Partial Default*

Cristina Arellano
Federal Reserve Bank of Minneapolis,
arellano.cristina.gmail.com

Xavier Mateos-Planas
Queen Mary University of London,
x.mateos-planas@qmul.ac.uk

José-Víctor Ríos-Rull
University of Minnesota, FRB Mpls,
CAERP, CEPR, NBER
vr0j@.umn.edu

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PRELIMINARY AND INCOMPLETE

Abstract

In this paper we produce a theory of partial default applicable to sovereign debt. We document that contrary to standard theory, countries always default on only part of their debt and continue to borrow while having debt on arrears. In our model default acts as expensive debt which curtail the countries' productive capabilities. Debt in arrears is carried until it is finally repaid. The model is consistent with the large heterogeneity in defaults observed across countries and years.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System

1 Introduction

Developing countries often stop payment on their debt obligations. These sovereign defaults are commonly thought as discrete events, where countries either repay of default in full.[1] This view pervades the existing theories that study default (Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008) to name a few). They assume that sovereign default is only full and the country goes over an extended period without access to financial markets after defaulting. In this paper, we document that sovereign defaults are far from binary events and are always partial. Countries carry over their debt in arrears until the default is resolved. Some defaults are large with lengthy resolutions and others are minor with fast resolutions. While in default, countries continue to pay some of their debt and to borrow. We propose a model of partial default to incorporate the empirical heterogeneity of default events where defaulted debt is not dissipated but is carried until resolution and countries continue to pay some debt and borrow while in default.

Using a panel of 99 countries since 1975, we document that countries often have some debt in arrears and that default is always partial. We find that countries default on average on 50% of what they owe and have positive arrears about 53% of the time. When arrears are positive, countries continue to service the debt paying about 4% of output and continue to borrow, about 1.7% of output. We also find that deeper recessions are associated with more complete defaults.

A typical default starts with a partial default when interest payment on some bonds are missed. Then the country might continue to default on other bonds but not always. Sometimes the default becomes wide spread as in the case of Argentina 2001; sometimes it remains contained to a subset of bonds as in the case of Russia 1998; and in some other cases, like Venezuela 2005, the default is only on 1 or 2 interest payments that are just made up later.

The paper develops a Markovian theory of debt without commitment that accounts for the heterogeneity of default events. The model consists of a small open economy that faces a stochastic income stream. The economy issues bonds and can default on them to a chosen degree at all times. The theory uses explicitly the observation that while there is no international legal system to enforce contracts, there is enough of a legal system to difficult the economic activity of countries with outstanding unpaid debt. We model this difficulty as a loss of output where the size of the loss is an increasing function of the unpaid debt. In our

[1]As individuals, in the U.S. and Canada for example, can file for bankruptcy when they hold debt and in a dire economic situation to have their debts condoned and have a fresh start (Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), Livshits, MacGee, and Tertilt (2007)).

model, missing debt payments acts as an alternative form of borrowing, that in some circumstance provides more advantageous terms. Unpaid debt does not disappear, it accumulates at an institutionally determined rate $\overline{R}$.

Our model consists of one large agent that borrows and many small lenders. Lending always exists as long as is profitable like in Krueger and Uhlig (2006). Hence, the country has access to borrowing both when it has paid the debt in full or when it has defaulted on some of it. Default is partial in the sense that only some of the debt is unpaid and by the fact that in any case the debt does no disappear. The debt is eventually repaid when circumstances get better.

In our model there is no need to keep track of the seniority of the debt, each period the borrower just decides how much to repay and how much to default on out of the total debt due in the period, which includes the coupon payments due as well as the interest on the coupons unpaid previous periods.

Bond prices compensate lenders for the expected loss from defaulting on some of the debt obligations in the future. The bond price function depends on the total debt due next period, the total unpaid debt, and the shock because the size of the defaults in all future periods depend on these three variables. Larger unpaid debt increases default incentives because the paying capacity is reduced by the output costs. Our model generates endogenously that spreads during default events are high because the bond price function is tighter when the economy has unpaid debt. Moreover, bond prices always have a long term component (even in the case of one period bonds) because it might take several periods for the defaulted debt to be paid back.

In our model the borrower most times repays in full. When income starts to fall spreads rise and the borrower reduces borrowing because of a tight bond price schedule. If income continues to fall, the borrower starts to partially default on the debt. Here the borrower issues only a modest amount of new bonds at high spreads. If income recovers, then the defaulted debt is paid back and the default event ends. If income further deteriorates, then default becomes full, and new borrowing is severely restricted. The borrower takes longer to recover after full defaults.

We calibrate our model to the data in Argentina. In the model, as in the data of Argentina, default is partial, the economy continues to borrow and pay some of the debt during default, and spreads are higher during default events. As models with full default, our model generates business cycles that fit the data with high volatility of consumption and countercyclical interest rates. The model also generate a positive correlation between spreads and partial defaults which is a feature of the data.

Our paper is related to the literature that studies sovereign default and renegotiation. The work of Yue (2010)) and D'Erasmo (2008) extend the model of complete default by Aguiar and Gopinath (2006) and Arellano (2008) to allow for renegotiation of debt through bargaining. They show that allowing for renegotiation improves the models' ability to match the frequency of defaults and debt levels in emerging markets. Benjamin and Wright (2009) study inefficient delays in debt renegotiation that arises because it is worthwhile to wait until default risk is low enough to renegotiate. In contrast to these works, in our model we allow for default to be partial in the first place and allow the economy to continue to borrow during defaults. Default events end in our model endogenously when the debt in arrears is paid back.

