alpha_t \tau_t)k_t^{\alpha_t}} = \alpha_{t+1} \beta k_{t+1}^{\alpha_{t+1}-1} \frac{1 - \tau_{t+1}}{(1 - \varphi_{t+1})(1 - \alpha_{t+1} \tau_{t+1})k_{t+1}^{\alpha_{t+1}},$$

which can be simplified to

$$\frac{\varphi_t}{(1 - \varphi_t)} = \alpha \frac{1 - \tau_{t+1}}{1 - \alpha_{t+1} \tau_{t+1}} \frac{1}{(1 - \varphi_{t+1}).$$
The transversality condition is
\[ \lim_{t \to \infty} \beta^t \frac{k_{t+1}}{c_t} = \lim_{t \to \infty} \beta^t \frac{\varphi_t}{(1 - \varphi_t)} = 0 \]

The Euler and transversality conditions must hold in the competitive equilibrium of the original economy. By defining \( \alpha_t = \frac{1 - \tau_{t+1}}{1 - \alpha \tau_{t+1}} \), there exists a unique sequence of saving rates that satisfies them in the social planner’s problem. As a result, there exists a unique competitive equilibrium.

E.2 Proof of Proposition 7

First, we prove existence by showing that we can find a sequence \( \{\tau_t\}_{t=0}^\infty \) satisfying the three conditions of Proposition 4. An organizational equilibrium, given the sequence of tax rates, must give rise to a competitive equilibrium with constant value \( V \) given by
\[ V = \sum_{j=0}^\infty \beta^j \left\{ \log(1 - \alpha \tau_{t+j} - s_{t+j}) + \gamma \log \tau_{t+j} + \frac{\alpha \beta(1 + \gamma)}{1 - \alpha \beta} \log s_{t+j} \right\} \]

\[ = \log(1 - \alpha \tau_t - s_t) + \gamma \log \tau_t + \frac{\alpha \beta(1 + \gamma)}{1 - \alpha \beta} \log s_t \]

\[ + \beta \sum_{j=0}^\infty \beta^j \left\{ \log(1 - \alpha \tau_{t+j+1} - s_{t+j+1}) + \gamma \log \tau_{t+j+1} + \frac{\alpha \beta(1 + \gamma)}{1 - \alpha \beta} \log s_{t+j+1} \right\} \]

\[ = \log(1 - \alpha \tau_t - s_t) + \gamma \log \tau_t + \frac{\alpha \beta(1 + \gamma)}{1 - \alpha \beta} \log s_t + \beta V. \]

This leads to
\[ (1 - \beta) V = \log(1 - \alpha \tau_t - s_t) + \gamma \log \tau_t + \frac{\alpha \beta(1 + \gamma)}{1 - \alpha \beta} \log s_t, \] (42)

Meanwhile, the private sector’s Euler equation needs to be satisfied as well, which is
\[ \frac{s_t}{(1 - s_t - \alpha \tau_t)} = \frac{\alpha \beta(1 - \tau_{t+1})}{1 - s_{t+1} - \alpha \tau_{t+1}}. \] (43)

along with the transversality condition (22).

In what follows, it is more convenient to adopt the following change in variables:
\[ x_t := \frac{s_t}{1 - s_t - \alpha \tau_t} \]
and
\[ y_t := \frac{\alpha \beta(1 - \tau_t)}{s_t}. \]

The mapping from \( (s_t, \tau_t) \) to \( (x_t, y_t) \) is continuous and one to one. The domain of the mapping is \( \tau_t \in (0, 1/\alpha), s_t \in (0, 1 - \alpha \tau_t) \), and its range is \( x_t \in (0, \infty), y_t \in (-\infty, \alpha \beta(1 + x_t)/x_t) \).
With this change in variables, equation (42) becomes

$$(1 - \beta) V = V(x_t, y_t) := \frac{\alpha \beta (1 + \gamma)}{1 - \alpha - \beta} \log x_t - \frac{(1 + \gamma)}{1 - \alpha \beta} \log [\beta + x_t (\beta - y_t)] + \gamma \log [\alpha \beta + x_t (\alpha \beta - y_t)] - \gamma \log \alpha + \frac{(1 + \alpha \beta \gamma (1 - \alpha \beta)}{1 - \alpha \beta}$$

and the Euler equation (43) becomes

$$x_t = x_{t+1} y_{t+1}.$$ 

Except possibly for the initial period, the Euler equation requires $\tau_t < 1$, which corresponds to $y_t > 0$.

