Intergenerational Redistribution in the Great Recession

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Introduction

- Features of the Great Recession:
  1. Large fall in output and labor income
  2. Larger fall in asset prices (stocks, houses)
- What are the distributional consequences for households at different stages of the life-cycle?

Motivating Facts

1. Wealth varies substantially by age.
2. Portfolio composition (risky versus riskless assets) varies substantially by age.
3. Earnings losses vary by age.

- What is the net effect of these forces in allocating welfare losses across age groups?

Figure: Labor Income and Net Worth by Age, SCF 2007 ($1,000)

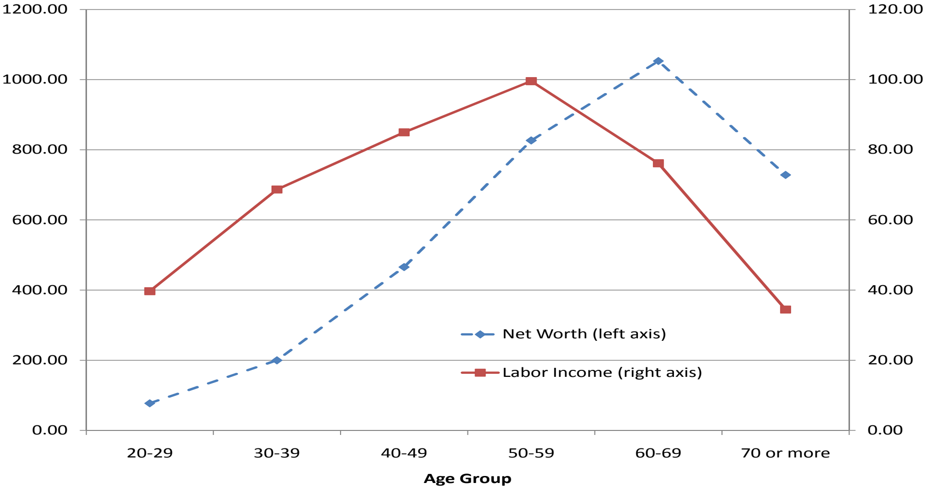
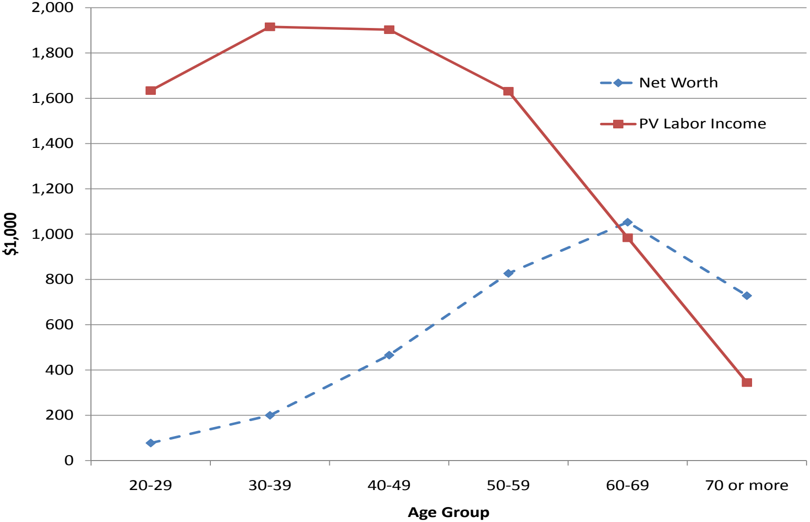


Figure: Present Value Labor Income and Net Worth by Age



Portfolio Shares, SCF 2007

Age of Head	% Risky	% Safe	Total ($1,000)
20-29	135%	-35%	77
30-39	140%	-40%	200
40-49	104%	-4%	466
50-59	92%	8%	827
60-69	85%	15%	1053
70+	79%	21%	728
All	94%	6%	555

