Credit Lines

Xavier Mateos-Planas
University of Southampton,
F.X.Mateos-Planas@soton.ac.uk

José-Víctor Ríos-Rull*
University of Minnesota, CAERP
University of Pennsylvania, CEPR, NBER
FRB Mpls, vr0j@.umn.edu

December 31, 2007

Abstract

In this paper we develop and solve a model of credit lines. Credit lines are long term relations between banks and households that prespecify a credit limit and have a fixed interest rate. Households can unilaterally default according to U.S. Bankruptcy law, and can switch credit lines at will, albeit at a (utility) cost. Banks issue costly credit lines to households and they can either have to commit to them or not (we look at both cases). We solve and characterize the equilibria. We find that this model replicates the main properties of typical lending contracts and that it holds a lot of promise for quantitative work.

Extremely Preliminary

*Ríos-Rull thanks the National Science Foundation for Grant SES-0351451.

1 Introduction

In recent times, unsecured consumer debt on credit cards has been gaining importance in the U.S. and other developed economies. One characteristic of this form of revolving credit is that contracts specify pre-approved credit limits and interest rates. For all the increasing presence of credit card lending, there has been little attempt at analyzing its significance for understanding important macroeconomic variables such as consumer debt, bankruptcy and the distribution of wealth. This paper takes a step forward by providing such an analysis.

The paper seeks to investigate the significance of the features of the credit card market for the macroeconomic analysis of personal debt and bankruptcy. The first objective is to introduce in a quantitative macroeconomic model contracts that exhibit the main traits of actual credit lines. The second objective is to use this model to study bankruptcy and the distribution of wealth in the economy.

The analytical framework that we develop extends the model of consumer idiosyncratic risk and incomplete markets of the kind that has become common fare in quantitative macroeconomic analysis (e.g., Imrohoroğlu (1989), Huggett (1993), Aiyagari (1994)). A main distinctive feature is that households have access to credit in the form of credit lines, each defined by a credit limit and interest rate. The set of credit lines available to a consumer of a certain class type depends on its initial withdrawal. Switching to a different credit card with different conditions is costly for the household. Consequently, consumers may in general wish to stick to the present credit card even if their circumstances change. Credit lines are issued by competitive financial intermediaries. Two alternative assumptions about intermediaries are considered. First, the basic model will assume commitment on the side of intermediaries so that contracts last for as long as the consumer's action does not terminate the relationship. Second, banks can decide to discontinue the contract if circumstances change in a way that make it unprofitable. Consumers can exercise the option to declare bankruptcy in a way that encompasses the main provisions of Chapter 7 of the US bankruptcy code. The different conditions across credit lines reflect their different default risk. The model in equilibrium delivers profiles of wealth/debt, credit card limits and interest rates, and default across households.

Notice that the properties of the contracts of our model agree with the typical credit card contracts. The only issue is exclusivity, but this as we argue in Section 2 is implicity in the existing arrangements. Banks know the asset position of its customers every few months and rearrange the terms of the contract accordingly.

There is an emerging literature analyzing bankruptcy and credit in quantitative general equilibrium models. In Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) there is no switching cost and credit conditions adjust instantaneously. Livshits, MacGee, and Tertilt (2007b) and Livshits, MacGee, and Tertilt (2007a) and Athreya and Simpson (2006) share similar features. In contrast, the present paper has the realistic feature that, due to the cost of switching, households can keep the same credit conditions for loans of varying size and over changing personal circumstances. Mateos-Planas (2007) studies the determination of an endogenous credit limit that is binding for some households in a model where banks confront an adverse selection problem. The existence of fixed costs and scale effects in banking in a model with one-period loan contracts buys the existence of a single limit and interest rate serving all borrowers. The present paper instead relies on a cost to switching contract for the household which delivers long-lasting contracts and a non-trivial distribution of credit limits and interest rates. While more complex, this is probably a more natural description of actual credit card arrangements. A new strand of the literature (Chatterjee, Corbae, and Rios-Rull (2005)) attempts to provide a theory of credit without exogenous punishment.