$V(x_t, y_t)$ is continuous and it achieves a unique maximum at

$$(x_t, y_t) = \left( \frac{\alpha \beta (1 + \gamma)}{1 - \alpha - \beta}, 1 - \frac{(1 - \alpha \beta) \gamma}{\alpha (1 + \gamma)} \right).$$

If the planner were not constrained by the requirements of a competitive equilibrium, it would choose a constant allocation corresponding to equation (45). However, since this would require $x_t$ to be constant and $y_t < 1$, it would violate the household Euler equation.

At the boundaries of the domain of $(x, y)$, $V(x, y)$ diverges to $-\infty$. $V$ is continuously differentiable at any interior point.

Furthermore,

$$\frac{\partial V}{\partial x} = \frac{\gamma (\alpha \beta - y)}{\alpha \beta + x (\alpha \beta - y)} + \frac{\alpha \beta (1 + \gamma)}{x (1 - \alpha - \beta)} - \frac{(1 + \gamma) (\beta - y)}{(1 - \alpha \beta) [\beta + x (\beta - y)]} =$$

$$\frac{-(\beta - y) (\alpha \beta - y) (1 - \alpha \beta) x^2 - \beta [\alpha [1 - \alpha \beta (1 + \gamma)] (\beta - y) + \gamma y (1 - \alpha) - \alpha \beta (\alpha \beta - y)] x + \alpha x^2 \beta^2 (1 + \gamma)}{(1 - \alpha \beta) [\beta + x (\beta - y)] [\alpha x (\alpha \beta - y)]}.$$  

The denominator of (46) is positive over the entire domain. Given any value of $y$, the numerator is a quadratic expression in $x$. It is positive as $x \to 0$. If $y \leq \alpha \beta$, the upper bound for $x$ is $+\infty$, and the numerator is negative as $x \to \infty$. If $y > \alpha \beta$, then the upper bound for $x$ is $\alpha \beta / (y - \alpha \beta)$; at that point, the numerator simplifies to $-\alpha x^2 \beta^2 (1 + x) (1 - \alpha) (1 - \alpha \beta) (1 + x) < 0$. Hence, no matter what the value of $y$ is, there exists a cutoff value of $x$ (dependent on $y$) such that $V$ is strictly increasing in $x$ below the cutoff and strictly decreasing above it.

Similarly,

$$\frac{\partial V}{\partial y} = -\frac{\gamma x}{\alpha \beta + x (\alpha \beta - y)} + \frac{x (1 + \gamma)}{(1 - \alpha \beta) [\beta + x (\beta - y)]}.$$  

Fixing $x$, there exists a cutoff for $y$ such that $V$ is strictly increasing in $y$ below the cutoff and strictly decreasing above it.

The properties above imply that all the level curves of $V$ are closed smooth curves that intersect each horizontal
or vertical line at most two times, with one intersection only possible if it’s a tangency point. They also imply that \( V \) has a unique point that is stationary in both \( x \) and \( y \), which is the global maximum.

We know that the global maximum of \( V \) occurs for a value \( y < 1 \). Let \( V^* \) be the value for which \( V(x, y) = (1 - \beta)V^* \) is tangent to \( y = 1 \). The properties of the level curves and the range of \( V \) imply that this value exists and is unique.

To help with the intuition, Figure 7 displays this level curve for a specific choice of parameters.\(^{52}\)

**Figure 7**: Combinations of \( x \) and \( y \) attaining the value \( V^* \). Point SS represents the steady state to which an organizational equilibrium converges, point A is the minimum value of \( x \) in an organizational equilibrium, and point B represents the starting point of the organizational equilibrium that attains the highest initial saving rate.

