Sticky Wage Models and Labor Supply Constraints*

Zhen Huo
Yale University

José-Víctor Ríos-Rull
University of Pennsylvania
UCL, CAERP, CEPR, NBER

Tuesday 31st July, 2018

Abstract

In New Keynesian models with sticky wages à la Calvo, the quantity of labor is solely determined by the demand side. Unions with monopsony power set the wage above what it takes to make agents work. If wages are sticky, however, a change of circumstances may make the demand for labor higher than agents' willingness to work. The literature implicitly assumes that the markup of wages over the willingness to work is large enough to ensure that workers always comply with the quantity of labor demanded. In this paper, we explore the extent to which this is the case and find that workers are required to work against their will about 10% of the time. Moreover, when we use, as proposed by traditional theory Drèze (1975), the minimum of the demand and supply of labor instead of the demand-determined quantity, we find that the typical parametrization yields a variance of hours around 25% lower (depending on the particular model). We estimate an approximated Dreze equilibrium in a state-of-the-art DSGE model and find that it yields answers that are sharply different from the demand-determined allocation. We conclude that when working with sticky wage economies, the use of Calvo pricing and demand determined quantities is questionable.

Keywords: Sticky wages, New Keynesian model, Dreze equilibrium, Monetary policy

JEL classifications: E20, E32, E37, E52

*Ríos-Rull thanks the National Science Foundation for Grant SES-1156228. We are thankful for discussions with V.V. Chari, Jean-Jacques Herings, Patrick Kehoe, Ellen McGrattan and Frank Schorfheide.

1 Introduction

In New Keynesian models with sticky wages à la Calvo, the quantity of labor is solely determined by the demand side, implicitly assuming that households are always willing to work at whatever wage rate is specified. Unions with monopsony power set wages above agents' marginal willingness to work, which provides a cushion that accommodates the effects of various shocks on the demand and supply of labor. If the shocks to the economy are small, the cushion is sufficient to guarantee that households happily accommodate the quantity of labor required. In this paper we document that the cushion is too small: demand-determined labor often implies that some of the labor is provided against the will of workers, a violation of the principle of voluntary exchange. We then explore a natural alternative that has a strong tradition in the literature, the ***Drèze equilibrium*** (Drèze, 1975), where the amount of labor is the minimum of labor demand and labor supply. We characterize a tractable approximation to the Drèze equilibrium and estimate a version of the Christiano et al. (2005) and Altig et al. (2011) environment where there are no agents that work against their will. We find that the economic properties of this economy are very different from those that result from estimating the model under the demand-determined solution. The relative importance of the various shocks changes, with neutral technology shocks accounting for 71% of the variance of employment instead of the 13% obtained with the demand determined model. Moreover, neutral technology shocks tend to have larger but less persistent innovations, a necessary feature to induce workers to contribute more labor despite facing low wages. We conclude that the demand-determined solution, although convenient, gives answers that are too strongly shaped by agents working against their will.

Modern computational tools such as Dynare have made it very easy to pose and estimate macroeconomic models by means of log-linearization and Bayesian estimation methods. New Keynesian models with sticky wages a lá Calvo where the quantity of labor is demand determined are well suited for log-linearization which undoubtedly contributes to their popularity.[1] If wages are sticky, however, a change of circumstances may induce the demand for labor to become higher than agents' willingness to work. The literature ignores this possibility and implicitly assumes that the markup of wages over agents' willingness to work is large enough to accommodate the changed circumstances. But is it? This is the question that we address in this paper.

What is the equilibrium condition when prices are not market clearing? Drèze (1975), following the notion of disequilibrium modeling of Barro and Grossman (1971) and Malinvaud (1977), posed that the amount traded is the minimum of the quantities supplied and demanded and that the agents are aware of the limitation in the availability of the trades. We think that the Drèze equilibrium is the natural equilibrium when wages are deemed to be fixed, as they are in sticky wage à la Calvo models, and consequently we explore its implications in this paper.

[1]Similar reasoning could be made about New Keynesian models with sticky prices.

To see whether some agents are working against their will in demand-determined allocations, we start our analysis by first substituting the demand-determined quantity of labor that the log-linearization procedure delivers with the minimum of the quantity of labor demanded and the quantity that agents would like to work. We refer to this quantity as ***voluntary ex-post aggregate labor***.[2] This is not a Drèze equilibrium, since agents made their decisions based on the demand-determined quantity of labor, but it does give us a preliminary account of the extent to which the demand-determined allocation is consistent with agents not working against their will. We compare the properties of the demand-determined quantity of labor with those of the voluntary ex-post aggregate labor on two of the most standard models of the New Keynesian literature: Altig et al. (2011) and Smets and Wouters (2007). We find that the properties are quite different. In particular, we find that between 2% and 32% of the labor is supplied against the will of workers. Moreover, the variance of the voluntary ex-post aggregate labor is typically lower than that of the demand-determined quantity of labor, over 30% lower in our preferred specifications, although it varies across specific models, so much so that in some cases it is actually higher for the voluntary ex-post aggregate labor.

Next, we use global methods to solve for the Drèze equilibrium of a New Keynesian model where all agents, firms, unions, and households take into consideration that labor is determined by the minimum of supply and demand when they make decisions. Note that the Drèze equilibrium introduces the labor supply constraint, which is occasionally binding, and this constraint makes the model nonlinear and hence ill-suited for log-linearization. Computing the Drèze equilibrium in standard medium-size dynamic stochastic general equilibrium (DSGE) models presents two main difficulties. First, Calvo-style sticky wages introduce the wage distribution as state variables. While under the assumption that labor is demand-determined, one only needs to keep track the aggregate wage index. This is not the case when we want to compute the Drèze equilibrium where we have to know what each group of workers, characterized by their own fixed wage, wants to do. This feature dramatically increases the number of state variables. Second, medium-size New Keynesian models have a large number of other state variables because of adjustment costs, backward indexation, various shocks, and the like.

For these reasons, we solve for the Drèze equilibrium in a relatively simple New Keynesian model with Taylor-type staggered wage contracts that stretches the class of economies that we can solve exactly. We find that employment in the Drèze equilibrium behaves similarly to what we found in the medium-size New Keynesian models by the voluntary ex-post aggregate labor: the variance of employment is about 25% smaller than under demand-determined employment.

We then propose a suitable approximated solution to the exact Drèze equilibrium, which we label as ***approximated Drèze equilibrium***. It requires much less computation cost and can be applied to medium-size DSGE models. As in our calculation of the voluntary ex-post aggregate labor, we also employ the

[2] Because various wages are coexisting at any point in time depending on the exact period when the wage was last set, the calculation of how much labor agents want to provide is not trivial.

log-linearized solution of the demand-determined allocation and then impose the ex post labor supply constraint. But unlike in the construction of the voluntary ex-post aggregate labor, the approximated Drèze equilibrium reconstructs all the main aggregate variables recursively, including capital, output, interest rate, and so on, guaranteeing that the resources constraints are satisfied. Comparing this approximation with the exact Drèze equilibrium, we find that the allocations are very similar. As a result, we argue that the approximated Drèze equilibrium can be used to address questions in medium-size DSGE models where computing the exact Drèze equilibrium is extremely hard.

Finally, as an application, we estimate the approximated Drèze equilibrium in a version of the Christiano et al. (2005) and Altig et al. (2011) model. We find that the unwillingness of agents in the Drèze equilibrium to contribute more labor leads to very different estimates, so that other pieces of the model can create the observed fluctuations. Notably, although the demand-determined solution implies a small role for the neutral technology shock in accounting for employment volatility (13% of the variance), the Drèze equilibrium estimates point to this shock in accounting for 70% of the labor variance, reducing the role of monetary shocks and investment-specific technology shock, and keeping it more in line with traditional real business cycle models. The Drèze equilibrium estimates of the process for the neutral technology shock become more volatile and less persistent, which simultaneously makes agents more responsive to shocks and less likely to violate the labor supply constraint. Finally, the Drèze equilibrium estimates imply higher wage rigidity, indicating that these rigidities play an important role in generating employment volatility even under the Drèze equilibrium.

Our conclusions are clear: using demand-determined labor in economies with sticky wages yields answers that are quite different from those obtained by what we think is a more reasonable equilibrium condition, the Drèze equilibrium. We find that in general, labor moves about 25% less for a given parametrization, with the actual outcomes varying depending on features of the economies that we detail later on. We also find that an approximated solution, by imposing an ex-post labor supply constraint on an economy solved by log-linearization, yields properties very similar to those of the Drèze equilibrium. We conclude that in environments with sticky wages, the demand determination in the labor market generates outcomes that are inappropriate, and an alternative is required. In this paper, we explore the Drèze equilibrium as an alternative which has a strong tradition in the literature, and we show that the approximated Drèze allocation is both easy to implement and close enough to the true Drèze equilibrium to be used.

Though we document that the demand-determined allocation in the sticky wages environment is questionable, the Drèze equilibrium is not the only alternative to address this type of issues. One alternative is to assume that when the demand determined allocation is larger than the supply determined, wages could be reset (similar to Hall and Milgrom (2008)). Another alternative is to introduce search and matching frictions when dealing choices on the extensive margin. We are open to these alternatives and we believe they are interesting and relevant. In this paper, we focus on the Drèze equilibrium because it has an appealing feature that it maintains the same primitive environment as the original model, and we can directly explore the logical implications of wage stickiness to contrast with the demand-determined allocation. This exercise can be theoretically interesting by itself, and it extends the original Drèze equilibrium literature to a dynamic decision problem. The construction of voluntary ex-post labor and the approximated Drèze equilibrium can be quantitatively relevant because it provides practitioners of DSGE models with a simple tool to examine whether the laobr supply constraint is violated or not in their own models.

The central notion that we highlight in this paper is that agents should not work against their will, and that the labor supply constraint should be thought of as a participation constraint. A similar ideas has already been explored in the literature. Hall (2005) develops a search model with sticky wages to account for the observed employment fluctuations. The wage is reset only if it hits the boundary of the bargaining set which is between the minimum wage acceptable to the worker and the maximum wage acceptable to the employer. The workers' participation constraint has to be respected. In a similar fashion, Gertler et al. (2008) and Gertler and Trigari (2009) explore a search model with Calvo-style sticky wages, and whether the bargaining set is violated or not is checked ex-post. It is generally true that the bargaining set is large enough to accommodate Calvo-type sticky wages when agents are only subject to aggregate shocks, but it remains a question whether the bargaining set is large enough when agents also face idiosyncratic shocks. Recently, Christiano et al. (2013) develop a quantitative model in which the wage is determined by alternating-offers bargaining, a variant of Hall and Milgrom (2008). The alternating-offer-bargaining mechanism introduces wage inertia endogenously, and is free of the concern on violating the participation constraint. Our paper focuses on the willingness of agents to work, but a similar argument can also be made on the willingness of firms to produce goods at a fixed price. For example, Corsetti and Pesenti (2005) emphasize that firms should only produce if the ex-post price markup is larger than one. Bills (2004) and Alessandria et al. (2010) consider firms' inventory stockout problem, where firms' sales have to be the minimum of the goods demanded and their existing inventory. Michaillat and Saez (2015) combine nominal rigidity with matching frictions in both goods and labor markets, where the supply and demand jointly determines the outcome via affecting the market tightness.[3]

Related to Drèze (1975)'s original work, Van der Laan (1980), Kurz (1982), Dehez and Drèze (1984), Drèze (1997), and Citanna et al. (2001) study the properties of supply-constrained economy and explore the connection between price distortion and coordination failure. Herings (1996, 2014) extend Drèze (1975)'s work to settings with more flexible primitives and to dynamic environments. Bénassy (1993) compares the original Drèze equilibrium with other closely related disequilibrium concepts, and explore their implications in a static monetary economy with fixed prices and wage. Our paper differs from the previous literature in two ways: first, the market structure in our paper is monopolistic competition instead of perfect competition. Therefore, in periods where the wages can be reset, they will be set by forward-looking unions rather than the market.[4] Second, the previous literature emphasized equilibrium existence

[3] The Drèze equilibrium can be viewed where the matching function is the minimum operator.

[4] Bénassy (1993) also considers the case where private agents set the prices and wages in a static environment, and in

and multiplicity, while our paper explores the quantitative properties in a state-of-the-art DSGE model.

Models that study the quantity of labor in the economy sometimes look at employment and sometimes at total hours worked. Although workers do not have to work for a wage that is lower than their reservation wage, the argument can be made that workers may have to work longer hours than desired if they want to keep their jobs. For this reason, we look not only at the standard New Keynesian models concerned with total hours (see Smets and Wouters (2007) or Christiano et al. (2005)), but also at those New Keynesian models that are explicitly concerned with understanding movements in employment (Galí et al. (2011)), where it is clear that workers should not work if they do not want to.

We discuss the implicit assumption made in New Keynesian models when there is trade at non-market-clearing prices in Section 2. We proceed to explore in Section 3 the extent to which agents work against their will—what we jocularly label as slavery—in standard New Keynesian models (versions of Altig et al. (2011) and Smets and Wouters (2007)) and conclude that it happens too often to simply ignore. Section 4 discusses what we think is the appropriate equilibrium concept, the Drèze equilibrium (Drèze (1975)), and compares its properties with those of the demand-determined allocation used in New Keynesian models and with those of an approximation to the Drèze equilibrium in various economies that we can solve. We then proceed to estimate a version of Christiano et al. (2005) using the approximated Drèze equilibrium and we show that we obtain quite different estimates than those obtained when using demand-determined allocations in Section 5. Section 6 concludes by arguing that the approximation to the Drèze equilibrium should be used in lieu of the demand-determined equilibrium when studying environments with sticky wages. Various Appendices complete the paper: Appendix A provides the details of the estimation of Altig et al. (2011); Appendix B adds the details of the specification of the Smets and Wouters (2007) model; Appendix C discusses our approach to deal with the wage markup shocks; Appendix D discusses the problems associated with solving for an equilibrium with occasionally binding constraints when the constraint is a function of the state rather than predetermined; Appendix E provides the details of the global approximation to the Drèze Equilibrium in a staggered wage economy.

