Wealth, Wages, and Employment

Preliminary

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Preliminary

Introduction

- We want a theory of the joint distribution of employment, wages, and wealth, where
  - Workers are risk averse, so only use self-insurance.
  - Employment and wage risk are endogenous. (More concerned about whether people work than about how long they work.)
  - The economy aggregates into a modern economy (total wealth, labor shares, consumption/investment ratios)
  - Business cycles can be studied. In particular, we want to study employment flows jointly with the other standard objects.
- The most sophisticated version compares well with fluctuations data.

Literature

- The steady state of this economy has as its core Aiyagari (1994) meets Merz (1995), Andolfatto (1996) meets Moen (1997).
- Related Lise (2013), Hornstein, Krusell, and Violante (2011), Krusell, Mukoyama, and Şahin (2010), Ravn and Sterk (2016, 2017), Den Haan, Rendahl, and Riegler (2015).
- Specially Eeckhout and Sepahsalari (2015), Chaumont and Shi (2017), Griffy (2017).
- Developing empirically sound versions of these ideas compels us to
  - Add extreme value shocks as a form of accommodating quits and on the job search as choices.
  - Use new potent tools to address the study of fluctuations in complicated economies Boppart, Krusell, and Mitman (2018)

What are the uses?

- The study of Business cycles including gross flows in and out of employment, unemployment and outside the labor force
- Policy analysis where now risk, employment, wealth (including its distribution) and wages are all responsive to policy.
- Get some insights into the extent of wage rigidity
- Life-Cycle versions of these ideas (under construction) will allow us to assess how age dependent policies fare.

Today: Build the Theory Sequentially and discuss & Fluctuations from two types of shocks

1. No Quits: Exogenous Destruction, no Quits. Built on top of Growth Model. (GE version of Eeckhout and Sepahsalari (2015)): Not a lot of wage dispersion. Not a lot of job creation in expansions.
2. Add Endogenous Quits: Higher wage dispersion may arise to keep workers longer (quits via extreme value shocks).
3. On the Job Search workers may get outside offers and take them. (Similar but not the same as in Chaumont and Shi (2017)).
4. Outside of the Labor Force
5. All of the Above

- Employers commit both to either a wage or a wage schedule $w(z)$ that depends on the aggregate shock.

Key Findings

- If wages are fully fixed and committed (Drastic Wage rigidity)
    - Both endogenous quits and on-the-job yield counter factual procyclical unemployment and massive on the job search.
    - Allowing the wage of an already formed job match to respond some to aggregate shocks corrects this.
    - Getting the right relative volatility of old and new wages and the amount of job-to-job moves and quits provides a way to measure wage rigidity.
- With partial wage rigidity the model fares reasonably well with the data. A few things still to improve. (Excessive Job-to-JOB transitions)
- Similar behavior to that in the Shimer/Hagedorn-Manowski debate. Here we can try to move towards an accommodation of both points of view.

A Brief Look At Data

Relevant Properties in U.S. Data

	Mean Perc	St Dev Relt to Output	Correl w Output	Source
Average Wage	-	0.44-0.84	0.24-0.37	Haefke et al. (2013)
New Wage	-	0.68-1.09	0.79-0.83	Haefke et al. (2013)
Unemployment	4-6	4.84	-0.85	Campolmi&Gnocchi (2016)
Annual Quits (All)	10-40	4.20	0.85	Brown et al. (2017)
Annual Switches	25-35	4.62	0.70	Fujita&Nakajima (2016)
Consumption	75	0.78	0.86	NIPA
Investment	25	4.88	0.90	NIPA

Model 1: No (Endogenous) Quits Model

No (Endog) Quits: Precautionary Savings, Competitive Search

- Jobs are created by firms (plants). A plant with capital plus a worker produce one (z) unit of the good (z is the aggregate state of the economy).
  - Firms pay flow cost $\bar{c}$ to post a vacancy in market $\{w, \theta\}$.
  - Firms cannot change wage (or wage-schedule) afterwards.
  - Think of a firm as a machine programmed to pay w or $w(z)$
  - Plants (and their capital) are destroyed at rate δ^f.
  - Workers quit exogenously at rate δ^h.
- Households differ in wealth and wages (if working) but not in productivity. There are no state contingent claims, nor borrowing.
  - If employed, workers get w and save.
  - If unemployed, workers produce b and search in some $\{w, \theta\}$.
- General equilibrium: Workers own firms.

Order of Events of No Quits Model

1. Households enter the period with or without a job: $\{e, u\}$.

2. Production & Consumption: Employed produce z on the job. Unemployed produce b at home. They choose savings.

3. Firm Destruction and Exogenous Quits : Some Firms are destroyed (rate δ^f) They cannot search this period. Some workers quit their jobs for exogenous reasons δ^h. Total job destruction is δ.

4. Search: Firms and the unemployed choose wage w and tightness θ.

5. Job Matching : $M(V, U)$: Some vacancies meet some unemployed job searchers. A match becomes operational the following period. Job finding and job filling rates $\psi^h(\theta) = \frac{M(V,U)}{U}$, $\psi^f(\theta) = \frac{M(V,U)}{V}$.