Note first that there exists no competitive equilibrium that attains a value higher than \( V^* \). Such a competitive equilibrium would require \((x_t, y_t)\) to remain on a level curve higher than \( V^* \) in all periods. However, along those curves \( y < 1 \). The Euler equation then implies that \( x_{t+1}/x_t \) is bounded away from one (from above). Since the level curve is closed, there exists a maximum value for \( x \) compatible with staying on the curve, and eventually the competitive equilibrium would violate it, establishing a contradiction.

Let \((x^*, 1)\) be the point at which the level curve that attains \( V^* \) is tangent to \( y = 1 \); this is point SS in Figure 7. The steady state \((x_t, y_t) = (x^*, 1)\) is a competitive equilibrium that attains the value \( V^* \).\(^{53}\) This

\(^{52}\) As in the example of the main text, we pick \( \alpha = .36, \beta = .9 \) and \( \gamma = 0.5 \).

\(^{53}\) In addition to the Euler equation, the transversality condition is satisfied by the steady state.
proves that the value \( V^\ast \) is the highest constant value that can be attained in a competitive equilibrium; to satisfy the no-restarting and optimality conditions of Proposition 4, a sequence must achieve this value. If we express the payoff in terms of \( s \) and \( \tau \) rather than \( x \) and \( y \) and we impose a steady-state allocation and competitive equilibrium (that is, that the Euler equation is satisfied), we obtain that this steady state satisfies equation (24).

We now show that the steady state itself does not satisfy the no-delay condition. If the economy is expected to be in steady state from period 1 onwards, the discounted value that the government attains from period 1 onwards is \( \beta V^\ast \) independently of the current choice. If the government starts at steady state, then its one-shot action payoff is \( V(x^\ast, 1) = (1 - \beta)V^\ast \). Consider next what happens if the government deviates and the households expect \( (x^\ast, 1) \) in period 1. In this case, the Euler equation implies \( x_0 = x^\ast \). By its choice of \( \tau_0 \in (0, 1/\alpha) \), the government is free to choose \( y_0 \in (-\infty, \alpha \beta (1 + x^\ast)/x^\ast) \), and the saving rate \( s_0 \) will adjust so as to keep \( x_0 = x^\ast \). Since the steady state does not coincide with the maximum of the function \( V \), the fact that it is a maximum along the \( x \) dimension implies that it is not a maximum along the \( y \) dimension;\(^{54}\) hence, as in Section 3, it cannot be an organizational equilibrium, because it does not satisfy the no-delay condition.

We now seek characterize other competitive equilibria that achieve the value \( V^\ast \). First, note that no such competitive equilibrium can feature \( x_t > x^\ast \) at any time \( t \): if this were the case, we would have \( x_{t+1} > x_t \) (since \( x_{t+1} y_{t+1} = x_t \) and \( y_{t+1} < 1 \) except at \( x^\ast \)); as is the case for values greater than \( V^\ast \), this would result in an increasing sequence that would eventually violate the upper bound for \( x \) on the level curve \( V^\ast \). Next, we focus on the upper arch of the level curve, between point \( A \) (with coordinates that we define to be \( (x^4, y^4) \)) and \( (x^\ast, 1) \). Point \( A \) attains the minimum value of \( x \) on the level curve \( V^\ast \). Figure 8 magnifies the relevant region of the level curve.

We prove that, along this arch, there exists a continuum of competitive equilibria with constant value \( V^\ast \), indexed by their initial value \( x_0 \). These equilibria satisfy (42) and (43) in each period and can be constructed recursively using \( x \) as the state variable. We distinguish two cases:

- **Case 1:** \( y^A > 0 \). This is the case in our picture.\(^{55}\) Starting from an arbitrary point \( x_0 \), we know that \( (x_1, y_1) \) must lie on the level curve, and also that \( x_1 y_1 = x_0 \). Let \( y_0 \) be such that \( V(x_0, y_0) = V^\ast \). We know \( x_0 y_0 \leq x_0 \), with equality only if \( x_0 = x^\ast \). We also know that \( x^\ast \cdot 1 \geq x_0 \), again with equality only if \( x_0 = x^\ast \). Along the arch, both \( x \) and \( y \) are strictly increasing as we move \( x \) from \( x^A \) to \( x^\ast \), and hence the same is true for \( xy \). It follows that there exists a unique point along the arch at which \( x_1 y_1 = x_0 \), and this point is such that \( x_1 \in (x_0, x^\ast) \) if \( x_0 < x^\ast \) (and equal to \( x^\ast \) if \( x_0 = x^\ast \)). We can then proceed in the same way to compute \( (x_2, y_2) \), \( (x_3, y_3) \) and so on. We obtain a sequence that converges monotonically to \( (x^\ast, 1) \), and which then satisfies (22) as well. This sequence is thus a competitive equilibrium.