2 The Labor Market in New Keynesian Models à la Calvo

We pose a typical New Keynesian model with sticky wages, first introduced by Erceg et al. (2000). There is a continuum of differentiated labor varieties n_i, $i \in [0,1]$, which firms combine into a final labor input n for production using a Dixit-Stiglitz aggregator with elasticity of substitution ϵ_w:

$$n = \left[\int n_i^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}. \tag{1}$$

our paper agents need to solve a more complicated dynamic pricing problem.

Wage w_i is set by unions that are specific to each labor variety i. Firms take all wages as given. Cost minimization, given wages and total employment n, yields a set of demand schedules for each labor variety i that is

$$n_i = \left(\frac{w_i}{w}\right)^{-\epsilon_w} n, \tag{2}$$

where w is an aggregate wage index $w = \left[\int w_i^{1-\epsilon_w} di\right]^{\frac{1}{1-\epsilon_w}}$ that satisfies $\int w_i \, n_i \, di = wn$.

A representative household consists of a continuum of workers, each one with different labor variety i that enjoys the same consumption level. The household's utility is given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \int_i v(n_{i,t}) di \right) \right\}. \tag{3}$$

The union sets the wage to maximize agents' utility. The opportunity to reset the wage occurs with probability $1 - \theta_w$ (à la Calvo) every period. The union's problem is

$$\max_{w_{i,t}^*} \quad E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[u'(c_{t+k}) \frac{w_{i,t}^*}{p_{t+k}} n_{i,t+k} - v(n_{i,t+k}) \right] \right\}, \qquad \text{subject to} \tag{4}$$

$$n_{i,t+k} = \left(\frac{w_{i,t}^*}{w_{t+k}}\right)^{-\epsilon_w} n_{t+k}. \tag{5}$$

The first-order condition is

$$E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \left[n_{i,t+k} u'(c_{t+k}) \left(\frac{w_{i,t}^*}{p_{t+k}} - \frac{\epsilon_w}{\epsilon_w - 1} \frac{v'(n_{i,t+k})}{u'(c_{t+k})} \right) \right] \right\} = 0. \tag{6}$$

Implicit behind these equations is the fact that firms can acquire any quantity that they want of all labor varieties, which implies that workers comply. Note that the worker is not choosing how much to work. If it did, it would choose ℓ_i to equate the real wage to the marginal rate of substitution (the standard intratemporal Euler condition):

$$\frac{w_{i,t}}{p_t} = \frac{v'(\ell_{i,t})}{u'(c_t)}. \tag{7}$$

We refer to the ℓ_i that solves equation (7) as the optimal labor supply under wage w_i.

In the absence of wage rigidity ($\theta_w = 0$), the union sets the wage every period and condition (6) becomes

$$\frac{w_{i,t}^*}{p_t} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{v'(n_{i,t})}{u'(c_t)}. \tag{8}$$

Marginal revenue equals the marginal rate of substitution, or using standard parlance, the real wage is set to equal the marginal rate of substitution multiplied by the wage markup $\frac{\epsilon_w}{\epsilon_w - 1}$. The standard values of the elasticity of substitution ensure that $\ell_i \geq n_i$, agents would like to work more than the quantity chosen by firms, and the determination of the equilibrium quantity of labor via the quantity demanded is justified.

Under wage stickiness, however, the wage set by equation (6) may imply an optimal supply of labor $\ell_{i,t} < n_{i,t}$. In this case, the assumption that labor is demand determined implies that workers are working against their will (i.e., slavery).

What is the correct notion of equilibrium in the context of a non-market-clearing price? Drèze (1975), following the disequilibrium models of Barro and Grossman (1971) and Malinvaud (1977), argued that it should be the minimum of supply and demand: trades should be voluntary. This is the notion that we follow in this paper.

But is there anything really inappropriate about posing a model where agents work more than desired? Labor varies because of both changes in hours per worker and changes in the number of workers. The argument could be made that workers may not be free to choose the number of hours that they work without losing their jobs, and therefore our notion that workers should not work against their will only applies to the extensive margin. In that case, it is only when dealing with the extensive margin that the argument that the correct equilibrium condition is the minimum of the quantity supplied and the quantity demanded is really strong.

A recent wave of New Keynesian models (Galí (2011) and Galí et al. (2011)) have incorporated unemployment by looking explicitly at changes in the extensive margin. In these models, households have a continuum of workers represented by the unit square and indexed by a pair $(i, j) \in [0, 1] \times [0, 1]$. The i-dimension represents the type of labor service, while the j-dimension determines the worker's disutility from work, which equals j^γ if it is employed and zero if unemployed or outside the labor force. The household's utility is now given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \int_i \int_0^{n_{i,t}} j^\gamma \, dj \, di \right) = E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \int_i \frac{n_{i,t}^{1+\gamma}}{1+\gamma} di \right). \tag{9}$$

An individual worker (i, j) takes the household's consumption level and the labor market conditions as given and will find it optimal to participate in the labor market if and only if

$$u'_{ct} \frac{w_{i,t}}{p_t} \geq j^\gamma. \tag{10}$$

Hence, the measure of workers in sector i who want to work is ℓ_i, which solves[5]

$$u'_{ct}\,\frac{w_{i,t}}{p_t} = \ell_{i,t}^{\gamma}. \tag{11}$$

We could (as Galí (2011) and Galí et al. (2011) do) define the unemployment rate as $u_t = \ell_t - n_t$. Moreover, in the absence of wage rigidities or in a steady state, the natural rate of unemployment rate u^n and the union's market power are linked by

$$\frac{1}{\epsilon_w - 1} \approx \gamma u^n. \tag{12}$$

In these models, labor supply is defined by the number of agents willing to work. When labor demand exceeds labor supply (i.e., when $n_i > \ell_i$), some agents are required to work against their will (hence our use of the term *slavery*). More dramatically, if labor demand exceeds the total population, $n_i > 1$, firms would be hiring workers that do not exist. It is in this type of model where our argument that the appropriate equilibrium is the Drèze equilibrium is the strongest.

3 Are Agents Working Against Their Will in Popular New Keynesian Models?

We now turn to the quantitative exploration of the extent to which agents work against their will by comparing the properties of employment in our versions of the standard Altig et al. (2011) and Smets and Wouters (2007) environments with the level of employment in those economies that would be the minimum of supply and demand. Altig et al. (2011) augment Christiano et al. (2005) with neutral and embodied technology shocks, while Smets and Wouters (2007) also include preferences shocks, markup shocks and government spending shocks. We use both of these models because they are de facto the standard New Keynesian models (more details about these models are in Appendices A and B). In these two models, labor is interpreted as hours worked and it could be argued that workers are implicitly obliged to work some periods more than they wish. For this reason, we also provide a discussion in Appendix C of the Galí et al. (2011) model where a unit of labor has the meaning of a worker. As we show there, the quantitative findings are very much in line with the findings of this Section.

To find out the extent to which agents are working against their will, we start solving and simulating the models as in the New Keynesian literature by assuming that labor is determined only by the demand. With the simulated history of aggregate wages and the aggregate labor demand, we construct the cross-sectional desired labor supply and labor demand for different labor varieties. We then construct our notion of voluntary ex-post employment by computing the minimum of the quantity of labor demanded and the quantity that agents would like to work for each wage/cohort and then adding them up across cohorts every period. The larger the difference between the demand-determined labor and the voluntary ex-post

[5]Note that when the labor disutility function is $v(n) = \frac{n^{1+\gamma}}{1+\gamma}$, then equation (11) coincides with equation (7).

labor the more how severe the "slavery" problem is. Still, the voluntary ex-post labor is not the labor in a Drèze equilibrium because the latter requires that agents are aware of the equilibrium condition, and also of the implied adjustments in all the other model variables. However, this short-cut is useful in detecting whether we need to worry about this issue at all.

We discuss the details of how to construct the voluntary ex-post employment in Section 3.1. This is not a trivial issue, because at any point in time, economies with wage rigidities have a large number of different wages, each one of them affecting a different group of workers who have different preferred labor choices. The quantitative analysis is in Section 3.2.

3.1 The Determination of the Voluntary ex-post Employment

To determine the desired labor supply of workers we have to keep track, not only of the aggregate wage index of the economy, but also of the wages for all labor varieties i. Fortunately, this can be done by noting that all labor varieties that set the wage in a given period choose the same wage. We describe our procedure in three steps.

Step 1: Construct the cross-sectional wage distribution The measure of workers that can reset their wages in the current period is $\mu_0 = 1 - \theta_w$, while the measure of workers with wage reset τ periods before is $\mu_\tau = (1-\theta_w)\theta_w^\tau, \quad \tau = 0, 1, 2, \ldots$, which for τ large enough, μ_τ becomes negligible.

The simulation of the log-linearized model with demand-determined labor yields the sequence of the aggregate wage index $\{w_t\}$, which evolves according to

$$w_t = \left[\int w_{i,t}^{1-\epsilon_w} di\right]^{\frac{1}{1-\epsilon_w}} = \left[\theta_w (w_{t-1})^{1-\epsilon_w} + (1-\theta_w)(w_t^*)^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}, \tag{13}$$

where w_t^* is the newly set wage in period t. Since we already have the aggregate wage sequence $\{w_k\}_{k=0}^t$, we can easily calculate the sequence of newly set wages $\{w_k^*\}_{k=0}^t$ using Equation (13). The wages prevailing in period t are then $\{w_{t-\tau}^*\}$, with corresponding measure $\mu_\tau, \tau = 0, 1, 2, \ldots$.

Step 2: Construct cross-sectional labor Demand and labor Supply Given aggregate employment $\{n_t\}$, the labor demand for workers with wage rate $w_{t-\tau}^*$ is

$$n_{\tau,t} = \left(\frac{w_{t-\tau}^*}{w_t}\right)^{-\epsilon_w} n_t. \tag{14}$$

For workers with wage rate $w_{t-\tau}^*$, the optimal choice of labor is given by the $\ell_{\tau,t}$ that solves

$$\frac{w_{t-\tau}^*}{p_t} = \frac{v'(\ell_{\tau,t})}{u'(c_t)}. \tag{15}$$

Aggregating both series over cohorts or wage groups, we obtain the aggregate demand for labor[6], $n_t = \left[\sum_{\tau=0}^{\infty} \mu_\tau, n_{\tau,t}^{\frac{\epsilon_w - 1}{\epsilon_w}}\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$, and the aggregate supply of labor $\ell_t = \left[\sum_{\tau=0}^{\infty} \mu_\tau, \ell_{\tau,t}^{\frac{\epsilon_w - 1}{\epsilon_w}}\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$.

Step 3: Construct aggregate employment Voluntary ex-post labor, e_t^p (we use the superscript p to denote that it is an ex-post quantity), is the minimum of supply and demand at each wage,

$$e_t^p = \left[\sum_{\tau=0}^{\infty} \mu_\tau \left(\min\{n_{\tau,t}, \ell_{\tau,t}\}\right)^{\frac{\epsilon_w - 1}{\epsilon_w}}\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \qquad (16)$$

We want to emphasize that e_t^p is not an equilibrium object, both because when making decisions, neither firms nor unions or workers take this factor into consideration, and because the implied path of consumption, investment, and capital is that associated with the demand-determined allocation. However, it allows us to check whether the labor supply constraint is an issue or not. If $n_{\tau,t} < \ell_{\tau,t}$ all the time, then $n_t = e_t^p$ and it is correct to use demand-determined labor. If instead, $n_{\tau,t} > \ell_{\tau,t}$ happens frequently and the difference between $n_{\tau,t}$ and $\ell_{\tau,t}$ is large, then n_t will be substantially different from e_t^p and the answers obtained by models that use demand-determined quantities of labor are questionable.

3.2 Analysis of the Altig et al. (2011) and Smets and Wouters (2007) Models

The Altig et al. (2011) and Smets and Wouters (2007) models lack a straight identification of the steady-state markup. Altig et al. (2011) sets the wage markup to be 5%, and Smets and Wouters (2007) sets its value to 50%, which implies an elasticity of substitution $\epsilon_w = 3$, a much larger value than both the empirical estimates and the number used in the New Keynesian literature.[7]

We reestimate both of the two models, setting the mean wage markup to values more in accordance with the recent literature, ranging from 5% to 15%. Figure 1 displays sample paths of the aggregate employment obtained by demand-determined labor (n_t) and by the voluntary ex-post labor e_t^p constructed in the way discussed in Section 3.1 for different wage markups. Note that demand-determined labor is always greater than the voluntary ex-post aggregate employment by construction. The difference between these two series is noticeable. Both series coincide in recessions, but the voluntary ex-post labor does not expand as much as the demand-determined labor in expansions. In fact, for a 5% wage markup, the voluntary ex-post labor actually declines when the demand-determined labor is in a boom, because the original boom takes place by asking the low-paid workers to supply a huge amount of labor which is no longer possible if workers can choose not to meet the demand. Also, the smaller the wage markup, the

[6]Under log-linearization, an approximation error results in a negligible difference between aggregate employment and this expression.

[7]Lewis (1986) surveys the literature on the wage premium for workers in a union, which corresponds to the wage markup in the model, and the value is between 10% to 20%. The mean wage markup is 5% in Christiano et al. (2005), 15% in Chari et al. (2002), and 20% in Levin et al. (2006).

larger the differences between these two series as the average distance between labor demand and labor supply shrinks.

