

# Partial Default

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# Motivation

- Sovereign defaults are somewhat frequent in developing countries.
- Defaults are commonly thought as discrete events: country either repays or defaults on all its debt.

(As if they filed for bankruptcy like people.)

- But defaults are very heterogeneous events.
  - ▶ Some defaults have costly and lengthy resolutions.
  - ▶ Others defaults are minor with fast resolutions.

# Existing Theory

- Quantitative models of sovereign default have countries either repaying or defaulting in full.

(Aguiar and Gopinath 2006, Arellano 2008).

- With countries restructuring all of its debt after default.

(Yue 2010, Benjamin and Wright 2009, D'Erasmus 2012).

- Default as state contingent assets does not sit well with the evidence that default is costly.

(Trade costs, Rose 2002; financial crises, Reinhart and Rogoff 2010; lawsuits and sanctions, Hatchondo & Martinez 2013).

- The theory is Non-Markovian. It requires coordination among existing and prospective lenders.

# This paper

- We document the properties across heterogeneous sovereign defaults.
  - ① Sovereign defaults are partial.
  - ② During defaults sovereigns continue to receive foreign credit.
  - ③ Larger defaults in downturns.
- We develop a Markovian model of partial default.
  - ▶ The model promising for explaining the heterogeneity across defaults.
  - ▶ The environment requires output losses when debt is in arrears, and partial recovery of those debts.

# DATA

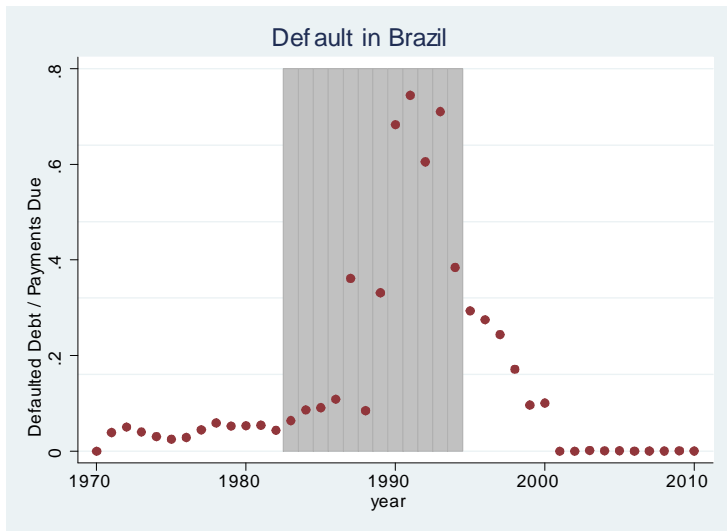
## Defaults in the Data

- Panel data for 99 developing countries from 1970-2010.
- Public debt data from World Development Indicators: debt in arrears and new loans.
- Default events from Standard & Poor and Trebesch and Cruces (2012).

$$\text{Partial Default} = \frac{\text{Debt in Arrears}}{\text{Debt Service} + \text{Debt in Arrears}}$$

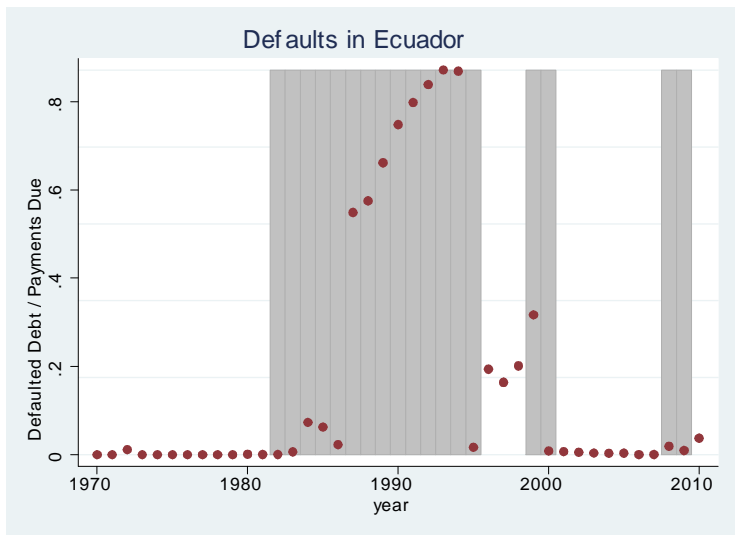
- Debt in Arrears = Interest and principals due this period but in arrears.

# Defaults in the Data: 1. Sovereign Default is Partial



Default events are associated with large arrears but default is partial

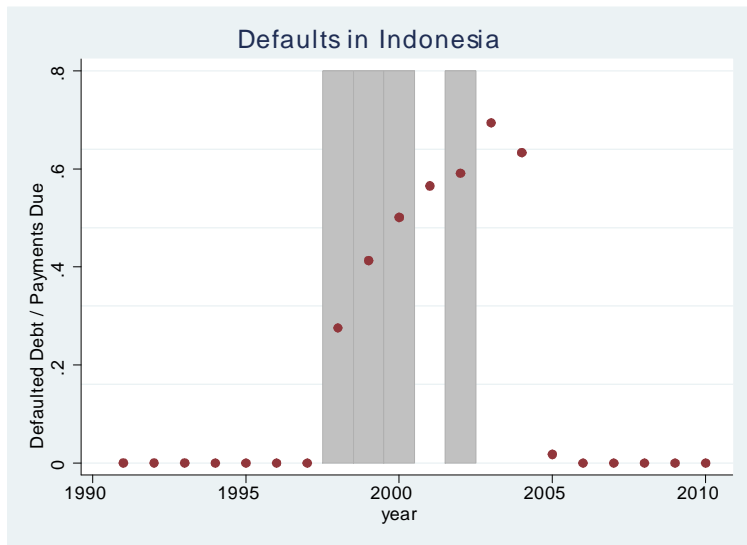
## Defaults in the Data: 1. Sovereign Default is Partial