No Quits Model: Household Problem

- Individual state: wealth and wage
  - If employed: (a, w)
  - If unemployed: (a)
- Problem of the employed: (Standard)

$$V^e(a,w) = \max_{c,a'} u(c) + \beta \; [(1-\delta)V^e(a',w) + \delta V^u(a)]$$

$$\text{s.t.} \quad c + a' = a(1+r) + w, \quad a \geq 0$$

- Problem of the unemployed: Choose which wage to look for

$$V^u(a) = \max_{c,a',w} u(c) + \beta \left\{ \psi^h[\theta(w)] \; V^e(a',w) + [1-\psi^h[\theta(w)]] \; V^u(a') \right\}$$

$$\text{s.t.} \quad c + a' = a(1+r) + b, \quad a \geq 0$$

$\theta(w)$ is an equilibrium object

Firms Post vacancies: Choose wages & filling probabilities

- Value of wage-w job: uses constant $\overline{k}$ capital that depreciates at rate δ^k ($\Omega = \overline{k}$)

$$\Omega(w) = z - \overline{k}\delta^k - w + \frac{1-\delta^f}{1+r}\left[(1-\delta^h)\,\Omega(w) + \delta^h\,\Omega\right]$$

- Affine in w: $\Omega(w) = \left[z + \overline{k}\left(\frac{1-\delta^f}{1+r}\delta^h - \delta^k\right) - w\right]\frac{1+r}{r+\delta^f+\delta^h-\delta^f\delta^h}$

Block Recursivity Applies (firms can be ignorant of Eq)

- Value of creating a firm: $\psi^f[\theta(w)]\ \Omega(w) + [1-\psi^f[\theta(w)]]\,\Omega$

- Free entry condition requires that for all offered wages

$$\bar{c} + \overline{k} = \psi^f[\theta(w)]\ \frac{\Omega(w)}{1+r} + [1-\psi^f[\theta(w)]]\ \frac{\Omega}{1+r},$$

No (Endog) Quits Model: Stationary Equilibrium

- A stationary equilibrium is functions $\{V^e, V^u, \Omega, g'^e, g'^u, w^u, \theta\}$, an interest rate r, and a stationary distribution x over (a, w), s.t.

1. $\{V^e, V^u, g'^e, g'^u, w^u\}$ solve households' problems, $\{\Omega\}$ solves the firm's problem.

2. Zero profit condition holds for active markets

$$\bar{c} + \overline{k} = \psi^f[\theta(w)] \; \frac{\Omega(w)}{1+r} + [1 - \psi^f[\theta(w)]] \; \frac{\overline{k}(1-\delta-\delta_k)}{1+r}, \quad \forall w \;\; \text{offered}$$

3. An interest rate r clears the asset market

$$\int a \;\; dx = \int \Omega(w) \;\; dx.$$

Characterization of a worker's decisions

- Standard Euler equation for savings

$$u_c = \beta\,(1 + r)\; E\,\{u_c'\}$$

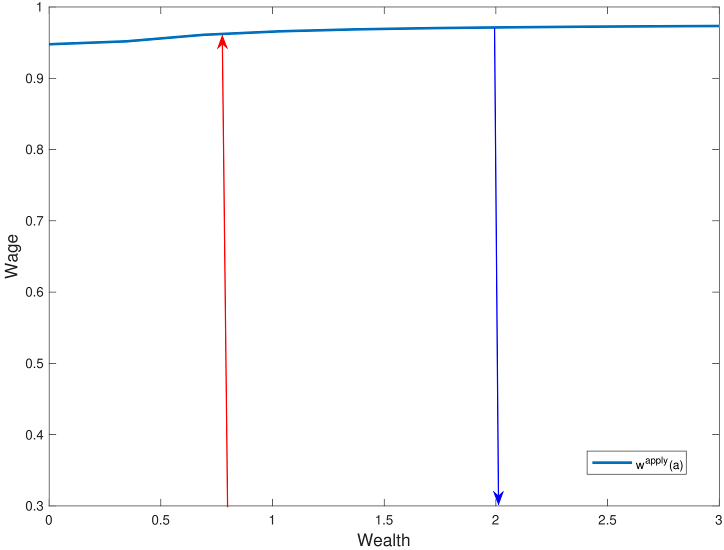
- A F.O.C for wage applicants

$$\psi^h[\theta(w)]\; V_w^e(a', w) = \psi_\theta^h[\theta(w)]\, \theta_w(w)\; [V^u(a') - V^e(a', w)]$$

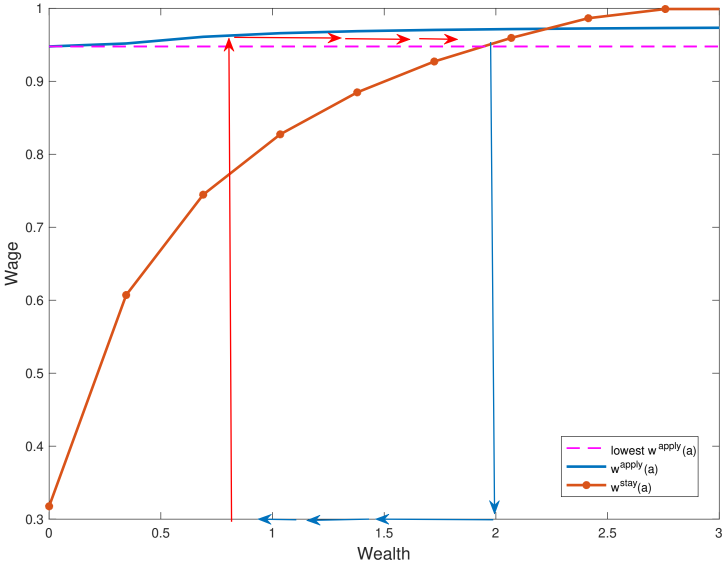
- Households with more wealth are able to insure better against unemployment risk.
- As a result they apply for higher wage jobs and we have dispersion

