Redistributive Shocks and Productivity Shocks*

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December 2009

Abstract

We document the dynamic effects of productivity shocks on labor share that we label as *overshooting*: a productivity innovation produces a reduction of labor share at impact, making it countercyclical, but it also subsequently produces a long-lasting increase in labor share that peaks above mean five years later at a level larger in absolute terms than the initial drop, after which time it slowly returns to average. We pose and estimate a bivariate shock to the production function that under competition in factor markets accounts for the dynamic overshooting response of labor share to productivity innovations. When we confront agents with this bivariate process in an otherwise standard real business cycle economy we find that the contribution of productivity innovations to the variance of hours is, independently of the Frischian elasticity of labor supply, 1% of that in the standard RBC model, which poses a univariate shock to the Solow residual and imposes constant factor shares. Our results drastically shift the assessment of previous findings regarding the contribution of productivity shocks to aggregate fluctuations: the dynamic overshooting response of labor share to productivity (almost entirely) eliminates the ability of standard RBC models to deliver model-generated hours that resemble actual data. We conclude that the modeling of aggregate fluctuations that account for—and use as discipline device—the overshooting property of labor share should become an important piece of business cycle research.

*We thank the attendees at the Macro Lunch at Wharton who made fundamental contributions to this project. We benefited from conversations with Urban Jermann, John Knowles, and especially Se Kyu Choi. We also thank Andrés Erosa, Jonas Fisher, Gary Hansen, Claudio Michelacci, Josep Pijoan-Mas, and seminar participants at the Bank of Canada, the Bank of Portugal, the Federal Reserve Bank of Atlanta, the Federal Reserve Bank of Minneapolis, the London School of Economics, the Economic Dynamics Conference in Honor of Edward Prescott at the University of Tokyo, Seoul National University, University of California Santa Bárbara, Universitat de València, and the 2008 ADRES/EDHEC conference on "Labor Market Outcomes: A Transatlantic Perspective". Ríos-Rull thanks the National Science Foundation (Grant SES-0079504) and the Penn Research Foundation for support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, the Federal Reserve Bank of St. Louis, or the Federal Reserve System.

1 Introduction

Models driven by productivity shocks have been successful in accounting for a broad set of business cycle phenomena. In particular, the cyclical volatility of output and hours, and the majority of the movements in hours worked can be attributed to such shocks (Prescott (1986), Kydland and Prescott (1991), Prescott (2006)). The gist of this research is to pose optimizing agents that respond optimally to changes in the environment: a productivity shock is an opportunity to produce more than normal, and agents take advantage of it by increasing hours worked and investment as well as consumption. Over the past 25 years, these results have been robust to a wide range of abstractions of the standard real business cycle model via the introduction of complementary sources of fluctuations and alternative propagation mechanisms (see a comprehensive review in King and Rebelo (1999) and Rebelo (2005)). Yet, an ingredient common to (almost all) real business cycle (RBC) models is the assumption that the factor distribution of income is constant at all frequencies, a direct implication of Cobb-Douglas technology and competitive factor pricing. Implicit therein is the premise of unimportant implications for the business cycle of the seemingly small fluctuations we observe in the factor shares of income (which move within a range of 5–6%, U.S. 1954.I–2004.IV) and, most importantly for us, of the dynamic interaction these fluctuations have with the impulse mechanism of these models, the productivity residual.

Although few papers (see our review that follows) that have documented and addressed the cyclicality of the labor share, our contribution corresponds to: $i)$ bringing a new piece of empirical evidence documenting the dynamic effects of productivity shocks on labor share described by an overshooting response of labor share to productivity innovations; and $ii)$ investigating whether this overshooting property of labor share matters for our understanding of business cycles.

First, we document for the first time, to the best of our knowledge, the dynamic effects of productivity shocks on labor share identified as impulse response functions from data: labor share overshoots. Our empirical finding is that a productivity innovation produces a reduction of labor share at impact, making it countercyclical, but it also subsequently produces a long-lasting increase of labor share that overshoots its long-run average after five quarters and peaks above mean five years later at a level larger in absolute terms than the initial drop, after which it slowly returns to average. We also find that innovations to labor share, on the contrary, do not have any effect on productivity at any period. Further, we review and update the set of cyclical properties of labor share that previous literature has invariably focused on: labor share (a) is quite volatile (its variance is 18% that of output[1] and 64% that of the Solow residual); (b) is countercyclical

[1]Although it is customary to describe what models account for in terms of the percentage of the standard deviation in the model relative to that in the data, we report the percentage of the variance. We do this because

(a contemporaneous correlation of -.24 with output and of -.48 with the Solow residual); (c) is highly persistent (with a first-order autocorrelation of .78, whereas this coefficient is .85 for output and .71 for the Solow residual); and (d) lags output (the correlation between current output and next year's labor share is .47).[2]

Second, we explore, as parsimoniously as we can with respect to the standard RBC model, whether the dynamic effects of productivity on labor share—the novel empirical evidence that we document—matter for our understanding of the business cycle. To implement the joint dynamics of labor share and the productivity residual that replicate those in the U.S. economy, we pose a bivariate shock to a Cobb-Douglas production function that, when factor markets behave competitively, generates in an otherwise absolutely standard RBC model both the overshooting response of labor share to productivity innovations and, at the same time, the properties of labor share studied by previous literature as in (a)–(d) above. Specifically, the two sources of fluctuations in our model are a productivity shock that is essentially identical to the Solow residual, and a redistributive shock constructed from the deviations of the labor share with respect to its mean. We estimate a joint process for these two shocks, which we feed back into the model. Importantly, we preserve the productivity residual as the main driving force of the business cycle and treat innovations in labor share as purely redistributive in nature, that is, without productivity level effects.

We find that in such an environment, the response of hours and output is dramatically reduced: the size of the fluctuations of HP-filtered hours falls to about one-tenth in terms of the variance of its univariate counterpart; the volatility of HP-filtered output falls to about one-half in terms of the variance; and the correlation of hours with output drops to one-fifth. Further, our findings are not sensitive to the Frisch elasticity of labor supply—Hansen-Rogerson preferences do not alter our results. Most importantly, we find that the specific contribution of productivity innovations alone is essentially zero. More precisely, accounting for the overshooting response of labor share to productivity conduces productivity innovations to generate 1.25% of the variance of hours of its univariate counterpart—that is, the bivariate process generates 13.5% of the variance of a univariate RBC process, but of that percentage, 12.25% is generated by innovations to the redistributive shock. In terms of the data, when we incorporate the dynamic effect of productivity on labor share into an otherwise standard RBC model, productivity innovations generate 0.21% of the variance of hours observed in the U.S. economy.

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variances are additive, whereas standard deviations are not. In this sense, if one mechanism accounts for 70% of the standard deviation, what other (orthogonal) mechanisms have to account for is not 30% as one might think, but 71%. This is not the case for the variance: if one mechanism accounts for 70% of the variance, then other (orthogonal) mechanisms account for the remaining 30%.

[2] These statistics are computed from logged and hp-filtered series for the U.S. 1954.I–2004.IV. See Section 2.

The reason for the decline in the volatility of hours can be described with the standard tools of wealth and substitution effects: when we incorporate the dynamic overshooting effect of productivity on labor share, the response of factor prices to productivity innovations generates a larger wealth effect than in the standard univariate shock economy that induces a very small response of hours. The differential substitution effects have a small impact tending to delay, rather than reduce, the response of hours worked first intratemporally through the relative price of the labor input and then through the intertemporal price of consumption. That the reduction in hours occurs in economies without wealth effects (those with Greenwood, Hercowitz, and Huffman (1988) preferences) further attests that wealth effects are the main culprit of the low hours volatility with cyclically moving labor share.

Our findings are independent of the Frisch elasticity of labor, since they also hold in Hansen-Rogerson indivisible-labor economies. Most importantly, we find the margin that explains our results and shapes the smaller volatility of hours is the overshooting response of labor share to productivity innovations, that is, the large positive effect of the current productivity shock in next periods' labor share that we document here rather than the set of cyclical properties of labor share—as in (a)–(d) above—analyzed by previous literature that do not significantly alter (or even increase) the equilibrium fluctuations of hours. We now turn to reviewing this literature.

Related Previous Literature: Exogenous Labor Share. Our paper is related to two main sources of independent evidence for invoking labor share movements exogenously. Castañeda, Díaz-Giménez, and Ríos-Rull (1998) pose an exogenous process for the labor share coefficient that moves one to one with the productivity shock in its study of the cyclical behavior of income distribution. Young (2004) introduces a sole univariate process for the coefficients in the Cobb-Douglas production function. Hence, both studies omit the dynamic effect of productivity on labor share, the fundamental margin of our analysis.

Endogenous Labor Share. A first set of papers builds on the cyclical allocation of risk and optimal labor contracts. Gomme and Greenwood (1995) study a complete markets economy with workers and entrepreneurs that insure against business cycle income losses through two different financial arrangements: first, workers' Arrow securities are directly included in the wage bill, and second, workers buy bonds issued by the entrepreneurs and only the insurance component net of workers' savings is added to the wage bill. Either wedge counterbalances the procyclical marginal product of labor and generates a countercyclical labor share of income. Importantly for us, the labor choice in Gomme and Greenwood (1995) is not affected by movements in the labor share. Boldrin and Horvath (1995) use contract theory in a model with workers and entrepreneurs, where workers are not allowed to self-insure through savings and are more risk averse than entrepreneurs.

The optimal contract trades a provision of insurance from entrepreneurs to workers for a more flexible labor supply that generates a negative correlation of the labor share with output.[3]

A second important set of papers that considers endogenous cyclical movements in labor share uses models with binding capacity constraints. Hansen and Prescott (2005) introduce variable capacity utilization and idle resources in a real business cycle model to study asymmetries generated by *occasionally* binding capacity constraints. In this model, small plants face decreasing returns to scale and operate if they satisfy a minimum labor input requirement. Aggregate output is then determined by labor, capital, and "location" capital. At full capacity, the labor share of income is lower than when some plants remain idle, because in the latter case, the "location" capital is not a scarce factor and does not earn income. Since the capacity constraint binds in expansions, the model obtains a countercyclical labor share of income (of -.51). The changes in the cyclical behavior of the real variables are minor with respect to the standard model; in particular, hours are 90% of that of the standard RBC model. This is also the case when capacity constraints *always* bind; see Cooley, Hansen, and Prescott (1995), whose results yield a negative correlation between output and labor share of -.91.

A third relevant strand of the literature that deals with endogenous cyclical variations in the factor shares is that which includes an explicit role for markups. With increasing returns to scale, a fixed number of firms in monopolistic competition, and a constant markup, Hornstein (1993) obtains a labor share that is half as volatile as what is observed in the data and that is perfectly and negatively correlated with output. He also finds that productivity shocks increase output volatility but lower that of hours with respect to the standard RBC model and argues that for not unreasonably large returns to scale and markups, the contribution of productivity shocks remains almost unchanged. Finally, Ambler and Cardia (1998) allow for the not simultaneous entry and exit of firms and obtain similar results.

What Makes Our Paper Different? Taking stock, the view underlying this literature on RBC models with endogenous labor share is that extra fluctuations of the distribution of income across capital and labor leaves the role of productivity shocks unchanged (or perhaps, even enhances it), completely at odds with our findings. The key differential element that identifies our paper and results is the inclusion in our analysis of the new empirical evidence that we provide regarding the dynamic effects of productivity shocks on labor share: labor share overshoots. We show that we can attribute to this previously omitted margin the differences between the results of previous models of labor share and ours. That is, shutting down the overshooting response of labor

[3] Danthine, Donaldson, and Siconolfi (2007) introduce a distribution of risk calibrated to generate the cyclical variation of the factor shares. However, their model is silent about the allocation of hours because agents supply labor inelastically.

share while keeping the cyclical properties of labor share addressed by previous literature (as in (a)–(d) above), we obtain—as this literature did—equilibrium allocations similar to those of the standard RBC model. In this context, we argue that a fundamental feature an RBC model with time-varying factor shares must incorporate is the joint dynamics that factor shares—the element under study—have with the productivity residual—the main driving source of fluctuations in those models—described by the overshooting response of labor share that we document. We also argue that more effort is needed in this endeavor and, in particular, we point to the endogenous characterization of the overshooting property of labor share. In this pursuit, as we discuss later, we consider (among others) several versions of the mechanisms suggested by previous literature to study (a)–(d), also promising to understand the dynamic overshooting effects of productivity on labor share. Whether the role of productivity shocks resuscitates in alternative (old and new) business cycle models with richer propagation mechanisms when we confront these with the overshooting property of labor share is still a very open question, and we call for such an evaluation.

The Empirical Debate. Recently, a new discussion has taken place about the role of productivity shocks in generating business cycles. Galí (1999) identifies technology innovations as the only ones that can have a permanent impact on average labor productivity and finds that, under this identifying assumption, non-stationary hours do not empirically respond positively to productivity innovations. These results have been challenged by several authors, and an active ongoing debate remains.[4] Although this debate is important in shaping our understanding of productivity shocks, it focuses on margins that are substantively very different from ours: we do not identify productivity shocks with their permanent impact on long-run labor productivity and then analyze the empirical response of hours and output that result from an estimated system identified with this (or other additional circular) assumption(s); instead, we note the central role played by the overshooting response of labor share to productivity innovations in determining the volatility of hours—and the contribution of technology shocks—as an equilibrium outcome of standard RBC models, and we call for business cycle models where this overshooting property of labor share is incorporated. In other words, Galí's criticism does not challenge the ability of standard real business cycle models to deliver model-generated hours that resemble actual data, which is what we do.

The reminder of the paper is structured as follows. We begin in Section 2 by documenting

[4]Most notably, Fisher (2006), Canova, López-Salido, and Michelacci (2007), Ravn and Simonelli (2008), and Chang and Hong (2006). See also Christiano, Eichenbaum, and Vigfusson (2003) and Dedola and Neri (2007) regarding robustness with respect to variable specification; Francis and Ramey (2005) and Uhlig (2004) for alternative identification strategies; and Chari, Kehoe, and McGrattan (2008), Erceg, Guerrieri, and Gust (2005), and Galí and Rabanal (2005) regarding misspecification.

the overshooting response of labor share to productivity innovations and reviewing the cyclical properties of labor share. Section 3 describes how we construct the stochastic process that creates both shocks to productivity and shocks to labor share (redistributive shocks), and we estimate these shocks. In Section 4 we incorporate the bivariate shock into the standard RBC model to derive our results and report our findings. Section 5 discusses why such a small and parsimonious change to the standard RBC model has such dramatic implications. Section 6 concludes. In the Appendix we lay out in detail how we construct labor share (Section A); we replicate our analysis of the dynamic properties under alternative definitions of the labor share (Section B); we explore the sensitivity to an alternative identification scheme of the joint dynamics of productivity and the labor share (Section C); and we derive the Slutsky decomposition of hours (and consumption) that we use to discuss our results (Section D).

2 The Behavior of Labor Share

The ratio of all payments to labor relative to output is labor share. Its exact value depends on the details of the definition of output and its partition into payments to labor and payments to capital. Perhaps the more standard definition of labor share, which is the one we take as the baseline, is that proposed by Cooley and Prescott (1995), which assumes that the ratio of ambiguous labor income to ambiguous income is the same as the ratio of unambiguous labor income to unambiguous income. Alternative definitions that we explore expand capital stock and capital services to include durables and also add government afterward, while a fourth definition sets labor share equal to the ratio of compensation of employees (CE) to gross national product (GNP), which renders all ambiguous income to capital. A detailed analysis of the construction of labor share of income data series is given in Appendix A.2.

Fig 1.

Fig 2.

The baseline definition of labor share for the period U.S. 1954.I–2004.IV is plotted in Figure 1. It oscillates between a maximum value slightly above 0.71 in 1970 and a minimum value about 0.66 in 1997 without discernible trend. The other definitions, while differing on their average, have very similar properties. This can be seen in Figure 2, which plots their deviations with respect to the mean.

From the point of view of the study of business cycles, what matters is not whether labor share moves but whether it does so in any systematic way with respect to the main macroeconomic aggregates. In this context, our analysis of the behavior of labor share U.S. 1954.I–2004.IV has two distinct dimensions: first, we document the dynamic effects of productivity innovations on labor share, a novel piece of empirical evidence; and second, we review and update the cyclical properties of labor share previously addressed by the business cycle literature. The following are our empirical findings.

Fig 3.