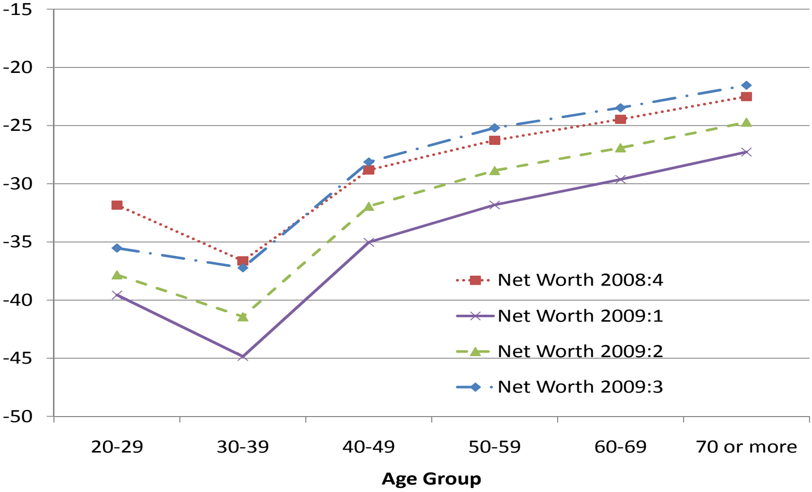
Risky NW: Stocks, Real Estate, Non-Corp. Bus.

Safe NW: Bonds, Cars, Other Assets, Debt

Percentage Decline in Net Worth from 2007:2 to 2009:1

Age of Head	Total ($1,000)	% of NW	% of Income
20-29	31	40%	79%
30-39	90	45%	128%
40-49	163	35%	175%
50-59	263	32%	223%
60-69	311	30%	286%
70+	199	27%	345%
All	177	32%	213%

Figure: **Decline in net worth by age relative to 2007:2 (percent)**



Percentage Decline in Labor Income, 2007-2009 (CPS, relative to trend GDP p.c.)

Age of Head		
20-29		-11.0%
30-39		-11.9%
40-49		-8.8%
50-59		-8.9%
60-69		-6.2%
70+		+1.6%
GDP p.c. (NIPA)		-8.3%

Goals for Theory

- Welfare consequences of downturn depend on future paths for wages and asset prices and on behavioral response
- $\Rightarrow$ Need a model to evaluate welfare effects
- General equilibrium delivers joint process for wages and endogenous prices
- Can the model generate a great recession?
  - wealth declines 3 times as much as output
- How are welfare losses distributed across households of different ages?
  - Can the young gain from a recession? How much do the old lose?

Related Literature

- OLG economies with aggregate risk:
  - Asset pricing: Huffman (1987), Constantinides, Donaldson and Mehra (2002), Storesletten, Telmer and Yaron (2007), Kubler and Schmedders (2010)
  - Allocations: a) Business cycles: Rios-Rull (1994, 1996), b) Intergenerational risk sharing: Smetters (2006), Krueger and Kubler (2006), Miyazaki, Sato and Yamada (2009).
- Redistributional consequences across age cohorts of other aggregate shocks:
  - Inflation: Doepke and Schneider (2006a,b), Meh, Rios-Rull and Terajima (2010)
  - Demographics: Rios-Rull (2001), Attanasio, Kitao and Violante (2007), Krueger and Ludwig (2007).
- Consumption disasters: Barro (2006, 2009), Nakamura, Steinsson, Barro and Ursua (2010).

The Model: Production

- Production function

$$Y(z) = z \;\; K^{\theta} \; L^{1-\theta}.$$

- Labor income and asset prices driven by same shock, $z \in Z$,
- z follows Markov process with transition matrix $\Gamma_{z,z'}$
- Total supply of labor $L = 1$
- Supply of fixed factor (land, capital) $K = 1$
- Wage (labor income) is $w(z) = (1 - \theta)z$
- Capital income is θz

The Model: Households

- Mostly OLG economies (also a representative agent economy)
- Households live for I periods
- Endowed with 1 unit of time supplied to the market inelastically
- Labor efficiency units $\{\varepsilon_i(z)\}_{i=1}^{I}$
- Zero initial wealth, no bequests
- Time discount factors $\{\beta_i\}_{i=1}^{I}$ vary with age
- Period utility function is CRRA $\quad u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}, \quad \sigma \neq 1$

The Sequence of Models

- Representative agent economy
- Simple OLG models with $I = 2$ and $I = 3$. Households trade equity (claims to capital income)
- Calibrated OLG models with $I = 6$.