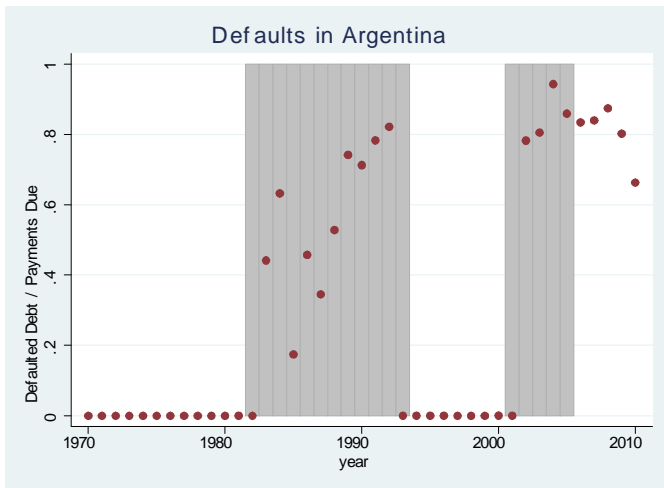
Defaults can be very small as in 2008.



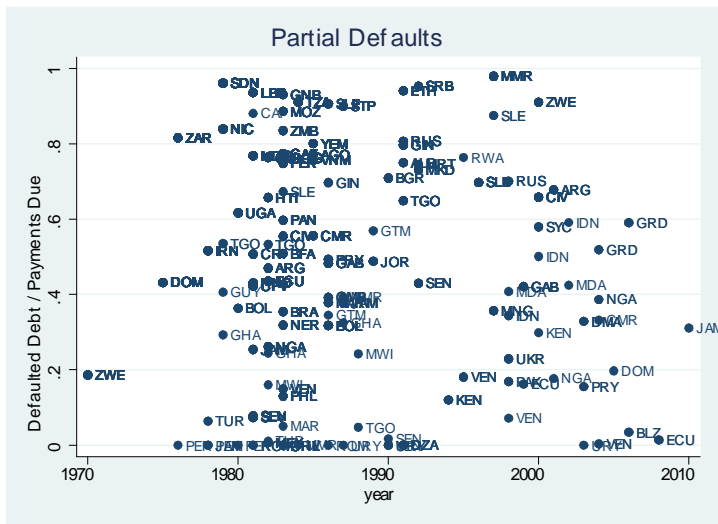
# 1. Sovereign Default is Partial



# Defaults in the Data: 1. Sovereign Default is Partial

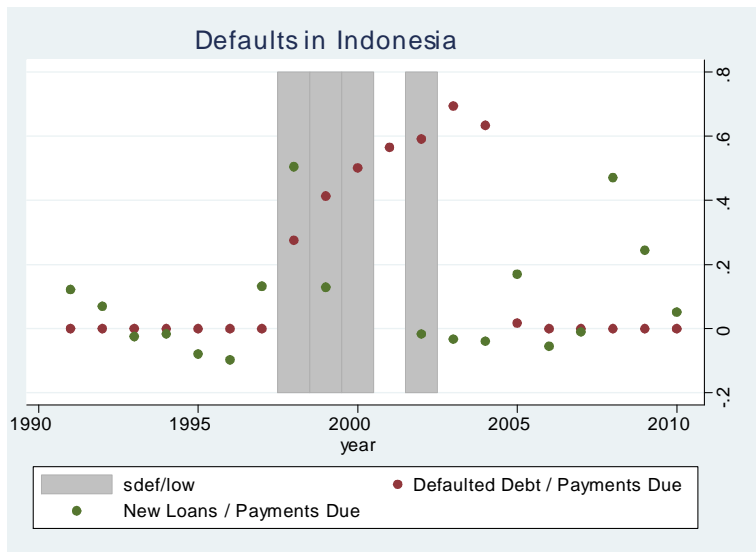


# Defaults in the Data: 1. Sovereign Default is Partial



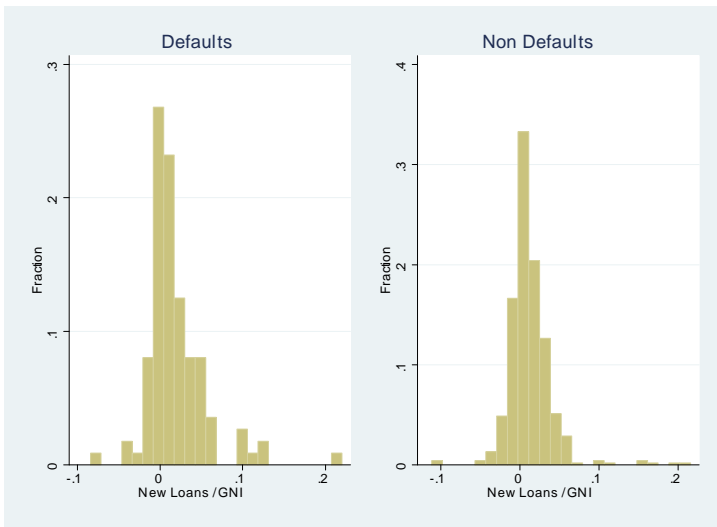
Across all S&P default : countries default on average on 59% of what is due.