1. **Dynamic effects of productivity on labor share: Labor share overshoots**. This is a property of the impulse response of labor share with respect to the cycle. Figure 3 shows labor share's response to (orthogonalized) output innovations (left panel) and to (orthogonalized) Solow residual innovations [5] (right panel) with asymptotic (analytic) standard errors. Both panels display similar behavior: after falling below -.2% from average upon impact, labor share continuously rises in a concave fashion, it overshoots its long-run average after five quarters, and it peaks at the fifth year at a level larger in absolute terms (about .27% above average) than the initial drop, after which labor share slowly returns to its long-run average—seven years after the peak labor share is still halfway toward (.13% above) its average. The asymmetric hump-shaped pattern observed in Figure 3 arises from the fact that labor share increases more rapidly after its initial drop than it declines after the peak.[6]

2. Updated and reviewed cyclical properties of labor share:

   (a) **Labor share is quite volatile**. Table 1 displays the main business cycle statistics of output, the Solow residual, and the various definitions of labor share (all variables are logged and HP-filtered).[7] As we can see, the variance of labor share is 64% of that of the Solow residual and 18% of that of output. The values for the alternative definitions are even larger.

   (b) **Labor share is countercyclical**. The (baseline) labor share is negatively correlated with output with a coefficient of -.24. Similar figures are attained under alternative definitions of the labor share. Moreover, this negative correlation is much larger with respect to the Solow residual, where the value is -.48.

[5] To compute labor share's responses in Figure 3, we run two bivariate vector AR(n), where the first endogenous variable is either linearly detrended logged series of real GNP or the Solow residual and the second endogenous variable is demeaned logged labor share. Lag length order criteria suggest $n = 1$ for both systems. The impulse response functions (IRFs) are obtained with an identification strategy that assumes that redistributive innovations affect only labor share. Please see Section 3.3 for a much more detailed discussion on the estimation procedure and identification scheme.

[6] These dynamic properties of labor share are robust to alternative definitions of labor share, see Appendix B

[7] As in Gomme and Greenwood (1995) and Young (2004), we logged the labor share.

(c) **Labor share is highly persistent**. The first-order autocorrelation coefficient of (baseline) labor share is .78; that is, labor share displays almost as much persistence as output and slightly more than the Solow residual.

(d) **Labor share lags output.** To see this, observe Table 2 which shows the phase-shift of labor share with respect to output. We observe that although labor share is negatively correlated with output, the correlation between current labor share and leads of output takes large positive values above .4 about one year after the peak of output. In fewer words, labor share lags output by one year or so.

Table 1.

Table 2.

We think that the large size of the fluctuations in labor share, as well as its systematic covariation with the other macroeconomic variables, indicates that the cyclical movements of labor share cannot be due just to measurement error.

3 The Specification of the Shocks

Recall that we want to construct a stochastic process of shocks capable of generating the dynamic effects of productivity on labor share and the set of cyclical properties of labor share that we have described in order to feed them into a standard business cycle model. Consequently, both labor share and productivity have to be directly affected by the stochastic process. To do so, in Section 3.1 we start by recalling how in a standard business cycle model the Solow residual is given a structural interpretation as a univariate shock. In Section 3.2, we then turn to our specification of a joint process that yields both labor share and a productivity residual as a bivariate process. In Section 3.3, we estimate the process for the standard univariate productivity shock as well as for our bivariate process.

3.1 The Standard Specification: Solow Residuals as Shocks

The Solow residual, which we denote S_t^0, is computed from time series of real output Y_t, real capital K_t, and labor input N_t[8] (see Kydland and Prescott (1993) or King and Rebelo (1999)):

$$\ln S_t^0 = \ln Y_t - \zeta \ \ln K_t - (1-\zeta) \ \ln N_t, \tag{1}$$

[8] Real output is obtained from NIPA-BEA Table 1.7.6. The construction of the real capital (extended with durables and government capital services) and labor input (employment times hours per worker) series is explained in Appendix A.

where ζ is a relative input share parameter chosen to match the long-run average of the capital share of income.

But S_t^0 has trend, and we want a trendless object. Consider now a detrending procedure that uses the following linear regression:

$$\ln X_t = \chi_x + g_x t + \widetilde{x}_t, \tag{2}$$

where X_t is any economic variable, and where χ_x and g_x are the mean and trend parameters and $\widetilde{x}_t$ are the residuals. To detrend the Solow residual, we can either apply (2) directly to the Solow residual or, equivalently, apply (2) to the series of output, capital, and labor input to obtain the following detrended Solow residual:[9]

$$s_t^0 = \widetilde{y}_t - \zeta\, \widetilde{k}_t - (1-\zeta)\, \widetilde{n}_t. \tag{3}$$

3.1.1 A structural interpretation of the Solow residual

To see that in the standard business cycle model the Solow residual has a structural interpretation, consider the following Cobb-Douglas technology with constant coefficients and multiplicative shocks to productivity:

$$Y_t = e^{z_t^0} A\, K_t^{\theta} \left[(1+\lambda)^t\, \mu\, N_t\right]^{1-\theta} = e^{z_t^0} A\, K_t^{\theta} \left[(1+\lambda)^t\, (1+\eta)^t\, \mu\, h_t\right]^{1-\theta}, \tag{4}$$

where z_t^0 represents a shock that follows a univariate process, and λ is the rate of labor-augmenting (Harrod-neutral) technological change. The labor input, N_t, is the product of the number of agents in the economy, L_t, and the fraction of time agents devote to market activities, $0 \leq h_t \leq 1$. Population grows deterministically according to $L_t = (1+\eta)^t$. Parameters A and μ are just unit parameters (it will be clear later why we are posing two different unit parameters).

Note that in the balanced growth path, output Y_t and capital K_t grow at rate (approximately) $\gamma \approx \lambda + \eta$, and with constanta relative risk aversion (CRRA) preferences, the model economy generates paths for capital and output that can be written as $K_t = (1+\lambda)^t(1+\eta)^t\, k_t$ and $Y_t = (1+\lambda)^t(1+\eta)^t\, y_t$, where both k_t and y_t are stationary. Denote steady-state values by x^*, and let lowercase-hat variables be log deviations from steady state, i.e., $\widehat{x}_t = \log(\frac{x_t}{x^*})$. Then we

[9] The equivalence relies on a capital-output ratio that fluctuates around a constant long-run value. This is sustained by very similar growth rates of real output and capital—both about 3.2% annually over the 1954–2004 period. We use this second procedure because it nicely highlights the importance of the units of the model for the case when we deal with a bivariate shock, as we discuss later.

can write the equilibrium paths as

$$
\begin{aligned}
Y_t &= (1+\lambda)^t (1+\eta)^t \, y^* \, e^{\widehat{y}_t}, &(5)\\
K_t &= (1+\lambda)^t (1+\eta)^t \, k^* \, e^{\widehat{k}_t}, &(6)\\
N_t &= (1+\eta)^t \; h^* \, e^{\widehat{h}_t}. &(7)
\end{aligned}
$$

If we plug these paths (5)–(7) into the production function (4), cancel trend terms, take logs, and rearrange variables, we yield

$$
z_t^0 = \widehat{y}_t - \; \theta \widehat{k}_t - \; (1-\theta)\widehat{h}_t + \ln \frac{y^*}{A k^{*\theta} \left(\mu h^*\right)^{1-\theta}} \; = \; \widehat{y}_t - \; \theta \widehat{k}_t - \; (1-\theta)\widehat{h}_t, \tag{8}
$$

where the second equality follows directly from the fact that the denominator of the fourth term is steady-state output. But this is exactly the Solow residual as calculated in (3), allowing us to structurally identify the Solow residual generated by the data as the multiplicative shock to the production function.

3.2 The Bivariate Specification: Redistributive Shocks and Productivity Shocks

We now specify a bivariate stochastic process for the labor share and a productivity residual (slightly different from the Solow residual) that explicitly considers the fact that factor input shares change over time. We provide these two data series with a structural interpretation in Section 3.2.1.

Productivity Residual. This productivity residual is different from the Solow residual in Section 3.1 only in one regard: we now use the time-varying relative input share, ζ_t, instead of a constant share parameter, ζ. We define this productivity residual as

$$
\ln S_t^1 = \ln Y_t - \zeta_t \; \ln K_t - (1-\zeta_t) \; \ln N_t, \tag{9}
$$

which we detrend as

$$
s_t^1 = \widetilde{y}_t - \zeta_t \, \widetilde{k}_t - (1-\zeta_t)\, \widetilde{n}_t, \tag{10}
$$

where, as above, $\widetilde{y}_t$, $\widetilde{k}_t$, and $\widetilde{n}_t$ are the corresponding residuals of a fitted linear trend to the logged original series of output, capital, and labor.

Labor Share. Labor share is a unitless ratio. Rather than the level of the labor share, we are interested in deviations from its mean, which are

$$s_t^2 = (1 - \zeta_t) - (1 - \zeta), \tag{11}$$

where the average of labor share is $1 - \zeta = \sum_t \frac{1-\zeta_t}{T}$. The data series s_t^2 extracted from various definitions of labor share are depicted in Figure 2.

We note that the productivity residual s_t^1 is extremely similar to the Solow residual s_t^0 (see Figure 4).[10] This can also be seen by noting that we can write an expression that links the two residuals s_t^0 and s_t^1 as follows:

$$s_t^1 = s_t^0 + s_t^2(\widetilde{k}_t - \widetilde{n}_t)$$

and that the last term, $s_t^2(\widetilde{k}_t - \widetilde{n}_t)$, is very small.

Fig 4.

3.2.1 A structural interpretation of labor share and associated productivity residual

We now pose a production function with stochastic factor shares, which is otherwise the standard Cobb-Douglas technology,

$$Y_t = e^{z_t^1}\ A\ K_t^{\theta - z_t^2}\ \left[(1+\lambda)^t\ (1+\eta)^t\ \mu\ h_t\right]^{1-\theta+z_t^2}, \tag{12}$$

where z_t^1 and z_t^2 are the two elements of a bivariate stochastic process. We refer to these elements as the productivity shock and the redistributive shock, respectively. We use parameters μ and A to determine the units of effective labor and to normalize output to one. However, unlike in the standard specification, μ and A now play an important role.

First, under competitive markets, the labor share of income in the model is given by

$$\frac{\frac{\partial Y_t}{\partial N_t} N_t}{Y_t} = (1 - \theta) + z_t^2. \tag{13}$$

But this implies that with the choice $\theta = \zeta$, the redistributive shock in the model is the deviation from mean labor share in the data: $z_t^2 = s_t^2$.

[10] Interestingly, although the empirical definition of the Solow residual that has become standardly used is one that keeps factor shares constant, the original residual constructed in Solow (1957) uses a time series for the factor shares of income, as we do in our specification (10).

Second, we turn to the model counterpart of the residual (10). Similarly to what we did in Section 3.1.1, we look at the production function along the equilibrium path. Divide both sides of (12) by $(1+\lambda)^t(1+\eta)^t$, take logs, and rearrange to get

$$z_t^1 = \widehat{y}_t - (\theta - z_t^2)\widehat{k}_t - (1-\theta+z_t^2)\widehat{h}_t + z_t^2 \ln\left(\frac{k^*}{\mu h^*}\right), \tag{14}$$

where we have used steady-state output,

$$y^* = Ak^{*\theta}\left(\mu h^*\right)^{1-\theta}. \tag{15}$$

Then, note that by using the model-generated data, we can compute the productivity residual s_t^1 as

$$s_t^1 = \widehat{y}_t - (\theta - z_t^2)\widehat{k}_t - (1-\theta+z_t^2)\widehat{h}_t = z_t^1 - z_t^2 \ln\left(\frac{k^*}{\mu h^*}\right), \tag{16}$$

which means that the units matter. Formally, the productivity residual is the productivity shock, i.e., $s_t^1 = z_t^1$, only if the units in the model are chosen so that the ratio of capital to effective labor in the steady state is one, $\frac{k^*}{\mu h^*} = 1$. This explicitly implies a calibration strategy that uniquely identifies μ and A and that we follow. First, we calibrate μ such that the productivity shock in the model is the Solow residual in the data; that is, we solve for μ in $\frac{k^*}{\mu h^*} = 1$. Second, we calibrate A to ensure that, given $\frac{k^*}{\mu h^*} = 1$, the steady-state capital-output ratio in the model is the long-run average of the capital-output ratio in the data; that is, we solve for A in (15). We further discuss our calibration strategy in Section 4.2.

We now turn to estimating a parameterization to represent the univariate process z_t^0 and another one for the bivariate process $\{z_t^1, z_t^2\}$.

3.3 Estimation of a Process for the Shocks

We start by discussing a univariate process for the Solow residual in Section 3.3.1, and then we move to a bivariate process for the productivity residual and labor share in Section 3.3.2.

3.3.1 A univariate process for the Solow residual

We assume the Solow residual follows an AR(1) process with normally distributed innovations. For the whole sample 1954.I–2004.IV, the *full* maximum-likelihood estimation delivers[11]

$$z_t^0 = \underset{(.020)}{.958}\, z_{t-1}^0 + \epsilon_t^0, \qquad \epsilon_t^0 \sim N\left(0, \underset{(.000)}{.00672}\right).$$

Notice that the volatility of the innovations is lower than the value of .00763 originally estimated in Prescott (1986) and lower than the value of .007 used in Cooley and Prescott (1995). This difference in values is due to the sample period; there has recently been a reduction in volatility.[12]

3.3.2 A bivariate process for the productivity residual and labor share

We now pose a statistical model to find an underlying stochastic process that generates the joint distribution of z_t^1 and z_t^2 described in Section 3 using the residuals obtained. In particular, we aim to capture both the dynamic effects of productivity on labor share and the cyclical properties of labor share as described in Section 2. We assume the processes to be weakly covariance stationary so that classical estimation and inference procedures apply.

For estimation purposes, we specify a vector AR(n) model. Thus, we express each variable z_t^1 and z_t^2 as a linear combination of n-lags of itself and n-lags of the other variable. Information criteria (Akaike, Schwarz, and Hannan-Quinn) suggest that the correct specification is a vector AR(1), which we write compactly as

$$z_t = \Gamma\, z_{t-1} + \epsilon_t, \qquad \epsilon_t \sim N\left(0, \Sigma\right), \tag{17}$$

where $z_t = (z_t^1, z_t^2)'$ and Γ is a 2-by-2 square matrix with generic element γ_{ij}. The innovations $\epsilon_t = (\epsilon_t^1, \epsilon_t^2)'$ are serially uncorrelated and follow a bivariate Gaussian distribution with unconditional mean zero and a symmetric positive definite variance-covariance matrix Σ. Thus, this specification has seven parameters: the four coefficient regressors in Γ, and the variances and covariance in Σ.

The regressors of the endogenous variables z_t^1 and z_t^2 are the same; thus, we can separately apply the ordinary least squares method to each vector AR equation and yield consistent and

[11] The OLS estimation yields a (biased) regressor coefficient of .951 and a standard deviation of .0067. Despite the high persistence of the process, we do not find substantial differences between these estimates and the *full* maximum-likelihood estimates in terms of the equilibrium fluctuations of the RBC model.

[12] For instance, using a similar sample (1955.III–2003.II), Arias, Hansen, and Ohanian (2006) calibrate the autocorrelation coefficient to 0.95 and the volatility of the innovations to .0065.

efficient estimates. Also, with normally distributed innovations, these OLS estimates are equivalent to the *conditional* maximum likelihood estimates. Using the whole quarterly 1954.I–2004.IV sample, the estimated parameters associated with the baseline labor share are[13]

$$\widehat{\Gamma} = \begin{pmatrix} .952 & -.004 \\ \scriptstyle (.023) & \scriptstyle (.043) \\ .050 & .931 \\ \scriptstyle (.011) & \scriptstyle (.019) \end{pmatrix}, \qquad \widehat{\Sigma} = \begin{pmatrix} .00675^2 & -.1065E-04 \\ -.1065E-04 & .00304^2 \end{pmatrix}. \tag{18}$$

This generates a negative contemporaneous correlation between innovations ϵ_t of -.51. Notice that all parameters except γ_{12} are statistically significant. We also reject the joint null hypothesis that $\gamma_{12} = \gamma_{21} = 0$. The fact that γ_{12} is not significantly different from zero implies that current shocks to labor share do not have an impact on future productivity; that is, labor share shocks are purely redistributive.[14]

To get a better idea of the dynamics of the vector AR system, we use impulse response functions and forecast error variance decompositions. First, we check that the estimated vector AR is stable with eigenvalues .951 and .925 so that we can have a moving average representation of it. Second, since our innovations ϵ_t are contemporaneously correlated, we transform ϵ_t to a set of uncorrelated components u_t according to $\epsilon_t = \Omega\, u_t$, where Ω is an invertible square matrix with generic element ω_{ij}, such that

$$\widehat{\Sigma} = \frac{1}{n}\sum_t \epsilon_t \epsilon_t^{'} = \Omega \left(\frac{1}{n} \sum_t u_t u_t^{'} \right) \Omega^{'} = \Omega\Omega'$$

and we have normalized u_t to have unit variance. Notice that while $\widehat{\Sigma}$ has three parameters, the matrix Ω has four: there are many such matrices. We further impose the constraint that innovations to labor share are purely redistributive, i.e., innovations u_t^2 have a contemporaneous effect on z_t^2 but not on z_t^1; that is, we set Ω to be a lower triangular matrix.[15] Our factorization

[13]See Appendix B for the estimation results under alternative definitions of labor share.