1. Trade in equity only
2. Trade in leveraged (risky) stocks and (safe) bonds. Portfolio shares exogenous
3. Trade in leveraged stocks and bonds. Portfolio shares endogenous

Simple Example I: Representative Agent

- Exogenous net supply of bonds B
- Bond price $q(z)$, stock price $p(z)$
- Stock dividends

$$d(z) = \theta z - (1 - q(z))B$$

- Total start of period wealth given by

$$W(z) = p(z) + d(z) + B = p(z) + \theta z + q(z)B$$

Budget Constraints and Market Clearing

- Let a be share of total wealth owned by a household
- Chooses consumption $c(z,a)$, $y(z,a)$, fraction of savings in equity $\lambda(z,a)$:

$$\begin{aligned} c(z,a)+y(z,a) &= (1-\theta)z+W(z)\ a \\ a'(z',a)W(z') &= \left(\frac{\lambda(z,a)\left[p(z')+d(z')\right]}{p(z)}+\frac{(1-\lambda(z,a))}{q(z)}\right)y(z,a) \end{aligned}$$

- Market clearing

$$\begin{aligned} c(z,1) &= z \\ \lambda(z,1)y(z,1) &= p(z) \\ (1-\lambda(z,1))y(z,1) &= q(z)B \end{aligned}$$

Pricing in the Representative Agent Model

- Suppose $z \in \{z_L, z_H\}$

- Can solve exactly for $\widetilde{p} = \frac{p_H}{p_L}$ as a function of $\widetilde{z} = \frac{z_H}{z_L}$:

$$\widetilde{p} = \widetilde{z}\left(\frac{(1-\Gamma_{HH})\,\widetilde{z}^{\sigma-1} + \beta + \Gamma_{HH} - \beta\Gamma_{HH} - \beta\Gamma_{LL}}{(1-\Gamma_{LL})\,\widetilde{z}^{1-\sigma} + \beta + \Gamma_{LL} - \beta\Gamma_{HH} - \beta\Gamma_{LL}}\right)$$

- If z *iid* or $\beta = 1$ or $\sigma = 1$, then $\widetilde{p} = \widetilde{z}^{\sigma}$

- Let ξ^{RA} denote elasticity of relative prices to relative output:

$$\xi^{RA} = \frac{d \ln \widetilde{p}}{d\ \ln \widetilde{z}}$$

- In our favorite parameterization $\sigma = 3 \Rightarrow \xi^{RA} = 3$

The OLG Models: Notation

- State space (i, a, z, A),
  - $A = (A_1, \ldots, A_N)$ is the distribution of start of period wealth across age cohorts
  - a is the number of own shares
- Bond price $q(z, A)$, stock price $p(z, A)$, total wealth $W(z, A)$

Recursive Problem of Household

$$v_i(a,z,A) = \max_{c,y,\lambda,a'} \left\{ u(c) + \beta_{i+1} \sum_{z' \in Z} \Gamma_{z,z'} v_{i+1}(a',z',A') \right\}$$

$$\begin{aligned} c + y &= \varepsilon_i(z) w(z) + W(z,A)a \\ a' W(z',A') &= \left(\frac{\lambda \left[p(z',A') + d(z',A') \right]}{p(z,A)} + \frac{1-\lambda}{q(z,A)} \right) y \\ A'(z') &= G(z,A,z') \end{aligned}$$

- Policy functions $c_i(a,z,A)$, $y_i(a,z,A)$, $\lambda_i(a,z,A)$, $a'_i(a,z,A,z')$

The OLG Models: Consistency and Market Clearing

- Aggregate law of motion: $A_1'(z') = 0$ and

  $A_{i+1}'(z') = G_{i+1}(z, A, z') = a_i'(A_i, z, A, z')$ for all $i = 1, \ldots, I-1$

- Labor market: $w(z) = (1 - \theta)z$

- Financial Markets: $d(z, A) = \theta z - [1 - q(z, A)]\, B$

$$\begin{aligned}
\sum_{i=1}^{I} \lambda_i(A_i, z, A) y_i(A_i, z, A) &= p(z, A) \\
\sum_{i=1}^{I} (1 - \lambda_i(A_i, z, A)) y_i(A_i, z, A) &= q(z, A)B
\end{aligned}$$

The Model: Computation

- Even for moderate number of generations state space is large: $I - 2$ continuous state variables (plus z).
- We use both log-linearization and global methods based on Smolyak sparse grids (Krueger-Kubler-05, Krueger-Kubler-Malin-10).