## Defaults in Data: 2. Borrowing during Sovereign Default



During default countries continue to borrow

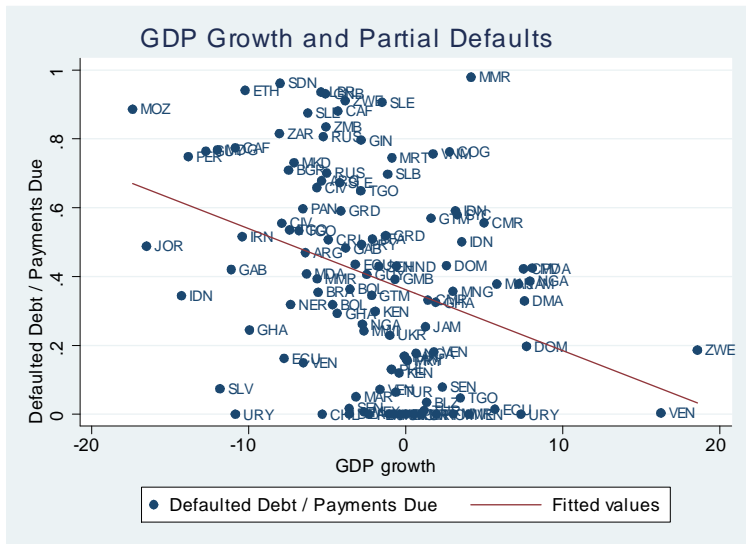
## Defaults in Data: 2. Borrowing during Sovereign Default



Countries get new loans during defaults almost as much as in normal times,  
Caveat: Data on new government loans contains many missing observations.

# Defaults in Data: 3. Larger Default in Downturns

## Partial Default and GDP Growth



# THEORY

# Ingredients of our theory

- Limited, but not inexistent, legal system in the world allows for sovereign default. However,
  - ▶ Creditors of defaulted debt create some havoc. Costs increasing in the level of defaulted debt.
  - ▶ Defaulted Debt does not disappear, it remains in the balance sheet until repayment (like Venezuela 2005) or renegotiated (with a wide range of haircuts- 0-100%). Today a constant fraction of debt in arrears survives.
- Inability of lenders to coordinate to exclude further future lending (free entry in lending markets (Krueger and Uhlig (06))).
- Markov Equilibria (when non multiple equilibria in the static counterpart, it is the limit of equilibria in finite horizon economies).



# Model

## Dynamic model of borrowing and default

- Small open economy with stochastic endowment  $z$  which is Markov with transition  $\Gamma_{z,z'}$ .
- The small open economy trades bonds with international lenders (often, but not always, borrows, hence borrower) and can default on them.
- Cost of defaulting reduces next period output and is increasing with the level of defaulted debt.
- Lenders are risk neutral.

## Borrower

- Trades perpetuity bonds that decay at rate  $\delta$ .
- Has coupons  $A$ , defaults on  $D$ , borrows  $B$ .

$$c = y - (A - D) + q(z, A', D) B$$
$$D \leq A$$

- Total coupon obligations tomorrow.

$$A' = \delta A + B + (\bar{R} - \delta) D$$

- $D$  remains as future obligations annuitized at rate  $\bar{R}$ .
- $D > 0$  has direct costs on endowment with  $y' = z' \psi(D)$ .
- Price functions  $q(z, A', D)$  describe access to credit.

## Recursive Problem: Borrower

- State:  $(z, A, y)$ . Nature, what it owes, what it has.
- Choose consumption, new loans, and default.

$$V(z, A, y) = \max_{c, B, D} u(c) + \beta E \{ V'(z', A, y') | z \}$$

s.t.

$$\begin{aligned} c &= y - (A - D) + q(z, A', D) B \\ A' &= \delta A + B + \bar{R} D \\ y' &= z' \psi(D), \quad 0 \leq D \leq A. \end{aligned}$$

- Resulting policy functions:  $B(z, A, y)$ , and  $D(z, A, y)$ .

## Recursive Problem: Lenders

- Take as given policy functions and discount at world's interest rate  $r$ .
- Value to a claim of one unit.

$$H(z, A, y) = \left(1 - \frac{D(z, A, y)}{A}\right) \quad \text{Today}$$

$$\text{Tomorrow} \quad + \frac{1}{1+r} \left( \delta + (\bar{R} - \delta) \frac{D(z, A, y)}{A} \right) E\{H(z', A', y')|z\}.$$

- $A'$  and  $y'$  are determined by borrower's functions.

## Bond Price and Equilibrium

- Zero profit condition determines price functions

$$q(z, A', D) = \frac{1}{1+r} E\{H(z'\psi(D), A', z')|z\}.$$

- Compensates for expected loss in default.
- Partial defaults give price of debt a long-term component.
- Markov equilibrium is the obvious thing. The small country maximizes given prices and the free entry condition given the expected return of loans.

## Default as Expensive Debt

- Transfer future resources towards present with  $B$  or  $D$ .
- Let  $w = y - A$  denote cash in hand.
- Standard consumption-savings trade-off:
  - ▶ Increase in consumption with  $B$  or  $D$