[14]We could impose $\gamma_{12} = 0$ as an identification restriction to generate redistributive shocks. We find that such a constrained estimation yields almost identical results.

[15]Because $\widehat{\Sigma}$ is positive definite symmetric, it has a unique representation of the form $\widehat{\Sigma} = ADA'$, where A is a lower triangular matrix with diagonal elements equal to one and D is a diagonal matrix. A particularization of this is to set $\Omega = AD^{1/2}$, as we do, which is the Cholesky factorization. Also, our vector AR system allows for reverse ordering. That is, we can alternatively implement an identification scheme that lets the contemporaneous innovations to the factor shares of income affect productivity, but not the opposite. In this case, factor share innovations are not purely redistributive. We explore the resulting dynamics under either identification assumption and find similar responses of the endogenous variables in our economic model (see Appendix C). In any case, note that the equilibrium business cycle moments of the economic model (the total effects), which is what we are interested in, remain exactly the same.

of $\widehat{\Sigma}$ results in

$$\begin{pmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{pmatrix} = \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{12} & \omega_{22} \end{pmatrix} \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix} = \begin{pmatrix} .00675 & .0 \\ -.00157 & .00260 \end{pmatrix} \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix}, \tag{19}$$

where $\omega_{11} = \sigma_{\epsilon^1}$, $\omega_{21} = E[\epsilon_t^2 | \epsilon_t^1]$, and ω_{22} is the standard error of the regression of ϵ_t^2 on ϵ_t^1.

Fig 5.

Fig 6.

Figure 5 illustrates the consequences for z_t^1 and z_t^2, within a band of one asymptotic (analytic) standard error, from a one-time productivity innovation—that is, u_t^1 increases by one at $t = 0$ and is set to zero afterward. We find that z_t^1 reacts promptly and positively to this perturbation in its own innovations and that it slowly dies out afterward, very similar to (if not exactly as) the univariate process z_t^0 in response to a onetime one standard deviation of ϵ_t^0.[16] More importantly, Figure 5 displays the dynamic effects of productivity innovations on z_t^2. We find that the labor share of income immediately drops from its long-run average by -.157% at $t = 0$; it strongly rises to overshoot its average after the fifth quarter; it continuous to rise, reaching a maximum of about .178% above mean (a deviation larger in absolute terms than the initial drop) after twenty quarters; and finally, after its peak, it monotonically and slowly returns to its unconditional mean. The rapid rise (after the initial drop z_t^2 rises from -.157% to .178%, a total increase of .334%, in the first five years) and slow decline (seven years after its peak, z_t^2 is still halfway above mean, .089%) build the asymmetric hump-shaped pattern response of z_t^2 to productivity innovations.[17]

We see the time-path of z_t^1 and z_t^2 derived from a onetime redistributive innovation $u_0^2 = 1$ in Figure 6. This perturbation results in a labor share above average that monotonically decreases from a maximum attained at $t = 0$. Perhaps more importantly, we obtain that the response of productivity, z_t^1, to redistributive innovations, u_t^2, is negligible—not only contemporaneously at t

[16] Further, we find the response of productivity shocks in response to their own productivity innovations in the bivariate economy, $z_t^1(u_t^1)$, falls within a band of one asymptotic (analytic) standard error of the univariate economy; that is, they are not significantly different from each other. See the supplementary material at `http://rsantaeulalia.wustl.edu` or at `http://www.econ.umn.edu/~vr0j`. On perhaps more important terms, if we feed the standard univariate model with $z_t^1(u_t^1)$ instead of $z_t^0(e_t^0)$, we obtain (almost) identical equilibrium allocations.

[17] Here, note that these figures result from a specification of labor share in *level*-deviations from mean, as posed in our production function (24), which are slightly different from those presented in Section 2 that result from a specification of labor share in *log*-deviations from mean. It is straightforward to convert one into the other by a slight modification in the coefficients of the production function.

but also—at all future periods.[18] This results from an estimated γ_{12} (in equation (18)) that is not significantly different from zero—that is, we find that current labor share does not have an impact on tomorrow's productivity.

Finally, we decompose the variance of z_t^1 and z_t^2 and find with a long-run horizon that the fluctuations in z_t^1 are 100% due to its own innovations, u_t^1. Perhaps more importantly, we find that 66.7% of the variation in z_t^2 is due to productivity innovations, u_t^1, and 33.3% to its own innovations, u_t^2.

4 The Implications of the Specification of the Shocks for Output and Labor Fluctuations

In this section, we explore the implications of the two alternative specifications of shocks to the production function for the behavior of standard real business cycle models. Since it is wellknown that the answer to how important productivity shocks are in generating business cycle fluctuations depends on labor elasticity, we explore two different sets of preferences with different values for this elasticity. We start by specifying the model economies in Section 4.1.

4.1 The Model Economies

The economy is populated by a large number of identical infinitely lived households with the following preferences:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \, L_t \, u(c_t, 1-h_t) \right\}, \tag{20}$$

where c_t is per capita consumption and h_t denotes the proportion of time devoted to work. The population grows at rate η, $L_t = (1+\eta)^t$. Agents discount the future with a factor β, and E_0 is the expectations operator conditioned by the initial information. We choose standard momentary utility functions that imply balanced growth paths. One parameterization that fulfills this requirement is the log-log utility function used in Cooley and Prescott (1995):

$$u(c_t, 1-h_t) = (1-\alpha)\log(c_t) + \alpha \, \log(1-h_t) \tag{21}$$

[18] This is due to the assumption discussed earlier on the contemporaneous purely redistributive nature of z_t^2 that eliminates the effect of u_t^2 on z_t^1, $\omega_{12} = 0$ in (19).

This specification has a Frisch labor elasticity of 2.2, given that we set the fraction of substitutable time working to .31.[19] This is a closed economy where output Y_t is used either for consumption or for investment I_t. The aggregate stock of capital K_t evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t = (1-\delta)K_t + Y_t - C_t, \tag{22}$$

where δ is the geometric depreciation rate.

The production function is as described in Section 3: it is Cobb-Douglas with labor-augmenting technical change where we consider model economies with univariate shocks z_t^0 and model economies with bivariate shocks z_t^1 and z_t^2. The specification we posed to obtain the Solow residual as a univariate process with both productivity and population growth was

$$Y_t = e^{z_t^0} A\, K_t^{\theta} \left[(1+\lambda)^t\, (1+\eta)^t\, \mu\, h_t\right]^{1-\theta}. \tag{23}$$

In this model economy, the units are irrelevant. Still for consistency across models, we choose A and μ so that steady-state output is one and the ratio of steady-state capital k^* to steady-state effective labor μh^* is also set to one.

The production we posed to model the bivariate process with productivity and redistributive shocks is

$$Y_t = e^{z_t^1} A K_t^{\theta - z_t^2} \left[(1+\lambda)^t (1+\eta)^t\, \mu\, h_t\right]^{1-\theta+z_t^2}. \tag{24}$$

As we saw in Section 3.2, the units matter for this specification. Again, we set A and μ so that both steady-state output and the steady-state capital to effective labor ratio are one. In this fashion, z_t^2 does not have implications for productivity, since it is a pure redistributive shock.

We can ensure stationarity in the model economies by taking into account population and technological growth. As before, we use lowercase letters to denote detrended variables, and we use lowercase-hat variables to denote detrended log deviations from steady state. With log-log utility, in the transformed economy the planner's problem is to solve[20]

$$\max_{\{c_t, k_{t+1}, h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t\, (1+\eta)^t \left[(1-\alpha)\log(c_t) + \alpha \log(1-h_t)\right] \tag{25}$$

[19] We also use the Rogerson (1988) log-linear utility function popularized by Hansen (1985), which is $u(c_t, 1-h_t) = \log(c_t) + \kappa\,(1-h_t)$, where the linearity in leisure arises from nondivisibilities and the use of lotteries and generates a very high aggregate labor elasticity (in fact, its Frisch labor elasticity is infinity).

[20] In our economies, the welfare theorems hold so we can use the planner's problem in lieu of solving for the competitive equilibrium.

subject to $c_t + k_{t+1}(1+\eta)(1+\lambda) = y_t + (1-\delta)k_t$ and either $y_t = e^{z_t^0}\ A\ k_t^{\theta}\ \ (\mu\ h_t)^{1-\theta}$ or $y_t = e^{z_t^1}\ A\ k_t^{\theta - z_t^2}\ \ (\mu\ h_t)^{1-\theta+z_t^2}$ .The aggregate shocks, either z_t^0 or $\{z_t^1, z_t^2\}$, follow the processes described in Section 3.3.

4.2 Calibration

Calibration is very simple in this model. We have four standard parameters, θ, δ, β, and α, in addition to the productivity growth rate λ and the population growth rate η, which we choose according to the estimated trends g_y and g_h, respectively 3.29%[21] and 1.79% in annual terms. Again using x^* to denote the steady-state value of x (with the shocks set to zero—their unconditional mean), the equilibrium satisfies a system of four equations:

$$(1-\theta)\frac{y^*}{c^*} = \frac{\alpha}{1-\alpha}\frac{h^*}{1-h^*} \tag{26}$$

$$(1+\lambda) = \beta\left[\left(1-\delta+\theta\frac{y_*}{k_*}\right)\right] \tag{27}$$

$$\delta = \frac{i^*}{k^*} - (1+\eta)(1+\lambda) + 1 \tag{28}$$

$$1-\theta = \text{Labor Share}^* \tag{29}$$

that when solved yield the value of the four parameters for four targets of the steady-state values.

Table 3.

The targets we choose are as follows:

1. The fraction of time devoted to market activities: $h^* = 0.31$.
2. The steady-state consumption-output ratio: $c^*/y^* = 0.75$.
3. The capital-output ratio in yearly terms $k^*/y^* = 2.31$.[22]
4. Labor share = 0.679.[23]

[21] The measure of output that includes durables grows at an annual rate of 3.28%, and when we also add government capital, output grows at an annual rate of 3.23%.

[22] This is the target only for the baseline model economy; it includes only fixed private capital. When we extend measured output with durables, this ratio goes to 2.40, and after adding government capital we get 2.81.

[23] This is the target only for the baseline model economy. When we extend measured output with durables, this share is 0.625, and 0.58 when we also consider the stock of government capital. It is 0.57 when we use the narrowest definition of labor share that includes only compensation of employees as labor income.

We also calibrate μ and A. Our calibration strategy uniquely identifies μ and A and has two steps. First, μ is calibrated to ensure that the productivity shock in the model is the Solow residual; that is, we choose μ to solve $\frac{k^*}{\mu h^*} = 1$. See also our previous discussion in Section 3.3.2. Formally, for any $\psi > 0$, if output at the steady state is $y^* = \psi$, then capital at the steady state is $k^* = \psi \frac{k^*}{y^*} = \psi 9.24$ (given an annual baseline $\frac{k^*}{y^*} = 2.31$) and the μ that solves $\frac{k^*}{\mu h^*} = 1$ is $\mu = \psi \frac{k^*}{y^*} \frac{1}{h^*} = \psi 9.24 \frac{1}{.31} = \psi 29.86$ (given $h^* = .31$). In our exercise we choose $\psi = 1$; that is, we normalize steady-state output to 1, and then $\mu = 29.86$.[24] Second, note that in addition to the system (26)–(29), steady-state output must satisfy $y^* = Ak^{*\theta} (\mu h^*)^{1-\theta}$, as in (15). Then, given $\frac{k^*}{\mu h^*} = 1$, we can solve for A in (15), that is, we calibrate A targeting the inverse of the capital-output ratio, $A = (\frac{k^*}{\psi})^{-1} = (9.24)^{-1} = .108$, ensuring that the capital-output ratio in the model matches the data.[25]

The implied value of the parameters in quarterly terms is reported in Table 3. For the sake of completion, we report the values used in the original sources.

4.3 Findings

We now turn to discussing the main finding of the paper: posing productivity shocks as a bivariate process that affects factor shares implies a striking reduction in the volatility of the business cycle. The volatility of hours is ten times smaller in terms of the variance in the bivariate shock economy relative to the univariate shock economy.

4.3.1 Business cycle properties of the model economies

Table 4 reports the business cycle statistics for the main economic variables and factor prices 1954.I–2004.IV in the United States and in the model economies with standard log-log preferences. In the univariate model economy, productivity shocks account for 67.2% of the variance of output in the data. In the bivariate model economy, shocks account for 32.4% of the variance.

Table 4.

However, the most important statistic to measure the model's ability to generate fluctuations is the variance of hours, since output moves both because of hours and because of the shocks. In this respect, the univariate model accounts for 16.5% of the variance of the data. The striking finding is that the variance of hours in the bivariate model is 13.5% of that in the univariate model,

[24] This normalization of steady-state output to one is without loss of generality because the results of the model, as in the univariate RBC model, are independent of the steady-state value of output, ψ.

[25] For the Hansen-Rogerson version of the model (with indivisible labor), the only equilibrium condition that changes is (26), which is substituted with $(1-\theta)\frac{y^*}{c^*} = \kappa h^*$.

an enormous reduction. Thus, the bivariate model accounts for only 2.2% of the variance of hours in the data. However, we note that not all the small fluctuations in the bivariate economy are due to productivity innovations, as is the case in the univariate economy. We properly compare the contribution of productivity innovations to fluctuations in both univariate and bivariate economies in Section 5.1 below.

The behavior of hours in the bivariate economy is also very different in terms of its correlation with output. Although it is very high in the data (.88) and in the univariate shock economy (.98), it is much lower in the bivariate shock economy (.19).

With respect to the other aggregate variables, the relative volatility of consumption and investment is quite noteworthy. In the data, the variance of consumption is 3% that of investment, in the univariate shock economy it is 1.2%, and in the bivariate economy it is 16.4%. In fact, consumption moves more in the bivariate shock economy than in the univariate shock economy despite the opposite behavior of output (the ratio of variances is 265.0%). Finally, the behavior of both residuals is very similar, and they are highly correlated with output (recall that the residuals are virtually identical across economies, but output is not). Although the univariate model economy does not display movements in labor share, the bivariate economy does, and as in the data, they are negatively correlated with output and with very similar size.

Table 5.

Table 5 shows the phase-shift of the variables in the data and in both model economies. The behavior of hours is quite different between the two economies: in the univariate shock economy, hours are highly procyclical and have a slight lead in the cycle, whereas in the bivariate shock economy, hours are quite flat and lag the cycle. In both economies, consumption lags the cycle and investment leads it, although not by much. The behavior of rates of return is also quite different. In the univariate shock economy, the rate of return is quite strongly correlated with output, leads the cycle, and does not become negative until a year after output peaks. In the bivariate shock economy, the rate of return is less correlated, also leads the cycle, but becomes negative three quarters after output peaks. The cyclical behavior of wages seems more similar across the two economies than that of the rates of return.

The Hansen-Rogerson Preferences. As is wellknown, the higher elasticity of hours of the Hansen-Rogerson log-linear model generates a larger response of hours to the shocks than the model that we have considered so far. It turns out that the economy with univariate shocks displays 66.2% of the variance of hours observed in the data and 119.4% of output. However, when we turn to the cyclicality of the bivariate model economy with Hansen-Rogerson preferences, the reduction of the volatility of hours is spectacular. The variance of hours is now 9.0% of that in the data; that is, the bivariate model generates 13.6% of the variance of the univariate model. As in the log-log economy, consumption is more volatile with the bivariate shock than with the univariate shock, and investment is less volatile.

5 Why Do Hours Move So Little in the Bivariate Economies?

The key question now is: why does such a seemingly small departure from the standard model generate such a large change in the behavior of aggregate hours? We find it useful to start by exploring the properties of the response of hours, wages and rates of return to innovations in both univariate and bivariate economies (Section 5.1). Then we interpret the behavior of hours using the implied notions of substitution and wealth effects (Section 5.2), and we finish by studying the role of the dynamic overshooting response of labor share to productivity innovations in determining our results (Section 5.3).

Our analysis reveals that the dampening of the response of hours is due to a strong wealth effect whereas the change in the shape of the response comes from the intra- and intertemporal substitution effects. More precisely, changes in the wage dampen the initial rise in hours but protract it, and changes in the rate of return are negligible for the first five quarters and thereafter pull the response upward.