In independent work, Drozd and Nosal (2007) have posed a model with credit lines and default in an environment with search type frictions. In it banks make offers to specific consumers of certain types (which include income, wealth and current credit line) who choose among the credit lines they are offered, if any; defaulting agents leave the economy and get a fixed utility level; default happens when households hit the borrowing constraint; banks commit to their offers. Our structure differs in various respects. We model default in line with U.S. law.[1] Default is an endogenous decision and borrowing constrained agents do not necessarily default. We have incomplete markets and the only frictions are a utility cost of switching contracts for the household that can be thought of the time spent in filling and signing forms and a resource cost for intermediaries of issuing contracts that can be thought of the cost of verifying and storing information. The terms of our credit lines depend on the initial withdrawal as well as whatever characteristics of the household are relevant for forecasting future income while. We allow banks to terminate the contract unilaterally, or to change it at a cost.

Section 2 discusses the type of contracts that lending companies and households engage in. Section 3 discusses our modeling choices and makes the case for its ease of computation in an environment that stretches the limits of current capabilities. Section 4 poses the model, the problem of both households and intermediaries, the equilibrium and the computation strategy. Section 5 poses the version of the model

[1] As in Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007).

where banks have no commitment. In Section 6 we discuss a few extensions that contemplate various relevants issues for credit, such as teasing rates, the role of credit cards in providing transaction services and the rationale for the payment of a minimum balance each month.

2 A discussion of credit contracts

To be written.

3 A discussion of modeling choices

To be enlarged.

3.1 Contracts depend on initial withdraw

The reason for this choice is completely technical. We do not want the contract to depend on many wealth levels.

3.2 For the household switching is costly in terms of utils and not resources

If a contract is costly in terms of resources then a switch to more favourable terms is attactive when the household is wealthy as a form of insurance even if borrowing is unlikely in the near future. This is not the case with a utility cost. There is no advantage in preempting switching when wealthy. Only when in a high income class.

We hope to proceed by making a guess and verify statement later that these assumptions are unnecessary and that these properties are implied from contracts that depend on initial cash in hand and income class. In particular we want to make sure that no contract that is not used in the first period can exist (at least for the worst income class).

4 The model

This is a model of an exchange economy with heterogeneous households and incomplete markets. Borrowing takes place through credit lines with pre-approved credit limits and interest rates. Credit relationships typically extend over various periods. In this section, we assume commitment on the side of the banks. Therefore a credit line terminates when the consumer defaults or switches to a new contract. The case without commitment will be studied in a later section. We start describing the households' problem in Section 4.1, then we look at intermediaries in Section 4.2 to move on to equilibrium in Section 4.3, and to computation in Section 4.4.

4.1 Households

A household starts a period with a certain level of assets brought from the past y and a level of earnings ε. Earnings $\varepsilon \in \mathcal{E}$ are random and distributed with probability measure $F(.|e)$. So conditional on what we label income class $e \in E$, earnings are *i.i.d.*. Income class e is Markov with transition matrix $\Gamma_{e,e'}$. We define cash in hand as $a = y + \varepsilon$.

A household has also a credit line ω which specifies a credit limit b^ω, an associated discount rate q^ω, and the initial borrowing taken against this credit line y^ω. The $\omega = 0$ denotes a no credit line. A household also has a credit history h that can be good (0) or bad (1). A household with a good credit history can choose to keep its credit line or to switch to those offered to him at utility cost χ. A household with a bad credit history is restricted to have the zero credit line. Bad histories switch to good with probability δ. Good histories switch to bad by defaulting on loans and filing for bankruptcy.

For the same reasons as in Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) there are relevant bounded sets $\mathcal{Y}$ and $\mathcal{A}$ that restrict where assets and cash in hand can lie. Let $\mathcal{Y}^-$ and $\mathcal{A}^-$ denote the respective subsets with negative values.

The individual state at the beginning of the period is $(y, \varepsilon, e, \omega, h)$ with $\omega = (q^\omega, b^\omega)$. It is easier to pose the problem by decomposing it in stages. First the decision is whether to default or not; second the decision of whether to switch contracts or not; third the decision of how much to save if no default and no switch occur. The decision to switch contracts or not is determined jointly with the savings decision as the new contract is associated to the initial borrowing amount. After the first stage there is no need to keep both y and ε. It is sufficient to keep track of cash in hand: if $d = 0$ then $a = y + \varepsilon$, while if $d = 1$ then $a = \varepsilon$. We now formalize these decision.