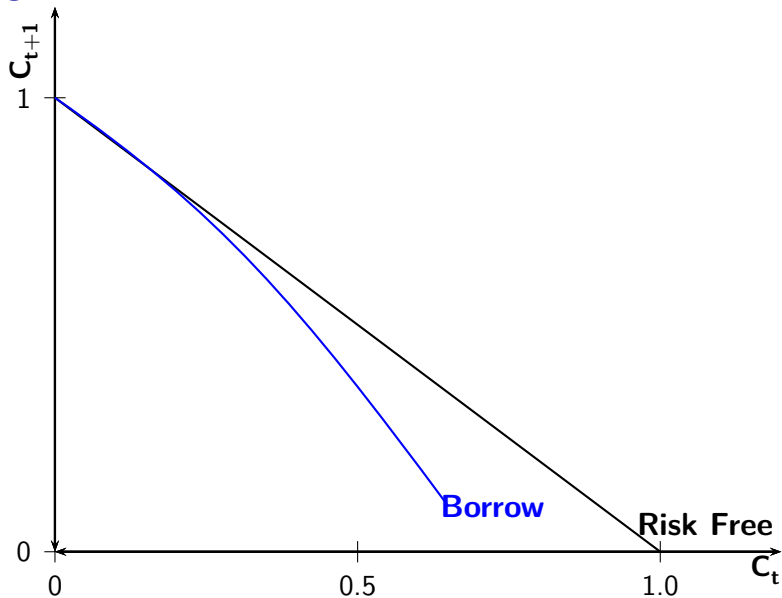
$$c - w = q(A', D, z)B + D$$

- ▶ By reduction in cash on hand tomorrow

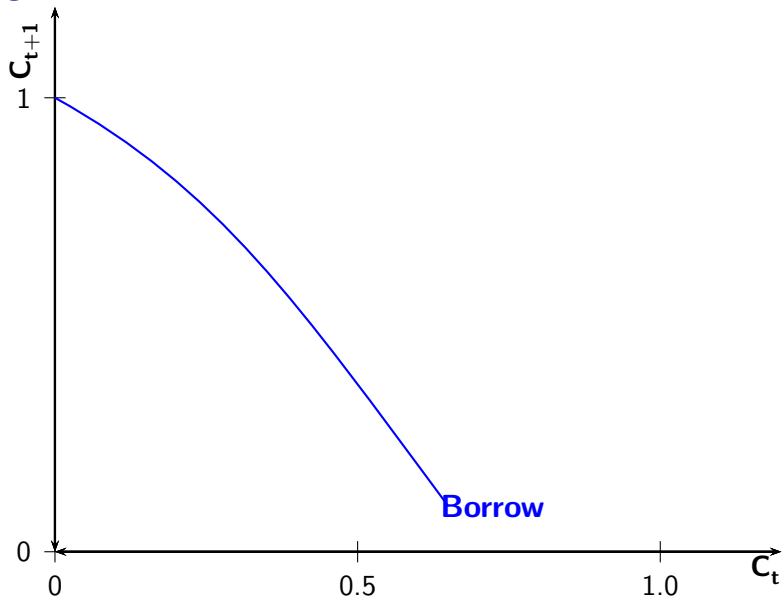
$$\begin{aligned} w' &= z'\psi(D) - A' \\ \text{with } A' &= B - \bar{R}D, \quad D < A \end{aligned}$$

- $B$  is restricted by  $q(A', D, z)$ .
- $D$  is restricted by  $A$  and carries additional cost through  $\psi(D)$ .

# Budget Constraint

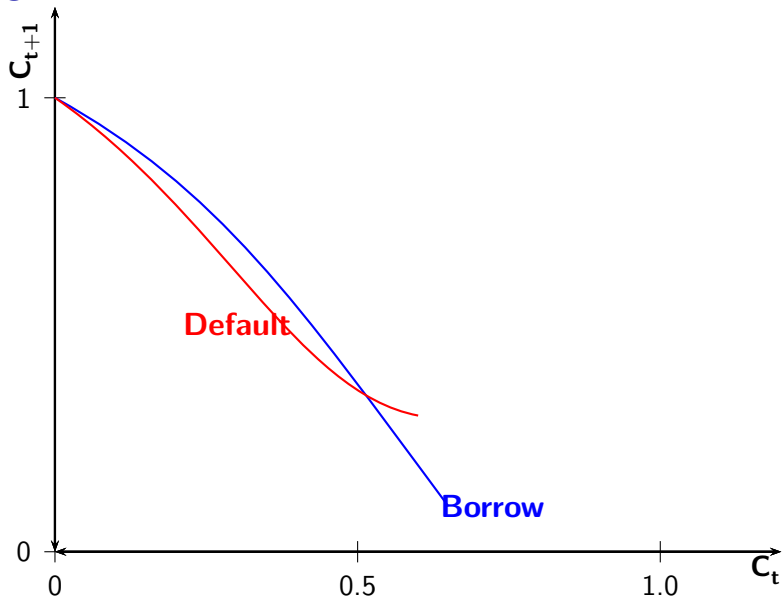


# Budget Constraint

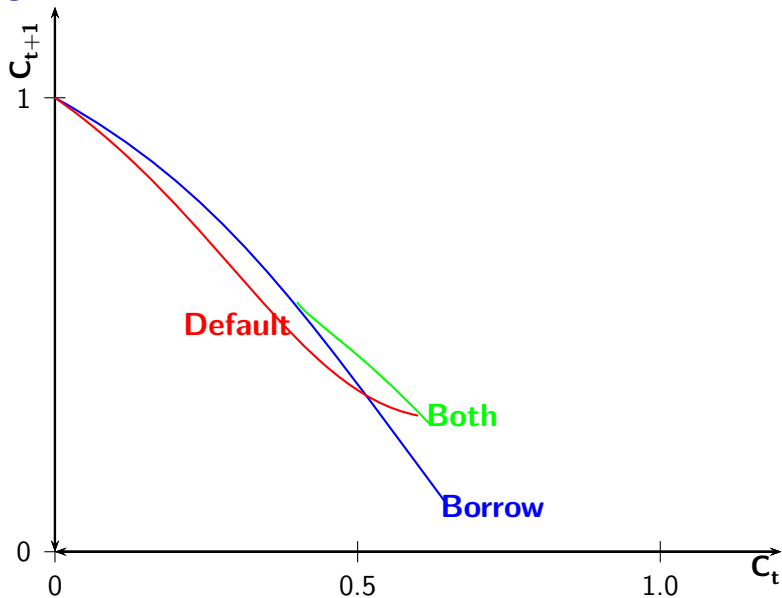




# Budget Constraint



# Budget Constraint



# Variety of Examples

- Explore the numerical properties of these economies.
- We look for the properties in the data that we documented
  - ① Sovereign defaults are partial.
  - ② During defaults sovereigns continue to receive foreign credit.
  - ③ Larger defaults in downturns.
- Designed to resemble developing countries with a year.
  - ▶ There is a fixed cost to default.
  - ▶ Various economies that differ in the size of debt and persistency of shocks.