Our work is also related to the literature on private defaultable debt. As in the literature of sovereign debt, the majority of the work has focused on full defaults and private bankrupcty (See for example, Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007)). Recently, however, analyzing partial defaults is gaining more attention because defaults outside formal bankruptcy procedures is substantial, as documented by Dawsey and Ausubel (2004) and Herkenhoff (2011). In the work of Mateos-Planas and Seccia (2007) for example, households default partially on their debts plays and such outcome gives rise to incomplete consumption insurance in an environment with a complete set of securities.

We start describing in some detail the nature of actual sovereign debt episodes in Section 2. We pose the model first without renegotiation in Section 3. Section 4 characterizes equilibria while Section 5 compares its properties with those of the data.

2 The empirical properties of sovereign defaults

Sovereign defaults are generally catalogued as discrete events. For example, the widely used Standard and Poor's dataset defines a country in default if it has failed to meet any payments on the due date. By the nature of such classification, default is a binary variable: a country is in default or not. Such description of defaults, however, masks an immense heterogeneity across default events. This section documents sovereign defaults using the WDI data on debt in arrears. We find that some defaults are small and resolve swiftly and some others are large, costly, and with lengthy resolutions.

2.1 Data

We use panel data for 99 developing countries from 1970-2010. The countries include all the countries that have experienced a default event as defined by Standard and Poor and the dataset by Trebesch and Cruces (2012) in addition to all emerging markets countries. We use information on public debt from the World Development Indicators which contains annual information on public debt in arrears, public debt service paid, and new public loans borrowed. We compute public arrears as the sum of public principal and interest in arrears. Arrears is the flow of funds that the country failed to pay during the year. Public debt service paid is the sum of principal and interest paid during the year.

We define partial default as the ratio of the arrears to the debt service due this period, which equals the sum of the debt service paid and arrears

$$\text{partial default } = \frac{\text{arrears}}{\text{arrears} + \text{debt service paid}} \tag{1}$$

Our definition of partial default essentially measures the fraction of payments missed. [2] Note that this measure of partial default differs from a debt haircuts. Debt haircuts are generally measured as the fraction of the value of debt in arrears that lenders lose after renegotiations. Partial default here measures the fraction of bonds in arrears.

A second variable of interest is the level of new borrowing for countries. The WDI dataset contains information for the net incurrence of external liabilities which measure new foreign liability financing. We define new loans as the ratio of net incurrence of external liabilities to gross national income

$$\text{new loans } = \frac{\text{net incurrence of foreign liabilities}}{\text{GNI}} \tag{2}$$

We also use data on debt to output ratios and GDP growth. Debt to output ratios are defined as the stock of public debt to GNI, where the stock of debt is measure at face value. The appendix contains the list of countries and variables in more detail.

2.2 Empirical Findings

The data on partial defaults and new loans provide a picture of sovereign defaults different from the standard discrete event view. Sovereign defaults are very heterogeneous and always partial. Countries continue to borrow during periods when they have positive arrears. Defaults are more complete in downturns.

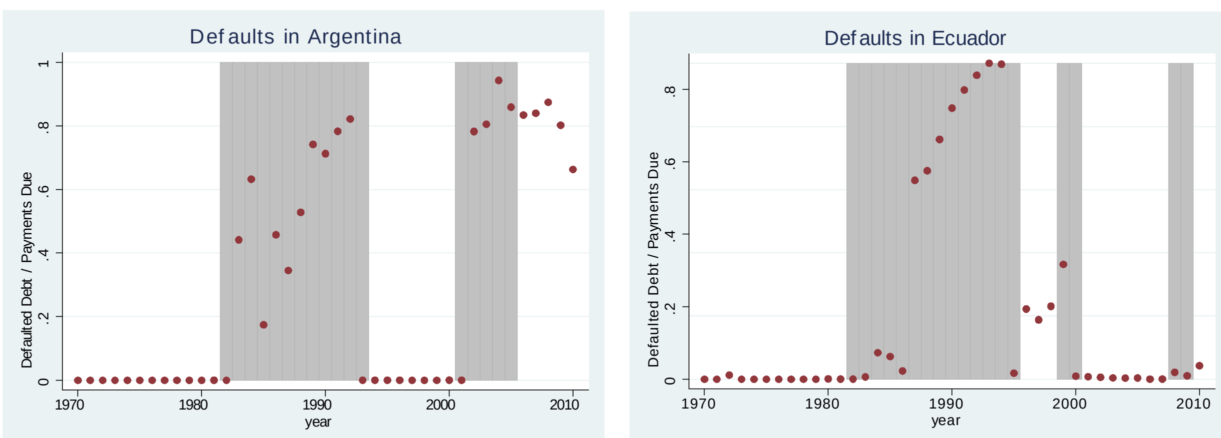
Table 1 reports the average statistics for partial default, frequency by which countries have positive arrears, and the debt ratios across all countries and years. The average partial

[2] A complementary measure of partial default could be the ratio of the market value of bonds in default to the market value of all debt. Unfortunately, the dataset does not have information on the market value of bonds in default.

default in the sample is 22%. Conditional on having positive arrears, partial default is on average 42%. The range for partial default however is large from close to 0 to close to 1. Countries frequency have positive arrears and about half of time. Debt stocks are large, on average about 54% of output. When countries have positive arrears they have larger debt ratios and on average 64% of output. Countries continue to borrow while having positive arrears, and average of 1.5% of output.