1. The credit history is updated to h'. This happens in three possible ways. A household with good credit standing, $h = 0$, and positive assets, $y \geq 0$, chooses nothing and remains with good credit standing. A household with good credit standing, $h = 0$ and negative assets $y < 0$ chooses whether to default, $d = 1$ and $h' = 1$, or not, $d = 0$ and $h' = 0$. A household with a bad credit history $h = 1$ switches to a good credit history, $h' = 0$, with probability δ or maintains its bad credit history, $h' = 1$, with probability $1 - \delta$.

   In this stage the inputs are the state $(y, \varepsilon, e, \omega, h)$ and the value function v^1 that results from the switching decision (if available) in stage 2. Note that $v^1(a, e, \omega, h')$ only depends on cash in hand, a, income class e, contract ω, and updated credit history h'. There is no need to separate assets and income.

   The decision problem at this stage is only for those with $h = 0$ and $y < 0$:

$$v(y, \varepsilon, e, \omega, 0) = \max_{d \in \{0,1\}} (1 - d)\, v^1(y + \varepsilon, e, \omega, 0)\; d\, [u(\varepsilon') + \beta\, v^d(y, e, 0, 1)] \quad (1)$$

   where we have introduced v^d to denote the expected utility of defaulting which is independent of assets and is

$$v^d(y, e, 0, 1) = \sum_{\varepsilon', e'} \Gamma_{e,e'}\, F(\varepsilon', e')\, v\,(0, \varepsilon', e', 0, 1) \quad (2)$$

   Function v is the solution to the problem and it denotes the beginning of period value function.

   The output of this stage is the policy function of whether to default $d(y, a, e, \omega, 0)$, the default set $D(y, e, \omega) = \{\varepsilon \in \mathcal{E} : d(y, y + \varepsilon, e, \omega, 0) = 1\}$, and the update of the credit history $h'(y, a, e, \omega, h)$ (the latter is a random variable when $h = 1$).

2. The second stage is only relevant for households with good credit $h' = 0$ who choose the credit line contract for the following period. The household can switch to another contract ω' which specifies its credit limit $b^{\omega'}$ its (inverse) interest rate $q^{\omega'}$ and its initial withdrawal $y^{\omega'}$ out of those that are available which we denote by $\Omega(e)$. To avoid cumbersome notation we do not explicitly write that for all contracts, if savings are positive, the interest rate is just the risk free rate. Note that the contract offered only depends on e because it is what is needed to forecast future income. The problem is

$$v^1(a, e, \omega, 0) = \max \{ \max_{\omega' \in \Omega(e)} v^2(a, y^{\omega'}, e, \omega', 0),\; v^3(a, e, \omega, 0)\} \quad (3)$$

where v^2 is the utility of borrowing $y^{\omega'}$ and switching to contract ω' or

$$v^2(a, y^{\omega'}, e, \omega', 0) = u\left(a - q^{\omega'}\, y^{\omega'}\right) - \chi + \beta\, v^e(y^{\omega'}, e, \omega', 0) =$$
$$u\left(a - q^{\omega'}\, y^{\omega'}\right) - \chi + \beta \sum_{\varepsilon', e'} \Gamma_{e,e'}\, F(\varepsilon', e')\, v\left(y^{\omega'}, \varepsilon', e', \omega', 0\right) \quad (4)$$

where we have introduced v^e to denote expected utility. The term v^3 is the result of solving for optimal savings, an option only available to those that did not switch plans.

The input in this stage is the "intra" state (a, e, ω, h'). The output is a policy function for the new credit line $\omega'(a, e, \omega, h')$. We also use a switching function

$$\nu(a, e, \omega, 0) = \begin{cases} 0 & if\ \omega'(a, e, \omega, 0) = \omega, \\ 1 & otherwise. \end{cases}$$

and let $\mathcal{V}(y, e, \omega) = \{\varepsilon \in \mathcal{E} : \nu(y + \varepsilon, e, \omega, 0) = 1\}$ denote the switching set.