- **Case 2:** \( y^A < 0 \). The proof is identical to Case 1, except that \( xy \) need not be strictly increasing while \( y < 0 \). However, \( y < 0 \) can only happen in period 0. From period 1 onwards, the Euler equation requires

\[^{54}\] We know that \( (x^\ast, 1) \) is a maximum along the \( x \) dimension because it is a tangency point of the level curve with the \( y = 1 \) line.

\[^{55}\] We conjecture that this is true in general and that Case 2 cannot arise, but we were unable to prove it analytically.
We study which of these competitive equilibria satisfy the no-delay condition. Let \((x_0, y_0)\) be the initial point for a competitive equilibrium along the upper arch, with \(x_0 \in [x^A, x^*]\). We show that this competitive equilibrium satisfies the no-delay condition if and only if \(x_0y_0 \leq x^A\).\(^{57}\) According to the no-delay condition, if the government chooses to deviate in period 0 and choose a different tax rate than the one implied by \((x_0, y_0)\), households expect that the economy will start from \((x_0, y_0)\) in period 1. In this case, their Euler equation implies that they will choose a saving level such that \(x = x_0y_0\) in the current period. By its choice of a tax rate, the government can attain the highest payoff conditional on this level of \(x\). When \(x_0y_0 \leq x^A\), the vertical line \(x = x_0y_0\) is outside of the level curve \(V^*\), so that, even by its choice of the best value of \(y\), the government attains a payoff that is lower than \(V^*\), and strictly so if \(x_0y_0 < x^A\). Hence, the government is better off not delaying and starting from \((x_0, y_0)\). Conversely, suppose that \(x_0y_0 > x^A\). In this case, the line \(x_0y_0\) intersects the level curve \(V^*\), and there exist values of \(y\) for which the government attains a strictly higher payoff than

\[ y > 0. \]

Over the arch of the level curve, \(y\) monotonically increases from \(y^A\) to 1 as \(x\) moves from \(x^A\) to \(x^*\). Hence, there exists a point \(C := (x^C, 0)\) such that \(V(x^C, 0) = V^*\). Given any initial value \(x_0\), not necessarily greater than \(x^C\), we can repeat the steps of Case 1 to find a sequence of points converging to \(x^*\).\(^{56}\)

\(^{56}\)Note that it is necessarily true that \(x_1 > x^C\).

\(^{57}\)Note that, by construction, \(x_0y_0 \leq x^*\).
In this case, the no-delay condition fails, since the government would have an incentive to take its one-shot best reply, counting on the economy to restart from \((x_0, y_0)\) in the next period.

As we already observed, \(xy\) is either strictly increasing in \(x\) on the entire upper arch, or is first negative, and then positive and strictly increasing as \(x\) grows from \(x^A\) to \(x^*\). Furthermore, \(x^Ay^A < x^A\) and \(x^*\cdot 1 = x^* > x^A\). Hence, there exists a unique point \((x^B, y^B)\) such that \(x^By^B = x^A\); this is point B in Figures 7 and 8. All competitive equilibria starting from points \(x \in [x^A, x^B]\) satisfy the no-delay condition and by Proposition 4 are therefore organizational equilibria, and all competitive equilibria starting from points \(x \in (x^B, x^*)\) fail the no-delay condition.