OLG Economies

- Two simple examples to get intuition.
  - Two period OLG $\rightarrow$ no endogenous state variables
  - Three period OLG
- We then get serious and map the model to data
  - One asset economy
  - Two asset economy with exogenous age-specific portfolios
  - Endogenous portfolios (complete markets)

Simple Example II: 2 Period OG

- $I = 2 \Rightarrow$ old own all assets $\Rightarrow z$ is only state
- No bonds: $B = 0$, $\lambda_i = 1$
- $\varepsilon_2 = 0$ (only the young work)
- Budget constraints:

$$\begin{aligned} c_1(z) &= (1-\theta)z - p(z) \\ c_2(z) &= \theta z + p(z) \end{aligned}$$

- Prices determined by inter-temporal FOC for the young:

$$p(z)c_1(z)^{-\sigma} = \beta \sum_{z'} \Gamma_{z,z'} \left[c_2(z')^{-\sigma} \left(\theta z' + p(z') \right) \right]$$

Local Price Elasticity

- Suppose z is *iid*
- First-order approximations around steady state:

$$\xi^{2p} \approx \sigma \frac{(1-\theta)}{1-\theta\frac{(R-\sigma)}{(R-1)}}$$

where R is the steady state stock return.

- For $\sigma > 1, \xi^{2p} > 1$, but $\xi^{2p} < \xi^{RA}$

Intuition

- Following a bad shock, because prices fall more than output ($\xi_{2p} > 1$), the consumption of the old falls more than output

- Thus the consumption of the young must fall by less than output

- Thus equilibrium stock prices need not fall so much to induce the young to be willing to buy the stocks

- Given calibrated θ, β and $\sigma = 3$, we find $\xi^{2p} = 1.97$.

Can the Young Gain from a Recession?

- NO: need a lower price for the young to gain, but if the young have more consumption, the price will rise
- $\Rightarrow$ For the young to potentially gain we need at least 3 generations
- Need middle-aged to price stocks and take a hit, so the young can buy stocks cheaply
- Next example illustrates how this can work

Simple Example III: 3 Period OG

- $\varepsilon_1 = 1, \varepsilon_2 = \varepsilon_3 = 0$ (only young work).
- No utility from cons. when young (young save everything)
- State: (z, A_3) (then $A_1 = 0$ and $A_2 = 1 - A_3$)
- Only middle-aged are price sensitive. Euler equation is

$$\left[(1 - A_3)(p(z, A_3) + \theta z) - a' p(z, A_3)\right]^{-\sigma} =$$

$$\beta \sum_{z'} \Gamma_{z,z'} \left[a'(p(z', A_3') + \theta z')\right]^{-\sigma} \left[p(z', A_3') + \theta z'\right] / p(z, A_3)$$

with $A_3' = a'(z, A_3)$ in equilibrium

Simple Example III: 3 Period OG

- Market clearing (plus budget constraint of young)

$$[1 - a'(z, A_3)] \; p(z, A_3) = (1 - \theta) \, z$$

- The more assets the middle-aged sell, the more the young must buy, and the lower must be stock prices

- Numerical examples:
  - Preferences: $\beta = 0.459$, various σ.
  - Technology: $\theta = 0.3008$, $z_L / z_H = 1/1.1$. Shocks *iid*.

- For $\sigma = 3$ and $A_3 = 0.342$, $\xi^{3p} = 1.234$

Figure: ξ^{3p} Elasticity of Asset Prices to Output

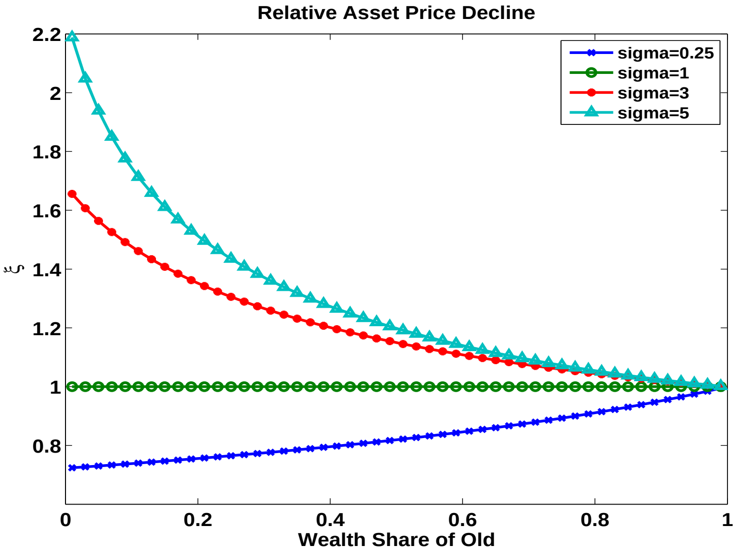
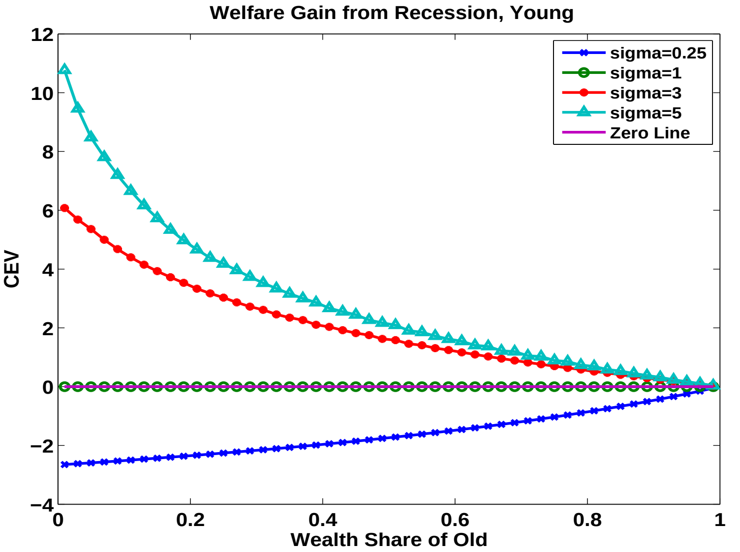