Table 1: Cross Country Data

Overall means	
Partial default	22%
Frequency of positive arrears	53%
Debt to Output	54%
Debt Service to Output	3.7%
Means when arrears>0	
Partial default	50%
Debt to Output	70%
Debt Service to Output	3.9%
Arrears to Output	18%
New loans to Output	1.7%



(a) Argentina (b) Ecuador

Figure 1: Time series for Partial Default

As an illustration Figure 1 plots the time series of the variable partial default for 2 classic defaulter countries, Argentina and Ecuador. The grey bars correspond to the Standard and Poors' years for when these countries were classified as in default. Generally the Standard and Poors' classification of default coincides with years when countries have positive arrears, but the correlation is not perfect. In the case of Argentina default was more partial in the

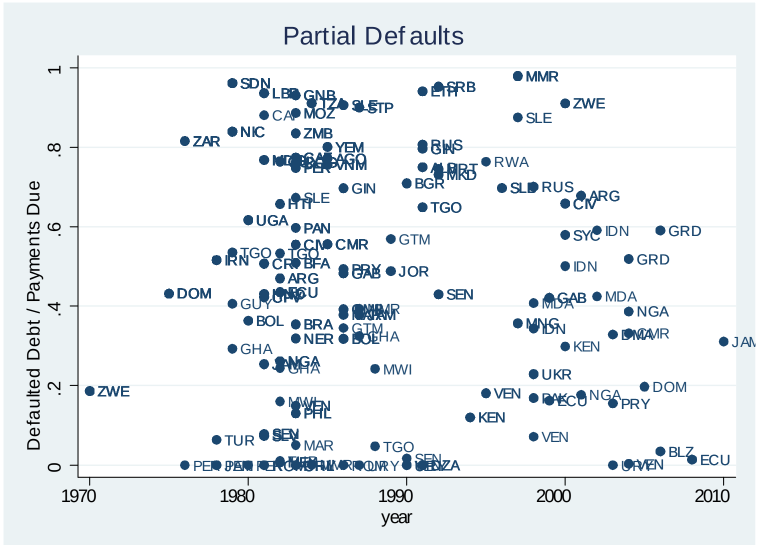


Figure 2: Defaults are Partial

late 1980s than in the early 2000s, and on average partial default in Argentina is 70%. After the renegotiation in 2005, Argentina continues to be in partial default due to positive arrears, while for S&P the country is no longer in default. The time series for Ecuador also tell a similar picture. During the three S&P defaults the country has positive arrears, although the 2008 default is very partial and on average less than 5%. In the late 1990s, Ecuador also had positive arrears while in the S&P dataset the country is not in default.

To illustrate the immense heterogeneity across defaults, in Figure 2 we plot the average partial default for each S&P default event. We average our annual partial default measure across all the years for each S&P default event and we plot that average in the year when the S&P default event starts and label it with the country indicator. The figure indicates that defaults are partial and vary across countries and years. The data indicates that in many S&P default events, arrears are small or in fact in some instances zero. The majority of default events are quite partial, and actually none of S&P default events is complete.

Figure 3 plots the average partial default for each S&P default event against the GDP growth in the beginning of the default event. The figure shows that the defaults are tend to be more complete when countries have downturns. The magnitude of this relation by running a linear regression implies that a 1% decline in growth increases partial default by about 1%.

Finally, we analyze the behavior of new loans during periods of positive arrears and compare them with periods when the country has no arrears. A word of caution for this data is that unfortunately we only have information of new loans for 17% of the country-year