3. For the households with bad credit and those that did not switch the last stage determines the optimal savings.

$$v^3(a, e, \omega, 0) = \max_{y' \geq b^\omega} u\left(a - q^\omega\, y'\right) + \beta\, v^e(y', e, \omega, 0) \quad (5)$$

For those with bad credit, the expected value takes into account the possible redemption:

$$v^3(a, e, 0, 1) = \max_{y' \geq b^\omega} u\left(a - q^0\, y'\right) + \beta\, \{\delta\, v^e(y', e, \omega, 0) + (1-\delta)\, v^e(y', e, \omega, 1)\} \quad (6)$$

The input at this stage consists of the "intra" state (a, e, ω, h'). The output is the policy function for savings for non switchers $y'(a, e, \omega, h')$.

4.2 The Intermediaries

Intermediaries issue contracts that specify the a credit limit and interest rate. The value of one such contract at ay time can be expressed as a function of the households class e and her level of borrowing y'. We then denote by $\Psi(\omega, y', e)$ the value of a contract ω with a household of type e who borrows the amount y'.

This value is the discounted sum of payoffs to the bank. The payoff associated with (ω, y', e) can be divided in two parts. In the current period of the contract, the cash flow from the bank is $q^\omega\, y'$. We use $m^0(\omega, y', e)$ to denote the current period cash flow from the bank to the household and $m^1(\omega, y', e)$ to denote the expected cash flow towards the bank of a credit line in the following period. In the current period then

$$m^0(\omega, y', e) = q^\omega \;\; y' \;\; 1_{\{y'<0\}}$$

In the next period, there are two possible outcomes. The household may default, that is $d(y', \varepsilon', e', \omega, 0) = 1$, in which case the intermediary gets zero, or she can pay back in full. The corresponding return is

$$m^1(\omega, y', e) = -y' \; 1_{y'<0} \; \Big(1 - \sum \Gamma_{e,e'}\, F(\varepsilon', e')\, d(y', \varepsilon', e', \omega, 0)\Big)$$

Under commitment on the part of the bank, the contract continues into the next period with an updated value $\Psi(\omega, y'(\varepsilon' + y', e', \omega, 0), e')$ as long as the household does not default nor switches to a different credit line. The value of the contract can then be expressed recursively as follows:

$$\begin{aligned}\Psi(\omega, y', e) = m^0(\omega, y', e) + q^0 m^1(\omega, y', e) + q^0 \\ \sum_{e'\varepsilon'} \; (1 - d(y', \varepsilon', e', \omega, 0)) \; (1 - \nu(y' + \varepsilon', e', \omega, 0)) \\ \Gamma_{e,e'} \; F(\varepsilon', e') \; \Psi(\omega, y'(\varepsilon' + y', e', 0), e'). \quad (7)\end{aligned}$$

Banks incur a cost π of issuing a contract.

4.3 Equilibrium

There is free entry in intermediation. An equilibrium is therefore a situation where new contracts have a zero net value when households make optimal decisions. The equilibrium thus determines the sets of contracts – that is triads of limits price, and initial withdrawal – available to each income class, $\Omega(e)$.

More formally, the analysis of households and intermediaries has produced the following objects. For households, given the available contracts $\Omega(e)$, one obtains decision rules for default $d(y, a, e, \omega, h)$, contract choice $\omega'(a, e, \omega, h')$, switching decision $\nu(a, e, \omega, h')$ and savings $y'(a, e, \omega', h')$, as well as value functions $v(y, a, e, \omega, h)$, $v^1(a, e, \omega, h)$, $v^2(a, e, \omega, h)$. For intermediaries, the given functions $d(.)$, $\omega'(.)$, $\nu(.)$ and $y'(.)$, yield the value of a contract $\Psi(\omega, y', e)$. For any arbitrary set of credit limits $\mathcal{B} \in \mathcal{Y}^-$ we can now define a $\mathcal{B}$-Equilibrium.