## Numerical settings

- Default cost  $y = z'\psi(D)$  decreasing and concave with lower bound

$$\psi(D) = \psi_0 \max \left\{ \frac{(D-\bar{D})^2(\gamma\bar{D}+D)}{(0-\bar{D})^2(\gamma\bar{D}+0)}, \underline{\psi} \right\}$$

- Explore 3 experiments: High debt, low debt, and persistent shocks
- Common parameters:  $\sigma = 2$ ,  $r = 1.7\%$ ,  $\delta = 0$ ,  $\bar{R} = 0.80$ ,  $\gamma = 0.5$ ,  $\underline{\psi} = 0.9$ .

Description	Parameter	Example 1 High Debt	Example 2 Low Debt	Example 2 Persistent
Shock process	$z$	iid with $\sigma_H$	iid with $\sigma_H$	Argentina
Penalty slope	$\bar{D}$	0.5	0.7	0.6
Fixed cost	$\psi_0$	0.99	0.99	0.995
Discount	$\beta$	0.85	0.85	0.94

## Average statistics

	Data
Partial default	59%
Frequency of default	51%
Debt /Output	49%
Spread	–
During defaults:	
Debt/GDP	87%
Spreads	–
Arrears/Output	6.2%
New loans/Output	1.07%
Output relt. to Mean	-1.4%

Large frequency of partial defaults

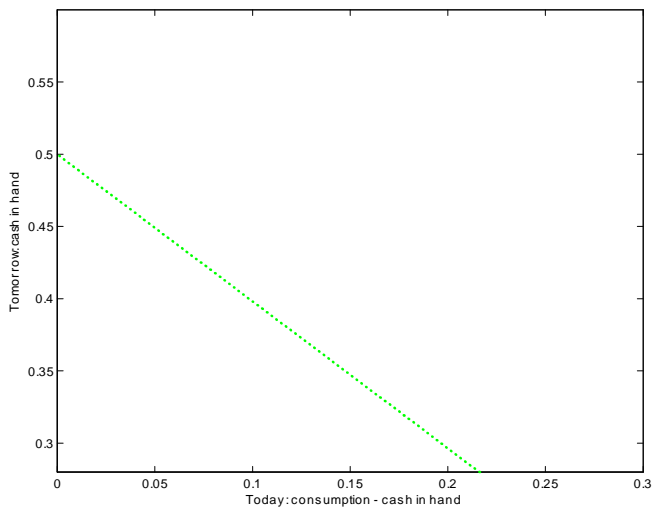
During defaults debt is large, output is low, countries continue to borrow

## Average statistics

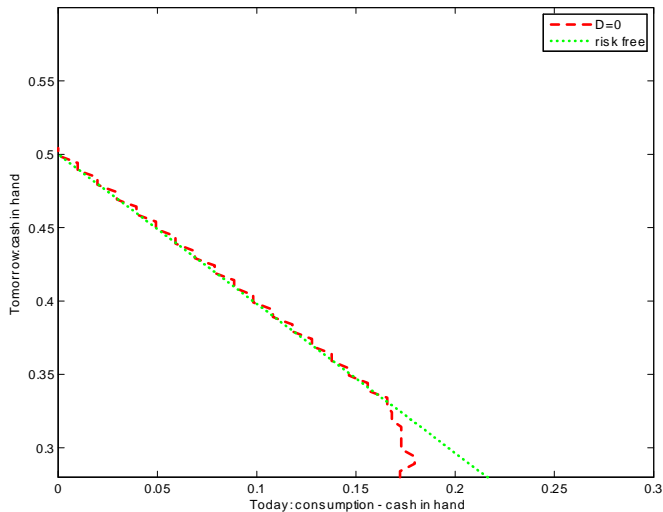
	Data	Examples		
		High Debt	Low Debt	Persistent
Partial default	59%	18%	100%	76%
Frequency of default	51%	11%	30%	74%
Debt /Output	49%	30%	1%	19%
Spread	–	0.5%	17%	1.5%
During defaults:				
Debt/GDP	87%	41%	5.5%	13%
Spreads	–	1.05%	43%	1.8%
Arrears/Output	6.2%	36%	5.5%	5.3%
New loans/Output	1.07%	7.3%	2%	8.6%
Output relt. to Mean	-1.4%	-15%	-8%	-4%

