Partial Default

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Sovereign defaults are somewhat frequent in developing countries. Defaults are commonly thought as discrete events: country either repays or defaults on all its debt. (As if they filed for bankruptcy like people.) But defaults are very heterogeneous events.  
- Some defaults have costly and lengthy resolutions.  
- Others defaults are minor with fast resolutions.
Existing Theory

- Quantitative models of sovereign default have countries either repaying or defaulting in full.  

- With countries restructuring all of its debt after default.  
  (Yue 2010, Benjamin and Wright 2009, D'Erasmo 2012).

- Default as state contingent assets does not sit well with the evidence that default is costly.  
  (Trade costs, Rose 2002; financial crises, Reinhart and Rogoff 2010; lawsuits and sanctions, Hatchondo & Martinez 2013).

- The theory is Non-Markovian. It requires coordination among existing and prospective lenders.
This paper

- We document the properties across heterogeneous sovereign defaults.
  1. Sovereign defaults are partial.
  2. During defaults sovereigns continue to receive foreign credit.
  3. Larger defaults in downturns.

- We develop a Markovian model of partial default.
  ▶ The model promising for explaining the heterogeneity across defaults.
  ▶ The environment requires output loses when debt is in arrears, and partial recovery of those debts.
DATA
Defaults in the Data

- Panel data for 99 developing countries from 1970-2010.

- Public debt data from World Development Indicators: debt in arrears and new loans.

- Default events from Standard & Poor and Trebesch and Cruces (2012).

\[
\text{Partial Default} = \frac{\text{Debt in Arrears}}{\text{Debt Service} + \text{Debt in Arrears}}
\]

- Debt in Arrears = Interest and principals due this period but in arrears.
Defaults in the Data: 1. Sovereign Default is Partial

Default events are associated with large arrears but default is partial
Defaults in the Data: 1. Sovereign Default is Partial

Defaults in Ecuador

Defaults can be very small as in 2008.
1. Sovereign Default is Partial

![Defaults in Indonesia](chart)

- Arellano, Mateos-Planas, Rios-Rull ()

**Chart Description:**
- The chart illustrates the defaulted debt/payments due in Indonesia over the years 1990 to 2010.
- The x-axis represents the years (1990 to 2010) and the y-axis represents the defaulted debt/payments due, ranging from 0 to 0.8.
- The data points indicate a notable increase in defaulted debt/payments due around the years 2000 and 2005.

**Note:**
- The chart highlights the trend of increased defaults, particularly around the turn of the millennium and later.
Defaults in the Data: 1. Sovereign Default is Partial

Defaults in Argentina

year

Defaul ted Debt / Payments Due

0
.2
.4
.6
.8
1

Defaults in the Data: 1. Sovereign Default is Partial

Across all S&P default: countries default on average on 59% of what is due.
Defaults in Data: 2. Borrowing during Sovereign Default

During default countries continue to borrow
Defaults in Data: 2. Borrowing during Sovereign Default

Countries get new loans during defaults almost as much as in normal times,
Caveat: Data on new government loans contains many missing observations.
Defaults in Data: 3. Larger Default in Downturns

Partial Default and GDP Growth

Arellano, Mateos-Planas, Rios-Rull ()
THEORY
Ingredients of our theory

- Limited, but not inexistent, legal system in the world allows for sovereign default. However,
  - Creditors of defaulted debt create some havoc. Costs increasing in the level of defaulted debt.
  - Defaulted Debt does not disappear, it remains in the balance sheet until repayment (like Venezuela 2005) or renegotiated (with a wide range of haircuts - 0-100%). Today a constant fraction of debt in arrears survives.

- Inability of lenders to coordinate to exclude further future lending (free entry in lending markets (Krueger and Uhlig 06)).

- Markov Equilibria (when non multiple equilibria in the static counterpart, it is the limit of equilibria in finite horizon economies).
Model
Dynamic model of borrowing and default

- Small open economy with stochastic endowment $z$ which is Markov with transition $\Gamma_{z,z'}$.

- The small open economy trades bonds with international lenders (often, but not always, borrows, hence borrower) and can default on them.

- Cost of defaulting reduces next period output and is increasing with the level of defaulted debt.

- Lenders are risk neutral.
Borrower

- Trades perpetuity bonds that decay at rate $\delta$.
- Has coupons $A$, defaults on $D$, borrows $B$.

$$c = y - (A - D) + q(z, A', D) B$$

$$D \leq A$$

- Total coupon obligations tomorrow.

$$A' = \delta A + B + (\bar{R} - \delta) D$$

- $D$ remains as future obligations annuitized at rate $\bar{R}$.
- $D > 0$ has direct costs on endowment with $y' = z' \psi(D)$.
- Price functions $q(z, A', D)$ describe access to credit.
Recursive Problem: Borrower

- **State:** \((z, A, y)\). Nature, what it owes, what it has.