5.1 The Response of Hours, Wages, and Rates of Return to Productivity Innovations

Figure 7 shows the impulse response of hours to innovations to all three shocks in percentage deviations from the steady state. A one-standard-deviation innovation to the only shock, e^0, in the univariate model increases hours by .48%, a response that subsequently dies out (this is the standard response). In the bivariate shock economy, things are very different. There is barely any immediate response of hours to a current innovation in the productivity shock, u^1; in fact, the little response that there is, a paltry peak at .09%, is delayed dramatically to 18 quarters after the productivity innovation occurs. A 1% innovation in the redistributive shock u^2 favoring labor increases hours initially by .16% (about a third of that of the level of a productivity shock in the univariate economy), and its effects die out quite slowly.

Fig 7.

We can assess the contribution of u_t^1 by setting the variance of u_t^2 to zero and then that of u_t^2 by setting the variance of u_t^1 also to zero. The business cycle statistics (HP-filtered model-generated data) of these economies are reported in Table 6, and Table 7 displays the cyclical variance decomposition of the main variables in the bivariate model by the source of the innovation.

Table 6.

Table 7.

When the bivariate economy is driven solely by productivity innovations, the volatility of hours falls to .005%, that is, $\frac{.005}{.054} \approx 9.26\%$ of the variance of hours in the bivariate model that receives both innovations (see Tables 6 and 7). Crucially, this result implies that productivity innovations in the bivariate economy generate significantly fewer fluctuations of hours than in the standard univariate economy. Precisely, productivity innovations generate $\frac{.005}{.40} \approx 1.25\%$ of the variance of hours in the univariate model (see Table 6 and recall Table 4), and about $\frac{.005}{2.43} \approx 0.00\%$ of the variance of hours observed in the U.S. economy (see Table 6 and recall Table 4). The correlation of hours with output is now .11 relative to .98 in the univariate model. It is clear, then, that it is the differential response to a productivity shock that is responsible for the lack of response of hours.

For most of the other variables, innovations to productivity account for the vast majority of the cyclical variance (see Tables 6 and 7), and their volatility resembles the bivariate model. The comovement with output and persistence of these variables also remains positive and high. With productivity innovations alone, there still are movements in the labor share through γ_{21} and ω_{21}. In this case, labor share is less volatile than in the data, and it is highly countercyclical, -.68. The negative impact on z_t^2 through ω_{21} is not counterbalanced by positive redistributive innovations, which strengthens the mechanisms that dampen the volatility of hours.

When only redistributive innovations are present in the bivariate economy, the volatility of all real allocations is largely dampened with respect to the bivariate model with both innovations except that of hours and the labor share,[26] and all variables display a high (either positive or negative) correlation with output. In this case, it is noteworthy that the labor share turns highly procyclical.

Fig 8.

Fig 9.

Figures 8 and 9 respectively plot the impulse response functions of real wages and tomorrow's rate of return to productivity innovations and redistributive innovations in percentage deviations from

[26] The model is linear in u^1 and u^2, and hence the variance of the endogenous variables in the bivariate economy with u^1 alone and u^2 alone adds up to the variance in the bivariate economy where both innovations are present.

the steady state. Wages show a clear hump-shaped pattern in the bivariate economy, whereas they are much less so in the univariate economy. After a productivity innovation, wages in the bivariate economy continuously rise for the following nine quarters from an initial deviation of .43% to .60% after, that is, 1.38 times the original deviation. In the univariate economy, however, wages remain almost flat for the first three years; they respond initially by deviating by .51% and barely increase to .55% after one and a half years. The rate of return increases initially in response to productivity innovations by .033% in the bivariate economy and .030% in the univariate economy, but it declines more steeply in the former. The rate of return falls below steady state about one year earlier in the bivariate economy (after the ninth quarter). Wages respond to redistributive innovations positively: they initially jump by .33% and die out monotonically afterward. The rate of return always remains below its steady state, initially dropping to -.022%.

5.2 Substitution and Wealth Effects

Our economies have two main sets of prices: the relative prices of labor and consumption within each period are responsible for the intratemporal substitution effects and the relative prices of consumption across periods that generate intertemporal substitution effects. These prices differ in the two economies, and they may also affect whether certain allocations are feasible, which is what generates wealth effects. We isolate the contribution of each of these effects by means of a Slutsky decomposition, where we give a lump-sum transfer to agents at $t = 0$ in order to control for the wealth effects by keeping the original equilibrium allocations just feasible at the new prices.[27]

Denote by $a\{w^i, r^j, T^\ell\}$ the allocation chosen with the intratemporal prices of shock i, intertemporal prices of shock j, and total resources in the exact amount required to acquire the choice made by the household in response to shock ℓ, for $i, j, \ell \in \{0, 1\}$, and where 0 and 1 respectively stand for the univariate and bivariate economies.

Intratemporal Substitution Effects (Figure 10) . An economy with the wealth and rates of return of the univariate economy but the wages of the bivariate has, relative to the univariate, upon impact, hours' movements dampened by more than one-third, but within a year, this effect disappears and the movement of hours is, in fact, larger.

Intertemporal Substitution Effects. (Figure 11) . An economy with the wealth and wages of the univariate economy but the rates of return of the bivariate economy has hours moving

[27]For a detailed derivation of the Slutsky decomposition of hours (and consumption), see Appendix D. Alternatively, King (1991) and King and Rebelo (1999) use a Hicksian decomposition that compensates agents by placing them back on their original indifference curve.

almost identically to those in the univariate economy for the first five or six quarters. After that time, this new economy displays much larger work effort than the univariate economy.

Wealth Effects (Figure 12) . We see that the wealth effects are very large. With the prices of the univariate economy, the bivariate economy allows a lot more consumption and leisure. The opposite also holds: with the prices of the bivariate economy, the univariate economy allows less consumption and leisure. As Figure 12 shows, the wealth effects reduce the response of hours in the bivariate economy by 60%.

Fig 10.

Fig 11.

Fig 12.

To see what generates this wealth effect, we can compute the total wealth change across economies. Let the sum of the total resources generated by the factor prices $\{w^i, r^j\}$ for $i, j \in \{0, 1\}$ up to period t be $T_t(w^i, r^j)$. Then, the total wealth change up to period t, $T_t(w^1, r^1) - T_t(w^0, r^0)$, is the sum of the wealth change due to the relative price of the labor input, $T_t(w^1, r^0) - T_t(w^0, r^0)$, and the wealth change due to the intertemporal prices, $T_t(w^0, r^1) - T_t(w^0, r^0)$. We are interested in the accumulation of total wealth, that is, a large t. For $t = 250$, we find $|T_t(w^0, r^1) - T_t(w^0, r^0)|$ is about nine times larger than $|T_t(w^1, r^0) - T_t(w^0, r^0)|$.[28] That is, it is the lower rate of return—the higher present value of future units of consumption—in the bivariate economy that is responsible for the rise in total wealth.

Fig 13.

Another View of the Role of Wealth Effects: Greenwood, Hercowitz, and Huffman (1988) (GHH) Preferences. We look at economies with GHH preferences, where wealth effects are absent, to further explore the role of wealth effects in shaping our findings.[29] In these economies, the response of hours is much larger (as is well known), and the introduction of the bivariate shock does not reduce the volatility of hours significantly: the variance of hours

[28] These results do not change in substance when we increase t.

[29] A useful formulation of these preferences is $u(c,h) = \frac{\left(c - \omega \frac{h^{1+\rho}}{1+\rho}\right)^{1-\sigma}}{1-\sigma}$, where ρ is the inverse of the Frisch elasticity of labor supply. For comparison purposes with our calibration in Section 4.2, we set $\rho = (2.2)^{-1}$. In addition, we have to bear in mind that since GHH preferences are inconsistent with a balanced growth path, we need to shut down growth in population and technological progress, that is, we set $\eta = \lambda = 0$. Then, our calibration exercise also requires us to reset $\beta = .991$ and $\delta = .025$. The results reported here take $\sigma = 1$; we get similar results for higher values of σ.

(output) in a bivariate GHH economy is 95% (98%) of that of a univariate GHH economy. The immediate response of hours in the bivariate economy is one-third smaller than in the univariate one, but over time the bivariate response increases drastically to peak four years after the initial productivity shock (Figure 13). We think that the behavior of these models stresses the role of wealth effects in shaping our main result and we should not interpret our findings as evidence in support of GHH preferences that have wellknown problems: not based on household experiences, they impose a perfect correlation of labor between wages and hours, and they are dramatically inconsistent with a balanced growth path.

Summary of Wealth and Substitution Effects. Perhaps the way to summarize why the bivariate economy induces a very small response of hours is to say that the substitution effects, both intra- and then intertemporally, induce an immediate reduction in the response of hours followed by an increase which after five quarters would imply more hours worked than the univariate economy. The overall reduction, however, is that which is due to the wealth effects. This can be seen by looking again at Figure 12 and seeing how to decompose the differences between the univariate shock effects ($\{w^0, r^0, T^0\}$) and ($\{w^1, r^1, T^1\}$) into the substitution effects ($\{w^1, r^1, T^0\}$) and the wealth effect ($\{w^0, r^0, T^1\}$).

5.3 The Role of the Dynamic Effects of Productivity Innovations on Labor Share

The key differential element that identifies our paper from previous RBC models of labor share is the inclusion of a new piece of empirical evidence in our analysis: the dynamic effects of productivity innovations on labor share described by an overshooting response of labor share. In this section, we argue that this overshooting response of labor share to productivity, which we document, is the margin that explains our findings, not the set of cyclical properties of labor share addressed by earlier RBC studies that yield equilibrium allocations that preserve (or even enhance) the contribution of technology shocks to aggregate fluctuations. The overshooting dynamics of labor share surfaces in our representation of the joint process of labor share and productivity by having a positive value of γ_{21} in the bivariate shock specification in equation (18). That is, only one parameter, γ_{21}, is responsible for the overshooting: because $\omega_{12} = -.157\% < 0$, setting $\gamma_{21} = 0$ in a bivariate economy implies that labor share does not display overshooting. Instead, after an initial drop in response to a productivity shock, labor share rapidly increases in a concave fashion to its long-run average. We investigate the contribution of this overshooting by looking at a bivariate economy with $\gamma_{21} = 0$ in the bivariate specification in equation (18). Table 8 shows that the cyclical allocations of output and hours are similar to the univariate economy (in terms of the variance, hours are 80% and output 91% as volatile as their univariate counterparts, and the correlation of hours and output is also high at .94).[30] The findings are clear: the effect of productivity on subsequent labor share is the crucial feature that shapes the smaller volatility of hours. The findings of this subsection support the argument that our exercise is entirely parsimonious in the sense that we are adding only one relevant—for the equilibrium allocations of the model—parameter, γ_{21}, to previous models of labor share. This is the parameter generates the dynamic effect of productivity on labor share.

Table 8.

5.4 The Components of Labor Share in the Data and in the Models

Labor share is the ratio of labor compensation to output, $\frac{W\,N}{Y} = \frac{W}{Y/N}$, or $\widehat{ls}_t = \widehat{w}_t - \widehat{(\frac{y_t}{n_t})}$, the log deviation of labor share is the difference between those of wages and labor productivity. Consequently, understanding the dynamics of labor share requires our understanding of the asynchronous behavior of these two variables. Figure 14 shows the impulse responses with respect to the productivity shock of these variables as well as total hours and labor share both in the data and in the two models that we have explored.

Recall that the two assumptions of Cobb-Douglas technology and competitive factor pricing that are a central ingredient in the standard RBC univariate model imply that both wages and labor productivity move identically. Clearly they do not in the data. Wages jump immediately and remain high for a long time while labor productivity jumps even more initially but slowly decays and becomes negative in its seventh year, the result of a decaying Solow residual and a long lasting increase in hours that takes some time to occur.[31] These features of the data imply some properties that models need to have in order to account for the facts.

Fig 14.

As we have said before, the obvious first message is an immediate departure of the simultaneous assumptions of competitive factor pricing and Cobb-Douglas technology. Choi and Ríos-Rull (2008) explore search and matching models with Nash bargaining and Cobb-Douglas technology but in that environment there is no overshoot. So departing from Cobb-Douglas technology at

[30] We also note that the correlation of consumption and output is lower when $\gamma_{21} = 0$. This is due to the fact that, while in both restricted and unrestricted bivariate economies, output starts to decrease toward trend immediately after a productivity innovation, it is only in the restricted bivariate economy that consumption keeps rising for three years following the impact of the productivity innovation—whereas this is not so in the unrestricted economy (i.e., consumption rises for about one year).

[31] Wages in the models are not too dissimilar to wages in the data, unfortunately, because hours are not, the general equilibrium nature of the models implies that they cannot simultaneously account for wages and hours.

least in the short run seems to be a necessary feature. Another feature that seems to be needed is the existence of mechanisms that generate a delayed and strong behavior of hours, something like treating labor as a quasi fixed factor. Another feature that is needed is a mechanism that while inducing long responses of wages it also induces a strong response of hours. Our bivariate model generates such strong response of wages but households do not choose to respond by working harder. Consequently, both preferences and technology seem to require a reassessment.

6 Conclusion

We have documented the dynamic effects of productivity on labor share: labor share overshoots in response to productivity innovations. An innovation to the Solow residual produces a reduction of labor share at impact, making it countercyclical, but it also produces a long-lasting subsequent increase of labor share that overshoots its long-run average after five quarters and peaks above mean five years later at a level larger in absolute terms than the initial drop, after which it slowly returns to average. To study the implications of the dynamic effects of productivity on labor share in our understanding of the business cycle, we have posed and estimated a bivariate shock to the production function. Under the assumption of competition in factor markets, this bivariate shock simultaneously accounts for the overshooting response of labor share to productivity innovations that we have documented and the cyclical properties of labor share addressed by previous RBC literature that we have updated and reviewed. We have then incorporated this bivariate process into an otherwise standard real business cycle model.

Our results are striking. First, we have found that, independently of the Frischian elasticity of labor supply, the volatility of hours worked in the bivariate shock economy is a lot smaller (about 13.5% of the variance) than that in the standard univariate shock economy. Further, we find that productivity innovations generate, in the bivariate economy, barely 1.25% of the variance of hours of the univariate counterpart. In other terms, when we incorporate the dynamic overshooting effect of productivity on labor share, we obtain in our baseline economy that productivity innovations account for 0.21% of the fluctuations of hours in the U.S. economy (2.43/.005). We show that our results can be described in terms of a very strong positive wealth effect in the bivariate shock economy relative to the univariate shock economy. The implied substitution effects, however—the core propagation mechanism in standard RBC models—tend to delay, first intra- and then intertemporally, the response of hours and do not mitigate the wealth effect.

We believe our findings put forth a broad and exciting avenue for future research. We consider our exercise as a first and necessary attempt to understand—as parsimoniously as we can with respect to the standard RBC model—the implications of the overshooting property of labor share for the equilibrium allocations of hours and output. Whether the role of productivity shocks resuscitates in alternative (old and new) business cycle models with richer propagation mechanisms when we confront these with the overshooting property of labor share is still a very open question, and we call for such an evaluation.

In this pursuit, we think research should focus on models that can potentially account for the *overshooting* property of labor share endogenously.[32] First, one promising avenue is noncompetitive models. In this line of attack, labor search models with various wage settings (see Andolfatto (1996), Merz (1995), Cheron and Langot (2004), Moscarini and Postel-Vinay (2009), and Gertler and Trigari (2009), among others) seem particularly well suited to this task, since they have the key feature that labor share drops in response to productivity innovations and employment lags productivity. Choi and Ríos-Rull (2008) explore explicitly the role of non-competitive factor prices in generating the properties of labor share described in this paper. They pose a variety of search and matching models with bargaining that generate a countercyclical labor share. However, the effects are short-lived and fail to overshoot. The wage-setting protocol used in these type of models (Nash bargaining) is not able to create a persistent wedge between real wages and labor productivity. Models with cyclical markups, as those that result from sticky prices and procyclical marginal costs, and that may include additional sources of fluctuations (for example, Rotemberg and Woodford (1999), Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), and Justiniano and Primiceri (2008)), present frameworks that can also potentially explore how to implement the dynamic effects of productivity on labor share. We note here that targeting the overshooting property of labor share (that is, matching the impulse response function of labor share to productivity) can help to impose discipline on the identification strategy of relevant parameters in these models. Alternatively, models with long-term labor contracts as in Gomme and Greenwood (1995)—that can, at least qualitatively, reproduce a cyclical labor share that lags output—enriched with additional features that may allow hours to be affected by the fluctuations of labor share, perhaps through deviations from the Arrow-Debreu allocations as in Boldrin and Horvath (1995), seem relevant. Second, we think it is also promising to investigate alternative technologies that—moving away from the Cobb-Douglas assumption—generate endogenous movements of factor shares. One such candidate are production functions with an elasticity of substitution between capital and labor that is different from one (see Blanchard (1997) and Krusell, Ohanian, Ríos-Rull, and Violante (2000)). Similarly, putty-clay technologies that display low capital-labor substitutability in the short run produce, with capacity constraints, interesting hump-shaped responses of hours (see Gilchrist and Williams (2000)) that can possibly be conducive to a labor share that overshoots. Finally, factor-saving technologies as in Boldrin

[32] In doing so, we find our decomposition of labor share in Section 5.4 provides further guidance on what may be promising avenues to overcome the challenges we pose in our exercise.

and Levine (2002)—which generate a labor share that increases in the recessions of growth cycles—extended to incorporate endogenous labor also seem worthwhile to explore.