# Examples confirm partial default is alternative credit

Intertemporal frontier: Only B



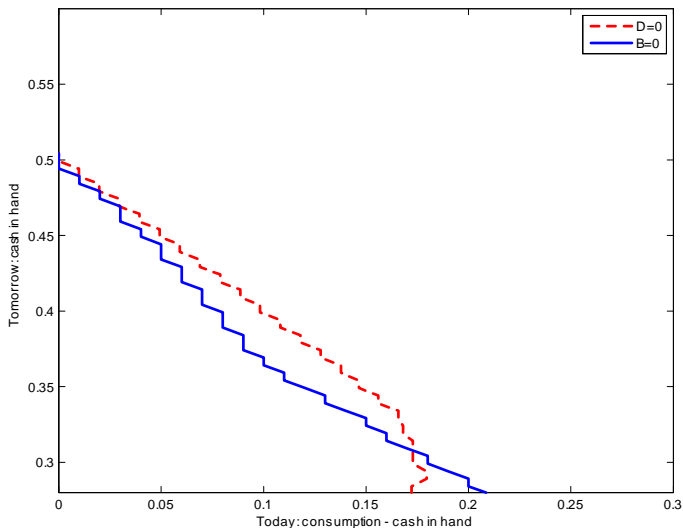
# Intertemporal frontier: Only B



Concave frontier via  $q(\cdot)$  due to increasing default risk

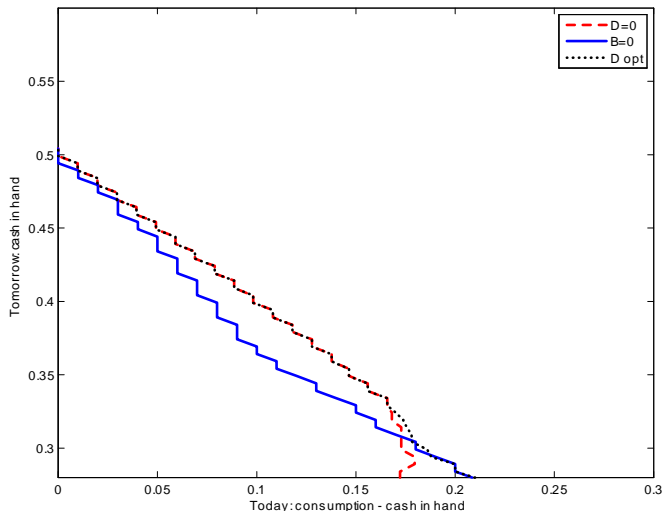


# Intertemporal frontier: Only D



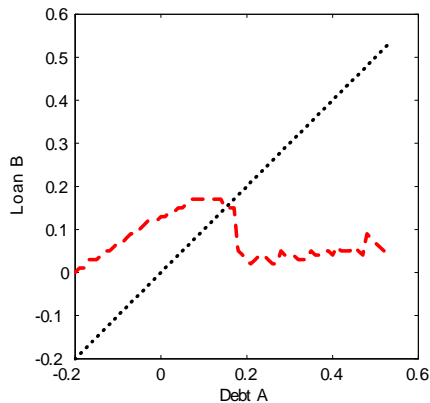
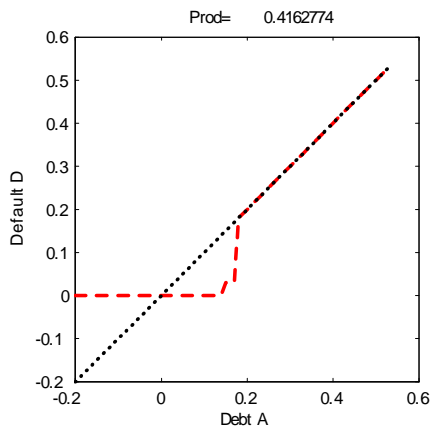
Shape of frontier depends on  $\psi(D)$  and  $\bar{R}$

# Intertemporal frontier: B and D



Smaller transfers with  $B$ , intermediate with  $B + D$ , large with  $D$

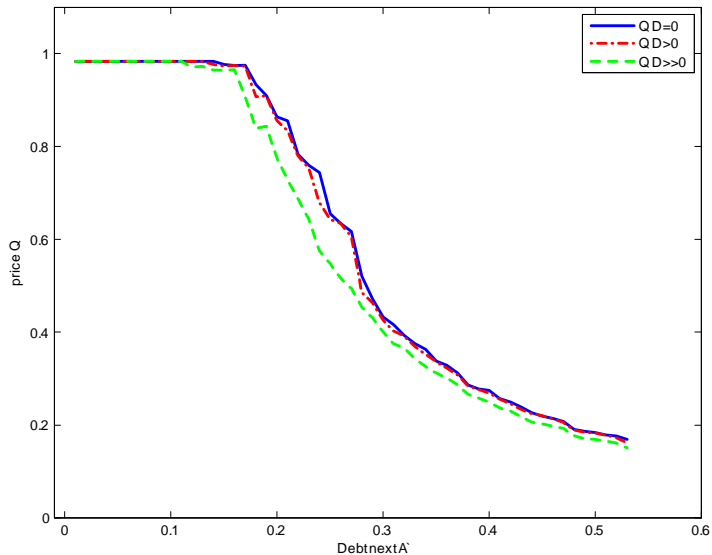
# Policy Functions: Borrow and Default



Small debt:  $B > 0$ , and  $D = 0$

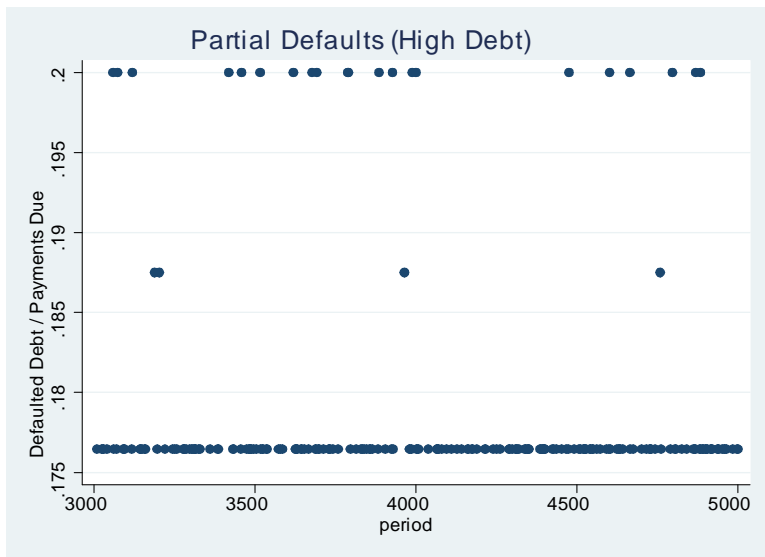
Large debt:  $B = 0$ ,  $D = A$ . Endogenously borrow less due to bad price.