- Choose consumption, new loans, and default.

\[
V(z, A, y) = \max_{c, B, D} u(c) + \beta E \left\{ V'(z', A, y') \mid z \right\}
\]

s.t.

\[
\begin{align*}
c &= y - (A - D) + q(z, A', D) B \\
A' &= \delta A + B + \bar{R} D \\
y' &= z' \psi(D), \quad 0 \leq D \leq A.
\end{align*}
\]

- Resulting policy functions: \(B(z, A, y)\), and \(D(z, A, y)\).
Recursive Problem: Lenders

- Take as given policy functions and discount at world’s interest rate $r$.

- Value to a claim of one unit.

\[
H(z, A, y) = \left( 1 - \frac{D(z, A, y)}{A} \right)
\]

Today

\[
+ \frac{1}{1+r} \left( \delta + (\bar{R} - \delta) \frac{D(z, A, y)}{A} \right) E\{H(z', A', y')\}.
\]

Tomorrow

- $A'$ and $y'$ are determined by borrower’s functions.
Bond Price and Equilibrium

- Zero profit condition determines price functions

\[ q(z, A', D) = \frac{1}{1 + r} \ E\{H(z'\psi(D), A', z')|z}\].

- Compensates for expected loss in default.

- Partial defaults give price of debt a long-term component.

- Markov equilibrium is the obvious thing. The small country maximizes given prices and the free entry condition given the expected return of loans.
Default as Expensive Debt

- Transfer future resources towards present with $B$ or $D$.
- Let $w = y - A$ denote cash in hand.
- Standard consumption-savings trade-off:
  - Increase in consumption with $B$ or $D$
    \[ c - w = q(A', D, z)B + D \]
  - By reduction in cash on hand tomorrow
    \[ w' = z'\psi(D) - A' \]
    with \( A' = B - \bar{RD}, \ D < A \)

- $B$ is restricted by $q(A', D, z)$.
- $D$ is restricted by $A$ and carries additional cost through $\psi(D)$.
Budget Constraint

\[ C_{t+1} \]

\[ C_t \]

Risk Free

0

1

0 0 .5 1 .0

Borrow
Budget Constraint

\[ C_{t+1} \]

\[ C_t \]

\[ 0 \]

\[ 1 \]

Borrow

Default

Both
Budget Constraint

\[ C_{t+1} + C_t = 0 \]

- Borrow
- Default
- Both
Variety of Examples

- Explore the numerical properties of these economies.

- We look for the properties in the data that we documented
  - Sovereign defaults are partial.
  - During defaults sovereigns continue to receive foreign credit.
  - Larger defaults in downturns.

- Designed to resemble developing countries with a year.
  - There is a fixed cost to default.
  - Various economies that differ in the size of debt and persistency of shocks.
Numerical settings

- Default cost $y = z'\psi(D)$ decreasing and concave with lower bound

$$\psi(D) = \psi_0 \max \left\{ \frac{(D-\bar{D})^2(\gamma\bar{D}+D)}{(0-\bar{D})^2(\gamma\bar{D}+0)}, \psi \right\}$$

- Explore 3 experiments: High debt, low debt, and persistent shocks

- Common parameters: $\sigma = 2$, $r = 1.7\%$, $\delta = 0$, $\bar{R} = 0.80$, $\gamma = 0.5$, $\psi = 0.9$.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Example 1 (High Debt)</th>
<th>Example 2 (Low Debt)</th>
<th>Example 2 (Persistent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock process</td>
<td>$z$</td>
<td>iid with $\sigma_H$</td>
<td>iid with $\sigma_H$</td>
<td>Argentina</td>
</tr>
<tr>
<td>Penalty slope</td>
<td>$\bar{D}$</td>
<td>0.5</td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>$\psi_0$</td>
<td>0.99</td>
<td>0.99</td>
<td>0.995</td>
</tr>
<tr>
<td>Discount</td>
<td>$\beta$</td>
<td>0.85</td>
<td>0.85</td>
<td>0.94</td>
</tr>
</tbody>
</table>
## Average statistics