The construction above shows that all organizational equilibria on the upper arch of the level curve, from the one starting at point A to the one starting at point B, can be constructed recursively using \(x\) as a state variable. Inverting the mapping from \((s, \tau)\) to \((x, y)\), we obtain

\[
\begin{align*}
  s &= \frac{\beta(1 - \alpha)}{\beta(1+x) - y} \\
  \tau &= \frac{\alpha \beta(1+x) - y}{\alpha (\beta(1+x) - y)}.
\end{align*}
\]

\(s\) is strictly increasing in both \(x\) and \(y\), and \(\tau\) is strictly decreasing in both \(x\) and \(y\). Hence, for all competitive equilibria described by sequences of \(x\) and \(y\) that stay on the upper arch of the level curve \(V^*\), \(x, y, s, \) or \(\tau\) are all connected by one-to-one mappings: as we move from point A towards the steady state, \(x, y, s, \) and \(\tau\) increase (strictly), and \(\tau\) decreases (strictly). We can thus equivalently express these equilibria recursively in \(x\), as we did in this appendix, or in \(\tau\), as we do in the main text. This completes the proof.

For completeness, we next study the points on the lower arch of the \(V^*\) level curve, from \(x = x^A\) to \(x = x^*\). All of these points can be starting points of a competitive equilibrium with value \(V^*\): the Euler equation \(x_1y_1 = x_0\) only depends on \(x_0\), and not on \(y_0\), so not on whether the economy is on the upper or lower arch. Whether points on the lower arch can be attained in subsequent periods depends on whether \(xy < x^A\) for these points. In our numerical examples, we find that this is the case, and so those points cannot occur after period 0. Since \(xy < x^A\), they do satisfy the no-delay condition and are organizational equilibria. If there is any parameterization for which there are points \((x, y)\) on the lower branch that satisfy \(xy \geq x^A\), these points would be valid continuation points in a competitive equilibrium that starts from \(x_0 = xy\) (and possibly from an even earlier point). Conversely, in this case, they would not satisfy the no-delay condition and would thus not be organizational equilibria. Regardless of the initial value of \(xy\) and whether it occurs on the lower or the upper branch of the \(V^*\) level curve, all competitive equilibria converge to \((x^*, 1)\) and are therefore asymptotically located on the upper branch: this is because \(y\) is bounded away from 1 on the lower branch, and sequences that stay there would eventually have to grow past \(x^*\), which we already know is impossible.

In our main text, we focus on the organizational equilibrium which has the highest saving rate: while the period-0 government is indifferent, the government in period 1 strictly prefers equilibria that features greater
capital accumulation. Along the upper arch, this is the equilibrium that starts from point B, the closest point to the steady state that is compatible with the no-delay condition. While we were not able to prove this analytically, we verify numerically that equilibria that start on the lower branch feature a lower saving rate in period 0.

E.3 Ramsey Outcome and Markov Equilibrium in the Taxation Problem

In this part, we specify the details in Section 5.1 and Section 5.2. We show how to derive the tax rates in the Markov equilibrium and the Ramsey outcome. We also show the separability of the environment in the quantitative model in Section 5.2.

E.3.1 Simple Taxation Example

First consider the Markov equilibrium. We use a guess-and-verify approach. The guess is all future governments will choose a constant tax rate $\tau'$ and future saving rate chosen by households is also a constant $s'$. The current government then chooses its own tax rate taking $\tau'$ and $s'$ as given

$$
\max_{\tau} \log(1 - \alpha \tau - s) + \gamma \log \tau + \frac{\alpha \beta (1 + \gamma)}{1 - \alpha \beta} \log s + \sum_{j=1}^{\infty} \beta^j \left\{ \log(1 - \alpha \tau' - s') + \gamma \log \tau' + \frac{\alpha \beta (1 + \gamma)}{1 - \alpha \beta} \log s' \right\},
$$

subject to

$$
s = \frac{\alpha \beta (1 - \tau')}{1 - s' - \alpha \tau'}.
$$

Because $\tau'$ and $s'$ are taken as given, the problem can be simplified to

$$
\max_{\tau} \log(1 - \alpha \tau - s) + \gamma \log \tau + \frac{\alpha \beta (1 + \gamma)}{1 - \alpha \beta} \log s.
$$

By the Euler condition, we have

$$
s = \frac{\alpha \beta (1 - \tau')}{1 - s' - \alpha \tau' + \alpha \beta (1 - \tau')(1 - \alpha \tau)}.
$$

It turns out that the current government always chooses a constant tax rate $\tau^M$ that is independent of $\tau'$,

$$
\tau^M = \frac{\gamma (1 - \alpha \beta)}{\alpha (1 + \gamma)},
$$

which verifies our guess.