How does the Model Work

Worker's wage application decision



Worker's saving decision



Shortcomings of this model

- Silent on Quits and Job-To-Job Movements.
- Low Wage Dispersion
- Small differences in volatility between average and new wages
- Low unemployment volatility

Summary: No (Endog) Quits Model

1. Easy to Compute Steady-State with key Properties
   - i Risk-averse, only partially insured workers, endogenous unemployment
   - ii Can be solved with aggregate shocks too
   - iii Policy such as UI would both have insurance and incentive effects
   - iv Wage dispersion small—wealth doesn't matter too much
   - v $\cdots$so almost like two-agent model (employed, unemployed) of Pissarides despite curved utility and savings
2. In the following we examine the implications of a quitting choice

Endogenous Quits

Endogenous Quits: Beauty of Extreme Value Shocks

- Temporary Shocks to the utility of working or not working: Some workers quit. (in addition to any intrinsic taste for leisure)
- Adds a (smoothed) quitting motive so that higher wage workers quit less often: Firms may want to pay high wages to retain workers.
- Conditional on wealth, high wage workers quit less often.
- But Selection (correlation 1 between wage and wealth when hired) makes wealth trump wages and those with higher wages have higher wealth which makes them quite more often: Wage inequality collapses.
- We end up with a model with little wage dispersion but with endogenous quits that respond to the cycle.

Quitting Model: Time-line

1. Workers enter period with or without a job: $\{e, u\}$.
2. Production occurs and consumption/saving choice ensues:
3. Exogenous job/firm destruction happens.
4. Quitting:
   - e draw shocks $\{\epsilon^e, \epsilon^u\}$ and make quitting decision.
     Job losers cannot search this period.
   - u draw shocks $\{\epsilon^u_1, \epsilon^u_2\}$. No decision but same expected means.
5. Search: New or Idle firms post vacancies. Choose $\{w, \theta\}$.
   Wealth is not observable. (Unlike Chaumont and Shi (2017)).
   Yet it is still Block Recursive
6. Matches occur

Quitting Model: Workers

- Workers receive i.i.d shocks $\{\epsilon^e, \epsilon^u\}$ to the utility of working or not
- Value of the employed right before receiving those shocks:

$$\widehat{V}^e(a', w) = \int \max\{V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u\}\, dF^\epsilon$$

  V^e and V^u are values after quitting decision as described before.

- If shocks are Type-I Extreme Value dbtn (Gumbel), then $\widehat{V}$ has a closed form and the ex-ante quitting probability $q(a, w)$ is

$$q(a, w) = \frac{1}{1 + e^{\alpha[V^e(a,w) - V^u(a)]}}$$

  higher parameter $\alpha \rightarrow$ lower chance of quitting.

- Hence higher wages imply longer job durations. Firms could pay more to keep workers longer.

Quitting Model: Workers Problem

- Problem of the employed: just change $\widehat{V}^e$ for V^e

$$V^e(a,w) = \max_{c,a'} u(c) + \beta \left[(1-\delta)\widehat{V}^e(a',w) + \delta V^u(a)\right]$$
$$\text{s.t.} \quad c + a' = a(1+r) + w, \quad a \geq 0$$

- Problem of the unemployed is like before except that there is an added term $\quad E\{\max[\epsilon_1^u, \epsilon_2^u]\}$

So that there is no additional option value to a job.

Quitting Model: Value of the firm

- $\Omega^j(w)$: Value with with j-tenured worker.
  Free entry condition requires that for all offered wages

$$\bar{c} + \overline{k} = \frac{1}{1+r} \left\{ \psi^f[\theta(w)]\, \Omega^0(w) + [1 - \psi^f[\theta(w)]]\, \Omega \right\},$$

- Probability of retaining a worker with tenure j at wage w is $\ell^j(w)$.
  (One to one mapping between wealth and tenure)

$$\ell^j(w) = 1 - q^e[g^{e,j}(a, w), w]$$

$g^{e,j}(a, w)$ savings rule of a $j-$ *tenured* worker that was hired with wealth a

- Firm's value

$$\Omega^j(w) = z - \overline{k}\delta^k - w + \frac{1-\delta^f}{1+r} \left\{ \ell^j(w)\Omega^{j+1}(w) + [1 - \ell^j(w)]\, \Omega \right\}$$

Quitting Model: Solving forward for the Value of the firm

$$\Omega^0(w) \quad = \quad (z - w - \delta^k k)\, Q^1(w) + (1 - \delta^f - \delta_k)k\, Q^0(w),$$