Figure 1: Demand-Determined and Voluntary Ex-Post Labor in Altig et al. (2011)

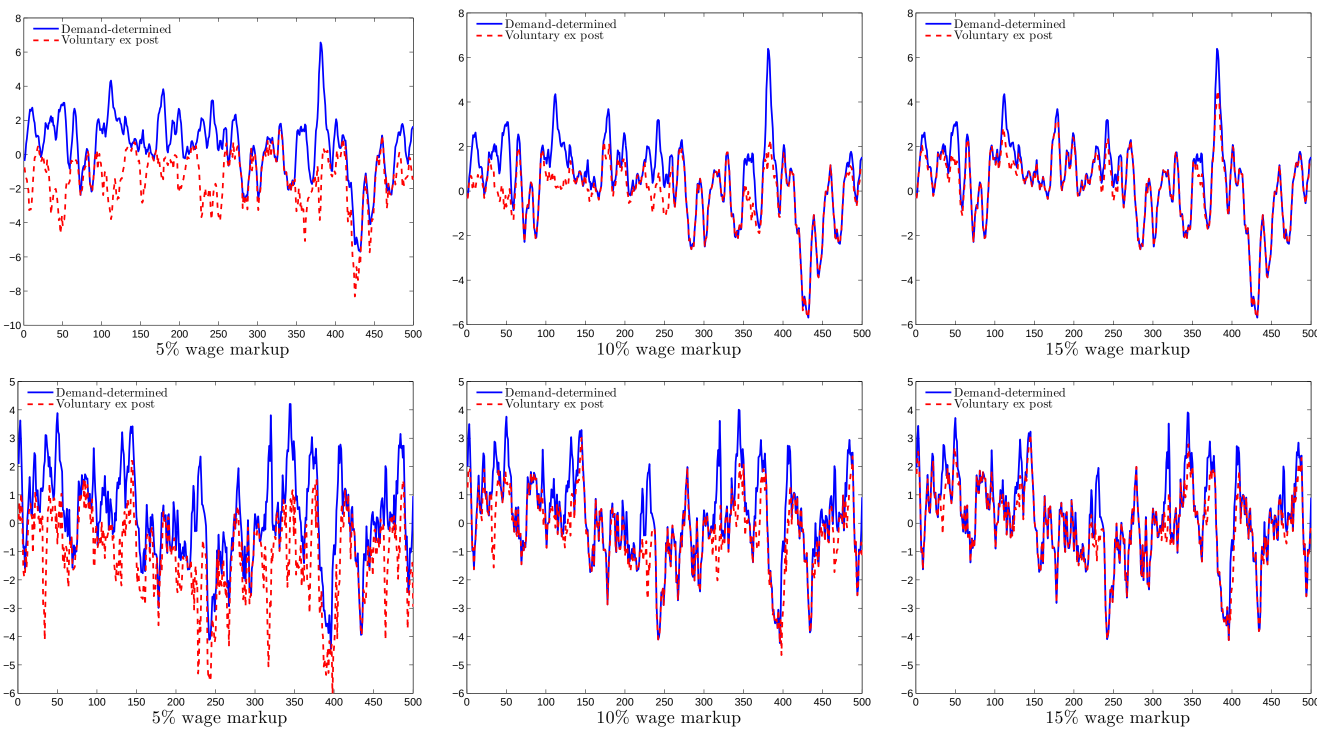


Table 1 summarizes the relevant statistics to compare both labor series. Qualitatively, the statistics for Altig et al. (2011) are similar to those in Smets and Wouters (2007). Quantitatively, the degree of violation of the labor supply constraint is more severe in Smets and Wouters (2007) about twice as often. In both models, the average voluntary ex-post labor is more than 1% lower than demand-determined labor in the economy with a 5% wage markup, and about 0.2% with a 15% wage markup. Column 4 of Table 1 lists the average excess of labor demand over labor supply (the extent of slavery). In Altig et al. (2011), for a 5% wage markup case, about 19% of the time, an agent is working more than what she would like (2% for a 15% wage markup). In Smets and Wouters (2007), this fraction is even larger, which ranges from 32% (5% wage markup) to 6% (15% wage markup). As one could have conjectured, the lower the markup, the larger the difference between the average labor of both series and the larger the fraction of labor performed against workers' will. We consider these differences to be large.

As a result of these differences, the business cycle statistics are also quite different. Table 1 displays the variance and correlation with demand-determined output of the two labor series. In most cases, the

Table 1: Demand-Determined and Voluntary ex-post Aggregate Labor

Altig et al. (2011) Model

Wage markup: 5%	Mean %	Var	Corr(N,Y)	Binding Freq. %
Demand determined	—	1.38	0.96	18.83
Voluntary ex-post	-1.41	0.97	0.42	—
Wage markup: 10%	**Mean%**	**Var**	**Corr(N,Y)**	**Binding Freq. %**
Demand determined	—	1.35	0.96	5.58
Voluntary ex-post	-0.45	0.94	0.84	—
Wage markup: 15%	**Mean %**	**Var**	**Corr(N,Y)**	**Binding Freq. %**
Demand determined	—	1.35	0.96	2.28
Voluntary ex-post	-0.15	1.18	0.93	—

Smets and Wouters (2007) Model

Wage markup: 5%	Mean %	Var	Corr(N,Y)	Binding Freq. %
Demand determined	—	1.03	0.80	31.57
Voluntary ex-post	-1.29	1.05	0.28	—
Wage markup: 10%	**Mean %**	**Var**	**Corr(N,Y)**	**Binding Freq. %**
Demand determined	—	0.99	0.79	12.88
Voluntary ex-post	-0.50	0.67	0.54	—
Wage markup: 15%	**Mean %**	**Var**	**Corr(N,Y)**	**Binding Freq. %**
Demand determined	—	0.97	0.79	5.73
Voluntary ex-post	-0.23	0.71	0.68	—

Notes: Numbers are in percentages except for the correlation. All the variables are logged and HP filtered. In the simulation, we include all the shocks except for the wage markup shock.

variance of voluntary ex-post labor is about 20% to 30% smaller than that of demand-determined labor. The voluntary ex-post aggregate labor mutes some booms, which explains why its variance is smaller. However, the relation between the frequency of binding of the labor supply constraint in the two series is not monotonic. The reason is that for a small wage markup, voluntary ex-post labor sometimes shrinks to the point of moving in the opposite direction, and hence what is an expansion under demand-determined labor is a recession in terms of voluntary ex-post labor (note the much lower correlation with output).

As we have seen demand-determined labor economies have very different properties than voluntary ex-post economies. Typically, labor is much more volatile and its correlation with output is much higher. The differences are usually, but not always, larger for small labor markups.

4 Drèze Equilibrium in a Staggered Wage Model

So far, we have made the case that the use of demand-determined labor as the equilibrium condition is inappropriate because households want to work less quite often: the minimum of the amount of labor demanded and supplied (as standard theory considers the appropriate equilibrium condition) behaves very differently from the amount of labor demanded. However, the series that we have constructed (voluntary ex-post labor) is not an equilibrium because it is constructed along a path defined by the demand-determined labor and its associated series: output, consumption, investment, prices, wages, and so on. Moreover, the forecasts of agents are those of the demand-determined allocation. We need to compute the Drèze equilibrium explicitly.

Unfortunately, log-linearization cannot be used to solve for the Drèze equilibrium. Global methods are needed given that the equilibrium condition is based on the min operator. Recent developments in computational economics that allow us to deal effectively with corner solutions, e.g. Guerrieri and Iacoviello (2015), cannot be applied either: these methods require the corners or temporarily binding constraints to be predetermined (like the zero bound of nominal interest rates or the lower and upper bound of hours worked), whereas in our economies, the min operator applies to two endogenous variables. We discuss this issue in detail in Appendix D. Moreover, the number of state variables is effectively infinite because the whole set of existing wages is part of the state vector (even if truncating this we would still need many state variables). Global methods can only be used with a limited number of variables, which presents a problem.

Our strategy here is to explore the properties of the Drèze equilibrium in an economy that we can solve with global methods (a simplified version of the Altig et al. (2011) economy with staggered wages à la Taylor), and to compare its solution with a suitable simple approximation (effectively one where we impose feasibility, maintain properties of the log-linearized solution but forgo the rationality of agents' expectations). We claim that the global solution and our approximation are close, and hence we argue that we can use the approximated solution to the Drèze equilibrium in New Keynesian macro models.

We first describe the simple model with staggered wage contracts (Section 4.1) and then describe an approximation to its solution that uses as a basis a log-linear approximation to the demand-determined equilibrium of the same economy (Section 4.2). We compare the quantitative properties of both objects in Section 4.3.

4.1 The Drèze Equilibrium

Consider an infinitely lived stochastic growth monetary economy. A representative household consists of a continuum of workers, each one with different labor variety i, that enjoy the same consumption level. The household's utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \phi \int_i \frac{e_{i,t}^{1+\gamma}}{1+\gamma} \, di \right), \tag{17}$$

where e_i is the labor of variety i. Households take prices and firms' profits as given, and their budget constraint is

$$p_t \left[c_t + k_{t+1} - (1-\delta) k_t \right] + \frac{1}{R_t} b_{t+1} = r_t^k k_t + \int_i w_{i,t} e_{i,t} di + b_t + \Pi_t. \tag{18}$$

Firms are competitive with Cobb-Douglas production technology $y_t = z_t k_t^{\alpha} e_t^{1-\alpha}$, where e_t is the final labor used in production aggregated via a Dixit-Stiglitz technology

$$e_t = \left[\int e_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} \, di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{19}$$

and total factor productivity (TFP) follows an AR(1) process

$$\log z_t = \rho_z \log z_{t-1} + \zeta_t^z, \quad \zeta_t^z \sim \mathbb{N}(0, \sigma_z^2).$$

Firms take the price of the final good, the capital rental rate, and the wage rate as given and solve

$$\max_{k_t, e_t, e_{i,t}} \; p_t z_t k_t^{\alpha} e_t^{1-\alpha} - r_t^k k_t \; - \; \int w_{i,t} \, e_{i,t} \, di \qquad \text{subject to} \tag{20}$$

$$e_t = \left[\int e_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} \, di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{21}$$

$$e_{i,t} \leq \Phi(w_{i,t}), \tag{22}$$

where $\ell_{it} = \Phi(w_{i,t})$ is the maximum amount of labor the firm can obtain of labor of variety i under the wage rate $w_{i,t}$ (what we referred to as ℓ_i earlier). In standard New Keynesian models, this last constraint is absent.[8] The solution to the firms' problems satisfies

$$\frac{r_t^k}{p_t} = \alpha \; z_t \; k_t^{\alpha-1} \; e_t^{1-\alpha}, \tag{23}$$

$$e_{i,t} = \min \left\{ \left[\frac{w_{i,t}}{(1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha} p_t} \right]^{-\epsilon_w} e_t, \; \Phi(w_{i,t}) \right\}, \tag{24}$$

$$e_t = \left[\int e_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} \, di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}. \tag{25}$$

[8] Constraint (22) implicitly assumes that all firms internalize are treated equally when facing limited labor supply. We can see this as the result of assuming that firms send bids and that the available workers are equally distributed between all firms. That firms understand this is consistent with the model. An alternative that would add a lot of complexity without any substance is to pose a randomization mechanism.

It is convenient to denote by $n_{it} = \Psi(w_{i,t})$ the desired labor demand in the absence of the quantity constraint. Functions Ψ and Φ depend on the aggregate state, but to simplify the exposition we write Ψ_t and Φ_t. We have

$$\Psi_t(w_{i,t}) = \left[\frac{w_{i,t}}{(1-\alpha)z_t k_t^{\alpha} e_t^{-\alpha} p_t}\right]^{-\epsilon_w} e_t. \tag{26}$$

In this economy, there is a continuum of labor unions, each setting the wage of the type of labor that they represent, that maximize households' welfare given the behavior of all other parts of the economy. Workers cannot be made to work against their will, and the union takes into account that there is an upper bound on the amount of labor that will be provided in their sector. The union chooses a nominal wage that will be effective for T^w periods:

$$\max_{w_t^*} \ \mathbb{E}_t \left\{ \sum_{k=0}^{T^w-1} \beta^k u'(c_{t+k}) \frac{w_t^*}{p_{t+k}} e_{i,t+k} - \frac{e_{i,t+k}^{1+\gamma}}{1+\gamma} \right\} \tag{27}$$

subject to

$$e_{i,t+k} = \min \left\{ \left(\frac{u'(c_{t+k})}{\phi} \frac{w_t^*}{p_{t+k}} \right)^{\frac{1}{\gamma}}, \Psi_{t+k}(w_t^*) \right\}, \tag{28}$$

where $\Psi_{t+k}(\cdot)$ is the desired labor demand from the firm's side and $\Phi(w_{i,t})$ is given by

$$\Phi(w_{i,t}) = \left(\frac{u'(c_{t+k})}{\phi} \frac{w_{i,t}}{p_t} \right)^{\frac{1}{\gamma}}. \tag{29}$$

In the standard model, the constraint for the union is simply

$$e_{i,t+k} = \Psi_{t+k}\left(w_t^*\right). \tag{30}$$

Note the three objects that we have defined: labor supply in variety i,

$$\ell_{i,t} = \Phi(w_{i,t}) = \left(\frac{u'(c_{t+k}) w_{i,t}}{\phi p_t} \right)^{\frac{1}{\gamma}}, \tag{31}$$

labor demand in variety i,

$$n_{i,t} = \Psi(w_{i,t}) = \left[\frac{w_{i,t}}{(1-\alpha)z_t k_t^{\alpha} e_t^{-\alpha} p_t}\right]^{-\epsilon_w} e_t, \tag{32}$$

and employment in variety i,

$$e_{i,t} = \min\{\ell_{i,t}, n_{i,t}\}. \tag{33}$$

To complete the model, we include a simple Taylor type monetary policy rule:

$$\log R_t = \log \frac{1}{\beta} + \phi_\pi \pi_t + \phi_y \log \frac{y_t}{y^*} + \eta_t, \tag{34}$$

where $\pi_t = \log \frac{p_t}{p_{t-1}}$ and y^* is the steady-state output level. The shock to the monetary policy rule follows an AR(1) process,

$$\eta_t = \rho_m \eta_{t-1} + \zeta_t^m, \quad \zeta_t^m \sim \mathbb{N}(0, \sigma_m^2). \tag{35}$$

The details of the numerical solution via global methods of this economy can be found in Appendix E.