Now turn to the Ramsey problem, which can be written as

$$
\max_{\{s_t\}, \{\tau_t\}} \sum_{t=0}^{\infty} \beta^t \log(1 - \alpha \tau_t - s_t) + \gamma \log \tau_t + \frac{\alpha \beta (1 + \gamma)}{1 - \alpha \beta} \log s_t,
$$

68
subject to
\[ \frac{s_{t-1}}{1 - s_{t-1} - \alpha \tau_{t-1}} = \frac{\alpha \beta (1 - \tau_t)}{1 - s_t - \alpha \tau_t}, \quad \text{for } t > 0. \]

Note that
\[ \log(1 - \alpha \tau_t - s_t) + \gamma \log \tau_t + \frac{\alpha \beta (1 + \gamma)}{1 - \alpha \beta} \log s_t = \mathcal{H}(s_t, \tau_t). \]

Denote \( \beta^t \lambda_t \) as the multiplier associated with the implementability constraint. The first-order conditions are
\[ \mathcal{H}_s(t) = \lambda_t \frac{\alpha \beta (1 - \tau_t)}{(1 - s_t - \alpha \tau_t)^2} - \beta \lambda_{t+1} \frac{1 - \alpha \tau_t}{(1 - s_t - \alpha \tau_t)^2}, \]
\[ \mathcal{H}_\tau(t) = \lambda_t \frac{-\alpha \beta (1 - s_t - \alpha)}{(1 - s_t - \alpha \tau_t)^2} + \beta \lambda_{t+1} \frac{\alpha s_t}{(1 - s_t - \alpha \tau_t)^2}. \]

The economy converges to a steady state in which \( \lambda_t = \lambda_{t+1} \) and
\[ \mathcal{H}_s = \frac{1}{\alpha} \mathcal{H}_\tau. \]

### E.3.2 Quantitative Taxation Model

In this appendix, we describe the case in which there is no tax allowance for capital depreciation.\(^{58}\) Let \( s_t \) denote the saving rate, i.e., \( k_{t+1} = s_t f(k_t, \ell_t) \). The allocation in the competitive equilibrium is
\[ k_{t+1} = \kappa k_t^{1-\delta} (s_t y_t)^\delta, \]
\[ g_t = (\alpha (\tau_k^t + \tau_t) + (1 - \alpha)(\tau_\ell^t + \tau_t)) y_t, \]
\[ c_t = (1 - s_t - \alpha (\tau_k^t + \tau_t) - (1 - \alpha)(\tau_\ell^t + \tau_t)) y_t, \]
where \( y_t = f(k_t, \ell_t) \) is the total output. The household’s inter and intra Euler conditions satisfy
\[ \frac{u_c(t)}{\delta k_{t+1}^{1-\delta}} \frac{1}{s_t} = \beta u_c(t+1) \left\{ (1 - \tau_{t+1} - \tau_k^t) f_k(k_{t+1}, \ell_{t+1}) + \frac{1 - \delta}{\kappa} \right\}, \]
\[ u_c(t) = -u_c(t)(1 - \tau_\ell^t - \tau_t) f_\ell(k_t, \ell_t), \]

\(^{58}\)When the allowance is present, the households' budget constraint is
\[ c_t + i_t = w_t \ell_t + r_t k_t - (\tau_k^t + \tau_t) w_t \ell_t - \left( \frac{\tau_k^t + \tau_t}{\tau_\ell^t + \tau_t} \right) r_t k_t. \]

However, this specification would break separability property. Instead, we approximate the depreciation allowance and rewrite the budget constraint as
\[ c_t + i_t = w_t \ell_t + r_t k_t - (\tau_k^t + \tau_t) w_t \ell_t - \left( \frac{\tau_k^t + \tau_t}{\tau_\ell^t + \tau_t} \right) r_t k_t, \]
where \( r_t \) is the steady state interest rate.
which can be written as
\[
\begin{align*}
\gamma_t \ell_t &= \frac{(1 - \alpha)(1 - \tau_t) - (1 - \alpha)(\tau_t^f + \tau_t)}{1 - \tau_t + \alpha(\tau_t^k - \tau_t + \tau_t)} \left\{ \alpha(1 - \tau_t + \tau_t + s_{t+1} + \frac{1}{1 - \delta} \right\}, \\
\delta &\frac{1}{1 - \delta} \\
\end{align*}
\]