Definition 1 *A $\mathcal{B}$-Equilibrium is a set of functions that solve the households' problem and where:*

$$\Psi(\omega^*, y^{\omega^*}, e) - \pi = 0 \qquad \forall \omega^* \in \Omega(e), \quad \textit{and all } b^{\omega^*} \in \mathcal{B}. \tag{8}$$

Note that free entry in banking requires that only newly issued contracts have values equal to the issuing costs. Under commitment on the part of the banks, old contracts can have any value.

We now characterize the equilibria. We show that for any borrowing limit there is only one equilibrium interest rate. To see this, define $\widehat{\Psi}(\omega^*, y^{\omega^*}, e, q^{\omega})$ as the value of a contract $\omega^* = \{b^{\omega^*}, q^{\omega^*}, y^{\omega^*}\}$ where the bank is charging q^{ω} to households that act as if they were charged q^{ω^*}.

Lemma 1 *Function $\widehat{\Psi}$ is monotonically decreasing in q^w.*

Define the limit price $q^{\omega}(\omega^*, y^{\omega^*}, e)$ as the zero-profit on contract $\omega^* = \{b^{\omega^*}, q^{\omega^*}, y^{\omega^*}\}$ when it is issued to an $\{e\}$ household.

$$\widehat{\Psi}(\omega^*, y^{\omega^*}, e, q^{\omega}(\omega^*, y^{\omega^*}, e)) - \pi = 0. \tag{9}$$

Lemma 2 *The limit price $q^{\omega}(\omega^*, y^{\omega^*}, e)$ exists and is unique for any $\Omega(e)$ and solution to the household problem.*

Note that for any contract set $\Omega(e)$, and any $\omega^* \in \Omega(e)$, $\{b^{\omega^*}, q^{\omega}(\omega^*, y^{\omega^*}, e), y^{\omega^*}\} \notin \Omega(e)$ violates the free entry condition either because some firm would undercut it or because nobody will offer.

The next result establishes that the equilibrium price must coincide with the limit price.

Lemma 3 *In Equilibrium, if $\omega^* \in \Omega(e)$, then $\{b^{\omega^*}, q^{\omega}(\omega^*, y^{\omega^*}), y^{\omega^*}\} \in \Omega(e)$ and $\nexists\, q^w \neq q^{\omega^*}$ such that $\{b^{\omega^*}, q^{\omega}), y^{\omega^*}\} \in \Omega(e)$.*

Finally an Equilibrium requires no free entry. Consequently,

Definition 2 *An Equilibrium is a $\mathcal{B}$-Equilibrium where $\mathcal{B} = \mathcal{Y}^-$.*

4.4 Computation of equilibria

For solving for equilibria it is easier tp build an increasing set of $\mathcal{B}_m$, $\mathcal{B}_M = \mathcal{Y}$ and to look for $\mathcal{B} - equilibria$. Specifically, the steps are

1. For each $b^w \in \mathcal{B}_m$, each $b^\omega < y' < 0$ guess the profile of prices across types $q_0^{\omega'}(e, b^\omega, y')$. This defines $\Omega_{m,0}(e)$
2. Solve the household problem obtaining decision rules.
3. Compute $q_{m,1} = q^\omega(\omega, y', e)$ by solving

$$\widehat{\Psi}[\omega, y', e, q^\omega(\omega, y', e)] - \pi = 0.$$

   Note that even if a contract is not chosen by households, its value can still be computed and hence its $q^\omega_{m,1}(\omega, y', e)$ found.

4. Update $q^\omega_{m,0}(\omega, y', e)$ using $q^\omega_1(\omega, y', e)$ and if different go back to 2. If equal, move on to $\mathcal{B}_{m+1}$ using as initial guesses the ones we just obtained and go to 1 if $\mathcal{B}_{m+1} \neq \mathcal{Y}$.
5. Compute all the relevant moments of the model economy.

5 Banks have no commitment

In the absence of commitment on the side of the bank, the bank will not hold a contract that has negative value. We look first at the simpler case when the bank just drops a household under a contract with a negative value.