Figure: Welfare Consequences of Recessions for Young Households



Recap So Far

- In the Rep Agent world a large price change is needed to induce households to bite the recession bullet

- In 2 and 3 Period OLG the price response is smaller, because the old take a disproportionate hit.

- In 2 Period OLG, neither young nor old can gain from a recession.

- In some 3 Period OLG economies, if $\sigma > 1$, then the young win

- What what about the world we live in?

Quantitative Model: Calibration

- $I = 6$ (one period is 10 years)
- Risk aversion $\sigma = \{1, 3, 5\}$
- Endowments $\{\varepsilon_i(z_H)\}$ to match SCF labor income profile
- $\{\varepsilon_i(z_L)/\varepsilon_i(z_H)\}$ to match CPS recession declines by age
- Discount factors $\{\beta_i\}$ so that SS matches SCF asset profile
- Capital's share $\theta = 0.3008$, debt supply $B = 0.048$
- $\Rightarrow r^e = 4.75\%$, $r^b = 0.75\%$ in exog. portfolios economy
- z *iid*, and $\Gamma_H = 0.85$
- $z_L/z_H = 0.917$: matches fall in GDP pc relative to trend between 2007:4 and 2009:2

Calibration, Alternative Market Structures

- One asset economy: $B = 0$, $\{\lambda_i\} = 1$
- Two asset economy, exogenous portfolios: $\{\lambda_i\}$ to match age profiles from SCF
- Two asset economy, $\{\lambda_i\}$ endogenous: agents choose how much risk to bear
  - 2 values for the shock + 2 assets $\Rightarrow$ markets are complete
  - Simplify computation by assuming assets traded are state-contingent shares, then reconstruct equivalent portfolios in terms of stocks and bonds

Figure: Implied Discount Factors (2 assets, exogenous portfolios)

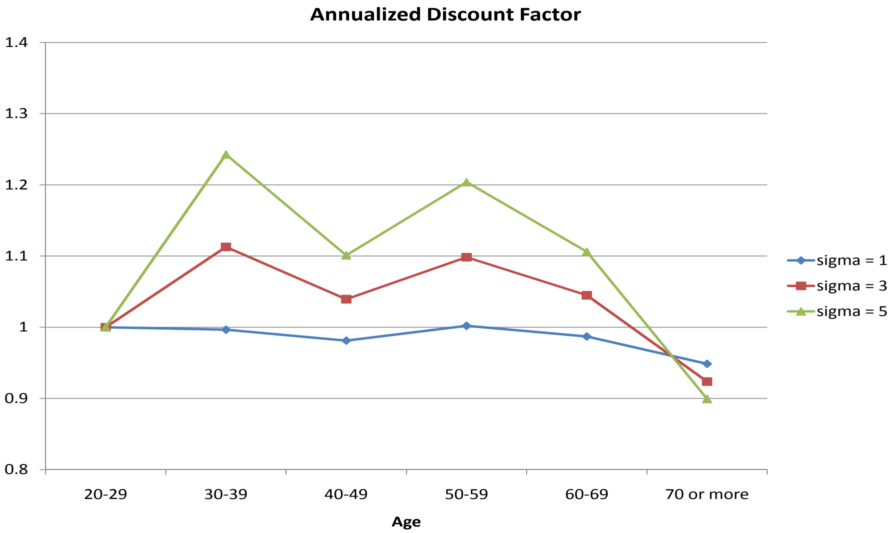
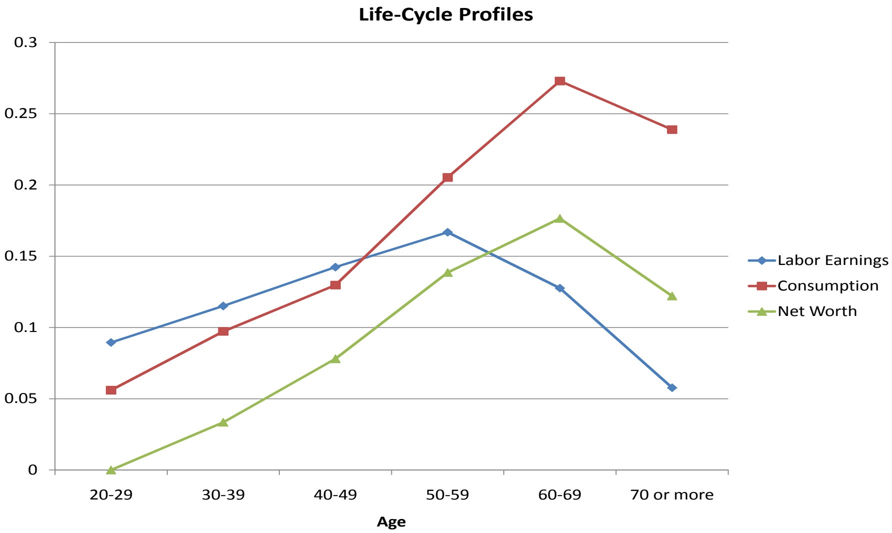