<table>
<thead>
<tr>
<th>Data</th>
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<tbody>
<tr>
<td><strong>Partial default</strong></td>
</tr>
<tr>
<td><strong>Frequency of default</strong></td>
</tr>
<tr>
<td><strong>Debt /Output</strong></td>
</tr>
<tr>
<td><strong>Spread</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>During defaults:</th>
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<tbody>
<tr>
<td><strong>Debt/GDP</strong></td>
</tr>
<tr>
<td><strong>Spreads</strong></td>
</tr>
<tr>
<td><strong>Arrears/Output</strong></td>
</tr>
<tr>
<td><strong>New loans/Output</strong></td>
</tr>
<tr>
<td><strong>Output relt. to Mean</strong></td>
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</tbody>
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Large frequency of partial defaults  
During defaults debt is large, output is low, countries continue to borrow
## Average statistics

<table>
<thead>
<tr>
<th></th>
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<th>High Debt</th>
<th>Examples</th>
<th>Low Debt</th>
<th>Persistent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partial default</td>
<td>59%</td>
<td>18%</td>
<td>100%</td>
<td>76%</td>
<td></td>
</tr>
<tr>
<td>Frequency of default</td>
<td>51%</td>
<td>11%</td>
<td>30%</td>
<td>74%</td>
<td></td>
</tr>
<tr>
<td>Debt /Output</td>
<td>49%</td>
<td>30%</td>
<td>1%</td>
<td>19%</td>
<td></td>
</tr>
<tr>
<td>Spread</td>
<td>–</td>
<td>0.5%</td>
<td>17%</td>
<td>1.5%</td>
<td></td>
</tr>
<tr>
<td>During defaults:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt/GDP</td>
<td>87%</td>
<td>41%</td>
<td>5.5%</td>
<td>13%</td>
<td></td>
</tr>
<tr>
<td>Spreads</td>
<td>–</td>
<td>1.05%</td>
<td>43%</td>
<td>1.8%</td>
<td></td>
</tr>
<tr>
<td>Arrears/Output</td>
<td>6.2%</td>
<td>36%</td>
<td>5.5%</td>
<td>5.3%</td>
<td></td>
</tr>
<tr>
<td>New loans/Output</td>
<td>1.07%</td>
<td>7.3%</td>
<td>2%</td>
<td>8.6%</td>
<td></td>
</tr>
<tr>
<td>Output relt. to Mean</td>
<td>-1.4%</td>
<td>-15%</td>
<td>-8%</td>
<td>-4%</td>
<td></td>
</tr>
</tbody>
</table>
Examples confirm partial default is alternative credit

Intertemporal frontier: Only B

![Graph showing the relationship between tomorrow's cash in hand and today's consumption-cash in hand.](image)

Arellano, Mateos-Planas, Rios-Rull ()
Intertemporal frontier: Only B

Concave frontier via $q(.)$ due to increasing default risk
Intertemporal frontier: Only D

Shape of frontier depends on $\psi(D)$ and $\bar{R}$
Smaller transfers with $B$, intermediate with $B + D$, large with $D$
Policy Functions: Borrow and Default

Small debt: $B > 0$, and $D = 0$
Large debt: $B = 0$, $D = A$. Endogenously borrow less due to bad price.
Price decreases with larger debt and is worse when default $D > 0$. 
Implication 1: Partial Default

Default is always partial. Narrow range

Partial Defaults (High Debt)
Implication 1: Partial Default

Wide range of partial default

Arellano, Mateos-Planas, Rios-Rull ()
Implication 2. Borrowing during Sovereign Default

New loans are used much more actively than defaults.
Implication 2. Borrowing during Sovereign Default

New loans and defaults actively used. Large substitution between two
Implication 3: Larger Default in Downturns

Defaults only with the lowest income
Implication 3: Larger Default in Downturns

Larger defaults with lower income
Conclusion

- Sovereign default is partial and countries continue to borrow during defaults.

- Propose new (Markovian) theory consistent with these facts.

- Continuing work:
  - Take model to data. Move a bit out of examples.
  - Model as laboratory for recovering costs of default.

- Link it with partial individual default (Herkenhoff and Ohanian (13)).
Policy Functions: Persistent Case

Arellano, Mateos-Planas, Rios-Rull
Persistent case: Frontier low shock

![Graph showing the relationship between cash in hand tomorrow and consumption cash in hand (today). The graph includes lines for different scenarios: D=0, B=0, and D_opt. The productivity (Prod) is 0.4162774.](image)
Persistent case: Frontier high shock

![Graph showing consumption-cash in hand today as a function of cash in hand tomorrow with different scenario lines.]

Arellano, Mateos-Planas, Rios-Rull ()

Partial Default

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