## Policy functions: Prices



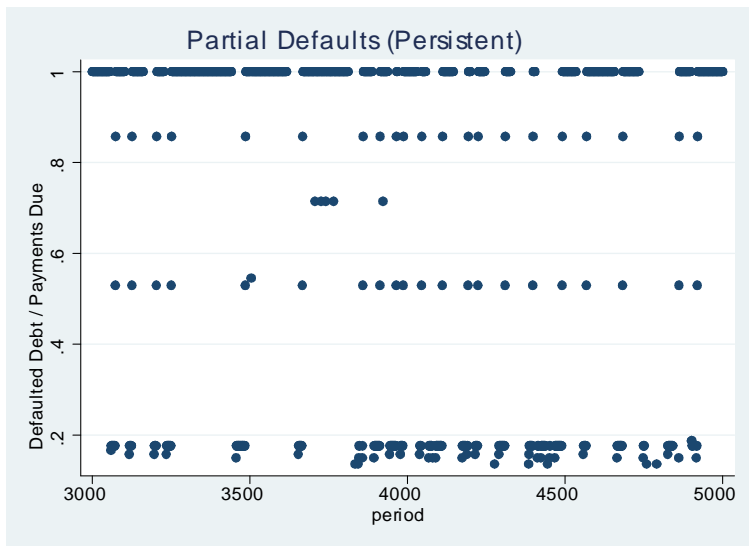
Price decreases with larger debt and is worse when default  $D > 0$ .

# Implication 1: Partial Default



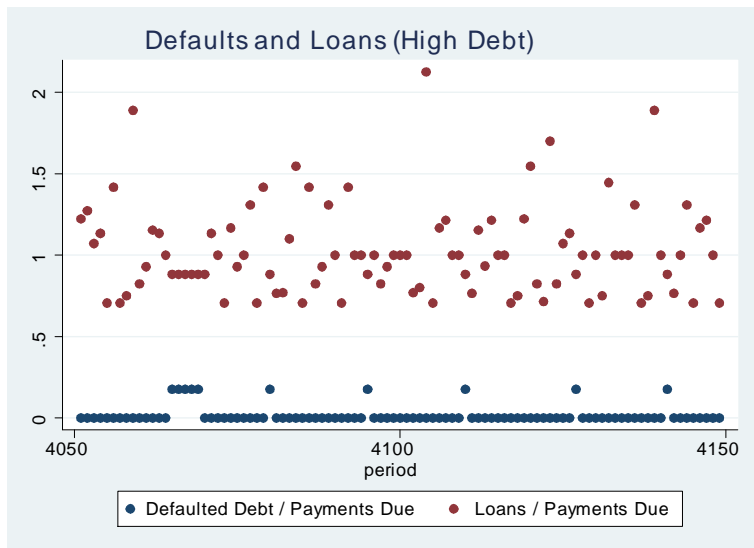
Default is always partial. Narrow range

# Implication 1: Partial Default



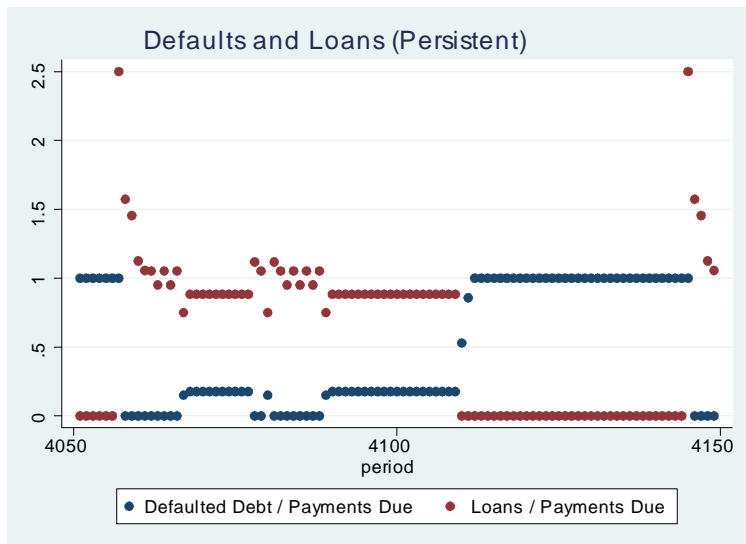
Wide range of partial default

## Implication 2. Borrowing during Sovereign Default



New loans are used much more actively than defaults

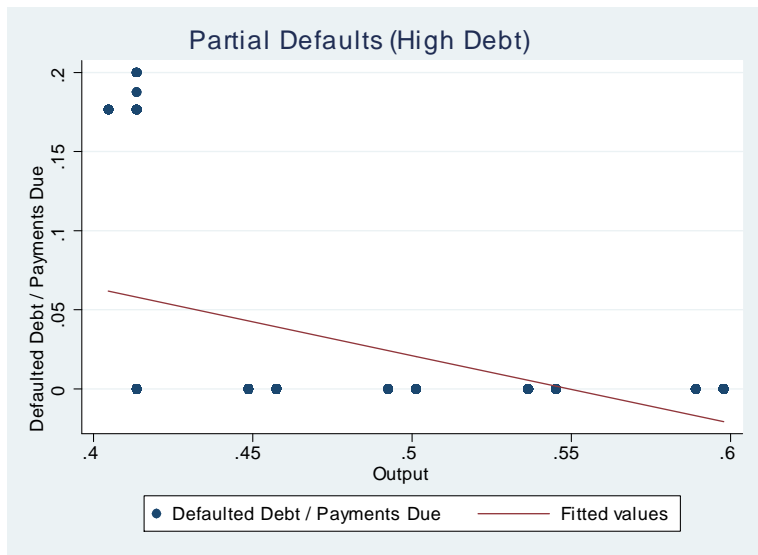
## Implication 2. Borrowing during Sovereign Default