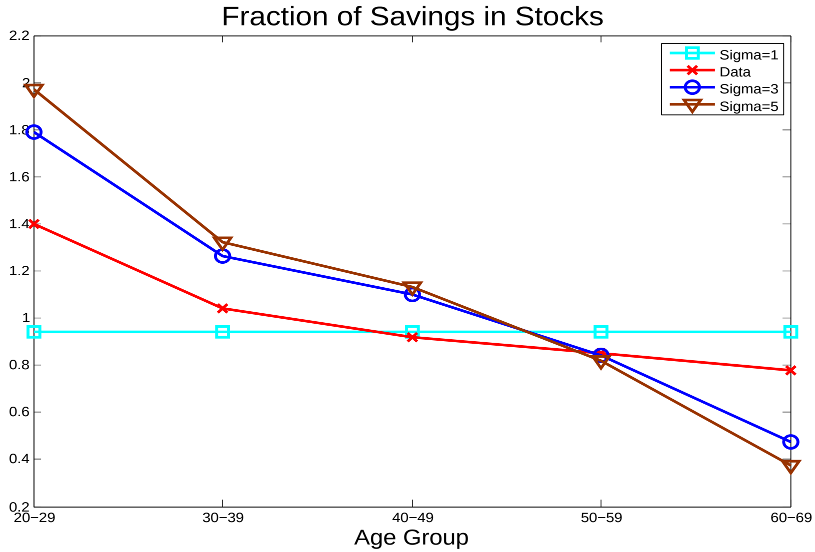
Figure: Life-Cycle Profiles, Data and Model (Implied for Cons)



Nature of the Experiments

- Let the economy enjoy $z = z_H$ for a long time until it settles down, $A = G(z_H, A)$
- Then look at dynamics along sequence $\{z_L, z_H, z_H, ...\}$
- Also look at a very long recession $\{z_L, z_L, z_L, ...\}$

Figure: Portfolios in Complete Markets Economy: $\varepsilon_i(z_H) = \varepsilon_i(z_L)$



Comments on Portfolios

- If $\sigma = 1$ and productivity shocks are age-neutral, then portfolios are age invariant
  - As in other models, prices are proportional to output
  - Thus age-invariant portfolios achieve perfect risk-sharing
  - This result requires shocks to be *iid*
- $\sigma > 1$ points to portfolio riskiness declining with age
  - Asset prices are more volatile than output and earnings
  - The old are more exposed to this risk