4.2 The Approximated Drèze Equilibrium

Even in the simplified staggered wage model, computing the exact Drèze equilibrium is computationally intense. We therefore consider an approximation to the Drèze equilibrium which does not require the global solution. Our approximation of the Drèze equilibrium consists of four logical steps.

Step 1: Log-linearize and solve the demand-determined equilibrium This is a standard step. The decision rules are required, not just a simulation.

Step 2: Recursively construct a voluntary ex-post measure of labor This step is what we described in Section 3.1. The key difference is that there we use the sequence of capital stocks yielded by the demand-determined equilibrium, which may not be feasible. Thus, at this stage we construct a measure of the voluntary employment one period at a time, denoted as e_t^a. In this step we keep track of historical wages, w_t^a, which also include the information about the cross-sectional wage distribution.

Step 3: Recursively construct the main aggregate variables Here we use the employment in period t, e_t^a, and the previous period series of capital k_t^a to calculate output y_t^a (which is also used to construct the output gap). We then use the same policy function as in the demand-determined equilibrium to determine the newly set wage and price level. This is an approximation, since in the true Drèze economy, agents will take into account the possibility that the labor supply constraint may be binding. The interest rate R_t^a is set by using the reconstructed output gap. This part is mechanical.

Step 4: Determine consumption, investment, and next period capital This step is not mechanical. We have considered two possibilities: use the same consumption-to-output ratio or the same consumption of the demand-determined solution (investment is set residually to satisfy the resource constraint). We finally chose the same consumption because choosing the consumption-to-output ratio sometimes leads to countercyclical consumption. More specifically, in the demand-determined economy, after a positive technology shock, the consumption-output ratio is below its steady-state level because agents understand that it is better to increase investment to take advantage of the temporary high productivity. In the Drèze equilibrium, however, the response of labor is much more subdued with the same positive technology shock, which may lead to a much smaller expansion. If we used the low consumption-to-output ratio of the demand-determined allocation, there would be a recession rather than an expansion.

We do not want to argue that our approximation strategy is conceptually ideal, and we are aware that the allocation obtained in this approximation is subject to the fact that agents are not fully rational. The usefulness of this approximated equilibrium is simply justified by its small distance to the true Drèze equilibrium as we will show next.

4.3 A Comparison between the Drèze Equilibrium and Its Approximation

We now specify the staggered wage model quantitatively and solve for the Drèze equilibrium and for its approximation. The model has a large number of state variables to keep track of the wage distribution (see Appendix E for more details). The model period is a quarter and the annual interest rate in the steady state is 4%. The implied Frisch elasticity is 0.75 ($\frac{1}{\gamma}$), similar to estimates in Heathcote et al. (2010). The labor share is 0.64, and the capital depreciation rate is 0.08 annually. The process for the TFP shock is similar to the one used in Ríos-Rull and Santaeulàlia-Llopis (2010). The monetary policy rule is the same as in Christiano et al. (2011). The persistence of the monetary shock is 0.5, the same as in Galí (2008). We set the standard deviation of the innovation to the monetary shock to be 0.004. As discussed earlier, the most important parameter is ϵ_w, which determines the wage markup. The one we use here implies a 10% wage markup. If we apply the logic of equation (12), our choice of ϵ_w and γ leads to a 6% average unemployment rate[9] (the parameterization is summarized in Table 2). We choose the duration of the wage contract to be four model periods, or one year.

Table 2: Baseline Parameters

β	σ	γ	ϕ_π	ϕ_y	α	δ	ϵ_w	ρ_z	σ_z	ρ_m	σ_m
0.99	1.0	1.5	1.5	0.0	0.36	0.02	11.0	0.95	0.006	0.50	0.004

We report in Table 3 the properties of employment in the simple economy using one shock at a time. We compare the statistics generated by the Drèze equilibrium and by its approximation. For further comparability, we also include those of the voluntary ex-post employment and the demand-determined solution.

The global solution and the approximated Drèze equilibrium are quite similar. This similarity can be visualized by Figures 2 and 3. As shown in Table 3, these two series share similar volatility and cyclicality.

[9] Following Galí et al. (2011), the unemployment rate in sector i (the economy-wide counterpart is immediate) is $u_{i,t} = \log \ell_{i,t} - \log e_{i,t}$.

The demand-determined allocation, however, is much more volatile than the others. More than 10% of agents work against their will in the demand-determined economy. Table 4 displays the business cycle statistics that show the same patterns for the other main macro variables. The main feature is that allocation in the demand-determined economy is much higher than that in the Drèze equilibrium, and the performance of our approximated Drèze equilibrium is fairly close to its true solution.

Figure 2: Employment in the Staggered Wage Model with TFP Shocks

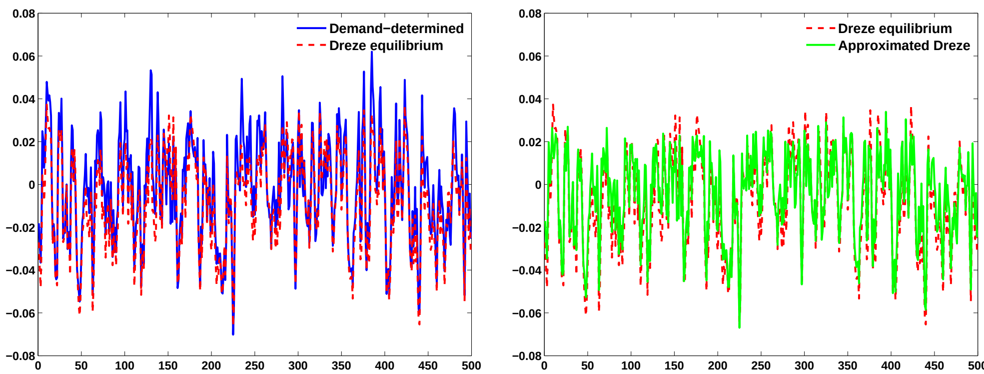
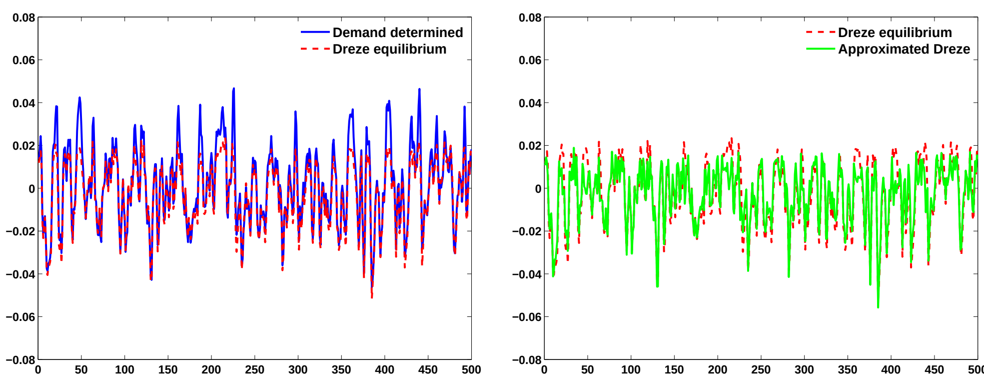


Figure 3: Employment in the Staggered Wage Model with Monetary Shocks



We conclude that the approximation to the Drèze equilibrium built via log-linearization of the demand-determined solution and the recursive imposition of the minimum of the amount of labor supplied and demanded is a good approximation to a global solution of the Drèze equilibrium where the condition that labor is the minimum of the amount supplied and demanded is imposed ex-ante.

Table 3: Properties of Employment in the Staggered Wage Model for Various Solutions

TFP Shock				
Solution Method	Mean	Var	Corr(N,Y)	Binding Freq.
Drèze equilibrium (global solution)	-0.10	2.67	0.93	—
Approximated Drèze equilibrium	-0.29	2.53	0.94	—
Voluntary ex-post employment	-0.33	2.60	0.88	—
Demand-determined employment	—	3.79	0.96	0.11
Monetary Shock				
Solution Method	Mean	Var	Corr(N,Y)	Binding Freq.
Drèze Equilibrium (global solution)	-0.52	1.60	0.94	—
Approximated Drèze equilibrium	-0.41	1.29	0.99	—
Voluntary ex-post employment	-0.45	1.38	0.77	—
Demand-determined employment	—	2.26	1.00	0.12

Notes: The numbers for variances are in percentages. All the variables are logged and HP filtered.

5 An application of the Drèze Equilibrium: the Estimation of the Altig et al. (2011) Economy

So far, we have argued that in New Keynesian models with sticky wages, the use of demand-determined labor yields allocations that are very different from those that the same parameterized model yields when labor is determined by the Drèze equilibrium where labor is the minimum of the amount supplied and the amount demanded. But this is not what really matters; perhaps different values of parameters yield similar properties between the two ways of determining the quantity of labor, and hence the answers that we obtain are the same. To settle this issue, we have to estimate the models under both types of labor determination.

The estimation of Smets and Wouters (2007) uses modern Bayesian methods that rely on the linearity of the model. Although demand-determined models are not linear, they are very well approximated by log-linear approximations and hence are extremely well suited for Bayesian or maximum likelihood estimation. The combination of the linearity and the Gaussian shock structure permits a relatively easy mapping from model parameters to its implied likelihood. The key feature of the Drèze equilibrium is its nonlinear nature, which unfortunately prevents us from applying the standard linear Kalman filter technique in evaluating the model's likelihood. The alternative nonlinear Kalman filter requires large computational power, which is only feasible for models with a relatively small number of state variables.

Table 4: Business Cycle Statistics in the Staggered Wage Model with Various Solutions

	TFP Shock			Monetary Policy Shock		
	Drèze Equilibrium	Approximated Drèze Equil.	Demand Determined	Drèze Equilibrium	Approximated Drèze Equil.	Demand Determined
		Variance			*Variance*	
Output	3.12	2.95	3.90	0.65	0.52	0.92
Employment	2.67	2.53	3.79	1.60	1.29	2.26
Consumption	0.17	0.17	0.19	0.01	0.01	0.02
Investment	42.82	38.10	52.11	10.12	7.96	13.95
		Correlation with output			*Correlation with output*	
Employment	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.61	0.57	0.62	0.61	0.57	0.62
Investment	1.00	0.99	1.00	1.00	0.99	1.00

Notes: The numbers for variances are in percentages. All the variables are logged and HP filtered.

We can, however, estimate the approximated Drèze equilibrium in Altig et al. (2011), the other central model in the New Keynesian literature. Altig et al. (2011) and its precedent Christiano et al. (2005) estimate a medium-scale DSGE model by matching the impulse responses of various variables to different shocks. The impulse responses are recovered from the estimation of a certain structural vector autoregression (VAR) model. The parameters of the model are chosen in such a way that the model's impulse responses to the structural shocks match their counterpart estimated from the data. In particular, three structural shocks are considered: a monetary shock, a neutral technology shock, and an embodied investment technology shock. The estimation method is generalized method of moments (GMM), which only requires the impulse response of the model.[10] Because the likelihood of the model is not required, we can apply this estimation method to the approximated Drèze equilibrium. In the baseline estimation, we keep the exogenously calibrated parameters and the estimation procedure the same as Altig et al. (2011), but use our approximated Drèze equilibrium instead. We discuss the robustness our results in the next subsection.

Table 5 shows the properties of the estimates of the approximated Drèze equilibrium and of the demand-determined allocation in the Altig et al. (2011) model. Our interpretation of these very different sets of estimates is that the unwillingness of households in the Drèze equilibrium to work a lot under some circumstances requires that other pieces of the model have to do a lot more work to create the observed fluctuations. This is accomplished in a particular way, according to the estimates:

[10] The weighting matrix of GMM is diagonal with the inverse of the standard deviations of the impulse responses estimated in the structural VAR.

1. The neutral technology shock is dramatically affected. Without any change, the employment tends to move less in the approximated Drèze equilibrium. To induce more movement in employment, the estimated shock is now both much more volatile and less persistent: the overall variance of the neutral technology shock is 0.039 in the Drèze equilibrium relative to 0.024 in the demand-determined allocation. A larger shock induces more circumstances to which households respond. A less persistent shock makes households more engaged in responding to the innovation of the shock, since households are less likely to have the same opportunities in the future.

2. The rigidity of wages and prices is larger. The lower response of employment in the Drèze equilibrium also requires, perhaps a bit counterintuitively, larger rigidities in the model to generate more fluctuations. This is true both for wages, where the Drèze equilibrium is imposed, and for prices, where it is not.

3. Two other pieces of the model are now larger. Variable capital utilization has a larger value, as does the investment adjustment cost parameter. Still, these two parameters are somewhat imprecisely estimated and we should not insist on them.

4. The effects of the shocks on monetary policy are adjusted. This is a minor, technical, and relatively unimportant change.