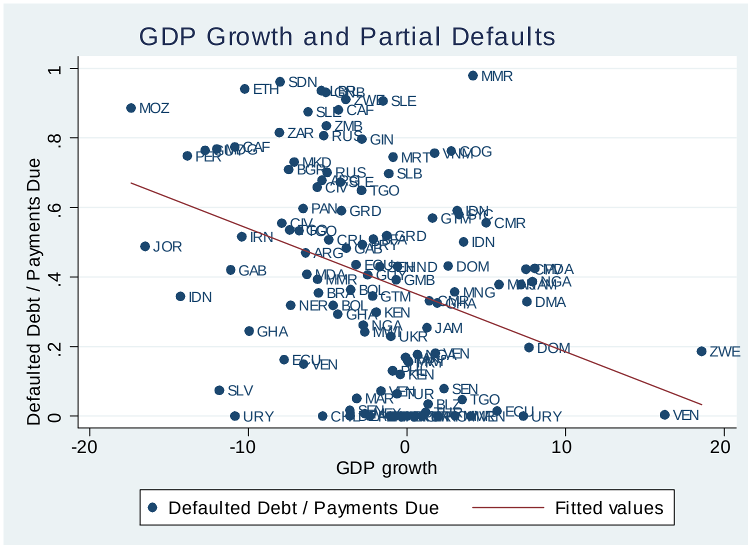


Figure 3: Partial Default and Growth

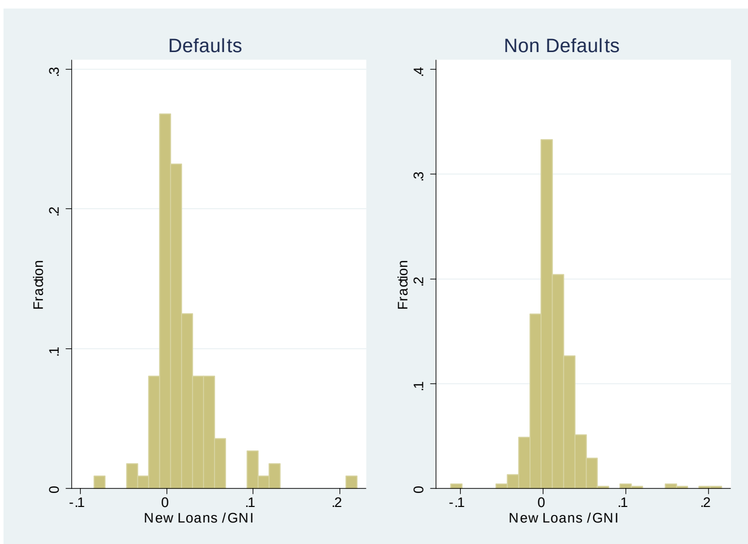


Figure 4: New Loans

observations. Figure 4 plots the histograms of new loans relative to GNI for the country-years when arrears is positive and when arrears is zero. The patterns of new loans are remarkably similar across these subsamples with similar averages of about 1%. [3]

3 The model

We consider a dynamic model of debt with partial default. A small open economy has a stochastic endowment stream. The economy borrows long term bonds and can partially default on its debt coupon payments at any time. The long bond is a perpetuity which coupon payments decay at rate δ. The defaulted coupon becomes delinquent and κ fraction is annuitized for future repayment at rate $\overline{R} - \delta$. Default entails a cost that is increasing in the amount of the delinquent debt. The economy can also pay off its delinquent debt at any time.

We use capital letters to denote the fact that the borrower is large, say a sovereign country, while the lenders may be small.

3.1 The borrower

The small open economy - henceforth the borrower - receives utility from consumption, C and discounts time at rate β. The borrower receives an endowment $\widehat{y}$ which is drawn from a distribution Γ_z. The index of the distribution z is itself a discrete Markov process with transition probabilities $\pi(z'|z)$.

Each period the borrower has total coupon obligations A, and chooses how much of these coupons to default on, D, and how much to borrow in the form of new loans B.

The choices of D and B imply that the next period total debt obligations are $A' = \delta A + B + \kappa(\overline{R} - \delta)D$. Delinquent debt $D > 0$ carries a direct cost on the endowment $\widehat{y}'$ that is increasing in the level of delinquent debt and is given by the function $\Psi(D)$. Moreover, κ fraction of delinquent debt remains as the borrowers future debt obligations and is annuitized at rate $\overline{R}$. For simplicity we annuitize the defaulted debt into perpetuities that also decay at rate δ. New loans carry a price $q(z, A', D)$ that compensates for the possible losses due to default.

The state vector of the borrower consists of three variables, (z, A, y). The Markovian

[3] Part of the reason why countries with positive arrears appear to borrow so much might be that countries might experience multiple partial renegotiations during default episodes and these fresh cash injections could be counted as new loans. Nevertheless, complete renegotiations are not included in periods with positive arrears because by construction arrears would be zero then. We are further exploring these issues.

structure of the shocks require us to keep track of z, the index of the distribution of the endowment. A is the total debt due today. Finally, the effective income of the borrower is $y = \widehat{y}\Psi(D)$ and it incorporates today's idiosyncratic shock $\widehat{y}$, and the difficulties of having a delinquent position $\Psi(D)$. The recursive problem of the borrower whose state is $\{z, A, y\}$ is

$$V(z, A, y) = \max_{c,B,D} u(c) + \beta \; E\{V(z', A', y') \mid z\} \tag{3}$$

$$\text{s.t.} \qquad c = y - (A - D) + q(z, A', D)\; B, \tag{4}$$

$$A' = \delta A + B + \kappa(\overline{R} - \delta)D, \tag{5}$$

$$y' = \widehat{y}' \; \Psi(D), \tag{6}$$

$$0 \leq D \leq A. \tag{7}$$

Here $q(z, A', D)$, the price of the new loan B, is an equilibrium object to be determined. We assume that $\Psi'(D) < 0$, $\Psi(0) = 1$, $\lim_{D\to 0} \Psi(D) < 1$, and $\lim_{D\to\infty} \; \Psi(D) > 0$.

This problem gives the optimal debt and default functions $B(z, A, y)$, $D(z, A, y)$, and $A'(z, A, y)$ as well as the consumption function $c(z, A, y)$.

3.2 The lenders

There are many identical lenders who lend to the borrower. They discount time at rate $1/R$. While the aggregate state of the arrangement is $\{z, A, y\}$, the individual state of any measure zero lender is $\{a\}$ which is its coupon debt holding. Each lender takes as given the borrowers decision rules $B(z, A, y)$, $D(z, A, y)$, and $A'(z, A, y)$

The value function of a lender is given by $\Omega(z, A, y, a)$ as follows

$$\Omega(z, A, y, a) = a \left(1 - \frac{D(z, A, y)}{A}\right) + \frac{1}{R} \, E\left\{ \, \Omega(z', A', y', a') \mid z\right\}, \qquad \text{with} \tag{8}$$

$$A' = \delta A + B(z, A, y) + \kappa(\overline{R} - \delta)D(z, A, y), \tag{9}$$

$$y' = \widehat{y}\Psi(D(z, A, y)), \tag{10}$$

$$a' = \delta a + \frac{\kappa(\overline{R} - \delta)D(z, A, y)}{A} \, a. \tag{11}$$

We can establish that $\Omega(z, A, y, a)$ is linear in a and solve it explicitly by guess-and-verify. Specifically, it takes on the form:

$$\Omega(z, A, y, a) = a \; H(z, A, y), \tag{12}$$

where the aggregate-state-dependent coefficient $H(.)$ solves

$$H(z,A,y)=\left(1-\frac{D(z,A,y)}{A}\right)+\frac{1}{R}\left(\delta+\kappa(\overline{R}-\delta)\frac{D(z,A,y)}{A}\right)\ E\left\{H(y',A',z')\mid z\right\}. \quad (13)$$

with A' and y' determined as above.