(48)

Given an initial capital level \( k_0 = k \), a sequence of saving rates, and a sequence of labor supply choices, the implied sequence of capital is
\[
k_t = k_0^{(1 - \delta + \alpha \delta)^{1 - j}} \prod_{j = 0}^{\infty} \left( (1 - \alpha)(1 - \delta + \alpha \delta)^{1 - j} \right) \ell_j (1 - \alpha)(1 - \delta + \alpha \delta)^{1 - j} \ell_j + k_1 \delta^{1 + (1 - \delta + \alpha \delta)^{1 - j}}.
\]

(49)

Given a sequence of tax rates, the action payoff for the government is
\[
V(s, \tau^k, \tau^f) = (\gamma_c + \gamma_g) \left\{ \frac{\alpha \beta s_{j-1} \mu_2}{1 - (\mu_1 + \alpha \mu_2) \beta} \sum_{j=0}^{\infty} \beta^j \log s_j + \frac{(1 - \alpha)(1 - \beta \mu_1)}{1 - (\mu_1 + \alpha \mu_2) \beta} \sum_{j=0}^{\infty} \beta^j \log \ell_j \right\} + \\
+ \sum_{j=0}^{\infty} \beta^j \gamma_c \log (1 - s_j - \alpha(\tau_j^k + \tau_j) - (1 - \alpha)(\tau_j^f + \tau_j)) + \\
+ \sum_{j=0}^{\infty} \beta^j \gamma_c \log (1 - \ell_j) + \sum_{j=0}^{\infty} \beta^j \gamma_c \log (\alpha(\tau_j^k + \tau_j) + (1 - \alpha)(\tau_j^f + \tau_j))
\]

(50)

Organizational Equilibrium The steady-state of the organizational equilibrium maximizes the action payoff (50) with constant saving rate, labor, and tax rates subject to the private sector’s implementability constraints (48) and (49). For example, in the case of total tax rate \( \tau \), the problem is
\[
\max_{\tau} V(s, \tau) = \frac{\gamma_c + \gamma_g}{\alpha \beta s_{j-1} \mu_2 (1 - \beta \mu_1) \log \ell + \gamma_c \log (1 - s - \tau) + \gamma_c \log (1 - \ell) + \gamma_g \log (\tau)}
\]

subject to
\[
\begin{align*}
\delta &\frac{1}{1 - \delta} \left\{ \alpha(1 - \tau) + s \right\}, \\
\gamma_c \frac{\ell}{1 - \ell} &= \gamma_c \frac{(1 - \alpha)(1 - \tau)}{1 - s - \alpha \tau}.
\end{align*}
\]

Ramsey Outcome Let \( g_t = \mu_t f(k_t, \ell_t) \). The government budget constraint requires that
\[
\mu_t = \alpha(\tau_t^k + \tau_t) + (1 - \alpha)(\tau_t^f + \tau_t).
\]

Depending on the tax instrument used for financing public spending, it is easy to define the required tax rate as a function of \( \mu_t \). Denote \( \mathcal{T}^k(\mu), \mathcal{T}^f(\mu), \) and \( \mathcal{T}(\mu) \) as the capital income, labor income, and total income tax rate to achieve the government spending to output ratio \( \mu \).
By the primal approach of the Ramsey problem, the government effectively chooses the sequence of saving rates, labor supply, and government spending to output ratios to maximize the welfare of the initial government

$$\max_{\{s_t, \ell_t, \mu_t\}} \sum_{t=0}^{\infty} \beta^t \left( \frac{\gamma_c + \gamma_g}{1 - (1 - \delta + \alpha \delta) \beta} \left( \alpha \beta \delta \log s_t + (1 - \alpha)(1 - \beta(1 - \delta)) \log \ell_t + \gamma_c \log (1 - s_t - \mu_t) \right) + \gamma_\ell \log (1 - \ell_t) + \gamma_g \log (\mu_t) \right),$$