Whether these proposed models (or others) are able to quantitatively generate behavior consistent (in size and length) with the dynamic overshooting response of labor share to productivity innovations is yet to be explored. Perhaps more importantly, how much the necessary engineering to do so may distort the equilibrium properties—in particular, the cyclicality of output and hours and the contribution of technology shocks to the cycle—of the original models remains unanswered as well. In this sense, we are ultimately proposing the overshooting property of labor share as one useful criterion to select across business cycle models.

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Appendices

A Data Construction

A.1 Raw Data Series

All raw data series were retrieved from the Bureau of Economic Analysis (BEA; www.bea.gov) and the Bureau of Labor Statistics (BLS; www.bls.gov) for the period 1954.I–2004.IV. To save on notation, we drop the period subindex in all series.

National Income and Product Accounts (NIPA-BEA)

1. Table 1.7.5: Gross National Product (GNP), Consumption of Fixed Capital (DEP),[33] Statistical Discrepancy (SDis)[34]
2. Table 1.12: Compensation of Employees (CE), Proprietor's Income (PI), Rental Income (RI), Corporate Profits (CP), Net Interests (NI), Taxes on Production (Tax), Subsidies (Sub), Business Current Transfer Payments (BCTP), Current Surplus of Government Enterprises (GE)
3. Table 5.7.5: Private Inventories (Inv)

Fixed Asset Tables (FAT-BEA)

1. Tables 1.1 and 1.2: Private Fixed Assets (KP), Government Fixed Assets (KG), Consumer Durable Goods (KD)
2. Table 1.3: Depreciation of Private Fixed Assets (DepKP), Depreciation of Government Fixed Assets (DepKG), Depreciation of Consumer Durable Goods (DepKD)

Current Establishment Survey[35] (CES-BLS)

1. Employment (E): Series ID CES0000000081
2. Average Weekly Hours (AWH): Series ID CES0500000082, Series ID EEU00500005

[33] This amounts to the difference between Gross National Product and Net National Product.

[34] The Statistical Discrepancy corrects the difference between Net National Product and National Income.

[35] The primary sources of employment and average weekly hours series are the Current Establishment Survey (CES) and Current Population Survey (CPS), which have existed in some form since 1947. Our choice of the CES data set is driven by comparison with Cooley and Prescott (1995).

A.2 Constructed Data Series

Labor Share. The labor share of income is defined as one minus capital income divided by output. Several sources of income, mainly proprietor's income, cannot be unambiguously allocated to labor or capital income. To deal with this, we proceed as in Cooley and Prescott (1995) by assuming that the proportion of ambiguous capital income to ambiguous income is the same as the proportion of unambiguous capital income to unambiguous income, and we compute these series as follows:[36]

1. Unambiguous Capital Income (UCI) = RI + CP + NI + GE
2. Unambiguous Income (UI) = UCI + DEP + CE
3. Proportion of Unambiguous Capital Income to Unambiguous Income: $\theta_P = \frac{\text{UCI+DEP}}{\text{UI}}$
   Then we can use θ_P to compute the amount of ambiguous capital income in ambiguous income,
4. Ambiguous Income (AI) = PI + Tax - Sub + BCTP + SDis
5. Ambiguous Capital Income (ACI) = $\theta_P \times$ AI

Then, capital income (service flows of private fixed capital), Y_{KP}, is computed as the sum of unambiguous capital income, depreciation, and ambiguous capital income, that is,

$$Y_{KP} = \text{UCI} + \text{DEP} + \text{ACI}, \tag{30}$$

which we use to construct our baseline labor share[37] as

$$\text{Labor Share} = 1 - \frac{\text{UCI} + \text{DEP} + \text{ACI}}{\text{GNP}} = 1 - \frac{Y_{KP}}{\text{GNP}} = 1 - \theta_P. \tag{31}$$

To see the equivalence with Cooley and Prescott (1995), notice that

$$Y_{KP} = \text{UCI} + \text{DEP} + \text{ACI} = \theta_P \text{UI} + \theta_P \text{AI} = \theta_P \text{GNP} \tag{32}$$

Assuming that the return on capital is the same for fixed private capital, consumer durables, and government stock, we can extend the measure of output, capital income, and the labor share to include service flows from consumer durables and government stock as follows.

[36] The labor share is a ratio, and we use nominal series to compute it. Notice that unless the same price index is applied to all nominal variables, the use of real variables will not yield identical results.

[37] Our computation of the labor share differs from that in Cooley and Prescott (1995) in three respects: we add GE to UCI and Tax - Sub + BCTP to AI, so that UI + AI = GNP; we do not include the stock of land as private fixed assets; and we compute the depreciation rates of consumer durables and government stock differently, as we discuss later.

First, we determine the return on capital, i, by solving the following equation that relates capital income to capital stock:[38]

$$Y_{KP} = i \times (\mathsf{KP} + \mathsf{Inv}) + \mathsf{DEP} \tag{33}$$

Second, the depreciation rates of consumer durables and government stock are computed as[39]

$$\delta_D = \frac{\mathsf{DepKD}}{\mathsf{KD}} \qquad\qquad \delta_G = \frac{\mathsf{DepKG}}{\mathsf{KG}}. \tag{34}$$

This way, the flow of services from consumer durable goods and government capital can be derived as

$$Y_{KD} = (i + \delta_D) \times \mathsf{KD} \qquad\qquad Y_{KG} = (i + \delta_G) \times \mathsf{KG}. \tag{35}$$

Finally, the labor share with durables that extends measured output and capital income with flow services from consumer durables is

$$1 - \frac{Y_{KP} + Y_{KD}}{\mathsf{GNP} + Y_{KD}}, \tag{36}$$

and the labor share with durables and government that also includes flow services of government stock is

$$1 - \frac{Y_{KP} + Y_{KD} + Y_{KG}}{\mathsf{GNP} + Y_{KD} + Y_{KG}}. \tag{37}$$

Our last measure of the labor share is defined as the compensation of employees divided by GNP; that is, we consider labor income the only source that we can unambiguously allocate to labor and add all ambiguous income to capital income.

Aggregate Hours. We construct the series of aggregate hours by multiplying the series of employment and average weekly hours:[40] Hours = E $\times$ AWH.[41]

[38]We transform the annual capital stock and depreciation series provided by FAT-BEA to a quarterly series by interpolation.

[39]Cooley and Prescott (1995) use the perpetual inventory method and investment series to pin down δ_D and δ_G. Instead, we use the depreciation series for consumer durables and government stock reported in FAT-BEA, Table 1.3, and operate following (34). We find that our values for $\delta_D = .19$ and $\delta_G = .04$ are similar to those reported in Cooley and Prescott (1995), respectively, .21 and .05. Here, notice that we also have a different sample period; theirs runs from 1954 to 1992.

[40]The series of average weekly hours CES0500000082 is available from 1964.I onward. For the period before 1964, we retrieve the annual observations from the series EEU00500005, which we use as quarterly observations. This way, we attribute all quarterly variation in hours before 1964 to employment.

[41]Alternatively, the Productivity and Costs program office at the BLS also provides a quarterly index of aggregate hours since 1947, series ID PRS85006033, which is composed of CES and CPS data and has cyclical properties that are very similar to those of our constructed series of hours in terms of correlation with output (.88), but are slightly more volatile (1.77).

Real Capital. To construct the series of real capital, we use the chain-type quantity index from Table 1.2 in FAT-BEA and the current-cost net stock in 2000 from Table 1.1 in FAT-BEA.

B Univariate and Bivariate Estimations under Alternative Definitions of Labor Share

To be consistent in our computations of the Solow residual under each definition of the labor share, we take the corresponding extended measures of (deflated) output and extend the measure of the real capital stock series accordingly. This way, when the labor share includes consumer durables (and government stock), the real output and real capital series used to compute the Solow residual are respectively defined as (deflated) GNP + Y_{KD} (+ Y_{KG}) and KP + KD (+ KG). The series of the labor input remains the same in all computations. Table App-1 reports the univariate estimation of the Solow residual for the four definitions of the labor share and Table App-2 the bivariate estimation[42] of the modified Solow residual and the labor share.

Our estimations show a high persistence of the Solow residual and the labor share, a larger volatility of the productivity innovations when government stock is included, a larger volatility of the redistributive innovations in our narrowest definition of the labor share, a negative covariance between the productivity and redistributive innovations which is largest under our narrowest definition of the labor share, and negligible (statistically nonsignificant) marginal effects of z_{t-1}^2 on z_t^1 under all labor share definitions. The impulse response functions depicted in Figures App-1 and App-2 show properties very similar to our baseline labor share studied in Section 3.3.2.[43]

C Alternative Identification Scheme

Our identification scheme treats innovations to factor shares as purely redistributive, that is, without contemporaneous effects on productivity. Alternatively, we can reverse the order of the vector AR system to orthogonalize the innovations ϵ_t as

$$\begin{pmatrix} \epsilon_t^2 \\ \epsilon_t^1 \end{pmatrix} = \begin{pmatrix} .00304 & .0 \\ -.00349 & .00577 \end{pmatrix} \begin{pmatrix} u_t^2 \\ u_t^1 \end{pmatrix},$$

where $\sigma_{\epsilon^2} = .00304$, $E[\epsilon_t^1|\epsilon_t^2] = -.00349$, and the standard error of the regression of ϵ_t^1 on ϵ_t^2 is $.00577$. This orthogonalization has the identifying assumption that while innovations to the factor shares have a contemporaneous effect on productivity, productivity innovations do not alter the distribution of income at impact.

Most notably, we find that the response of hours to productivity innovations and to labor

[42] Although we do not report it here, information criteria suggest the use of a vector AR(1) for the bivariate estimation under all definitions of the labor share, as in our baseline case.

[43] We have also explored the sensitivity of our results to alternative definitions of labor share in model economies with log-log preferences and Hansen-Rogerson preferences. The equilibrium allocations that result from these economies with alternative definitions of labors share confirm our findings discussed in Section 4.3. The interested reader may find these supplementary materials at `http://rsantaeulalia.wustl.edu` or at `http://www.econ.umn.edu/~vr0j`.

share innovations are similar under both identification schemes as depicted in Figure App-3.[44]

D Slutsky Decomposition of Hours and Consumption

Productivity innovations alter the relative reward of the labor input (intratemporal substitution effects), introduce intertemporal substitution effects through the (inverse of the) rate of return that households use to discount the future, and also alter the total resources of the agents (wealth effects). We use the Slutsky transfer compensation to compute the correct measure of total resources and, this way, decompose all these effects. Although in our exercise we do this decomposition numerically, here, and for exemplifying purposes, we write out the labor supply and consumption functions explicitly in terms of present and future wages and interest rates for the case of the log-log utility function.

D.1 Hours

To derive the labor supply function, we first consolidate the budget constraint at $t = 0$,

$$\sum_{t=0}^{\infty} \frac{(1+\gamma)^t c_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + \sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t (1-h_t)}{\prod_{s=1}^{t}(1+r_s-\delta)} = \sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + (1+r_0-\delta)k_0, \quad (38)$$

where we have used the transversality condition, $\lim_{T\to\infty} \frac{k_T}{\prod_{s=t}^{T}(1+r_s-\delta)} = 0$. The left-hand side is the present value of all future expenditures on consumption and leisure, and the right-hand side is the present value of total resources (wealth) accumulated from period $t = 0$ onward. Total resources are composed by the sum of the human wealth and the initial capital income evaluated in units of $t = 0$ consumption. We use the first-order condition for labor to substitute out consumption c_t in the left-hand side of (38), and then we use the Euler equation to rewrite the present value of expenditures as

$$\frac{1}{\alpha} \sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t (1-h_t)}{\prod_{s=1}^{t}(1+r_s-\delta)} = \frac{1}{\alpha} \sum_{t=0}^{\infty} \beta^{t-1} w_0 (1-h_0) = \frac{w_0(1-h_0)}{\alpha(1-\beta)}. \quad (39)$$

Now, we can plug (39) into (38) and rearrange to find the initial response of leisure for a given forecast of wages and interest rates, $w_0(1-h_0) = \alpha\,(1-\beta)\,\left(\sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + (1+r_0-\delta)k_0\right)$, and using the Euler equation we can recursively find

$$\frac{(1+\gamma)^t w_t (1-h_t)}{\beta^t \prod_{s=1}^{t}(1+r_s-\delta)} = \alpha(1-\beta)\left(\sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + (1+r_0-\delta)k_0\right). \quad (40)$$

That is, the present value of the expenditure on leisure at period t is a constant share of the present value of total resources. This constant share is the marginal propensity to consume leisure, α, and per period, $1-\beta$.

[44]We have computed IRFs of consumption to productivity and to redistributive innovations, and obtained similar responses under our original and alternative idetification schemes, see the supplementary materials at `http://rsantaeulalia.wustl.edu` or at `http://www.econ.umn.edu/~vr0j`.

If we log-linearize (40) around the steady state, we find that the deviation of period-t hours from the steady state can be decomposed as a linear combination of the deviations of period-t wages, the present value of one unit of period-t consumption, and the present value of total resources:[45]

$$\widehat{h}_t = \left(\frac{1-h^*}{h^*}\right)\left[\widehat{w}_t + \left(\widehat{\frac{1}{\prod_{s=1}^{t}(1+r_s-\delta)}}\right) - \left(\widehat{\sum_{t=0}^{\infty}\frac{(1+\gamma)^t w_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + (1+r_0-\delta)k_0}\right)\right] \tag{41}$$

where the constant $\frac{1-h^*}{h^*} = 2.2$ is the Frischian elasticity of labor supply. The expression (41) (which is identical for both univariate and bivariate economies) decomposes the overall response of hours to all innovations into intratemporal substitution (wage) effects, intertemporal substitution (rate of return) effects, and wealth (total resources) effects with respect to the steady state. Next, we discuss how we obtain these effects when they arise from crossing the prices between the univariate and bivariate economies.

Intratemporal Substitution Effect. To see how bivariate wages change the response of hours in an otherwise univariate economy, we keep the univariate rate of return and add a Slutsky transfer compensation that sets agents' total resources equal to those generated by the univariate shock, T^0. This transfer compensation is

$$\Psi(w_t^1, r_t^0, T^0) = \sum_{t=0}^{\infty}\frac{(1+\gamma)^t(w_t^1 - w_t^0)}{\prod_{s=1}^{t}(1+r_s^0-\delta)}.$$

If we provide agents with this transfer at $t = 0$, we obtain the allocations for an economy with bivariate wages and univariate rate of return and wealth, $a\{w^1, r^0, T^0\}$, which we plot in Figure 10. A symmetric procedure yields $a\{w^0, r^1, T^1\}$.

Intertemporal Substitution Effect. The introduction of the bivariate interest rate in the univariate economy changes the present value of future units of consumption and, in turn, the present value of the total resources available at $t = 0$. To disentangle these two effects, we introduce a Slutsky transfer compensation that keeps total resources unchanged:

$$\Psi(w_t^0, r_t^1, T^0) = \sum_{t=0}^{\infty}\frac{(1+\gamma)^t w_t^0}{\prod_{s=0}^{t}(1+r_s^1-\delta)} + (1+r_0^1-\delta)k^* - \sum_{t=0}^{\infty}\frac{(1+\gamma)^t w_t^0}{\prod_{s=0}^{t}(1+r_s^0-\delta)} - (1+r_0^0-\delta)k^*.$$

With this transfer, if we keep univariate wages and introduce the bivariate rate of return, we obtain the allocations $a\{w^0, r^1, T^0\}$, depicted in Figure 11. A symmetric procedure yields $a\{w^1, r^0, T^1\}$.

[45] Notice that $\widehat{(1-h_t)} = -\left(\frac{h^*}{1-h^*}\right)\widehat{h}_t$.