The expression $H(z,A,y)$ is the value to a claim of one unit of debt a. This value is intuitive to interpret. When a lender owns a claim to one unit of coupon a, it gets this period the portion $\left(1-\frac{D(z,A,y)}{A}\right)$ of the coupon. A claim today also has value tomorrow because debt contracts are perpetuities and because a fraction of the defaulted debt is paid in the future. Tomorrow, the lenders' holdings of a pay δ plus κ of the annuitized value of defaulted debt $(\overline{R}-\delta)\frac{D(z,A,y)}{A}$ of the next period's value. The expected value for every unit of debt tomorrow is precisely $E\left[H(y',A',z')|z\right]$.

Competition among lenders imply that every new loan to the borrower makes zero profits in expected value. This means that the bond price function is

$$q(z,A',D)=\frac{1}{R}\ E\left\{H(z',A',\widehat{y}'\Psi(D))\mid z\right\}. \quad (14)$$

Using the definition for $H(z,A,y)$, the bond price function solves the functional equation

$$q(z,A',D)=\frac{1}{R}\ E\left\{\left(1-\frac{D(z',A',y')}{A'}\right)+\left(\delta+\kappa(\overline{R}-\delta)\frac{D(z',A',y')}{A'}\right)q'(z',A'',D')\mid z\right\},$$

where $y'=\widehat{y}'\Psi(\overline{R}D)$.

3.3 Equilibrium

Definition 1. *A Markov Perfect Equilibrium without renegotiation is a set of decision rules for $\{B,D,c\}$, value for the lenders Ω, and the bond price function q such that the decision rules solve problem (3), the value for the lenders satisfies (12) and the equilibrium price satisfies (14).*

4 Characterization of Equilibrium

We now analyze the tradeoffs for borrowing using the two securities available B and D with the first order conditions. We assume that the bond price function $q(z,A',D)$ and the value function $v(z,A,y)$ are differentiable and that debt is one period, $\delta=0$. We substitute equations (2), (3), and (4) into the objective function. The first order conditions with respect to B and D are

$$u_c(c)[q(z,A',D)+q_A(z,A',D)B] \\ + \beta E V_A(\overline{R}D+B,\widehat{y}'\Psi(\overline{R}D),z')=0$$

$$u_c(c)[1+(q_A(z,A',D)\overline{R}+q_D(z,A',D))B]+\lambda_0-\lambda_A+$$

$$\beta\overline{R}E\left\{V_A(\overline{R}D+B,\widehat{y}'\Psi(\overline{R}D),z')+\widehat{y}'\Psi'(\overline{R}D)V_y(\overline{R}D+B,\widehat{y}'\Psi(\overline{R}D),z')\right\}=0,$$

where λ_0 and λ_A and the multipliers on the constraints $D \geq 0$ and $A-D \geq 0$ respectively.

The envelope conditions are:

$$\begin{aligned} V_A(z,A,y) &= -u_c(c)+\lambda_A, \\ V_y(z,A,y) &= u_c(c). \end{aligned}$$

Combining these conditions with the first order conditions we get the following Euler equations

$$u_c(c)[q(z,A',D)+q_A(z,A',D)B]=\beta E[u_c(c')-\widehat{\lambda}'_A] \tag{15}$$

$$u_c(c)[1/\overline{R}+B(q_A(z,A',D)+q_D(z,A',D)/\overline{R})]+\widehat{\lambda}_0-\widehat{\lambda}_A=\beta E[u_c(c')(1-\widehat{y}\Psi'(\overline{R}D))-\widehat{\lambda}'_A] \tag{16}$$

where $\widehat{\lambda}_j=\lambda_j/\overline{R}$.

These Euler equations illustrate that the borrower can transfer future resources to the present by borrowing with B or by defaulting D. Default acts as expensive debt.

Equation (15) is standard in dynamic default models. The LHS contains the marginal benefit from borrowing one unit of B which increases consumption by q but is discounted by the decrease in the price with more borrowing, $q_A < 0$. The RHS contains the discounted marginal cost of repaying back which is discounted by the fact that an extra unit of D relaxes the $A'-D' \geq 0$ constraint. If the borrower has more debt A' tomorrow, it can default more on it. The shape of the price function determines the intertemporal frontier from transferring future resources to the present with B.

Equation (16) contains the tradeoff when defaulting on a extra unit of D. The LHS of this equation says that defaulting on D is beneficial as consumption is increased by 1 but this benefit is discounted by the fact that the price for B falls by $(q_A+q_D/\overline{R})<0$. The benefit is limited by the level of current debt A as $A-D \geq 0$. The RHS contains the cost of paying the delinquent debt with rate $\overline{R}$ as well as the marginal output cost encoded in $\Psi' < 0$. Defaulting on debt today also increases future obligations A' and hence relaxes the $A' - D' \geq 0$ constraint. The default cost and the rate $\overline{R}$ determine the intertemporal frontier from transferring future resources to the present with D. The larger the marginal cost, i.e. the more negative the derivative Ψ' and the higher $\overline{R}$ the worse is borrowing with D.