subject to the corresponding implementability constraint

$$\frac{1}{\beta} \frac{s_t}{1 - s_t - \mu_t} = \frac{\delta \alpha \left( 1 - (T^k(\mu_{t+1}) + T(\mu_{t+1})) \right)(1 - \delta) s_{t+1}}{1 - s_{t+1} - \mu_{t+1}},$$
$$\gamma_\ell \frac{\ell_t}{1 - \ell_t} = \gamma_c \left( \frac{(1 - \alpha)(1 - T^\ell(\mu_t) - T(\mu_t))}{1 - s_t - \mu_t} \right).$$

**Markov Equilibrium** In the Markov equilibrium, the current government takes future government’s policy as given. In our setting, this will be taking future government policy as a constant independent of current policy. Assume future tax rates are \(\{\tau^k_f, \tau^\ell_f, \tau^g_f\}\) and the current policy choice is \(\{\tau^k_0, \tau^\ell_0, \tau^g_0\}\). The current government’s action payoff is

$$M(\tau^k_0, \tau^\ell_0, \tau^g_0; \tau^k_f, \tau^\ell_f, \tau^g_f) = (\gamma_c + \gamma_g) \left\{ \frac{\alpha \beta \mu_2}{1 - (\mu_1 + \alpha \mu_2) \beta} \left( \log s_0 + \frac{\beta}{1 - \beta} \log s_f \right) + \frac{(1 - \alpha)(1 - \beta \mu_1)}{1 - (\mu_1 + \alpha \mu_2) \beta} \left( \log \ell_0 + \frac{\beta}{1 - \beta} \log \ell_f \right) \right\}$$
$$+ \gamma_c \left\{ \log \left( 1 - s_0 - \alpha(\tau^k_0 + \tau^g_0) - (1 - \alpha)(\tau^\ell_0 + \tau^g_0) \right) + \frac{\beta}{1 - \beta} \log \left( 1 - s_f - \alpha(\tau^k_f + \tau^g_f) - (1 - \alpha)(\tau^\ell_f + \tau^g_f) \right) \right\}$$
$$+ \gamma_\ell \left\{ \log \left( 1 - \ell_0 \right) + \frac{\beta}{1 - \beta} \log \left( 1 - \ell_f \right) \right\}$$
$$+ \gamma_g \left\{ \log \left( \alpha(\tau^k_0 + \tau^g_0) + (1 - \alpha)(\tau^\ell_0 + \tau^g_0) \right) + \frac{\beta}{1 - \beta} \log \left( \alpha(\tau^k_f + \tau^g_f) + (1 - \alpha)(\tau^\ell_f + \tau^g_f) \right) \right\}.$$  

The current government’s problem is

$$\max_{\tau^k_0, \tau^\ell_0, \tau^g_0} M(\tau^k_0, \tau^\ell_0, \tau^g_0; \tau^k_f, \tau^\ell_f, \tau^g_f),$$
subject to the implementability constraints

\[
    s_0 = \delta \beta \frac{1 - s_0 - \alpha (\tau^b_0 + \tau_0) - (1 - \alpha) (\tau^b_0 + \tau_0)}{1 - s_f - \alpha (\tau^b_f + \tau_f) - (1 - \alpha) (\tau^b_f + \tau_f)} \left\{ \alpha (1 - \tau^b_f - \tau_f) + sf_0 \frac{1 - \delta}{\delta} \right\},
\]

\[
    \gamma \ell_0 = \gamma_c \frac{(1 - \alpha)(1 - \tau^b_0 - \tau_0)}{1 - s_0 - \alpha (\tau^b_0 + \tau_0) - (1 - \alpha) (\tau^b_0 + \tau_0)},
\]

\[
    s_f = \delta \alpha \beta (1 - \tau^b_f - \tau_f)
\]

\[
    \gamma \ell_f = \gamma_c \frac{(1 - \alpha)(1 - \tau^b_f - \tau_f)}{1 - s_f - \alpha (\tau^b_f + \tau_f) - (1 - \alpha) (\tau^b_f + \tau_f)}.
\]

The Markov equilibrium is then the fixed point where future taxes and the current taxes are the same.