Table 6 shows what the different solutions yield for each of set of estimates obtained. The left panel of the table shows the effects of the processes estimated via the demand-determined solution on employment when we look both at the demand-determined solution and at the approximated Drèze equilibrium. The right panel shows the effects of the processes estimated with the approximated Drèze equilibrium when we both look at the demand determined solution and at the approximated Drèze equilibrium. The numbers in boldface are the properties of the economies when they are used to estimate the parameters. This table shows some other important features of the differences between the demand-determined solution and the approximated Drèze equilibrium:

5. That the estimates of the approximated Drèze equilibrium increase the role of the neutral technology shock can be seen in the higher variance of employment that both solutions display with these estimates relative to the demand-determined estimates. The contribution of the neutral technology shock to the variance of employment goes from 13% to 71%, that of the investment or embodied technology shock goes from 49% to 16%, and that of the monetary shocks goes from 33% to 15%.[11]

[11] As the quick-witted reader may have noticed, the contributions of the orthogonal shocks to the variance of employment add up to slightly above 100%. The reason for this is the nonlinear nature of the model. Fortunately for our analysis, the differences are quite small, and the contribution of each individual shock when the others are shut out gives a good picture of their overall contribution.

6. Under both sets of estimates, the variance of employment is much larger in the approximated Drèze equilibrium. The unwillingness of households to work under many circumstances generates recessions that are not present in the demand-determined solution.

7. The contribution of the other shocks is smaller for the Drèze equilibrium estimates, especially for the demand-determined solution. This is the result both of the different estimates for the processes of the shocks and of the values of other parameters estimated, such as the investment adjustment cost.

5.1 Robustness of the Findings

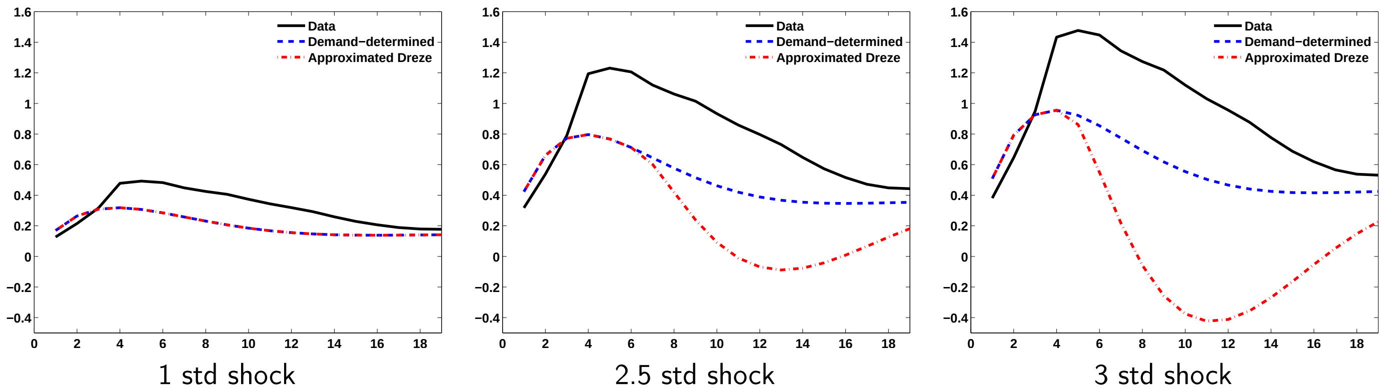
We now turn to two considerations about the robustness of our findings: the extent to which the nonlinearity of the Drèze equilibrium solution makes the estimates depend on how to specify the impulse response function (the size of shocks) and the specification of what wage markup to use (because the larger the markup, the larger the model's buffer to accommodate shocks without households becoming unwilling to work).

An important difference between the demand-determined solution and the Drèze equilibrium is that the latter is nonlinear with respect to the shocks. In the demand-determined solution, when the size of the shocks increases, the impulse response increases proportionally. As a result, when matching the empirical impulse response functions, setting the size of the shock to any value leads to the same estimates. In contrast, in the Drèze equilibrium (and in its approximation), the impulse response function is not proportional to the size of the shock. Moreover, even the sign of the impulse response can be reversed as the size of the shock increases. The intuition is simple. If the size of the shock is small, labor demand is typically smaller than the labor supply because of the wage markup, and the economy behaves just like the demand-determined one. As the shock becomes larger, labor demand can be larger than labor supply, and the labor supply constraint starts to be binding.

In our baseline estimation, we use 5% wage markup (the level used in Altig et al. (2011)) and a one standard deviation shock. This parameterization has already induces the labor supply constraint to be binding. It may be that for larger wage markups, a one standard deviation shock may not be able to trigger the labor supply constraint. To explore this issue, we show in Figure 4 the impulse responses to different sizes of the neutral technology shock where the wage markup is 10%, and the estimates are from the demand-determined solution. We can see clearly that as the size of the shock increases, the impulse response of the demand-determined solution adjusts proportionally, whereas the Drèze equilibrium implies a much smaller expansion than the demand-determined solution and may even turn the expansion into a recession within a couple of years.

If we estimate the approximated Drèze model using the one standard deviation shock, we would not be able

Figure 4: IRF of Labor to Neutral Tech. Shock: Estimation with Demand-Determined Model, 10% Markup



to capture this nonlinearity, and hence we would fail to match the actual response of the economy when the size of the shock is relatively large. Instead, when we estimate the approximated Drèze equilibrium, we use a shock that is three times standard deviations. Figure 5 shows the impulse response of the approximated Drèze equilibrium estimates with different shocks. Now the estimates have changed and with them the shape of the impulse responses. We see how the impulse response of the approximated Drèze equilibrium changes shape in its attempt to trace that of the data. The dive into recession two years after a shock of three standard deviations from the demand-determined estimates is now delayed and reduced by the different neutral technology shock, one that is larger and more temporary.

Figure 5: IRF of Labor to Neutral Tech. Shock: Estimation with Approximated Drèze Equilibrium, 10% Markup

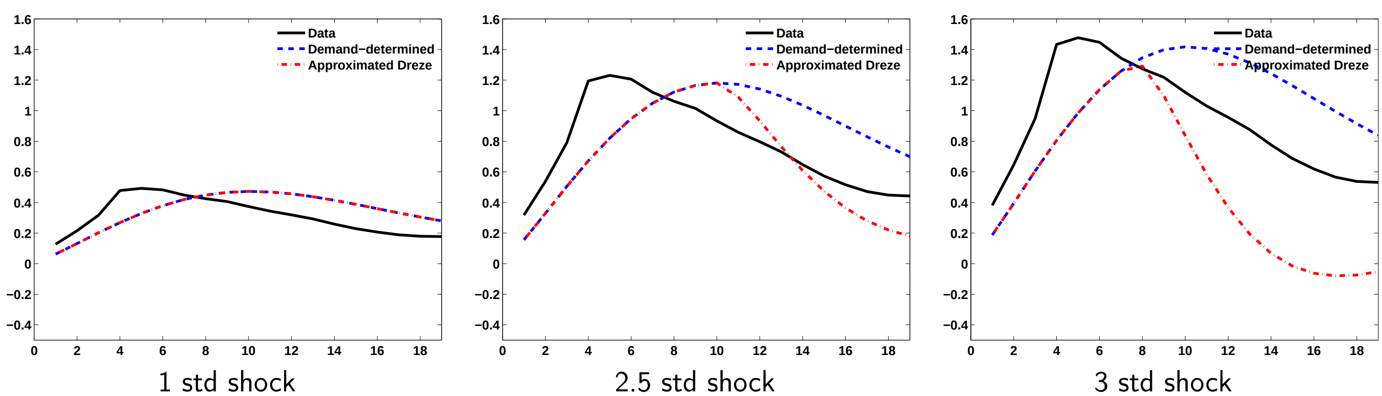


Table 7 shows the estimates for models with different wage markups and different sizes of the shock. We see again a high standard deviation and a low autocorrelation of the neutral technology shock as well as a larger wage and price rigidity.

In terms of the role of various shocks, we also find that the neutral technology shock becomes more important in accounting for the employment volatility. As shown in Table 8, with a 7% wage markup the contribution of neutral technology shock increases from 13% to 50% when using the approximated Drèze equilibrium. Table 9 displays the results with a 10% wage markup. Again, the contribution of the neutral technology shock is much larger, increasing this time from 13% to 37%.

We conclude that using larger markups and larger shocks to specify which impulse responses the model attempts to replicate generates patterns similar to those in our baseline specification of the Altig et al. (2011) model.

6 Conclusion

In this paper, we have explored what happens in the canonical New Keynesian models when the demand-determined solution for labor is replaced by the more theoretically sound Drèze equilibrium. That is, in these models, "slavery" is abolished and the quantity traded is determined by the short end of the market, in this case by the minimum of the quantity of labor that firms want to hire at the prevailing, staggered wages, and the amount of labor that agents are willing to provide at those wages.

We have shown that the differences are large. Typically between 5% and 25% of the labor force is working against agents' will during any given period in a demand-determined solution. Comparing the demand-determined solution with the Drèze equilibrium in standard models, we see substantially different employment volatilities, typically larger in the demand-determined models.

More importantly, when we estimate the Drèze equilibrium, it yields answers that are substantially different from those provided by the demand-determined solution estimates: in the context of the Christiano et al. (2005) and Altig et al. (2011) economy, the role of technology shocks rises from 13% to 70%, these shocks become larger and less persistent, and the estimates of the rigidities become larger.

We conclude by encouraging researchers to use the Drèze equilibrium (or an approximation of it) to eliminate the feature that agents work against their will, a notion that we find inappropriate. In this paper, we explain how, and why it matters.

Table 5: Estimated Parameter Values with 5% Wage Markup Using 1 St.d Impulse Response Functions

	Demand determined	Approximated Drèze
Std of neutral technology shock, $\sigma_{\mu z}$	0.068 (0.046)	0.140 (0.089)
Autocor neutral technology shock, ρ_{μ_z}	0.902 (0.102)	0.697 (0.240)
Std of monetary shock, σ_M	0.331 (0.084)	0.325 (0.078)
Autocor monetary policy shock, ρ_M	-0.037 (0.111)	-0.040 (0.130)
Std of embodied technology shock, $\sigma_{\mu\Upsilon}$	0.303 (0.042)	0.286 (0.046)
Autocor embodied technology shock, $\rho_{\mu\Upsilon}$	0.241 (0.224)	0.318 (0.176)
Wage rigidity, ξ_w	0.722 (0.123)	0.825 (0.043)
Price rigidity, γ	0.040 (0.029)	0.054 (0.039)
Variable capital utilization, σ_a	1.995 (2.222)	4.564 (7.070)
Investment adjustment cost, S''	3.281 (2.038)	4.752 (2.378)
Interest semi-elasticity of money demand, ϵ	0.808 (0.208)	0.779 (0.193)
Habit formation, b	0.706 (0.045)	0.698 (0.058)
Effects of neutral technology shock on policy, ρ_{xz}	0.343 (0.266)	0.195 (0.480)
Effects of embodied technology shock on policy, $\rho_{x\Upsilon}$	0.824 (0.154)	0.832 (0.132)
Scaling factor of neutral technology shock, c_z	2.997 (2.310)	1.027 (0.749)
Scaling factor of neutral technology shock, c_z^p	1.327 (1.381)	0.665 (0.650)
Scaling factor of embodied technology shock, c_Υ^p	0.135 (0.244)	0.107 (0.268)
Scaling factor of embodied technology shock, c_Υ	0.246 (0.244)	0.305 (0.266)

Table 6: Effects on Labor of the Shocks of Each Set of Parameters over Each Solution Concept

	Estimated with Demand Determined				Estimated with Approximated Drèze: 1 std shock			
Neutral technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	**0.18**	**0.87**	**15.09**	—	0.24	0.97	19.03
Approximated Drèze	-1.57	1.16	0.96	—	**-2.59**	**1.41**	**0.95**	—
Investment technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	–	**0.67**	**0.99**	**6.22**	–	0.52	0.99	7.89
Approximated Drèze	-0.42	0.34	0.98	—	**-0.55**	**0.32**	**0.99**	—
Monetary shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	**0.46**	**1.00**	**2.56**	—	0.33	1.00	1.15
Approximated Drèze	-0.07	0.33	0.99	—	**-0.01**	**0.30**	**1.00**	—
All shocks	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	**1.38**	**0.96**	**18.83**	—	1.15	0.95	22.63
Approximated Drèze	-2.28	2.06	0.98	—	**-3.41**	**1.99**	**0.96**	—

Notes: Numbers are in percentages except for the correlation with output.