In an interior solution with $\lambda_i = 0 \ \forall i$, we can obtain a standard portfolio equation by combining the two Euler equation

$$E[u_c(c')R^B] = E[u_c(c')R^D] \tag{17}$$

where R^B and R^D are the returns from one borrowing a unit of B and D defined by

$$R^B = \frac{1}{q(z, A', D) + q_A(z, A', D)B}$$

$$R^D = \frac{(\overline{R} - \overline{R}\widehat{y}\Psi'(\overline{R}D))}{[1 + (q_A(z, A', D)\overline{R} + q_D(z, A', D))B]}$$

The portfolio equation (17) says that at an interior optimal the expected returns from the two types of borrowing have to be equalized, with weights in each future state equal to the marginal utility of consumption.

The return of each security B or D equals to cost of repaying one unit of the security relative to the benefit in terms of total resources, $qB + D$, from borrowing one unit. Consider first the return of borrowing B. By borrowing an extra unit of B the marginal cost of repaying the security increases by one as cash on hand decreases by one. The marginal benefit equals the price q less the change in the price q_A with the extra unit of B. Total resources increase by less than the full price because prices decrease with more borrowing $q_A < 0$.

Now consider the return of issuing delinquent debt D. By issuing delinquent debt D the marginal cost of repaying the security increases by $\overline{R} - \widehat{y}\Psi'(\overline{R}D)$. Issuing one unit of D has two costs, first the direct reduction in cash on hand by $\overline{R}$ and second the reduction in cash on hand by the output cost $\overline{R}\widehat{y}\Psi'(\overline{R}D))$. The marginal benefit from issuing an extra unit of equals the marginal increase in total resources by 1 discounted also by the change in the bond price q. Defaulting affects the prices for borrowing in two ways. Prices for borrowing decrease with D because high D increases future obligations at the rate $\overline{R}$ as defaulted debt does not dissipate. Second, prices for borrowing decrease with D, $q_D < 0$ because the future costs in terms of output lower the paying capacity of the economy.

Note that when the relative cost of defaulting is too high, it might be optimal to set

$D = 0$. In addition, when the bond price schedule is too tight, it might be optimal to use mainly delinquent debt and set B close to zero because the term $(q_A\overline{R} + q_D)B$ can be eliminated with $B = 0$.

It is useful to illustrate the forces behind determining the choices of B and D by plotting the intertemporal frontiers of consumption today c and cash on hand tomorrow $y' - A'$. The budget set $\Gamma(y, A)$ for consumption today is given by

$$\Gamma(z, y, A) = \{(B, D) : c \leq y - A + q(z, A', D)B + D, c \geq 0\}$$

Expected cash on hand tomorrow is determined by D and B as follows

$$w(D, B, z) = E\hat{y}'\Psi(\overline{R}D) - (B + \overline{R}D)$$

The frontiers of Figure (5) plot the budget set for consumption against expected cash by varying B for a case when $D = 0$ and $D = A$.

When $D = 0$ reducing consumption today to zero increases expected cash on hand tomorrow by the total cash on hand today times the risk free rate $(1+r)(y-A)$. For low levels of B increasing B decreases cash on hand tomorrow one for one and increases consumption today at a rate $1/(1 + r)$. For sufficiently high B, default is a positive probability event tomorrow and $q < 1/(1 + r)$. The slope of the frontier becomes steeper by the derivative of the price functions with respect to A'. A main takeaway, which we share with other sovereign default models, is that default risk restricts the intertemporal transfer of resources through the shape of the bond price function. In this context, choosing full default today expands the budget set. The standard models of full default, such as Aguiar and Gopinath (2006) and Arellano (2008), expand the budget set by adding a point like X in the figure to the budget set. By defaulting on the debt due A the borrower can expand consumption today to the level of output at a cost of experiencing a reduction in output tomorrow.

In this model we expand the notion of delinquent debt relative to that in models of full default by keeping $\bar{R}$ of the debt as future obligations and by considering partial default. In this context, normal borrowing B is naturally allowed at all times. By expanding the notion of default, we essentially extend the budget sets further.

Consider now the case of full default when $D = A$ in our model. At low levels of consumption today the budget line is inside the case of $D = 0$ but parallel to it. The vertical distance equals the expected output costs associated with D. Allowing borrowing and default simultaneously further expands the budget set. When B is sufficiently high, nevertheless, the price of debt starts to fall because of default risk, and hence the slope of the frontier falls

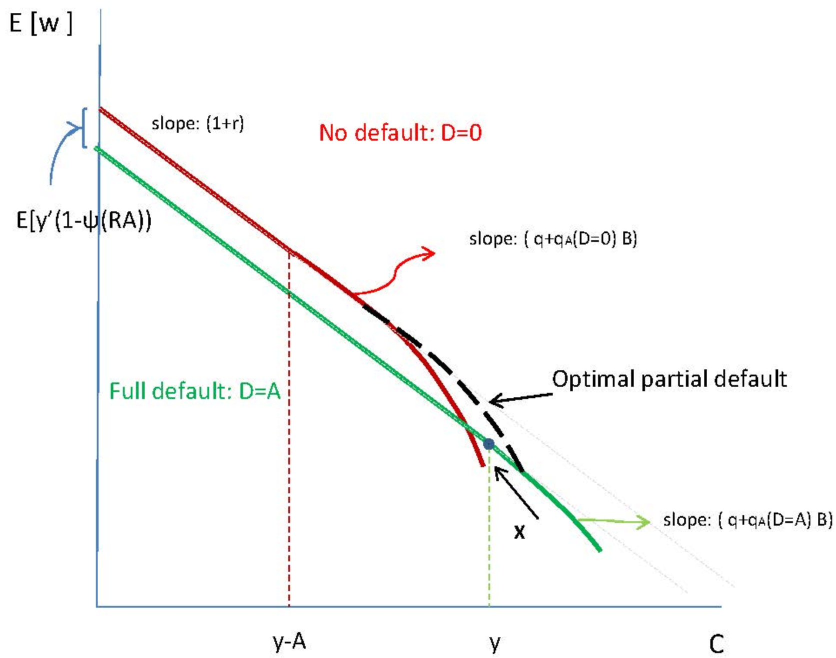


Figure 5: Intertemporal Frontiers

by the derivatives q_A.