5.1 Dropping the household

When the bank has the option of dropping an unsuitable household, ongoing contracts are restricted to be non negative at all times. In order to account for non-commitment the model has to be extended. First, for a bank holding a certain credit line ω, there is a new decision whether to continue or discontinue the contract. This decision will depend on the borrower's class type e and the amount she intends to borrow against this credit line y'. These decision will determine $\widehat{\Omega}(e)$, the set of combined contract terms, b^ω and q^ω, and borrowing levels, y', such that a bank will decide to continue serving the contract for a household of type e. By definition it must be true that an element in the set of new available contracts $(b^\omega, q^\omega, y^\omega) \in \Omega(e)$ must also be an element in the corresponding set of sustainable contracts, $\widehat{\Omega}(e)$. That is, $\Omega(e) \subset \widehat{\Omega}(e)$. Second, the household's saving/borrowing decisions will be further restricted by the possibility that they might induce the bank to end the credit facility.

More formally, the value of a contract ω for the intermediary accounts for the option to pull out. That is, with the notation introduced earlier, the value of a contract becomes.

$$\widehat{\Psi}(\omega, y', e) = \max\Bigg\{0, m^0(\omega, y', e) + q^0\,\widehat{m}^1(\omega, y', e) + q^0 \sum_{e,\varepsilon'} \Gamma_{e,e'}\; F(\varepsilon', e')$$
$$[(1 - d(y', \varepsilon', e', \omega, 0))(1 - \nu(y' + \varepsilon', e', \omega, 0))]\widehat{\Psi}(\omega, y'(\varepsilon' + y', e', 0), e')\Bigg\}. \quad (10)$$

In equilibrium, these banks' decisions will shape the sets of surviving terms $\widehat{\Omega}(e)$.

Regarding the household, the description of the default and switching decisions parallel that presented earlier. The savings decision however must be modified. The household with current class type e and holding contract ω, takes as given the set $\widehat{\Omega}(e)$ and, therefore, the viability of different borrowing levels y'. If the household intends to borrow to such an extent that $(b^\omega, q^\omega, y') \notin \widehat{\Omega}(e)$ then the credit facility would be withdrawn and she would be forced into a no-credit state, $\omega = 0$. However, reverting to this state requires non-negative savings, $y' \geq b^0$, and can never be superior to sticking to the current line ω with some credit facility. Consequently, the absence of commitment introduces a new constraint on savings for the household who has a good credit standing and decides not to switch contracts:

$$(b^\omega, q^\omega, y') \in \widehat{\Omega}(e).$$

Formally, the savings problem and the value at this stage become:

$$\widehat{v}^3(a, e, \omega, 0) = \max_{y' \geq b^\omega, (b^\omega, q^\omega, y') \in \widehat{\Omega}(e)} u\,(a - q^\omega\; y') + \beta\; \widehat{v}^e(y', e, \omega, 0) \quad (11)$$

We can now define an equilibrium without commitment. The households' decision rules and value function are conditional on the sets of available new contracts Ω and surviving contracts $\widehat{\Omega}$. These rules and the bank's termination decision determine the bank's value profile Ψ. In any $\mathcal{B}$-equilibrium, free entry and the termination decision determine the sets Ω and $\widehat{\Omega}$. For a set of borrowing limits $\mathcal{B}$, free entry requires, like before,

$$\Psi(\omega, y^\omega, e) - \pi = 0 \qquad \forall\, \omega \in \Omega(e), \quad \text{and all } \; b^\omega \in \mathcal{B}. \tag{12}$$

On the other hand, banks optimality requires positive continution values. That is,

$$\Psi(\omega, y', e) \geq 0 \qquad \forall\, (b^\omega, q^\omega, y') \in \widehat{\Omega}(e) \tag{13}$$

This added restriction on individual savings may or may not be effective, depending on the specific situation. Some households may still use the full available credit limit b^ω. However, other households may be deterred from doing so in order to keep their credit facility.