$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta^f}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta^f}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

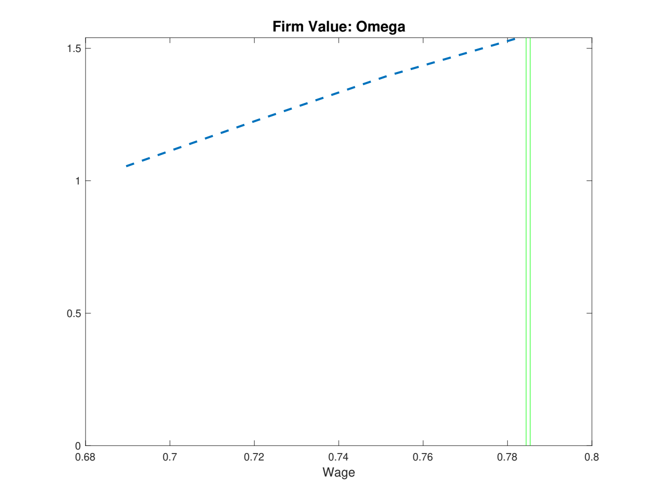
- New equilibrium objects $\{Q^0(w), Q^1(w)\}$. Rest is unchanged.
- It is Block Recursive because wealth can be inferred from w and j. (No need to index contracts by wealth (as in Chaumont and Shi (2017))).

Do we get More Wage Dispersion?

- This Model has the potential to get more wage dispersion
- Conditional on wealth higher wages lead to less quitting.
- So firms are willing to pay more to keep workers longer
- **BUT** we will see a problem

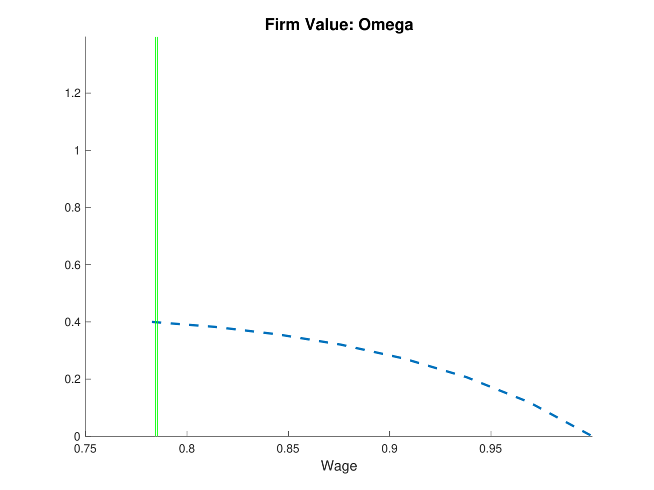
Value of the firm as wage varies: The Poor

- For the poorest, employment duration increases when wage goes up.
- Firms value is increasing in the wage



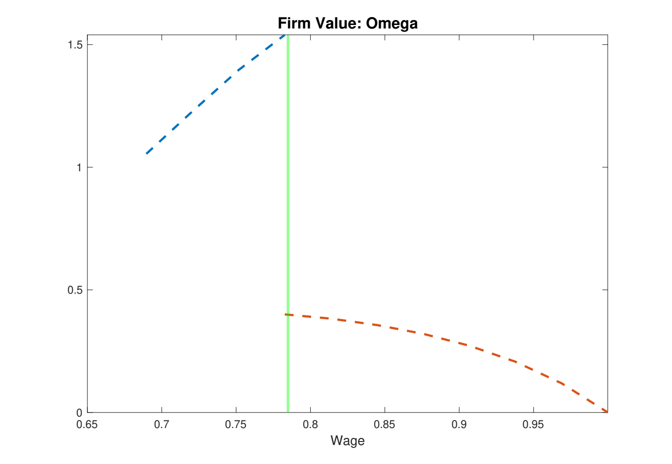
Value of the firm as wage varies: The Rich

- For the richest, employment duration increases but not fast enough.
- Firm value is slowly decreasing in wages (less than static profits).



Value of the firm: Accounting for Worker Selection

- Large drop from below to above equilibrium wages.
- In Equilibrium wage dispersion COLLAPSES due to selection.



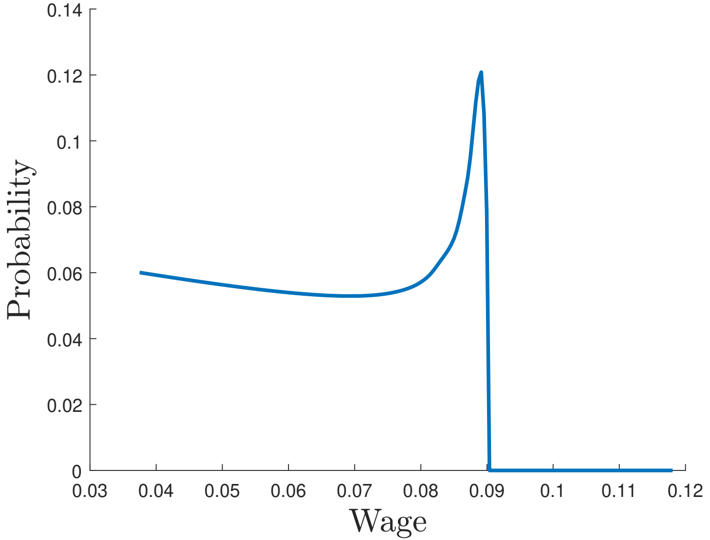
- Related to the Diamond dispersion paradox but for very different reasons.