Wealth Effect. The wealth effect is a number. The bivariate economy changes the amount of total resources with respect to the univariate economy by

$$\Psi(w_t^0, r_t^0, T^1) = \sum_{t=0}^{\infty} \frac{(1+\gamma)^t w^0}{\prod_{s=0}^{t}(1+r_s^0-\delta)} - (1+r_0^0-\delta)k^* - \sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t^1}{\prod_{s=0}^{t}(1+r_s^1-\delta)} + (1+r_0^1-\delta)k^*.$$

To measure the wealth effect of the bivariate prices on the univariate hours, we transfer $\Psi(w_t^0, r_t^0, T^1)$ and compute the response of hours to univariate prices. This yields $a\{w^0, r^0, T^1\}$ in Figure 12. A symmetric manipulation yields $a\{w^1, r^1, T^0\}$.

D.2 Consumption

Using the labor supply function and the log-linearization around the steady state of the first-order condition for labor, we can derive the consumption function as

$$\widehat{c}_t = -\left(\widehat{\frac{1}{\prod_{s=1}^{t}(1+r_s-\delta)}}\right) + \left(\widehat{\sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t}{\prod_{s=0}^{t}(1+r_s-\delta)}} + (1+r_0-\delta)\, k_0\right). \tag{42}$$

The deviations in consumption are driven by the price of future consumption evaluated in present units and the change in the present value of total resources. In our simulations we find productivity innovations cut the price of consumption in the bivariate and univariate economies very similarly during the first year (see Figure App-4). However, although future units of consumption become expensive more rapidly in the bivariate economy (which would favor a higher consumption in the univariate economy), the important wealth effect in the bivariate economy more than offsets the previous intertemporal substitution effect and sets consumption in the bivariate model above that of the univariate model, which explains the higher volatility of consumption in the bivariate economy.

Table 1: Variance and Correlation of Labor Share with Output, U.S. 1954.I–2004.IV

	σ_x^2	$\sigma_x^2/\sigma_{GNP}^2$	$\rho(x, GNP)$	$\rho(x, s^0)$	$\rho(x_t, x_{t-1})$
GNP	2.53	1.00	1.00	.74^a	.85^a
Solow Residual: s^0	.72	.29	.74^a	1.00	.71^a
Baseline Labor Share	.46	.18	-.24^a	-.48^a	.78^a
... with Durables	.50	.20	-.21^a	-.44^a	.77^a
... and Government	.71	.28	-.26^b	-.43^a	.78^a
CE/GNP	.66	.26	-.23^a	-.61^a	.71^a

Notes: All variables are logged and HP-filtered. CE/GNP is compensation of employees over GNP. Let a and b denote significance at 1% and 5%, respectively.

Table 2: Phase-Shift of Labor Share, U.S. 1954.I–2004.IV

	Cross-correlation of GNP_t with										
	x_{t-5}	x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}	x_{t+5}
Baseline Labor Share	-.20	-.26	-.32	-.34	-.33	-.24	.03	.25	.40	.47	.44
... with Durables	-.21	-.26	-.32	-.33	-.20	-.21	.07	.28	.41	.47	.42
... and Government	-.20	-.25	-.31	-.34	-.33	-.26	.03	.27	.42	.48	.44
CE/GNP[1]	-.24	-.30	-.35	-.38	-.31	-.23	.09	.31	.47	.49	.46

Table 3: Calibrated Parameters

	$1-\theta$	δ	β	α	A	μ	κ	λ
Baseline Labor Share	.679	.019	.988	.668	.108	29.86	2.92	.00367
... with Durables	.625	.018	.983	.649	.104	31.08	2.69	.00364
... with Government	.58	.014	.981	.632	.089	36.21	2.49	.00352
CE/GNP	.57	.019	.976	.628	.108	29.86	2.45	.00367
Cooley-Prescott (1995)	.60	.012	.987	.640	-	-	-	.00387
Hansen (1985)	.64	.025	.990	-	-	-	2.84	-

Table 4: Cyclical Behavior of U.S. Data 1954.I–2004.IV and Log-Log Utility Real Business Cycle Models with Univariate and Bivariate Shocks

	U.S. Data			Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x^2	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x^2	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x^2	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**2.53**	1.00	.85	**1.70**	1.00	.72	**.82**	1.00	.73
h	**2.43**	**.88**	.89	**.40**	**.98**	.71	**.05**	.19	.73
c	1.56	.87	.86	.20	.91	.80	.53	.92	.77
i	52.27	.91	.80	16.14	.99	.71	3.22	.89	.69
r	.01	.74	.78	.004	.96	.71	.004	.67	.70
w	.58	.08	.70	.48	.98	.75	.62	.88	.77
z^0, z^1	.72	.74	.70	.77	.99	.71	.77	.98	.71
ls	.46	-.24	.78	-	-	-	.40	-.27	.72

Notes: Data are obtained from NIPA-BEA: real GNP from Table (1.7.6) and real personal consumption expenditures and real gross private domestic investment from Table (1.1.6). The series of hours uses CES data; see Appendix A. The data series of factor prices are constructed as $w =$ *Labor Share* $\times$ *Output*/*Hours* and $r = (1 -$ *Labor Share*$) \times$ *Output*/*Capital*. All variables are logged (except the rate of return) and HP-filtered.

Table 5: Phase-Shift of the U.S. Data 1954.I–2004.IV and Log-Log Utility Real Business Cycle Models with Univariate and Bivariate Shocks

	Cross-correlation of y_t with										
	x_{t-5}	x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}	x_{t+5}
	U.S. Data 1954.I–2004.IV										
y	-.04	.14	.37	.63	.85	1.00	.85	.63	.37	.14	-.04
h	-.22	-.06	.15	.40	.67	.88	.91	.81	.63	.41	.21
c	.14	.33	.51	.70	.84	.87	.71	.50	.26	.03	-.14
i	.05	.20	.39	.60	.79	.91	.75	.51	.24	-.01	-.22
r	.16	.31	.48	.63	.73	.74	.47	.17	-.09	-.29	-.40
w	.18	.19	.19	.17	.10	.08	-.06	-.11	-.15	-.12	-.12
s^0, s^1	.25	.39	.54	.68	.73	.74	.39	.08	-.18	-.33	-.43
ls	-.20	-.26	-.32	-.34	-.33	-.24	.03	.25	.40	.47	.44
	Univariate Model $\{z_t^0\}$										
y	-.01	.11	.27	.46	.70	1.00	.70	.46	.27	.11	-.01
h	.08	.20	.34	.52	.73	.98	.63	.35	.14	-.03	-.15
c	-.21	-.09	.07	.29	.56	.91	.77	.63	.50	.37	.26
i	.05	.17	.32	.50	.72	.99	.65	.39	.18	.02	-.10
r	.12	.24	.37	.54	.73	.96	.58	.30	.08	-.09	-.20
w	-.10	.02	.19	.40	.66	.98	.75	.55	.37	.23	.10
z_t^0	.01	.13	.28	.48	.71	.99	.69	.44	.24	.08	-.04
	Bivariate Model $\{z_t^1, z_t^2\}$										
y	-.01	.11	.28	.48	.72	1.00	.72	.48	.28	.11	-.01
h	-.14	-.11	-.07	-.02	.07	.19	.22	.24	.23	.22	.20
c	-.11	.01	.17	.37	.62	.92	.74	.57	.41	.27	.16
i	.11	.21	.35	.50	.69	.89	.54	.27	.06	-.09	-.21
r	.16	.24	.33	.43	.55	.67	.35	.10	-.07	-.21	-.28
w	-.12	-.01	.15	.34	.59	.88	.71	.56	.41	.28	.17
z_t^1	.04	.16	.31	.50	.72	.98	.67	.41	.20	.04	-.08
ls	-.19	-.22	-.24	-.26	-.27	-.27	-.05	.10	.21	.28	.31

Table 6: Cyclical Behavior of Log-Log Utility Real Business Cycle Models with the Bivariate Shock with Both Innovations and Isolated Innovations

	Bivariate $\{u^1, u^2\}$			Bivariate with u_t^1 alone			Bivariate with u_t^2 alone		
	σ_x^2	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x^2	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x^2	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**.82**	1.00	.73	**.80**	1.00	.72	**.02**	1.00	.68
h	**.05**	.19	.73	**.005**	.11	.95	**.049**	.99	.69
c	.53	.92	.77	.43	.97	.78	.10	.99	.68
i	3.22	.89	.69	3.09	.96	.69	.13	-.99	.68
r	.004	.67	.70	.003	.88	.69	.001	-.99	.68
w	.62	.88	.77	.44	.95	.79	.18	.99	.68
z^1	.77	.98	.71	.77	.99	.70	.00	.00	.95
ls	.40	-.27	.72	.15	-.68	.75	.25	.99	.69

Table 7: Variance Decomposition (%), HP-Filtered

	y	h	c	i	r	w	z^1	lab. sh.
u^1	97.5	9.3	81.1	95.9	75.0	71.0	100	37.7
u^2	2.5	90.7	18.9	4.1	25.0	29.0	0	62.3

Table 8: Cyclical Behavior of Real Business Cycle Models with and without Overshooting

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$			Bivariate $\{z^1, z^2\}$ with $\gamma_{21=0}$		
	σ_x^2	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x^2	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x^2	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.70**	1.00	.72	**.82**	1.00	.73	**1.55**	1.00	.72
h	**.40**	**.98**	.71	**.05**	.19	.73	**.32**	**.94**	.71
c	.20	.91	.80	.53	.92	.77	.19	.54	.79
i	16.14	.99	.71	3.22	.89	.69	19.38	.97	.71
r	.004	.96	.71	.004	.67	.70	.004	.81	.70
w	.48	.98	.75	.62	.88	.77	.39	.76	.75
z^0, z^1	.77	.99	.71	.77	.98	.71	.77	.99	.71
ls	-	-	-	.40	-.27	.72	.34	-.40	.71

Table App-1: Univariate Estimation of the Solow Residual, z_t^0

	Baseline Labor Share	Plus Durables	Plus Government	CE/GNP
ρ	.954 (.020)	.951 (.022)	.937 (.019)	.951 (.021)
σ	.00668 (.000)	.00667 (.000)	.00726 (.000)	.00685 (.000)

Table App-2: Bivariate Estimation of the Solow Residual, z_t^1, and Labor Share Deviations, z_t^2

	γ_{11}	γ_{12}	γ_{21}	γ_{22}	σ_1	σ_2	σ_{12}
Baseline Labor Share	.946	.001	.050	.930	.00668	.00303	-.1045E-04
	(.023)	(.042)	(.010)	(.019)			
... with Durables	.941	-.012	.055	.930	.00665	.00287	-.1001E-04
	(.024)	(.043)	(.010)	(.019)			
... and Government	.927	-.041	.058	.953	.00723	.00313	-.139E-04
	(.025)	(.044)	(.011)	(.019)			
CE/GNP	.948	-.025	.051	.937	.00685	.00345	-.1696E-04
	(.023)	(.040)	(.011)	(.020)			

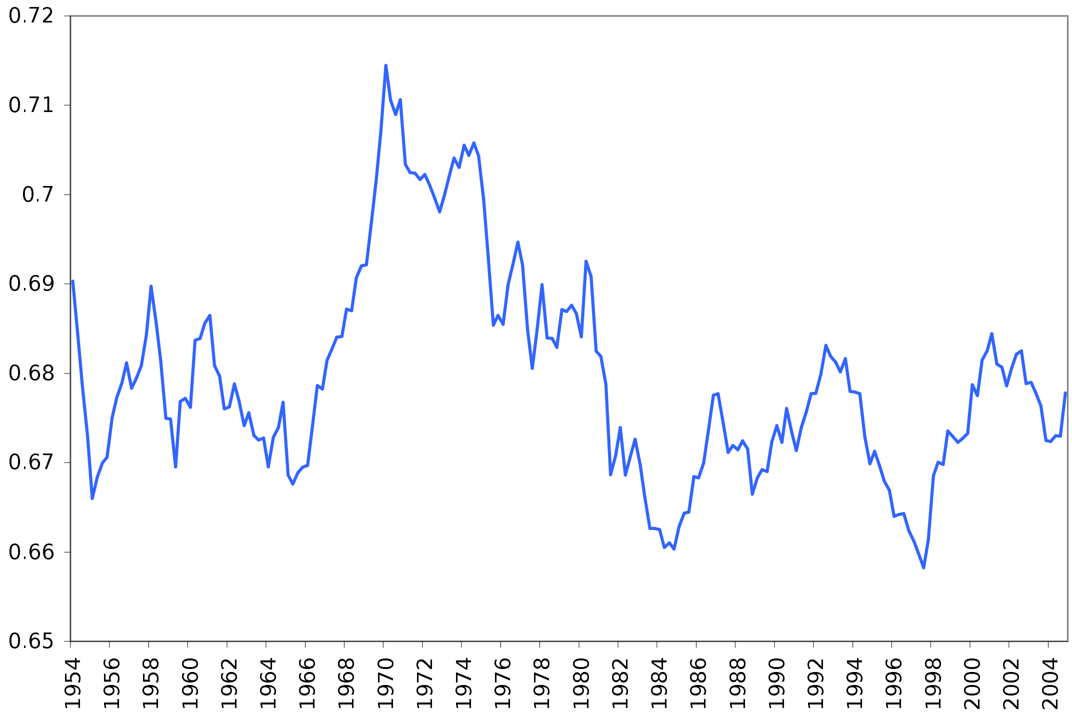

Figure 1: Labor Share, U.S. 1954.I–2004.IV

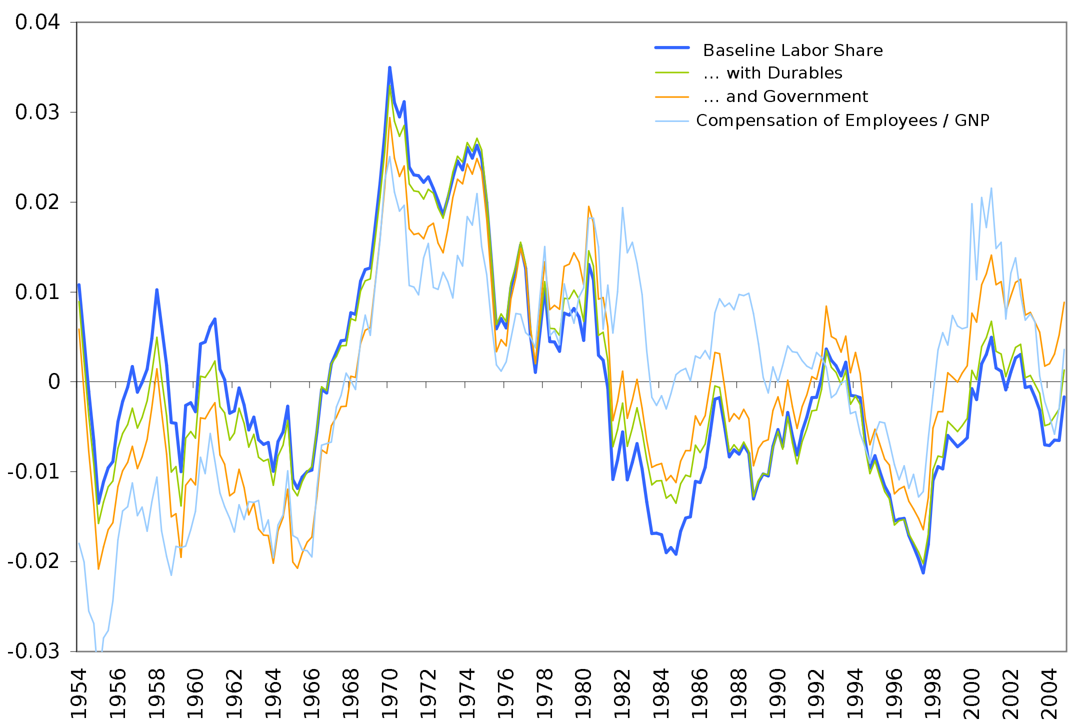

Figure 2: Deviations from Average Labor Share, U.S. 1954.I–2004.IV

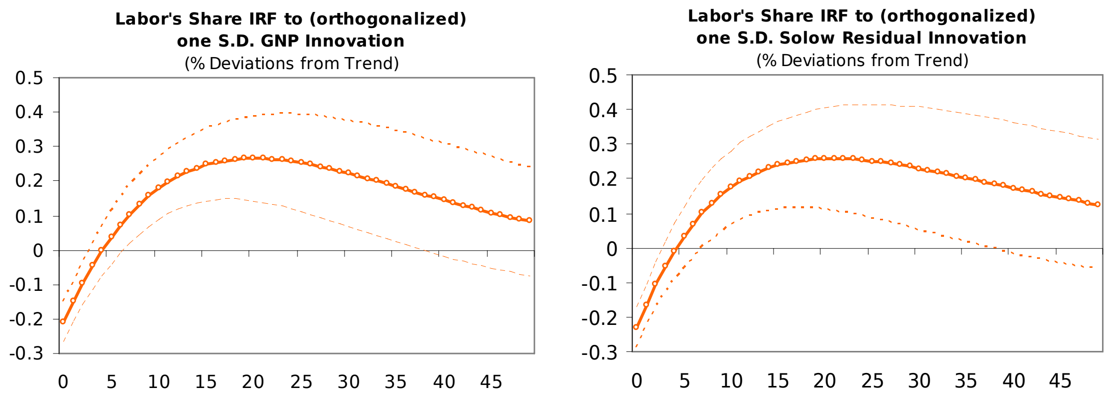

Figure 3: Labor Share's Impulse Response Functions (IRF) to GNP (left panel) and Solow Residual (right panel) Innovations