5.2 Renegotiating the contract

The lack of commitment on the parts of banks may be better modeled not as a context where the bank drops the household but as a situation where the bank restates the terms of the contract. Given the costly nature of contracts, there is an opportunity for the bank to take advantage of the implied holdup problem. We assume that in every period, the bank offers its customers a take it or leave it offer. In other words, there are two sets of contracts, introductory contracts and

Perhaps it is more interesting to renegotiate the contract. This is the bank can (perhaps after some initial period as in *teasing rates* contracts) change the terms of a contract, taking advantage of the reduced cost for the household if it does not switch. We denote by $\widetilde{\Omega}(e, \omega)$ the set of contracts that are posed to a household with income class e and current contract $\omega = \{b^\omega, q^\omega\}$. In order to characterize the solution let's define the best switch for a household $\overline{\overline{\omega'}}(a, e)$ and associated value $\overline{\overline{v}}^2$, which are the solution to

$$\overline{\overline{v}}^2(a, e) = \max_{\omega' \in \Omega(e)} u\left(a - q^{\omega'}\; y^{\omega'}\right) - \chi + \beta\, v^e(y^{\omega'}, e, \omega', 0) \tag{14}$$

Now the bank is constrained by $\overline{\overline{v}}^2$. So when offering alternatives, it solves

$$\overline{\overline{\Psi}}(\omega, a, e) = \max_{b^{\omega'}, y', q^{\omega'}} \Big\{ m^0(\omega, y', e) + q^0\, \overline{m}^1(\omega, y', e) + q^0 \sum_{e,\varepsilon'} \Gamma_{e,e'}\, F(\varepsilon', e') \\ [(1 - \overline{d}(y', \varepsilon', e', \omega, 0)]\, \overline{\Psi}(\omega, y'(\varepsilon' + y', e', 0), e'), \Big\}. \quad (15)$$

subject to:

$$u\left(a - q^{\omega'}\ y^{\omega'}\right) + \beta\ v^e(y^{\omega'}, e, \omega', 0) \geq \overline{\overline{v}}^2(a, e) \quad (16)$$

And obviously

$$\overline{\overline{\Psi}}(\omega, a, e) = \max\left\{0, \overline{\Psi}(\omega, a, e)\right\} \quad (17)$$

The rest of the terms of the definition follow immediately from these ones.

TO BE DETAILED

6 Extensions

6.1 Teasing rates

A teasing rate contract is a contract that has the feature that interest rates go up after a while. The model with lack of commitment is an environment that will deliver teasing rates for one period. Perhaps the best way of modeling teasing rates that last for longer amounts of time is to assume that the opportunity for a bank to change the terms of a loan arrives stochastically, this is to use a version of the model where access to commitment is random.

6.2 Cerdit cards are usefull for transactions

We can pose a simple shopping time type of utility function where the credit limit affects the utility function directly because it facilitates transactions.

6.3 Households have to pay a minimum balance each month

To keep the credit going intermediaries require some partial payment. We take this to be a signal of $e \notin \underline{E}$, an income class that indicates a disaster.

References

AIYAGARI, S. R. (1994): "Uninsured Idiosyncratic Risk, and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659–684.

ATHREYA, K., AND N. B. SIMPSON (2006): "Unsecured debt with public insurance: From bad to worse," *Journal of Monetary Economics*, 53(3), 797–825.

CHATTERJEE, S., D. CORBAE, M. NAKAJIMA, AND J.-V. RÍOS-RULL (2007): "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, 75(6), 1525–1589.

CHATTERJEE, S., D. CORBAE, AND J.-V. RIOS-RULL (2005): "Credit Scoring and Competitive Pricing of Default Risk," Mimeo, University of Pennsylvania, CAERP.

DROZD, L. A., AND J. B. NOSAL (2007): "Competing for Customers: A Search Model of the Market for Unsecured Credit," Mimeo, Nosal's job market paper.

HUGGETT, M. (1993): "The Risk Free Rate in Heterogeneous-Agents, Incomplete Insurance Economies," *Journal of Economic Dynamics and Control*, 17(5/6), 953–970.

İMROHOROĞLU, A. (1989): "The Cost of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political Economy*, 97(6), 1364–83.

LIVSHITS, I., J. MACGEE, AND M. TERTILT (2007a): "ccounting for the Rise in Consumer Bankruptcies," Mimeo, University of Western Ontario.

——— (2007b): "Consumer Bankruptcy: A Fresh Start," *American Economic Review*, 97(1), 402–418.

MATEOS-PLANAS, X. (2007): "A model of credit limits and bankruptcy with applications to welfare and indebtedness," Mimeo, University of Southampton.