Effect of Quitting: The Mechanism

- Two forces shape the dispersion of wages
  - Agents quit less at higher paid jobs, which enlarge the spectrum of wages that firms are willing to pay (for a given range of vacancy filling probability).
  - However, by paying higher wages, firms attract workers with more wealth.
- Wealthy people quit more often, shrink employment duration.
- In equilibrium, the wage gap is narrow (disappears?) and the effect of wealth dominates.

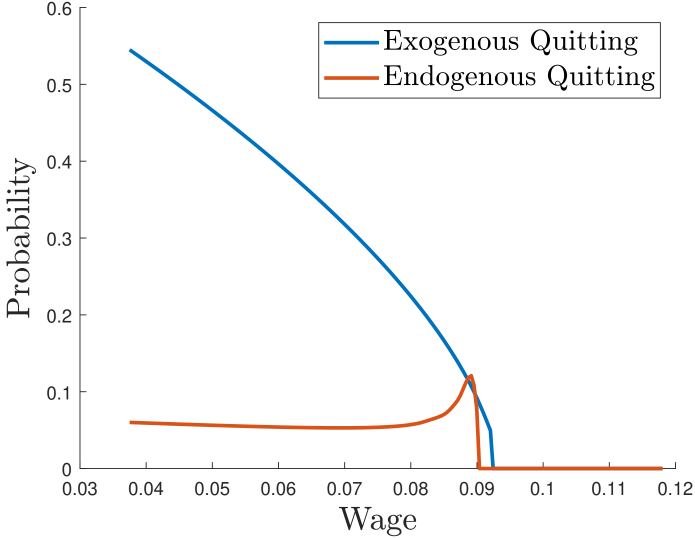
Value of the firm: Zero profit Job Finding Probability

- Increasing in Wage (up to Grid calculation): Unique wage.



Quitting Makes a Big Difference

- Job finding prob with Endo



Shortcommings

- Wage Dispersion Collapses
- Silent on Job-To-Job Movements.
- Unemployment Moves little (but more than the previous one) over the cycle
- No difference in volatility between average and new wages
- Correlation 1 between Wealth when starting to work and wage

A Detour on How to Improve the Correlation Between Wealth and Wages

- Pose *aiming* (extreme value) shocks).
- This reduces the correlation between wages and wealth when first hired.
- It will have many uses, we think.

On the Job Search

On the Job Search Model: Time-line

1. Workers enter period with or without a job: V^e, V^u.
2. Production & Consumption:
3. Exogenous Separation
4. Quitting? Searching? Neither?: Employed draw shocks $(\epsilon^e, \epsilon^u, \epsilon^s)$ and make decision to quit, search, or neither. Those who quit become u', those who search join the u, in case of finding a job become $\{e', w'\}$ but in case of no job finding remain e' with the same wage w and those who neither become e' with w. $\widehat{V}^E(a', w)$, is determined with respect to this stage.
5. Search : Potential firms decide whether to enter and if so, the market (w) at which to post a vacancy; u and s assess the value of all wage applying options, receive match specific shocks $\{\epsilon^{w'}\}$ and choose the wage level w' to apply. Those who successfully find jobs become e', otherwise become u'.
6. $\widehat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.
7. Match

On the Job Search: Household Probl

- After saving, the unemployed problem is

$$\widehat{V}^u(a') = \int \max_{w'} \left[\psi^h(w')V^e(a', w') + (1 - \psi^h(w'))V^u(a') + \epsilon^{w'}\right] \, dF^\epsilon$$

- After saving, the employed choose whether to quit, search or neither

$$\widehat{V}^e(a', w) = \int \max\{V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u, V^s(a', w) + \epsilon^s\} dF^\epsilon$$

- The value of searching is

$$V^s(a', w) = \int \max_{w'} \left[\psi^h(w')V^e(a', w') + [1 - \psi^h(w')]V^e(a', w) + \epsilon^{w'}\right] \, dF^\epsilon$$

On the Job Search: Household choices

- The probabilities of quitting and of searching

$$q(a', w) = \frac{1}{1 + \exp(\alpha[V^e(a', w) - V^u(a')]) + \exp(\alpha[V^s(a', w) - V^u(a') + \mu^s])},$$

$$s(a', w) = \frac{1}{1 + \exp(\alpha[V^u(a') - V^s(a', w)]) + \exp(\alpha[V^e(a', w) - V^s(a', w) - \mu^s])}.$$

$\mu^s < 0$ is the mode of the shock ϵ^s which reflects the search cost.