Table 7: Estimated Parameter Values for the Drèze Equilibria under Various Markups and Shock Sizes

	Demand Determined	Drèze, 5% markup		Drèze, 10% markup		Drèze, 15% markup	
		1 std	1.5 std	2.5 std	3 std	3 std	4 std
ρ_{μ_z}	0.902 (0.102)	0.697 (0.240)	0.579 (0.002)	0.824 (0.119)	0.736 (0.183)	0.902 (0.112)	0.898 (0.063)
$\sigma_{\mu z}$	0.068 (0.046)	0.140 (0.089)	0.110 (0.000)	0.112 (0.055)	0.132 (0.071)	0.070 (0.035)	0.089 (0.035)
ρ_M	-0.037 (0.111)	-0.040 (0.130)	-0.078 (0.121)	-0.019 (0.148)	-0.018 (0.125)	-0.030 (0.123)	-0.021 (0.131)
σ_M	0.331 (0.084)	0.325 (0.078)	0.319 (0.074)	0.339 (0.077)	0.331 (0.077)	0.334 (0.075)	0.332 (0.077)
ρ_{μ_Υ}	0.241 (0.224)	0.318 (0.176)	0.344 (0.377)	0.841 (0.138)	0.839 (0.146)	0.833 (0.178)	0.840 (0.143)
$\sigma_{\mu\Upsilon}$	0.303 (0.042)	0.286 (0.046)	0.287 (0.046)	0.296 (0.053)	0.297 (0.053)	0.304 (0.051)	0.296 (0.053)
ρ_{xz}	0.343 (0.266)	0.195 (0.480)	0.130 (0.553)	0.315 (0.380)	0.251 (0.413)	0.347 (0.285)	0.356 (0.338)
c_z	2.997 (2.310)	1.027 (0.749)	1.008 (0.704)	1.435 (0.875)	1.108 (0.717)	2.840 (1.823)	2.023 (1.124)
c_z^p	1.327 (1.381)	0.665 (0.650)	0.715 (0.724)	0.697 (0.551)	0.598 (0.427)	1.373 (0.974)	0.946 (0.717)
c_Υ^p	0.135 (0.238)	0.107 (0.244)	0.110 (0.270)	0.149 (0.247)	0.133 (0.247)	0.125 (0.247)	0.150 (0.244)
$\rho_{x\Upsilon}$	0.824 (0.154)	0.832 (0.132)	0.882 (0.066)	0.841 (0.138)	0.839 (0.146)	0.833 (0.178)	0.840 (0.143)
c_Υ	0.246 (0.244)	0.305 (0.266)	0.318 (0.276)	0.226 (0.253)	0.240 (0.250)	0.224 (0.230)	0.221 (0.245)
ϵ	0.808 (0.208)	0.779 (0.193)	0.722 (0.170)	0.823 (0.208)	0.799 (0.208)	0.818 (0.193)	0.796 (0.199)
S''	3.281 (2.038)	4.275 (2.378)	3.246 (2.030)	4.539 (2.462)	4.257 (2.252)	3.216 (1.758)	4.252 (2.623)
ξ_w	0.722 (0.123)	0.825 (0.043)	0.801 (0.135)	0.850 (0.036)	0.854 (0.050)	0.806 (0.096)	0.872 (0.040)
b	0.706 (0.045)	0.698 (0.058)	0.719 (0.078)	0.717 (0.051)	0.711 (0.055)	0.717 (0.042)	0.721 (0.046)
σ_a	1.995 (2.222)	4.564 (7.070)	0.932 (0.834)	3.373 (3.627)	3.688 (4.297)	1.744 (2.414)	3.075 (3.410)
γ	0.040 (0.029)	0.054 (0.039)	0.103 (0.144)	0.060 (0.036)	0.057 (0.039)	0.041 (0.026)	0.059 (0.037)

Table 8: Effects on Labor of Shocks over Each Solution Concept: 7% Markup, 2 Std Shock

	Estimated with Demand Determined				Estimated with Approximated Drèze: 2 std shock			
Neutral technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.18	0.87	7.72	—	0.26	0.97	10.10
Approximated	-0.90	0.52	0.92	—	-1.37	0.55	0.91	—
Investment technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	–	0.67	0.99	3.24	–	0.51	0.99	4.13
Approximated	-0.18	0.43	0.98	—	-0.27	0.38	0.99	—
Monetary shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.46	1.00	0.63	—	0.33	1.00	0.07
Approximated	-0.01	0.43	0.99	—	0.00	0.33	1.00	—
All shocks	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	1.38	0.96	10.96	—	1.17	0.96	13.54
Approximated	-1.41	1.26	0.96	—	-2.00	1.11	0.94	—

Notes: Numbers are in percentages except for the correlation with output.

Table 9: Effects on Labor of Shocks over Each Solution Concept: 10% Markup, 3 Std Shock

	Estimated with Demand Determined				Estimated with Approximated Drèze: 3 std shock			
Neutral technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.17	0.86	3.50	—	0.23	0.97	5.57
Approximated	-0.44	0.25	0.85	—	-0.83	0.35	0.87	—
Investment technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.62	0.99	0.90	—	0.47	0.99	1.04
Approximated	-0.03	0.55	0.99	—	-0.05	0.43	0.99	—
Monetary shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.50	0.99	0.05	—	0.36	1.00	0.00
Approximated	0.01	0.50	0.99	—	0.00	0.36	1.00	—
All shocks	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	1.35	0.96	5.58	—	1.12	0.95	7.59
Approximated	-0.74	0.99	0.94	—	-1.21	0.95	0.92	—

Notes: Numbers are in percentages except for the correlation with output.

Table 10: Effects on Labor of Shocks over Each Solution Concept: 15% Markup, 4 Std Shock

	Estimated with Demand Determined				Estimated with Approximated Drèze: 4 std shock			
Neutral technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.17	0.86	1.21	—	0.09	0.94	3.68
Approximated	-0.14	0.17	0.84	—	-0.71	0.23	0.83	—
Investment technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.62	0.99	0.06	—	0.47	0.99	0.11
Approximated	0.01	0.62	0.99	—	0.00	0.46	0.99	—
Monetary shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.50	0.99	0.00	—	0.36	1.00	0.00
Approximated	0.01	0.50	0.99	—	0.00	0.36	1.00	—
All shocks	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	1.35	0.96	2.28	—	0.98	0.94	4.92
Approximated	-0.25	1.11	0.95	—	-0.97	0.90	0.93	—

Notes: Numbers are in percentages except for the correlation with output.

References

Alessandria, G., Kaboski, J. P., and Midrigan, V. (2010). Inventories, Lumpy Trade, and Large Devaluations. *American Economic Review*, 100(5):2304–39.

Altig, D., Christiano, L. J., Eichenbaum, M., and Linde, J. (2011). Firm-specific capital, nominal rigidities and the business cycle. *Review of Economic Dynamics*, 14(2):225–247.

Barro, R. J. and Grossman, H. I. (1971). A general disequilibrium model of income and employment. *American Economic Review*, 61(1):82–93.

Bénassy, J.-P. (1993). Nonclearing markets: microeconomic concepts and macroeconomic applications. *Journal of Economic Literature*, 31(2):732–761.

Bills, M. (2004). Studying price markups from stockout behavior. Manuscript, University of Rochester, http://www.econ.rochester.edu/people/BilsPapers/st1.pdf.

Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2002). Can sticky price models generate volatile and persistent real exchange rates? *Review of Economic Studies*, 69(3):533–563.

Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2009). New keynesian models: Not yet useful for policy analysis. *American Economic Journal: Macroeconomics*, 1(1):242–266.

Chetty, R., Guren, A., Manoli, D. S., and Weber, A. (2011). Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins. *American Economic Review: Papers and Proceedings*, 101(3):471–475.

Christiano, L., Eichenbaum, M., and Rebelo, S. (2011). When is the government spending multiplier large? *Journal of Political Economy*, 119(1):78–121.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1):1–45.

Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2013). Unemployment and business cycles. Mimeo, Northwestern University.

Citanna, A., Crès, H., Drèze, J., Herings, P. J.-J., and Villanacci, A. (2001). Continua of underemployment equilibria reflecting coordination failures, also at walrasian prices. *Journal of Mathematical Economics*, 36(3):169–200.

Corsetti, G. and Pesenti, P. (2005). International dimensions of optimal monetary policy. *Journal of Monetary Economics*, 52(2):281–305.

Dehez, P. and Drèze, J. H. (1984). On supply-constrained equilibria. *Journal of Economic Theory*, 33(1):172–182.

Drèze, J. H. (1975). Existence of an exchange equilibrium under price rigidities. *International Economic Review*, 16(2):301–320.

Drèze, J. H. (1997). Walras keynes equilibria coordination and macroeconomics. *European Economic Review*, 41(9):1735–1762.

Erceg, C. J., Henderson, D. W., and Levin, A. T. (2000). Optimal Monetary Policy with Staggered Wage and Price Contracts. *Journal of Monetary Economics*, 46(2):281–313.

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.

Galí, J. (2011). The return of the wage Phillips curve. *Journal of the European Economic Association*, 9(3):436–461.

Galí, J., Smets, F., and Wouters, R. (2011). Unemployment in an estimated new keynesian model. CREI Working Paper.

Gertler, M., Sala, L., and Trigari, A. (2008). An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining. *Journal of Money, Credit and Banking*, 40(8):1713–1764.

Gertler, M. and Trigari, A. (2009). Unemployment fluctuations with staggered nash wage bargaining. *Journal of Political Economy*, 117(1):38–86.

Guerrieri, L. and Iacoviello, M. (2015). Occbin: A toolkit to solve models with occasionally binding constraints easily. *Journal of Monetary Economics*, 70:22–38.

Hall, R. (2005). Employment fluctuations with equilibrium wage stickiness. *American Economic Review*, 95(1):50–65.

Hall, R. E. and Milgrom, P. R. (2008). The Limited Influence of Unemployment on the Wage Bargain. *American Economic Review*, 98(4):1653–1674.

Heathcote, J., Storesletten, K., and Violante, G. L. (2010). The macroeconomic implications of rising wage inequality in the united states. *Journal of Political Economy*, 118(4):681–722.

Herings, P. J.-J. (1996). Equilibrium existence results for economies with price rigidities. In *Static and Dynamic Aspects of General Disequilibrium Theory*, pages 121–186. Springer.

Herings, P. J.-J. (2014). General equilibrium and the new neoclassical synthesis. *Economic Theory*, 57(3):437–477.

Kurz, M. (1982). Unemployment equilibrium in an economy with linked prices. *Journal of Economic Theory*, 26(1):100–123.

Levin, A. T., Onatski, A., Williams, J., and Williams, N. M. (2006). Monetary policy under uncertainty in micro-founded macroeconometric models. In Gertler, M. and Rogoff, K., editors, *NBER Macroeconomics Annual 2005*, volume 20, pages 229–312. MIT Press.

Lewis, H. G. (1986). Union relative wage effects. In Ashenfelter, O. and Layard, R., editors, *Handbook of Labor Economics*, volume 2, chapter 20, pages 1139–1181. North-Holland.

Malinvaud, E. (1977). *The Theory of Unemployment Reconsidered*. Yrjö Jahnsson Lectures. Wiley.

Michaillat, P. and Saez, E. (2015). Aggregate demand, idle time, and unemployment. *The Quarterly Journal of Economics*, 130(2):507–569.

Ríos-Rull, J.-V. and Santaeulàlia-Llopis, R. (2010). Redistributive shocks and productivity shocks. *Journal of Monetary Economics*, 57(8):931–948.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: a Bayesian DSGE approach. *American Economic Review*, 97(3):586–606.

Van der Laan, G. (1980). Equilibrium under rigid prices with compensation for the consumers. *International Economic Review*, pages 63–73.

Appendix

A Details of the Estimation of Altig et al. (2011)

The system of the log-linearized equations for estimation and simulation is the following:

$$\mathbb{E}\left[\widehat{\lambda}_{z^*,t+1} - \frac{1}{1-\alpha}\widehat{\mu}_{\Upsilon t+1} - \widehat{\mu}_{zt+1} + \frac{\rho\widehat{\rho}_{t+1} + (1-\delta)\widehat{\mu}_{t+1}}{1-\delta+\rho}\middle|\Omega_t^p\right] = 0 \tag{A-1}$$

$$\mathbb{E}\Big\{S''(\mu_\Upsilon\mu_{z^*})^2\left[\widehat{i}_t - \widehat{i}_{t-1} + \widehat{\mu}_{\Upsilon t} + \frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t} + \widehat{\mu}_{zt}\right] -$$
$$\beta S''(\mu_\Upsilon\mu_{z^*})^2\left[\widehat{i}_{t+1} - \widehat{i}_t + \widehat{\mu}_{\Upsilon t+1} + \frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t+1} + \widehat{\mu}_{zt+1}\right] - \widehat{\mu}_t\Big|\Omega_t^p\Big\} = 0 \tag{A-2}$$

$$\frac{\nu R}{\nu R + 1 - \nu}\widehat{R}_t + \widehat{w}_t + \frac{1}{1-\alpha}\left(\frac{y}{y+\phi}\widehat{y}_t - \widehat{k}_t + \frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t} + \widehat{\mu}_{zt} + \widehat{\mu}\Upsilon t\right) - \widehat{\rho}_t - \frac{1}{1-\alpha}\widehat{u}_t = 0 \tag{A-3}$$

$$[\mu_\Upsilon\mu_{z^*} - (1-\delta)]\widehat{i}_t - \left\{\mu_\Upsilon\mu_{z^*}\widehat{k}_{t+1} - (1-\delta)\left[\widehat{k}_t - \frac{1}{1-\alpha}\widehat{\mu}_{\Upsilon t} - \widehat{\mu}_{zt}\right]\right\} = 0 \tag{A-4}$$

$$\mathbb{E}[\beta(\widehat{\pi}_{t+1} - \pi_t) - \gamma\widehat{s}_t - (\widehat{\pi}_t - \pi_{t-1})|\Omega_t^p] = 0 \tag{A-5}$$

$$\widehat{c}_t - \frac{R}{(R-1)(2+\sigma_\eta)}\widehat{R}_t - \widehat{q}_t = 0 \tag{A-6}$$

$$\mathbb{E}\Big\{-\left(\frac{1}{c(1-b\mu_{z^*}^{-1})}\right)^2\left[c\widehat{c}_t - \frac{bc}{\mu_{z^*}}\widehat{c}_{t-1} + \frac{bc}{\mu_{z^*}}\left(\frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t} + \mu_{zt}\right)\right] +$$
$$\beta\left(\frac{1}{c(1-b\mu_{z^*}^{-1})}\right)^2\left[c\widehat{c}_{t+1} - \frac{bc}{\mu_{z^*}}\widehat{c}_t + \frac{bc}{\mu_{z^*}}\left(\frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t+1} + \mu_{zt+1}\right)\right] -$$
$$\lambda_{z^*}[(1+\eta(V)) + \eta'(V)V]\widehat{\lambda}_{z^*t} - \lambda_{z^*}\left[2 + \frac{\eta''(V)V}{\eta'(V)}\right]\eta'(V)V(\widehat{c}_t - \widehat{q}_t)\Big|\Omega_t^p\Big\} = 0 \tag{A-7}$$