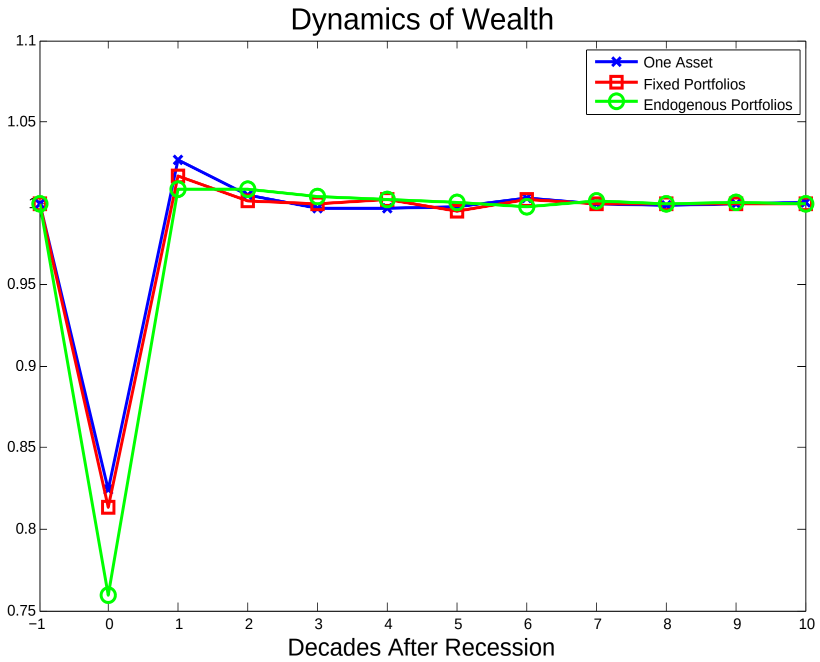

Dynamics of Wealth
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
−1
0
1
2
3
4
5
6
7
8
9
10
One Asset
Fixed Portfolios
Endogenous Portfolios
Decades After Recession

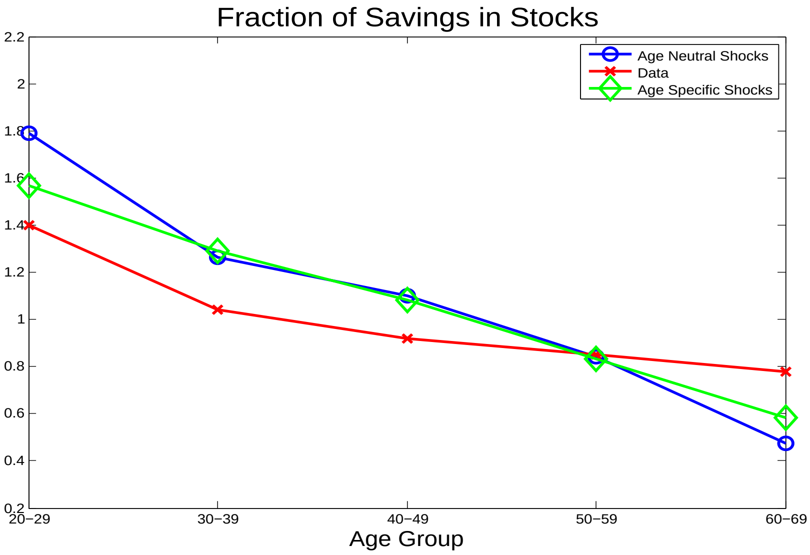
Table 1: **Expected Welfare Gain from One Period Recession**

Age i	$\sigma = 1$	$\sigma = 3$	$\sigma = 5$
	Single Asset Economy		
1	−1.43%	−0.78%	−0.38%
2	−1.72%	−1.19%	−0.82%
3	−2.14%	−1.29%	−0.67%
4	−2.85%	−2.75%	−2.17%
5	−4.24%	−6.26%	−6.57%
6	−8.30%	−12.66%	−15.16%
	Fixed Portfolio Economy		
1	−1.39%	−0.66%	−0.03%
2	−2.03%	−2.14%	−1.93%
3	−2.29%	−1.63%	−1.03%
4	−2.85%	−2.72%	−2.12%
5	−4.08%	−5.92%	−6.25%
6	−7.81%	−12.20%	−14.83%
	Endogenous Portfolio Economy		
1	−1.46%	0.33%	2.98%
2	−1.72%	−2.69%	−3.08%
3	−2.14%	−1.97%	−0.91%
4	−2.85%	−3.75%	−3.66%
5	−4.24%	−6.15%	−7.34%
6	−8.30%	−9.20%	−11.42%