- Households solve

$$V^e(a, w) = \max_{a' \geq 0} u[a(1+r) + w - a'] + \beta \left[\delta V^u(a') + (1-\delta)\widehat{V}^e(a', w)\right]$$

$$V^u(a) = \max_{c, a' \geq 0} u[a(1+r) + b - a'] + \beta \widehat{V}^u(a')$$

- The value of the firm is again given like in the Quitting Model

$$\Omega^0(w) \quad = \quad (z - w - \delta^k k)\; Q^1(w) + (1 - \delta - \delta_k)k\; Q^0(w),$$

$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

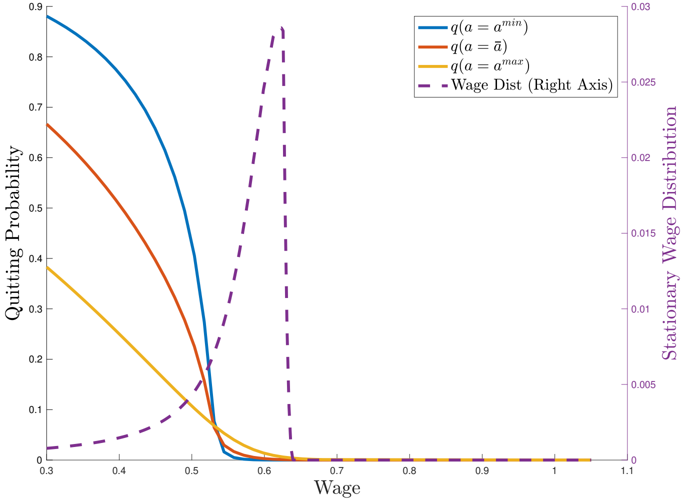
$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- Except that now the probability of keeping a worker after j periods is

$$\ell^j(w) = 1 - \int h(w;a)\; q[g^{e,j}(a,w), w]\, dx^u(a) -$$

$$\int h(w;a)\; s[w; g^{e,j}(a,w)] \left[\int \hat{h}[\widetilde{w}; g^{e,j}(a,w), w] \xi \phi^h(\widetilde{w})\; d(\widetilde{w}) \right] dx^u(a)$$

OJS Quitting Probabilities, Various wealths & Wage Density



- The rich pursue often other activities (leisure?)

Outside the Labor Force

Outside the Labor Force Model: Time-line

1. Workers enter period with or without a job: V^e, V^u.
2. In the beginning of the period non Workers get a shock to the utility of either searching or not searching. They then choose whether to sit out and not search or to search. It is an extreme value shock.

   Workers get a utility injection equal to the expected utility of the maximum of those two shocks to get no bias in the value of working versus not.
3. Production & Consumption:
4. Exogenous Separation
5. Quitting? Searching? Neither?:
6. Search
7. $\widehat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.
8. Match

Various Economies with added Life Cycle (live 50 years)

- Provides a mechanism for having poor agents

- Right now we have Four Economies

  1. Only Exogenous Quitting
  2. Endogenous Quitting
  3. Exogenous Quitting with On-the-job Search
  4. Endogenous Quitting and On-the-job Search
  5. ... and some agents do not want to work

- Today we will only look at the Economy with Endogenous quitting and On-the-Job-Search (4)

Quantitative Analysis: Steady States

Parameter Values

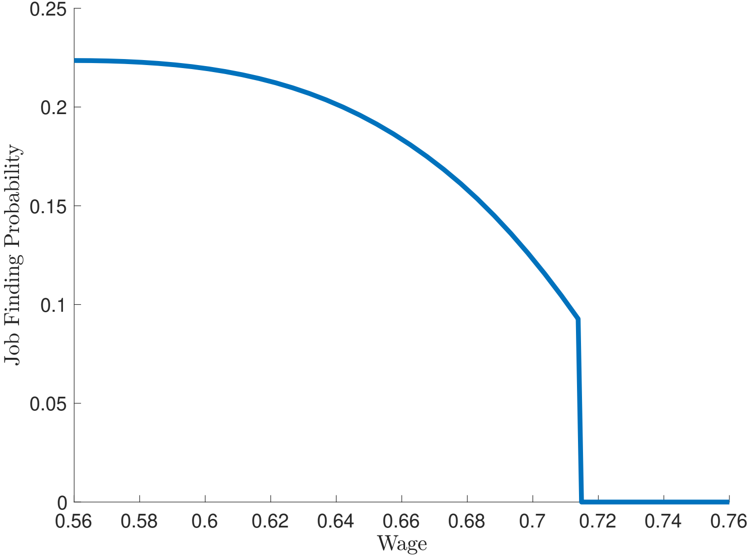
	Definition	Value in Yearly Units
r	interest rate	3%
K	fixed capital required	3
δ^f	firm destruction rate	2.88%
δ^k	capital maintenance rate	6.38%
δ^h	total worker quitting rate	8.56%
c^v	job posting cost	0.03
y	productivity on the job	1
b/w	productivity at home	0.4
σ	risk aversion	2
Matching function	$m = \chi u^\eta v^{1-\eta}$, non-OJS	$\chi = 0.15, \eta = 0.62$
	$m = \chi u^\eta v^{1-\eta}$, OJS	$\chi = 0.3, \eta = 0.5$

- We also explore a lower on the job search economy ()high value of leisure economy $b/w \sim 0.75$