The dash line in the figure considers is the upper contour for all possible budget lines for partial default with $0 < D < A$. Such contour determines the optimal partial default . Having partial default gives the borrower more flexibility in using the tradeoffs associated with each security and further expands the budget set.

5 Quantitative Analysis

We solve the model numerically to evaluate its performance against the empirical properties of sovereign defaults documented in Section 2. We parameterize the model to the data in Argentina, as a representative defaulter country. The patterns for partial defaults for Argentina are similar to the overall country averages.

5.1 Parameterization

To facilitate comparisons with the previous literature, we choose the following parameters from (Arellano (2008). The utility function for the country is $u(c) = c^{(1-\sigma)}/(1-\sigma)$. We set the coefficient of risk aversion to 2. The length of the period is one quarter and the risk free rate is set to 1.7%. We use data for Argentina's GDP to parameterize the process for output. We assume the stochastic process for output follows a lognormal AR(1) process and discretize the shocks into a seven-state Markov chain using quadrature.

Default costs are controlled by the function $\psi(D, z)$. We assume that the function takes the following flexible form:

$$\psi(D,z) = \psi_0(z)max\left(\frac{(D-\overline{D})^2(\gamma\overline{D}+D)}{(0-\overline{D})^2(\gamma\overline{D}+0)}, \underline{\psi}\right)$$

where $\psi_0(z) = \lambda_0(1-\lambda_1(z-\overline{z}))$ following (Chatterjee and Eyigungor (2011)). The function is decreasing in D and has a lower bound at $\underline{\psi}$. When $D = 0$ the function equals 1, i.e. no costs. The function also has a constant component that depends on z given by $\psi_0(z)$. Moreover, for the baseline results below we assume $\delta = 0$.

Our calibration strategy consists on choosing parameters for the default cost function $(\lambda_0, \lambda_1, \gamma, D, \underline{\psi})$, the discount factor β, and the fraction of arrears that are carried over $\overline{R}$ such that the model reproduces the following seven moments: an average spread of 13%, a standard deviation of spreads of 16%, an average debt service of 3%, a volatility of the trade balance relative to output of 0.23, a frequency of positive arrears of 0.70, and two conditional statistics for periods when arrears is positive, an average partial default of 0.70, and an average arrears relative to output of 9%. As in the empirical section partial default is defined as debt in arrears over total debt due D/A. Arrears relative to output is D/Y. We are still working on the calibration. Table (2) summarizes the current parameter values used in the quantitative results.

Table 2: Parameter Values

	Value	Target - Source
Borrowers' risk aversion	$\sigma = 2$	Arellano(2008)
Risk free rate	$r = 1\%$	
Process for shocks	$\rho = 0.945$, $\eta = 0.025$	
Calibrated parameters		
		Argentinian data:
Fixed default costs	$\lambda_0 = 1.0, \lambda_1 = 0.4$	spread: mean and volatility
Proportional default costs	$\gamma = 0.5$, $\overline{D} = 0.35$,	debt service
	$\underline{\psi} = 0.65$	trade balance volatility
Rate for Arrears	$\overline{R} = 0.3$	partial default rate
Borrowers' discount factor	$\beta = 0.94$	frequency of positive arrears
		average recovery

5.2 Main Results

Tables 3 and 4 report the averages and standard deviations of the variables of interest. We report the statistics for the Argentinian data, for our benchmark model, and for the full default in Arellano (2008).

Let's first consider the results in Table 3. As reported in the empirical section for the cross section of countries during periods of default (with positive arrears) Argentina had an average partial default of 70%, paid some of the debt with a debt service to output ratio of 3%, had high spreads of about 15%, and continued to borrow about 2.3% of output. The benchmark model can generate the richness in these variables during periods of default which is absence in the models of full default. During periods of default, the average partial default in the benchmark model is 90%, the economy pays about 3.6% of the debt, arrears are about 2% of output and new loans are positive but smaller than in the data and equal to 1/4 of a percent of output. Our model also contains market spreads during periods of default, which are absent by construction in models of full default and restricted borrowing. Spreads during positive arrears are higher in the model, just as in the data.

The benchmark model right now has too low frequency of positive arrears relative to the data due to a tension between a generating high enough debt to output levels and large frequency of positive arrears. In order to general substantial borrowing, the model requires a fixed cost in the cost of any default. However, with a substantial fixed cost, the frequency of arrears is low. We are currently exploring other cost functions and parameters.

In terms of business cycles, our model maintains many of the properties of models with full default which match many of the business cycles statistics well. Consumption is more volatile than output, spreads are countercyclical, and the correlation between the trade balance and spreads is positive. The benchmark model however does a better job than the full default model in generating a more volatile spread series. The reason is that in our model periods of defaults are periods of high spreads, just as in the data. By considering these periods, the model is able to generate a highly volatile spread series. Our model generates volatility in the series of partial default but the magnitude is lower than in the data. Default is more total when spreads are high as in the data which can be seen by the positive correlation between partial default and spreads.