New loans and defaults actively used. Large substitution between two

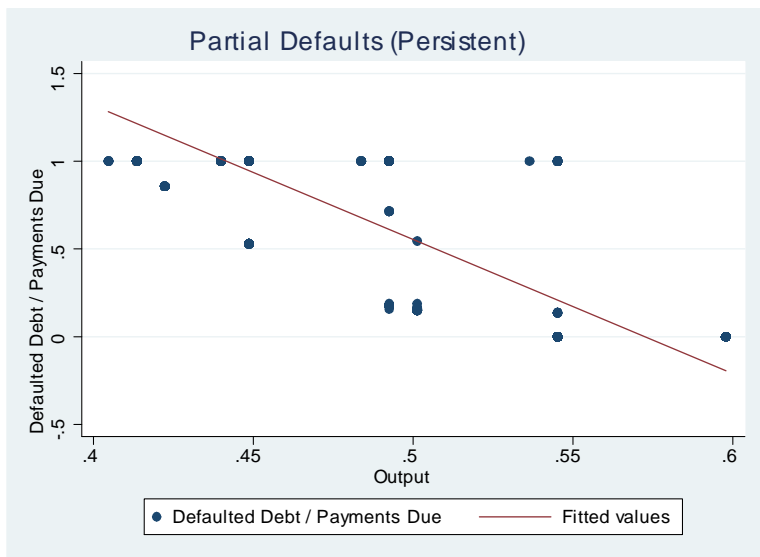


## Implication 3: Larger Default in Downturns



Defaults only with the lowest income

## Implication 3: Larger Default in Downturns

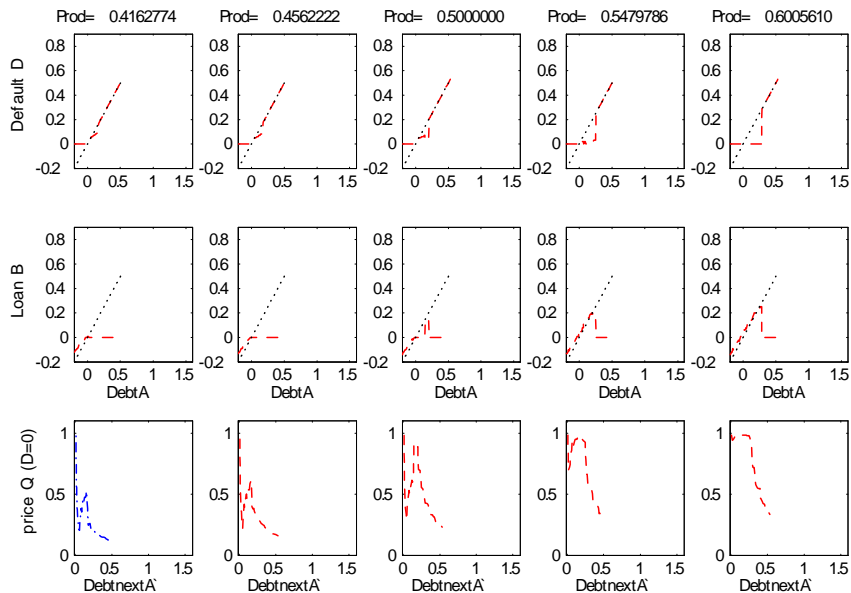


Larger defaults with lower income

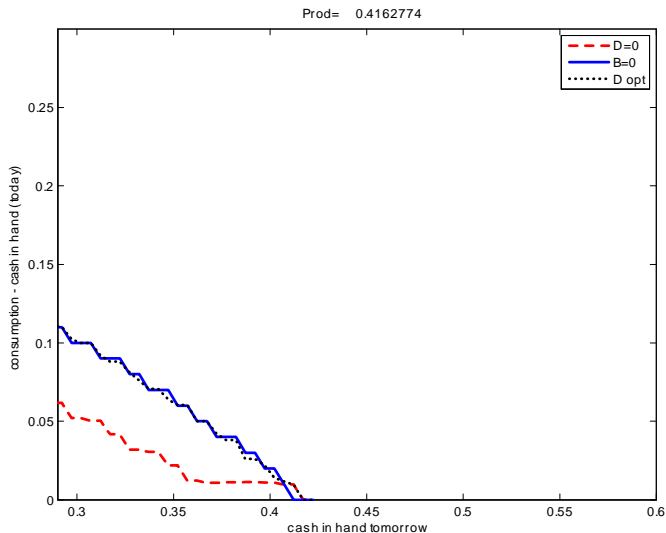
# Conclusion

- Sovereign default is partial and countries continue to borrow during defaults.
- Propose new (Markovian) theory consistent with these facts .
- Continuing work: .
  - ▶ Take model to data. Move a bit out of examples.
  - ▶ Model as laboratory for recovering costs of default.
- Link it with partial individual default (Herkenhoff and Ohanian (13)).

# Policy Functions: Persistent Case



# Persistent case: Frontier low shock



# Persistent case: Frontier high shock