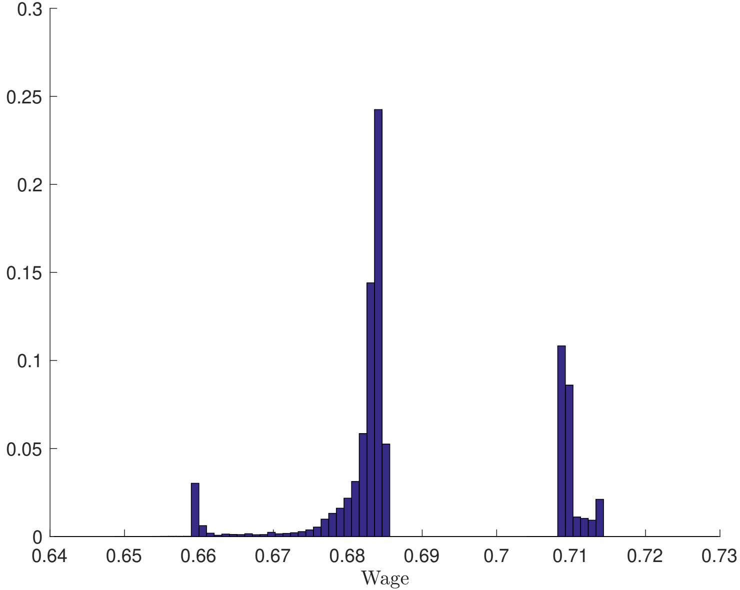
Steady State Allocations in Yearly Units: Endog Quits & OJS

interest rate	0.030
avg consumption	0.651
avg wage	0.689
avg wealth	3.041
stock market value	2.953
avg labor income	0.654
consumption to wealth ratio	0.225
labor income to wealth ratio	0.215
quit ratio	0.090
unemployment rate	0.097
job losers	0.117
wage of newly hired unemp	0.677
std consumption	0.011
std wage	0.002
std wealth	3.606
mean-min consumption	2.051
mean-min wage	1.058
UE transition	0.125
total vacancy	0.578
avg unemp duration	0.773
avg emp duration	7.228
avg job duration	1.898
OJS move rate	0.395