Figure 6 shows a plot of the time series generated by the model. We plot output, spreads, partial default, and new loans. The plots illustrate the dynamics around defaults in our model. The output panel shows the dynamics of a recession where output goes from 6% above trend to 12% below trend.

When output falls by 4% in period 2, spreads rise from about 1% to 25% because of a

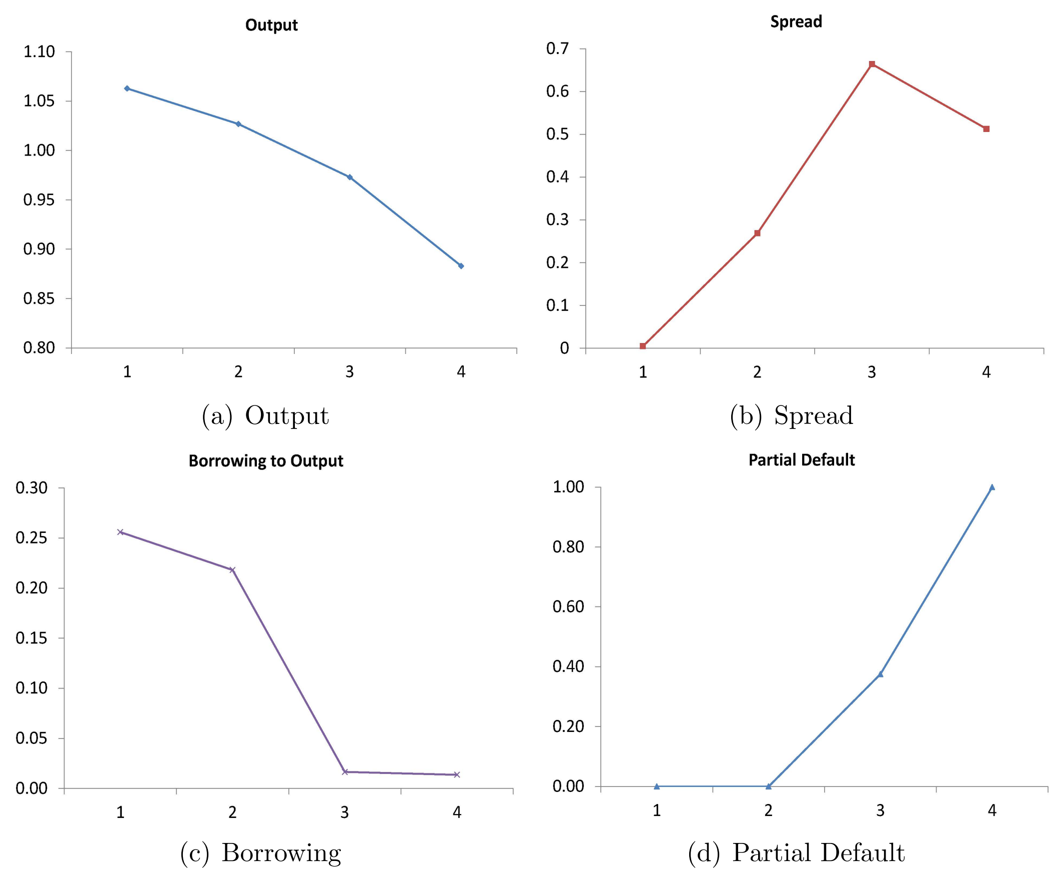
Table 3: Main Results

	Data	Benchmark	Full default
Means | Arrears >0 :			
Partial default	70%	90%	100%
Debt Service to Output	3.0%	3.6%	0%
Spreads	15%	11%	–
Arrears/Output	9.1%	2.2%	–
New loans/Output	2.3%	0.26%	0%
Overall Means			
Frequency of arrears >0	68%	3%	3%
Debt Service to Output	2.7%	6.4%	6%
Spread	13%	5%	3%

Table 4: Business Cycles

	Data	Benchmark	Full default
Standard deviations			
Consumption rel. to Output	1.10	1.06	1.10
Trade balance rel. to Output	0.23	0.22	0.26
Spread	16.2	13.2	3.2
Partial default	20.6	1.35	–
Correlations with spread			
Output	-0.88	-0.45	-0.29
Trade Balance	0.70	0.19	0.43
Partial default	0.24	0.59	–

rise in the default probability for period 3. In periods 1 and 2, the economy does not default yet but new borrowing falls from 25% of output to 21% because the bond price schedule is tighter in period 3. In period 3 output continues to fall, by about 5%. The economy here starts to default on about 38% of the debt due. New borrowing continues to be positive but tanks to about 2% as spreads rise to over 65%. In period 4 the economy experience another negative shock and output falls another 4%. Part of the reason for this decline, nevertheless, is due to the cost of having delinquent debt. The economy in a severe recession at this point, with output about 12% below trend. Default is here total, new borrowing continues to be small, about 1.4% of output and spreads are high, about 50%.



(a) Output (b) Spread

(c) Borrowing (d) Partial Default

Figure 6: Model Time Series

These plots show how that in the model, as in the data, many defaults start partial and end up complete after a sequence of bad shocks. If the economy were to recover in the middle of the default, however, the partial defaults would quickly be repaid and the default event would be end. The model also generates high spreads during default events, which is a feature of the data and absent in models of full default where spreads are not defined during default events because it is assumed that countries cannot borrow. Borrowing during defaults continues to be positive in the model, but restricted due to the high spreads.

5.3

5.4 Long Term Debt

To be completed

6 Conclusion

To be completed

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