$$\mathbb{E}\left[-\lambda_{z^*t} + \lambda_{z^*t+1} + \widehat{R}_{t+1} - \widehat{\pi}_{t+1} - \frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t+1} - \widehat{\mu}_{zt+1}\middle|\Omega_t^p\right] = 0 \tag{A-8}$$

$$\frac{\xi_w(\lambda_w\sigma_L - (1-\lambda_w))}{(1-\xi_w)(1-\beta\xi_w)}\Big\{\widehat{w}_{t-1} + \left[-\frac{1+\beta\xi_w^2}{\xi_w} + \sigma_L\lambda_w\frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w(\lambda_w\sigma_L - (1-\lambda_w))}\right]\widehat{w}_t + \beta\widehat{w}_{t+1} + \widehat{\pi}_{t-1} + \widehat{\pi}_t + \beta\pi_{t+1} +$$
$$\frac{(1-\xi_w)(1-\beta\xi_w)(1-\lambda_w)}{\xi_w(\lambda_w\sigma_L - (1-\lambda_w))}(-\sigma_L\widehat{h}_t + \widehat{\lambda}_{z^*t}) - (1-\vartheta)\widehat{\mu}_{zt} + \beta(1-\vartheta)\frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t+1} + \beta(1-\vartheta)\widehat{\mu}_{zt+1}\Big\} = 0 \tag{A-9}$$

$$(1+\eta)c\widehat{c}_t + \eta'\frac{c^2}{q}(\widehat{c}_t - \widehat{q}_t) + i\widehat{i}_t - (y+\phi)\left[\alpha\left(\widehat{u}_t\widehat{k}_t - \frac{1}{1-\alpha}\widehat{\mu}_{\Upsilon t} - \widehat{\mu}_{zt}\right) + (1-\alpha)\widehat{h}_t\right] + \rho\frac{k}{\mu_{z^*}\mu_\Upsilon}\widehat{u}_t = 0 \tag{A-10}$$

$$\widehat{w}_t + \widehat{h}_t - \frac{xm(\widehat{x}_t + \widehat{m}_t) - q\widehat{q}_t}{xm - q} = 0 \tag{A-11}$$

$$\widehat{x}_{zt} + \widehat{x}_{\Upsilon t} + \widehat{x}_{Mt} - \widehat{x}_t = 0 \tag{A-12}$$

$$\widehat{x}_{t-1} - \widehat{\pi}_t - \frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t} - \widehat{\mu}_{zt} + \widehat{m}_{t-1} - \widehat{m}_t = 0 \tag{A-13}$$

$$y\widehat{y}_t - (y+\phi)\left[\alpha\left(\widehat{u}_t + \widehat{k}_t - \frac{1}{1-\alpha}\widehat{\mu}_{\Upsilon t} - \widehat{\mu}_{zt}\right) + (1-\alpha)\widehat{h}_t\right] + \rho\frac{k}{\mu_{z^*}\mu_\Upsilon}\widehat{u}_t = 0 \tag{A-14}$$

$$\mathbb{E}\left[\widehat{u}_t - \frac{1}{\sigma_a}\widehat{\rho}_t\middle|\Omega_t^p\right] = 0 \tag{A-15}$$

In this system, there are three shocks, $\{\epsilon_{Mt}, \epsilon_{\mu_z t}, \epsilon_{\mu_\Upsilon t}\}$, which are shocks to monetary policy, neutral technology, and embodied investment technology. The processes for various shocks are

$$\widehat{\mu}_{zt} = \rho_{\mu z}\widehat{\mu}_{zt-1} + \epsilon_{\mu_z t} \tag{A-16}$$

$$\widehat{\mu}_{\Upsilon t} = \rho_{\mu\Upsilon}\widehat{\mu}_{\Upsilon t-1} + \epsilon_{\mu_\Upsilon t} \tag{A-17}$$

$$\widehat{x}_{Mt} = \rho_M \widehat{x}_{Mt-1} + \epsilon_{Mt} \tag{A-18}$$

$$\widehat{x}_{zt} = \rho_{xz}\widehat{x}_{zt-1} + c_z^p \epsilon_{\mu_z t-1} + c_z \epsilon_{\mu_z t} \tag{A-19}$$

$$\widehat{x}_{\Upsilon t} = \rho_{x\Upsilon}\widehat{x}_{\Upsilon t-1} + c_\Upsilon^p \epsilon_{\mu_\Upsilon t-1} + c_\Upsilon \epsilon_{\mu_\Upsilon t} \tag{A-20}$$

In the estimation, we choose the parameters to minimize the distance between model-implied impulse responses and their data counterparts, as in Altig et al. (2011).

B Details of the Estimation of the Smets and Wouters (2007)

The system of the log-linearized equations for estimation and simulation are the following (the only changes from Smets and Wouters (2007) are in equations (A-22) and (A-30)):

$$y_t = c_y c_t + i_y i_t + k_y \bar{r}^k u_t + \epsilon_t^g \tag{A-21}$$

$$c_t = \frac{h/\zeta}{1+h/\zeta} c_{t-1} + \frac{1}{1+h/\zeta}\mathbb{E}_t c_{t+1} - \frac{1-h/\zeta}{1+h/\zeta}(r_t - E_t \pi_{t+1} + \epsilon_t^b) \tag{A-22}$$

$$i_t = \frac{1}{1+\beta}\left(i_{t-1} + \beta \mathbb{E}_t i_{t+1} + \frac{1}{\zeta^2 \varphi} q_t\right) + \epsilon_t^i \tag{A-23}$$

$$q_t = \frac{\bar{r}^k}{1-\delta+\bar{r}^k} r_{t+1}^k + \frac{1-\delta}{1-\delta+\bar{r}^k}\mathbb{E}_t q_{t+1} - (r_t - \mathbb{E}_t \pi_{t+1} + \epsilon_t^b) \tag{A-24}$$

$$y_t = \Phi\left(\alpha(k_{t-1} + u_t) + (1-\alpha)\ell_t + \epsilon_t^a\right) \tag{A-25}$$

$$r_t^k = \frac{1-\psi}{\psi} u_t \tag{A-26}$$

$$k_t = \left(1 - \frac{1-\delta}{\zeta}\right) k_{t-1} + \frac{1-\delta}{\zeta} i_t + \epsilon_t^i \tag{A-27}$$

$$\pi_t = \frac{1}{1+\beta\iota_p}\left(\iota_p \pi_{t-1} + \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p((\Phi-1)\varepsilon_p+1)}(\alpha(k_{t-1}+u_t-\ell_t) - w_t + \epsilon_t^a)\right) + \epsilon_t^p \tag{A-28}$$

$$r_t^k = w_t + \ell_t - k_{t-1} - u_t \tag{A-29}$$

$$\begin{aligned} w_t = \frac{1}{1+\beta}\Bigg\{ & \beta w_{t+1} + w_{t-1} + \beta\pi_{t+1} - (1+\beta\iota_w)\pi_{t-1} - \iota_w \pi_{t-1} \\ & + \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\gamma\epsilon_w)\xi_w}\left(\gamma \ell_t + \frac{1}{1-h/\zeta} c_t - \frac{h/\zeta}{(1-h/\zeta} c_{t-1} - w_t\right)\Bigg\} + \epsilon_t^w \end{aligned} \tag{A-30}$$

$$r_t = \rho r_{t-1} + (1-\rho)(r_\pi \pi_t + r_y(y_t - y_t^p)) + r_{\Delta y}(y_t - y_t^p - (y_{t-1} - y_{t-1}^p)) + \epsilon_t^m \tag{A-31}$$

In this system, there are seven shocks, $\{\epsilon_t^a, \epsilon_t^i, \epsilon_t^b, \epsilon_t^p, \epsilon_t^w, \epsilon_t^g, \epsilon_t^m\}$, which are shocks to TFP, investment technology, risk premium, price markup, wage markup, government spending, and monetary policy. Some of the shocks are rescaled to facilitate estimation. The processes for various shocks are

$$\epsilon_t^g = \rho_g \epsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a \tag{A-32}$$
$$\epsilon_t^b = \rho_b \epsilon_{t-1}^b + \eta_t^b \tag{A-33}$$
$$\epsilon_t^i = \rho_i \epsilon_{t-1}^i + \eta_t^i \tag{A-34}$$
$$\epsilon_t^a = \rho_a \epsilon_{t-1}^a + \eta_t^a \tag{A-35}$$
$$\epsilon_t^m = \rho_m \epsilon_{t-1}^m + \eta_t^m \tag{A-36}$$
$$\epsilon_t^p = \rho_p \epsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p \tag{A-37}$$
$$\epsilon_t^w = \rho_w \epsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w \tag{A-38}$$

As in Smets and Wouters (2007), we fixed a few parameters, which are the depreciation rate $\delta = 0.025$, the average government spending to output ratio $g_y = 0.18$, the Kimball aggregator parameter $\varepsilon_p = 10$, and we use log utility function for consumption. Although Smets and Wouters (2007) set the wage markup to 50%, we estimate the model for various other wage markup levels and report them in Table A-1. The process for the shocks is reported in Table A-2.

The implied true wage markup shock $\widehat{\epsilon}_t^w$ is calculated based on equation (A-30) and is given by

$$\widehat{\epsilon}_t^w = \frac{(1+\beta)(1+\gamma\epsilon_w)\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\epsilon_t^w. \tag{A-39}$$

In Smets and Wouters (2007), the variance of the wage markup shock reported is 25.87%, quite a large value[12] that implies that the wage markup itself sometimes turns out to be negative, which does not make economic sense even if it can be handled by the log-linearization of the solution under demand-determined labor. Moreover, as we will see later, the approximation to the Drèze equilibrium requires positive wage markups to calculate the wage distribution for different sectors. Consequently, when we simulate the model, we abstract from this shock.

We make two further modifications to Smets and Wouters (2007), in line with most of the recent literature: the utility function is additively separable in consumption and leisure, and the aggregator of labor inputs is the standard Dixit-Stiglitz aggregator.

[12] That this is a huge value is also emphasized by Chari et al. (2009).

Table A-1: Smets and Wouters (2007) Model: Structural Parameters

Wage markup, $\frac{\epsilon_w}{\epsilon_w - 1}$	5%	10%	15%
Investment adjustment cost, φ	5.409	5.425	5.430
Habit formation, h	0.778	0.784	0.787
Wage Calvo probability, ξ_w	0.527	0.576	0.605
Inverse Frsich elasticity, γ	2.873	2.811	2.772
Price Calvo probability, ξ_p	0.681	0.667	0.659
Wage indexation, ι_w	0.571	0.583	0.589
Price indexation, ι_p	0.212	0.216	0.218
Utilization elasticity, ψ	0.535	0.537	0.538
Fixed cost, Φ	1.555	1.560	1.562
Share of capital in production, α	0.189	0.190	0.190
Monetary policy for inflation, r_π	1.895	1.907	1.913
Monetary policy for output gap, r_y	0.069	0.070	0.071
Monetary policy persistency, ρ	0.812	0.808	0.805
Monetary policy for output gap change, $r_{\Delta y}$	0.233	0.229	0.227
Trend, ζ	1.004	1.004	1.004
Discount rate, β	0.998	0.998	0.998

Notes: The specification of the prior distribution is the same as in Smets and Wouters (2007). We only report the posterior modes of the structural parameters in this table for various wage markup levels. The posterior means are not included here but are available upon request. When we simulate the model, we use the posterior modes of the structural parameters.

Table A-2: Smets and Wouters (2007) Model: Persistence and Standard Deviation of Various Shocks

Wage markup, $\frac{\epsilon_w}{\epsilon_w - 1}$	5%	10%	15%
Government spending shock, ϵ^g	0.517	0.517	0.518
Government spending shock, ρ_g	0.972	0.971	0.971
Government spending shock, ρ_{ga}	0.537	0.536	0.536
Risk premium shock, ϵ^b	0.218	0.223	0.226
Risk premium shock, ρ_b	0.373	0.347	0.333
Investment shock, ϵ^i	0.377	0.385	0.389
Investment shock, ρ_i	0.768	0.757	0.752
TFP shock, ϵ^a	0.459	0.458	0.458
TFP shock, ρ_a	0.950	0.950	0.949
Monetary policy shock, ϵ^m	0.239	0.240	0.240
Monetary policy shock, ρ_m	0.125	0.132	0.136
Price markup shock, ϵ^p	0.142	0.142	0.142
Price markup shock, ρ_p	0.901	0.906	0.908
Price markup shock, μ_p	0.748	0.746	0.746
Wage markup shock, ϵ^w	0.248	0.248	0.248
Wage markup shock, ρ_w	0.972	0.975	0.977
Wage markup shock, μ_w	0.927	0.916	0.908
Implied true wage markup shock, $\widehat{\epsilon}^w$	71.275	50.573	42.544

Notes: The specification of the prior distribution is the same as in Smets and Wouters (2007). We only report the posterior modes of the shock processes in this table for various wage markup levels. The posterior means are not included here but are available upon request. When we simulate the model, we use the posterior modes of the shock processes.

C Analysis of Galí et al. (2011)

Galí et al. (2011) set the steady-state wage markup level to 18%, which via equation (12) implies a Frisch elasticity of 0.25, a very low value, much lower than macroeconomists and even most microeconomists would use (see Chetty et al. (2011) for a recent discussion). Accordingly, and as for the Smets and Wouters (2007) model, we also explore lower values for the steady-state markup ranging from 5% to 15%, which imply much more reasonable Frisch elasticities.

Although the Galí et al. (2011) estimate of the standard deviation of the wage markup shock is much smaller than that of Smets and Wouters (2007) (0.04 versus 0.25), we still obtain frequently negative wage markups. Again, we simulate the model without the wage markup shock. Our choices should be thought of as a conservative benchmark. We have assessed economies with a positive standard deviation of the wage mark-up shock (0.015). In this case, the violation of the labor supply constraint becomes more severe.