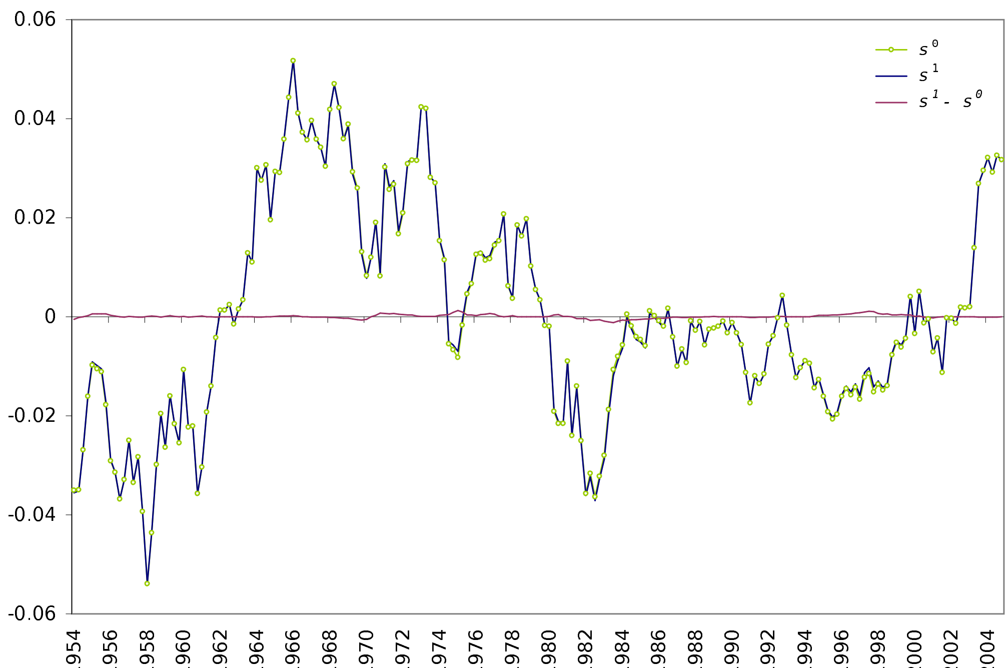

Figure 4: Productivity Residuals s_t^0 and s_t^1, U.S. 1954.I–2004.IV

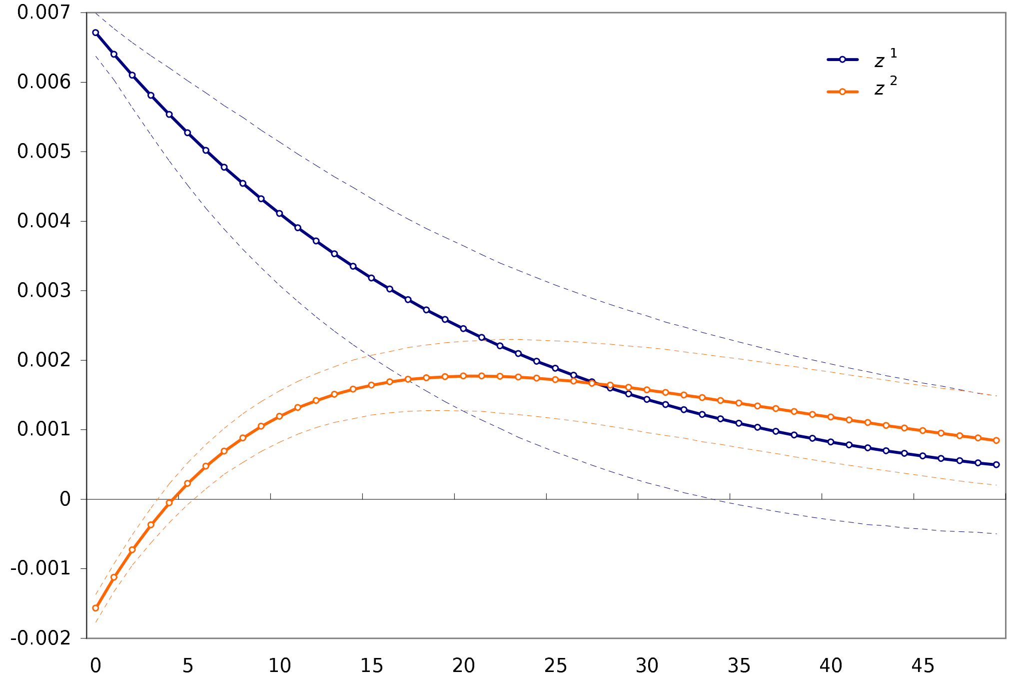

Figure 5: Impulse Response Functions to Orthogonalized Productivity Innovations u^1

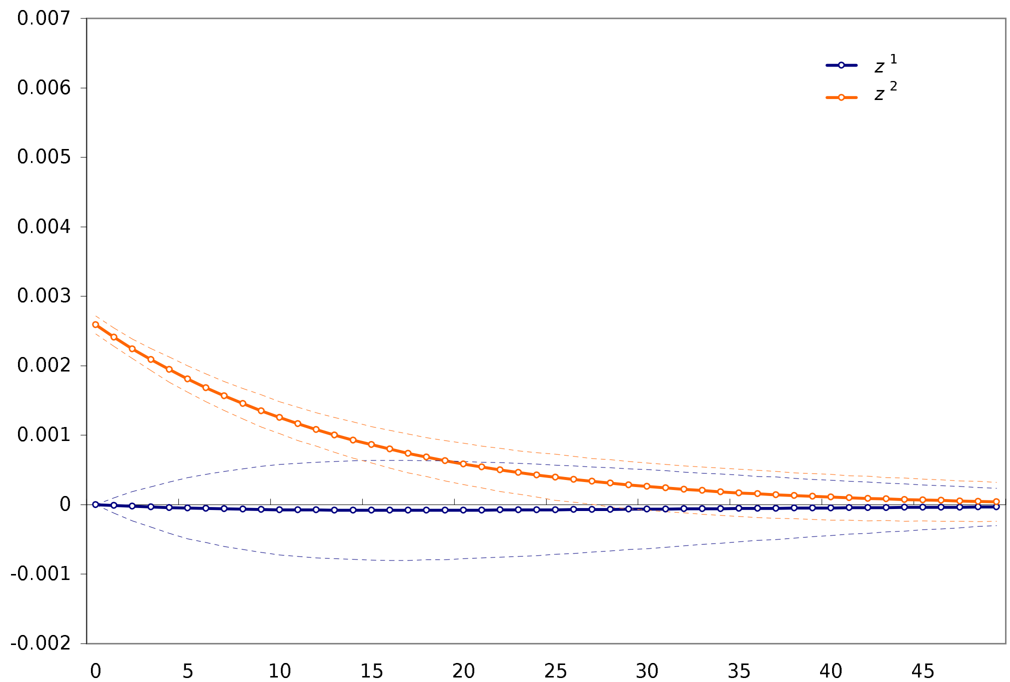

Figure 6: Impulse Response Functions to Orthogonalized Redistributive Innovations u^2

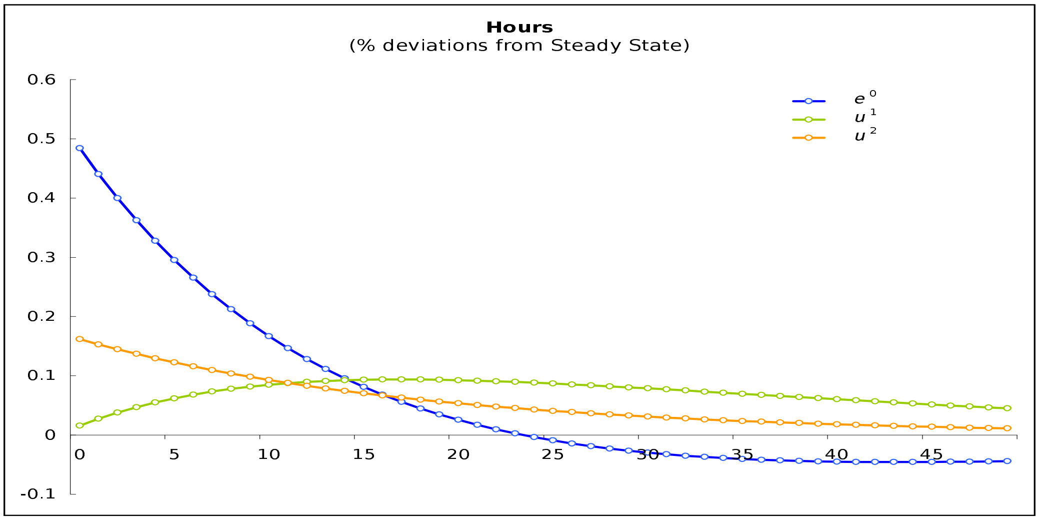

Figure 7: Hours Impulse Response Functions to Innovations to All Shocks

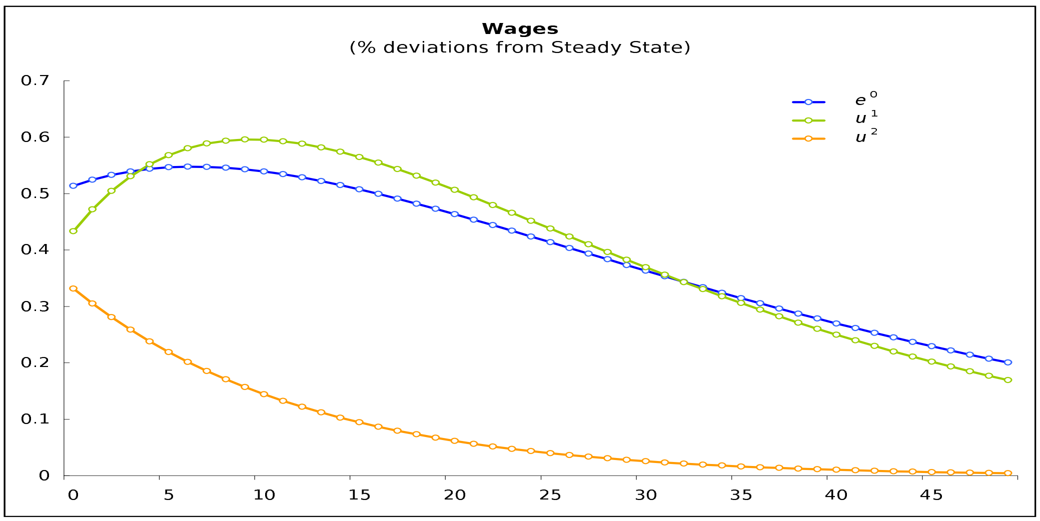

Figure 8: Wage Impulse Response Functions to Innovations to All Shocks

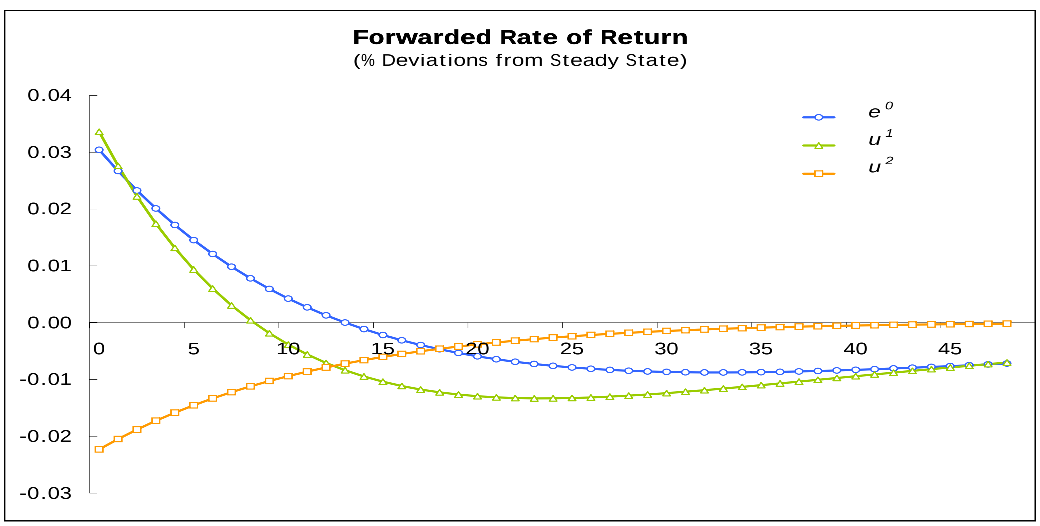

Figure 9: Forwarded Rate of Return Impulse Response Functions to Innovations to All Shocks