Table 1: **Realized Welfare Gain from 6-Period Recession**

Age i	$\sigma = 1$	$\sigma = 3$	$\sigma = 5$
Single Asset Economy			
1	-8.30%	-5.48%	-4.00%
2	-8.30%	-5.99%	-4.29%
3	-8.30%	-6.90%	-4.90%
4	-8.30%	-9.27%	-9.22%
5	-8.30%	-11.81%	-14.42%
6	-8.30%	-12.66%	-15.16%
Fixed Portfolio Economy			
1	-8.51%	-5.82%	-4.00%
2	-8.63%	-7.11%	-5.68%
3	-8.24%	-7.24%	-5.55%
4	-8.00%	-8.97%	-9.17%
5	-7.85%	-11.21%	-13.88%
6	-7.81%	-12.20%	-14.83%
Endogenous Portfolio Economy			
1	-8.30%	-4.51%	3.93%
2	-8.30%	-8.60%	-7.69%
3	-8.30%	-7.70%	-4.94%
4	-8.30%	-9.00%	-9.51%
5	-8.30%	-9.43%	-11.35%
6	-8.30%	-9.20%	-11.42%

Figure: Portfolios in Alternative Economies



Comments on Portfolios

- Larger earnings cyclicality for the young leaves the young more exposed to aggregate risk
- But the old are remain more exposed to volatile return risk
- With $\sigma = 3$ and age-varying earnings risk, model portfolios closely resemble those in the SCF

Asset Price Elasticities

Table: **Relative price decline** $\left(\frac{\%\Delta(p_0/p_{-1})}{\%\Delta(z_0/z_{-1})}\right)$ **for Each Economy**

Economy	$\sigma = 1$	$\sigma = 3$	$\sigma = 5$
Single Asset	1.13	2.29	2.94
Fixed Portfolios			
–Stock	1.18	2.45	3.19
–Bond	0.86	2.52	3.53
–Wealth	1.15	2.46	3.21
Endogenous Portfolios			
–Stock	1.07	2.98	5.00
–Bond	1.05	3.00	5.01
–Wealth	1.07	2.98	5.00

Table 1: **Expected Welfare Gain from One-Period Recession, Age-Specific Decline in Earnings**

Age i	$\sigma = 1$	$\sigma = 3$	$\sigma = 5$
	Single Asset Economy		
1	−2.03%	−1.30%	−0.80%
2	−2.55%	−2.05%	−1.60%
3	−2.15%	−1.13%	−0.44%
4	−3.02%	−2.96%	−2.37%
5	−3.92%	−6.05%	−6.47%
6	−6.50%	−11.34%	−14.08%
	Fixed Portfolio Economy		
1	−1.97%	−1.20%	−0.45%
2	−2.81%	−3.08%	−2.83%
3	−2.31%	−1.49%	−0.81%
4	−3.03%	−2.93%	−2.32%
5	−3.73%	−5.68%	−6.12%
6	−5.83%	−10.69%	−13.57%
	Endogenous Portfolio Economy		
1	−2.16%	−0.44%	2.26%
2	−1.55%	−2.79%	−3.48%
3	−2.81%	−2.80%	−1.83%
4	−2.92%	−3.94%	−4.04%
5	−3.86%	−5.36%	−6.45%
6	−8.22%	−8.05%	−9.73%

Table 1: **Realized Welfare Gain from Recession**

Age i	$\sigma = 1$	$\sigma = 3$	$\sigma = 5$
Single Asset Economy			
1	−9.52%	−7.28%	−6.30%
2	−9.02%	−7.48%	−6.87%
3	−8.02%	−7.30%	−7.06%
4	−7.90%	−8.67%	−9.17%
5	−7.26%	−10.03%	−11.61%
6	−6.50%	−11.34%	−14.08%
Fixed Portfolio Economy			
1	−9.79%	−7.65%	−6.60%
2	−9.42%	−8.27%	−7.77%
3	−8.03%	−7.31%	−7.00%
4	−7.65%	−8.31%	−8.76%
5	−6.78%	−9.48%	−11.10%
6	−5.83%	−10.69%	−13.57%
Endogenous Portfolio Economy			
1	−9.13%	−6.25%	1.73%
2	−8.30%	−9.7%	−9.82%
3	−8.53%	−8.66%	−6.62%
4	−7.67%	−8.48%	−9.15%
5	−7.34%	−8.19%	−9.86%
6	−7.65%	−8.05%	−9.73%

Conclusions

1. We explored the implications for asset prices of large recessions. Theory predicts price drops almost as large as those in the data (around 20%)

2. We explored the redistributive implications of such recessions. Old lose a lot, young lose less (and might even gain). If markets are complete the losses of the old are smaller

3. Our theory replicates observed portfolios well

4. Possible motivation for policies (TARP?, LSAP?) that boost asset prices and benefit the old