Table A-3: Demand determined and Voluntary ex-post Employment in the Galí et al. (2011) Model

Wage markup: 5	Mean	Binding Freq	Var	Corr(N,Y)
Demand Determined Employment	—	9.68	0.86	0.84
Voluntary ex-post Employment	-0.49	—	0.93	0.68
Wage markup: 10%	Mean	Binding Freq	Var	Corr(N,Y)
Demand Determined Employment	—	5.05	0.61	0.79
Voluntary ex-post Employment	-0.21	—	0.48	0.71
Wage markup: 15%	Mean	Binding Freq	Var	Corr(N,Y)
Demand Determined Employment	—	4.20	0.54	0.76
Voluntary ex-post Employment	-0.15	—	0.41	0.70
Wage markup: 18%	Mean	Binding Freq	Var	Corr(N,Y)
Demand Determined Employment	—	3.86	0.51	0.75
Voluntary ex-post Employment	-0.13	—	0.39	0.70

Notes: Numbers are in percentages except for the correlation with output. In the simulation, we include all the shocks except for the wage markup shock.

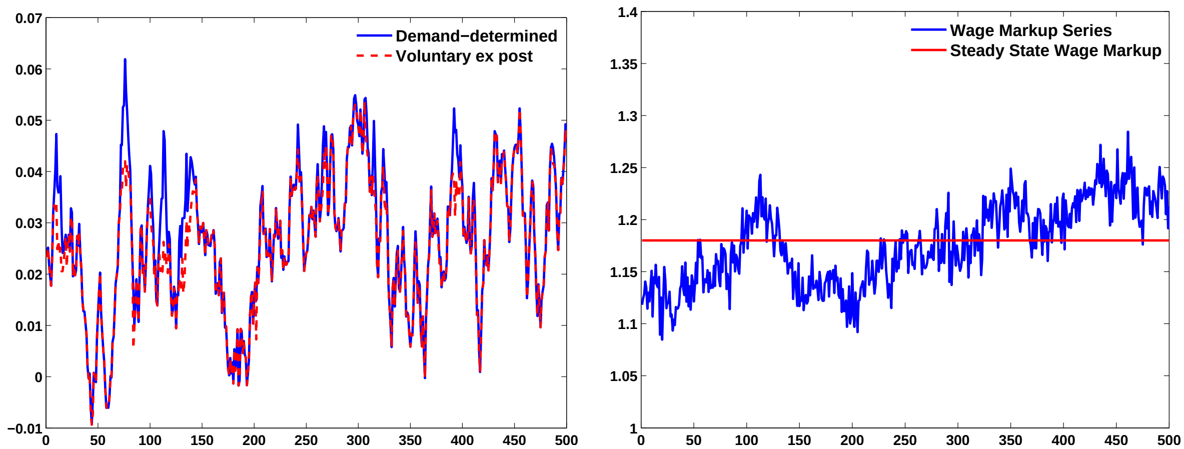
We also explore the Galí et al. (2011) model with shocks to the wage markup. To ensure that the wage markup is always positive, which is necessary to construct the voluntary ex-post employment series, we simulate the model with standard deviations of the wage markup shock ranging from 0.0 to 0.015. Table A-4 reports the findings. The larger the variance of the wage markup shock, the lower is mean employment and the larger the frequency of the violation of the labor supply constraint in the demand-determined allocation. The variance of the voluntary ex-post employment also increases with the volatility of the wage markup shock. The reason is that in periods when the value of the wage markup is low, the variance of cross-sectional labor demand becomes larger and the level of labor supply is lower, which makes the labor supply constraint be more likely to be violated. Figure A-1 shows a high wage markup sample where the demand-determined and the voluntary ex-post employment series are similar to each other. However, in Figure A-2 which shows a sample with a sequence of low wage markup shocks, the differences between the two series are apparent: the voluntary ex-post aggregate employment displays large reductions. We conclude that abstracting from wage markup shocks in the baseline analysis of the Galí et al. (2011) model, if anything, tends to bias toward making the demand-determined allocation look more similar to the Drèze equilibrium than it really is.

Table A-4: Galí et al. (2011) Model with voluntary Ex-Post Aggregate Employment

St.d of Wage Markup Shock: 0.000	Mean	Var	Corr(N,Y)	Binding Freq
Employment w/o constraint	—	0.51	0.75	3.86
Employment w/ voluntary ex-post aggregate employment	-0.13	0.39	0.70	—
St.d of Wage Markup Shock: 0.005	**Mean**	**Var**	**Corr(N,Y)**	**Binding Freq**
Employment w/o constraint	—	0.51	0.75	3.90
Employment w/ voluntary ex-post aggregate employment	-0.13	0.39	0.70	—
St.d of Wage Markup Shock: 0.010	**Mean**	**Var**	**Corr(N,Y)**	**Binding Freq**
Employment w/o constraint	—	0.51	0.75	4.94
Employment w/ voluntary ex-post aggregate employment	-0.14	0.41	0.69	—
St.d of Wage Markup Shock: 0.015	**Mean**	**Var**	**Corr(N,Y)**	**Binding Freq**
Employment w/o constraint	—	0.51	0.75	7.21
Employment w/ voluntary ex-post aggregate employment	-0.20	0.51	0.65	—

Notes: Numbers are in percentages except for the correlation with output. All the variables are logged and HP filtered. The standard deviation of the wage markup shock is 0.04 in Galí et al. (2011).

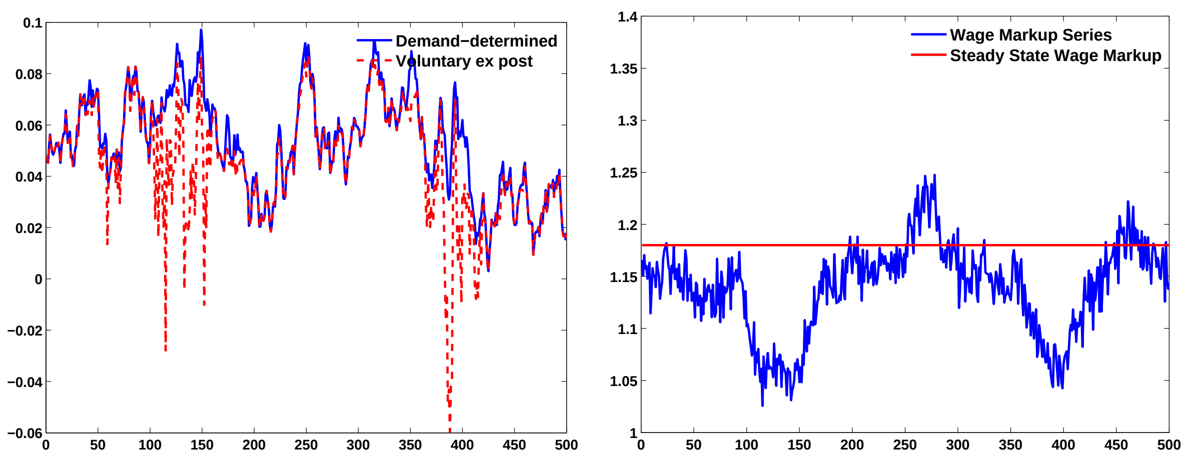
Figure A-1: Galí et al. (2011) Model with Wage Markup Shock: High Wage Markup Sample



Aggregate Employment

Wage Markup Series

Figure A-2: Galí et al. (2011) Model with Wage Markup Shock: Low Wage Markup Sample



Aggregate Employment

Wage Markup Series

D Note on Solving the Occasionally Binding Constraint Problem

Guerrieri and Iacoviello (2015) develop a Dynare toolkit that can solve DSGE models with occasionally binding constraints. However, this method cannot be applied in our model for the following reasons.

Guerrieri and Iacoviello (2015)'s method can handle problems with exogenous binding constraints such as non-negative investment, a zero-bound on the nominal interest rate, an exogenous borrowing constraint, and so on. In these cases, one can neatly partition the problem into two regions: in the first region, the constraint is not binding and one can use the first-order condition to characterize the solution. In the second region, the constraint is binding and one can simply set the variable to equal the constraint (for example, let the nominal interest rate be zero).

The problem in this paper is more involved. When the labor supply constraint is not binding, employment equals the labor demand, and the first-order condition can be applied as in the standard New Keynesian literature. When the labor supply constraint is binding, different from the examples listed earlier, the labor supply constraint is not an exogenous constraint because the level of the labor supply is endogenously determined. What makes this case even worse is that when the labor supply constraint is binding, the union's problem is not concave, which implies that we cannot use either the first-order condition or some exogenous value to determine the optimal wage (and hence employment).

To further illustrate the issue, consider the following example. A union needs to set the optimal wage for two periods, and it takes aggregate states as given. For simplicity, we abstract from price change and uncertainty:

$$\max_{w}\ U = u'(c_1)wn_1 - \frac{n_1^{1+\gamma}}{1+\gamma} + \beta\left(u'(c_2)wn_2 - \frac{n_2^{1+\gamma}}{1+\gamma}\right) \tag{A-40}$$

subject to

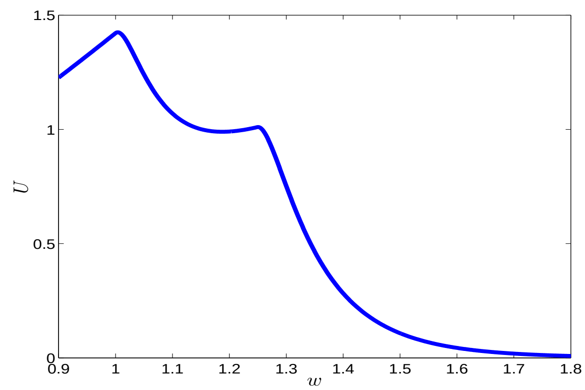
$$n_1 = \min\left\{(u'(c_1)w)^{\frac{1}{\gamma}}, \left(\frac{w}{\overline{w}_1}\right)^{-\epsilon_w}\overline{n}_1\right\}, \tag{A-41}$$

$$n_2 = \min\left\{(u'(c_2)w)^{\frac{1}{\gamma}}, \left(\frac{w}{\overline{w}_2}\right)^{-\epsilon_w}\overline{n}_2\right\}. \tag{A-42}$$

The following figure shows how the objective U changes with the choice w for a particular parametrization. It is obvious that the objective function is not concave, and the first-order condition cannot be used to solve this problem.

Finally, the computation cost to apply Guerrieri and Iacoviello (2015)'s method is not greatly affected by the number of state variables in the model but is increasing fast with the number of occasionally binding constraints. In Calvo-type sticky wage models, the number of occasionally binding constraints is

Figure A-3: Illustration of the Union's Problem



infinite because there are infinitely many cohorts. In our simple economy with Taylor-type staggered wage contracts, there are four occasionally binding constraints, which is still a relatively large number.

To summarize, in our case, there is no simple way to decide the optimal level of wage and employment when labor supply is binding. Therefore, we solve the Drèze equilibrium using a global method.

E Details of the Computation of the Staggered Wage Economy

We use a policy function iteration method to obtain the numerical solution. The system of equations that characterizes the solution is

$$c_t^{-\sigma} = \beta \mathbb{E}_t \left[c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right] \tag{A-43}$$

$$c_t^{-\sigma} = \beta \mathbb{E}_t \left[c_{t+1}^{-\sigma} \frac{1 + r_{t+1}^k - \delta}{\pi_{t+1}} \right] \tag{A-44}$$

$$\frac{r_t^k}{p_t} = \alpha \; z_t \; k_t^{\alpha-1} \; e_t^{1-\alpha}, \tag{A-45}$$

$$\log R_t = \log \frac{1}{\beta} + \phi_\pi \pi_t + \phi_y \log \frac{y_t}{y^*} + \eta_t, \tag{A-46}$$

$$c_t + k_{t+1} = y_t + (1-\delta) k_t \tag{A-47}$$

$$y_t = z_t \; k_t^{\alpha} \; e_t^{1-\alpha}, \tag{A-48}$$

$$e_t = \left[\sum_{i=0}^{T_w} e_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{A-49}$$

$$e_{i,t} = \min \left\{ \left[\frac{w_{t-i}^*}{(1-\alpha) z_t k_t^{\alpha} e_t^{-\alpha} p_t} \right]^{-\epsilon_w} e_t, \; \left(\frac{u'(c_t) w_{t-i}^*}{\phi p_t} \right)^{\frac{1}{\gamma}} \right\}. \tag{A-50}$$

In addition, the optimization problem (27) to (28) is also part of the system.

There are two differences between the demand-determined economy and the Drèze equilibrium: first, the employment is determined by the minimum of the demand and supply in the Drèze equilibrium (see equation (A-50)), whereas in the demand-determined economy, employment always equals to labor demand. Second, the choice of the optimal nominal wage, w_t^*, cannot be characterized by a simple first-order condition because of the potential binding labor supply constraint. In the computation, we have to use a global search method to find the optimal wage choice.

We look for policy functions for $\{k_{t+1}, c_t, y_t, e_{i,t}, e_t, w_t^*, \pi_t, R_t\}$. The state variables at period t include the following: the current technology shock z_t or the monetary shock η_t, the capital stock k_t, and the wages set in the previous three periods $\left\{\frac{w_{t-1}^*}{p_{t-1}}, \frac{w_{t-2}^*}{p_{t-1}}, \frac{w_{t-3}^*}{p_{t-1}}\right\}$. The real wage in period t can be obtained by dividing the current inflation rate:

$$\frac{w_{t-i}^*}{p_t} = \frac{w_{t-i}^*}{p_{t-1}} \frac{1}{\pi_t} = \frac{w_{t-i}^*}{p_{t-1}} \frac{p_{t-1}}{p_t}. \tag{A-51}$$