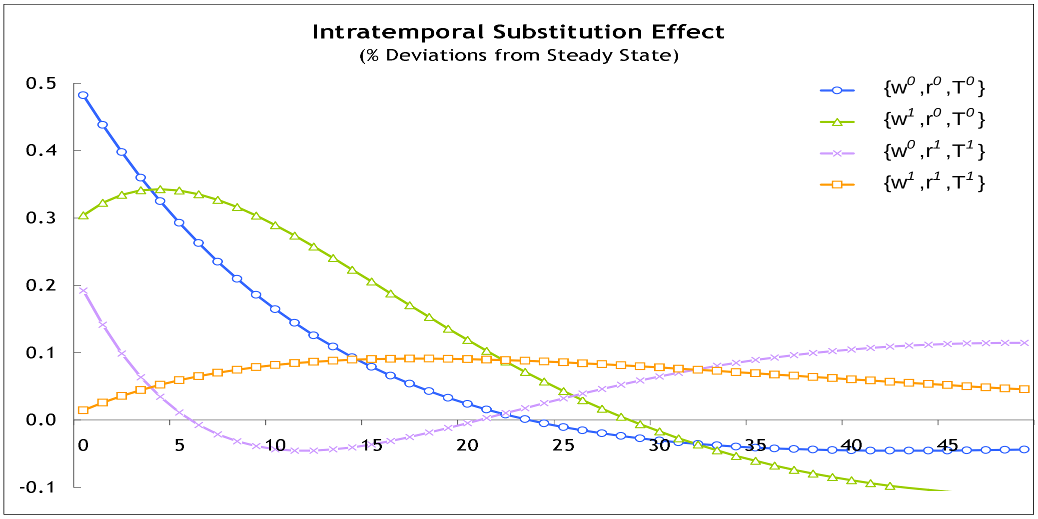

Figure 10: Hours Intratemporal Substitution Effects

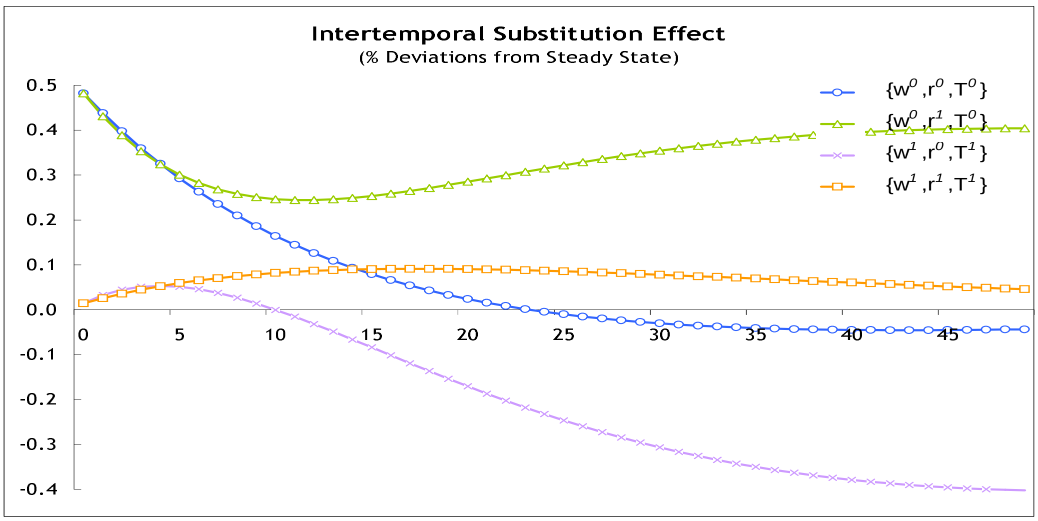

Figure 11: Hours Intertemporal Substitution Effects

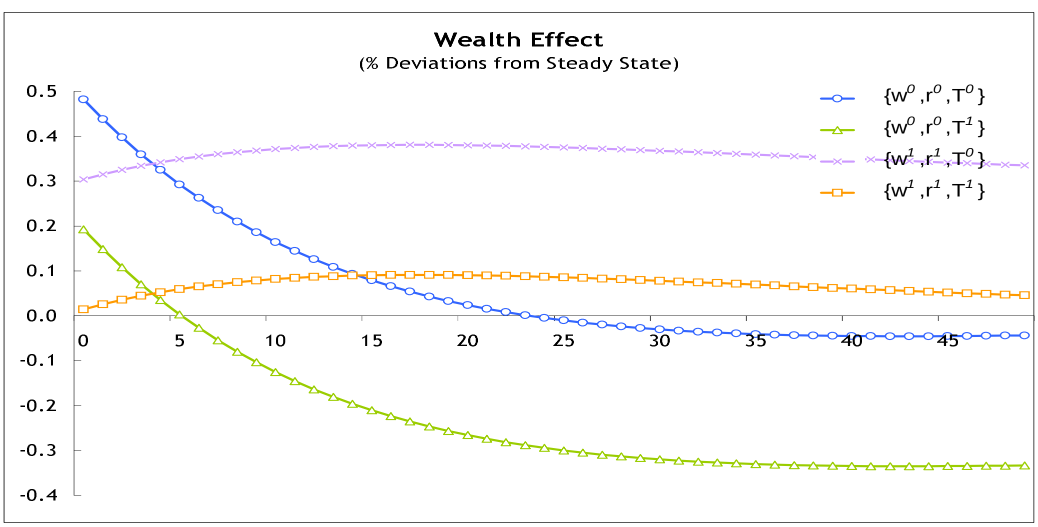

Figure 12: Hours Wealth Effects

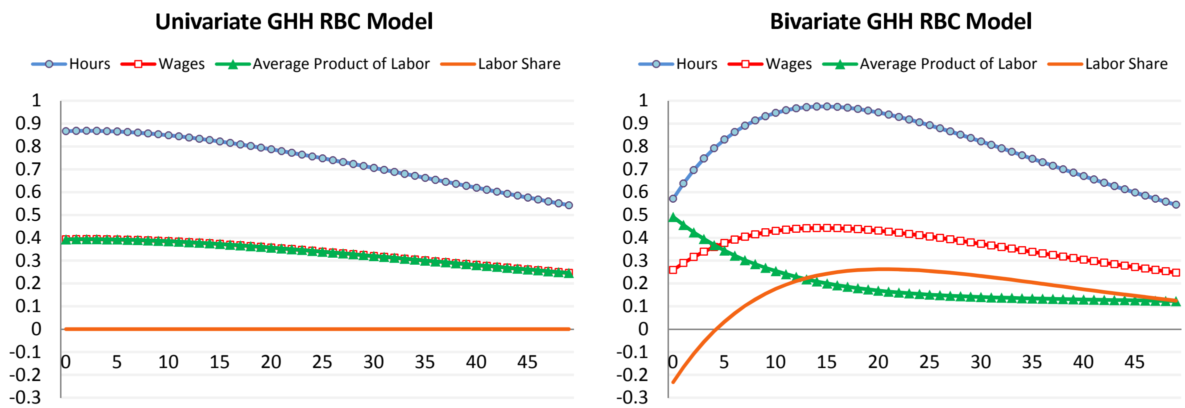

Figure 13: Univariate RBC Model (right panel) and Bivariate RBC Model IRFs with GHH Utility Functions

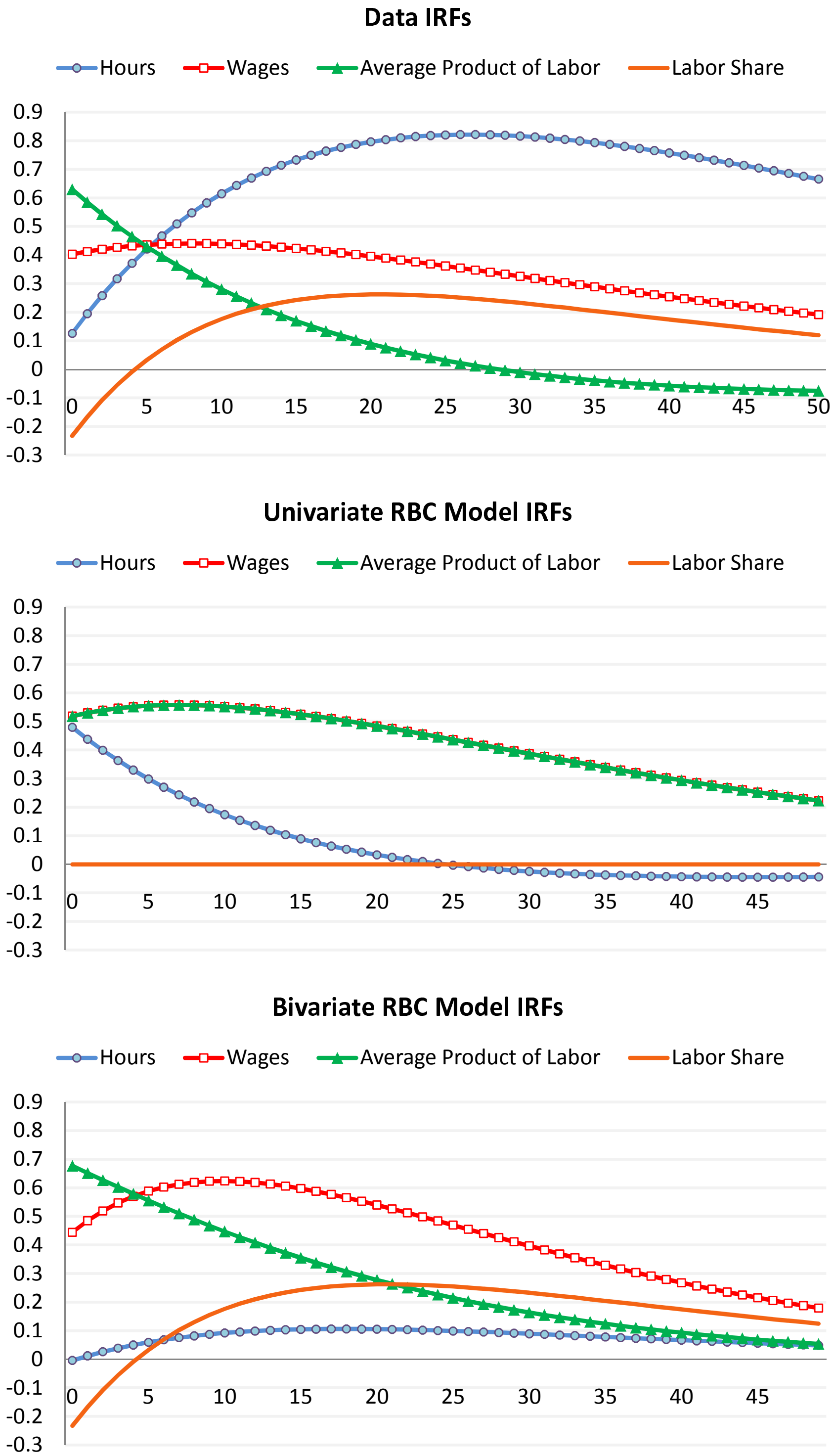

Figure 14: IRFs of Hours, Wages, Average Product of Labor and Labor Share to Productivity Innovations: Data (top panel), Univariate RBC Model (central panel) and Bivariate RBC Model (bottom panel).

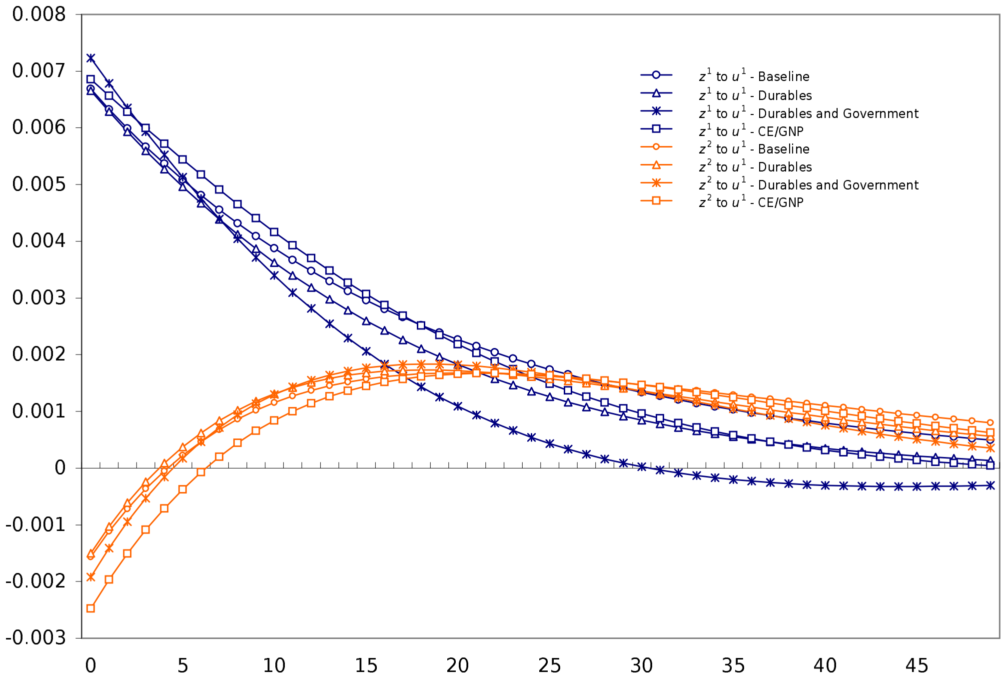

Figure App-1: Impulse Response Functions to Productivity Innovations u^1, All Labor Share Definitions

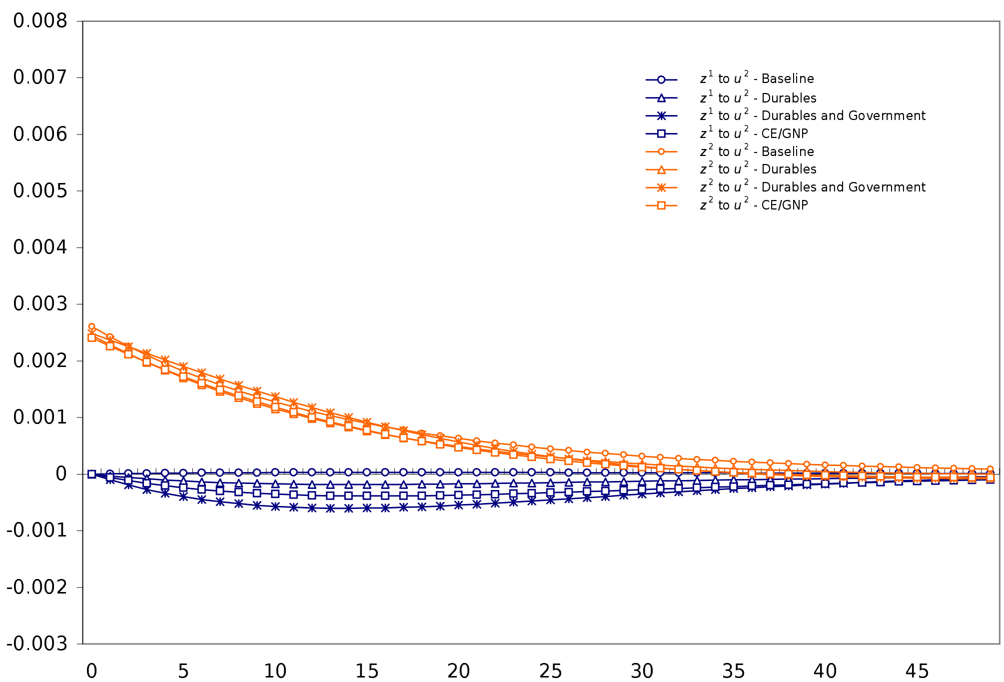

Figure App-2: Impulse Response Functions to Distributive Innovations u^2, All Labor Share Definitions

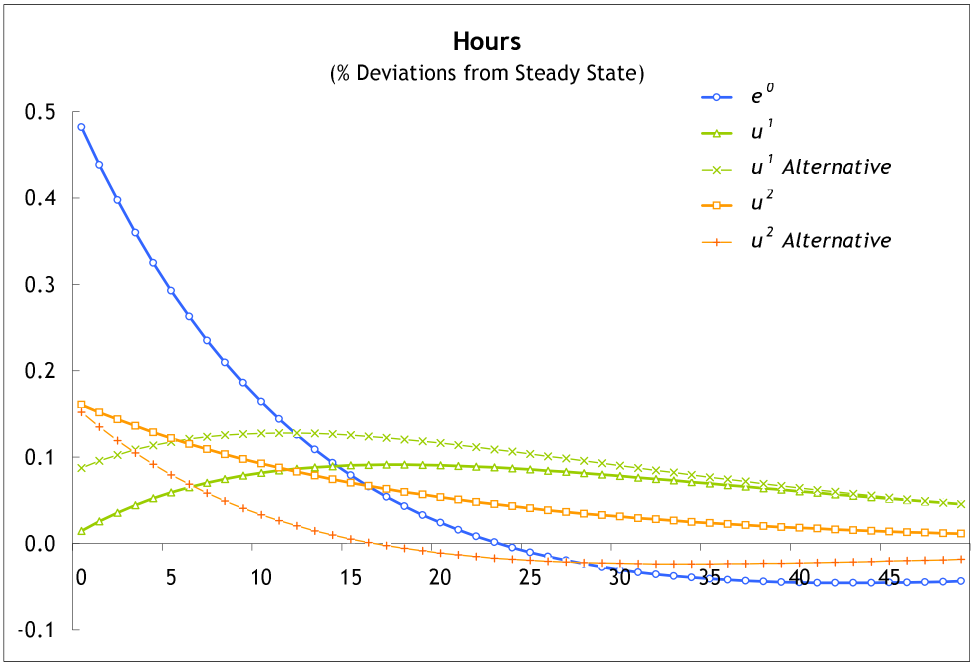

Figure App-3: Impulse Response Functions of Hours to All Innovations, Alternative Identification

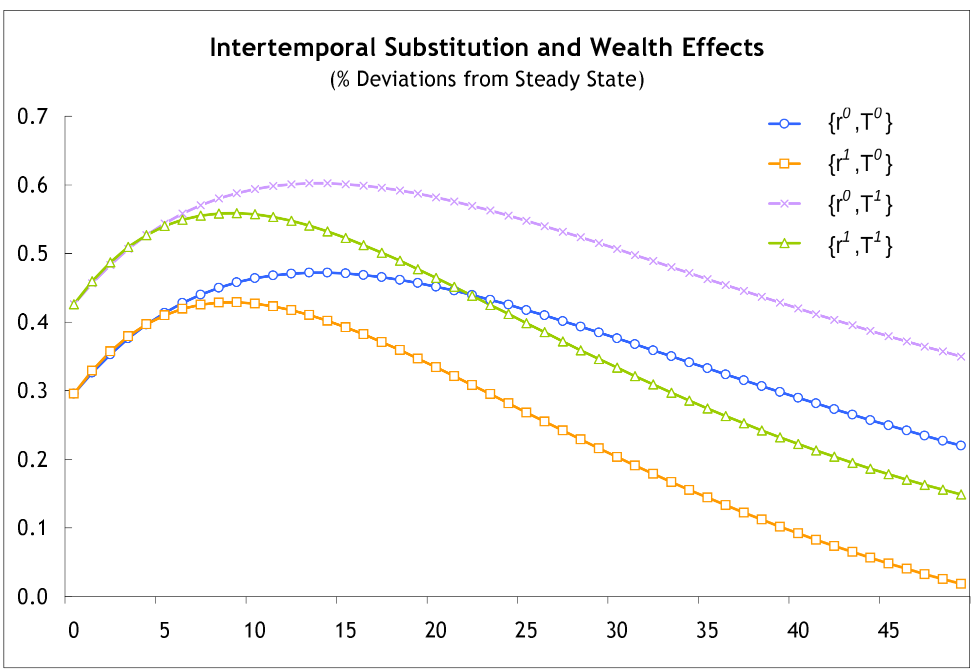

Figure App-4: Consumption Intertemporal Substitution and Wealth Effects