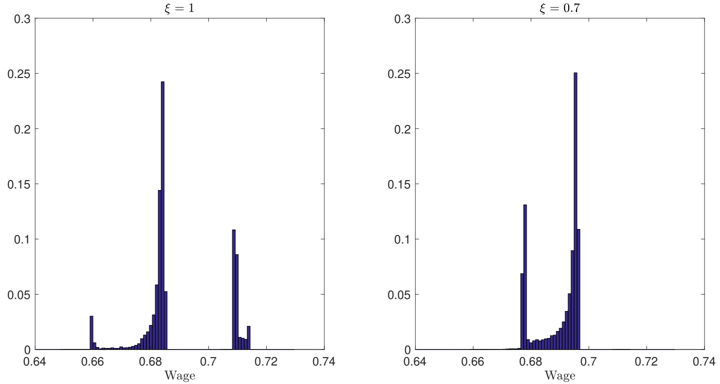
Job Finding Probability Curves



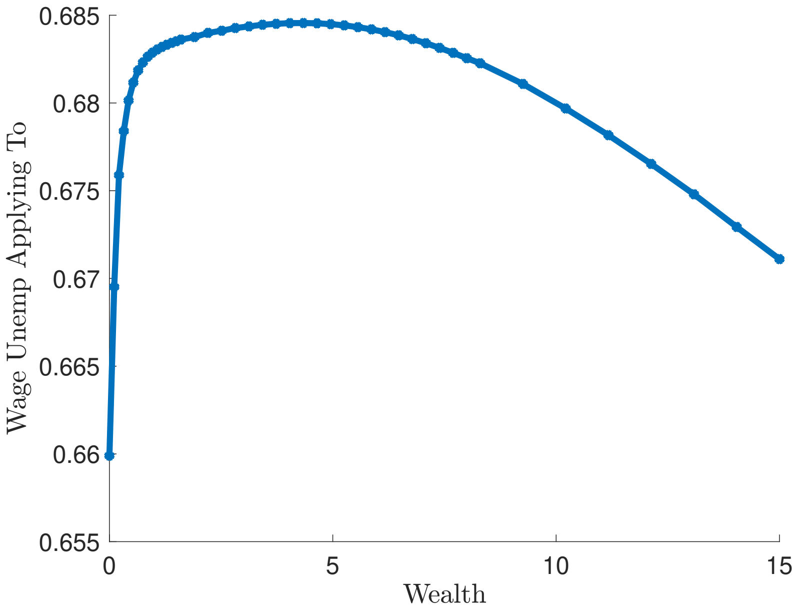
Wage Distributions: Baseline



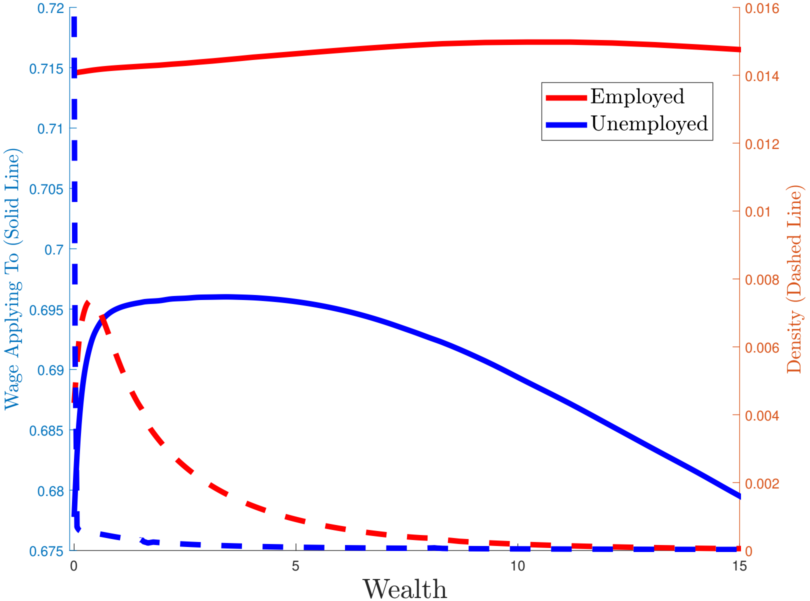
Wage Distributions: Comparing with lower OJS



Wage Applications of the Unemployed by Wealth



Wage Applications of U and $\overline{w}$ and densities of all



Aggregate Fluctuations

Introduce Aggregate Shocks

- We examine the model responses to two type of shocks
  1. Productivity shocks z_t: Output = EmpRate $\times (1 + z_t)$
  2. Firm destruction shocks d_t: Firm Destruction Rate $= \delta^f \times (1 - d_t)$
- We introduce a wage peg assumption:
  - To allow the wage of an already formed job match to respond to z_t shocks directly (by 50%) (but not to d_t shocks)
  - If wages were completely rigid there would be massive quits: counterfactual.

1% Productivity Shock ($\rho = .95$) [IRF]

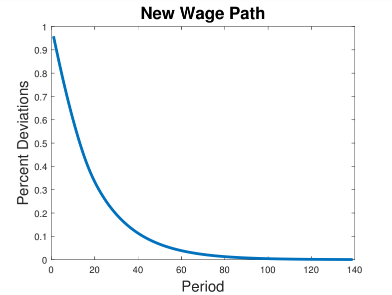


Fig. 1: Wages

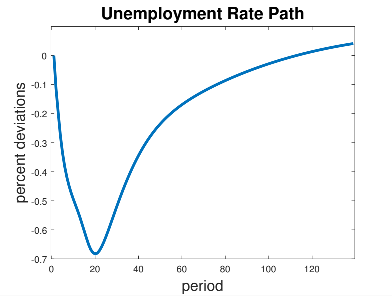


Fig. 2: Unemployment Rate

- Non-trivial response of wage and unemployment

1% Productivity Shock ($\rho = .95$) IRF

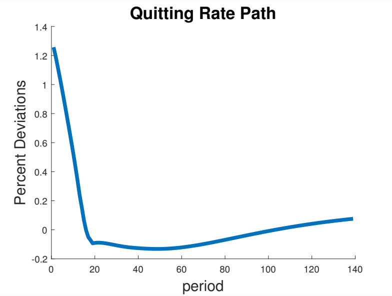


Fig. 3: Quits

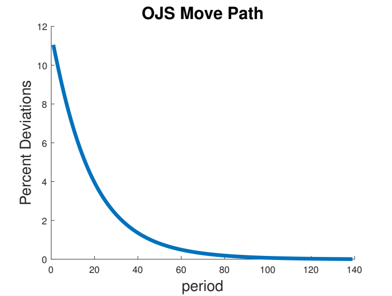


Fig. 4: Job-to-job Moves

- Quits are mildly responsive to the shock
- While on-the-job moves are much more responsive: (perhaps too much)

1% Delta Shock ($\rho = .95$)

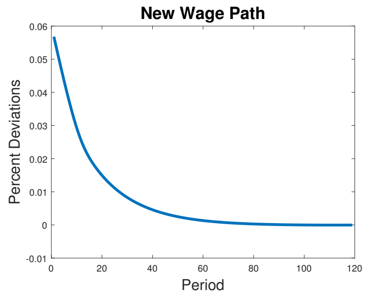


Fig. 5: Wages

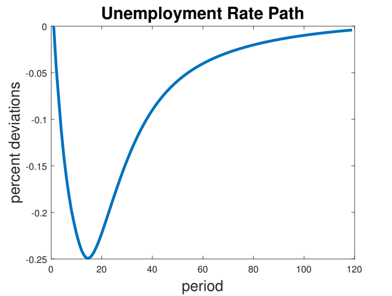


Fig. 6: Unemployment Rate

- Again 1% delta shock = 0.36 base points
- Large response of wage and unemployment to the delta shock
- Note wage is not pegged to the delta shock

M4: 1% Delta Shock ($\rho = .95$)

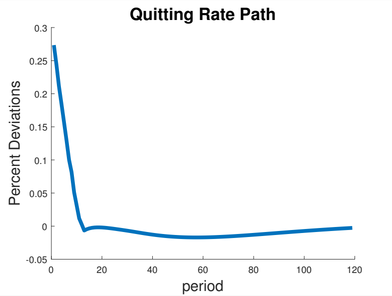


Fig. 7: Quits

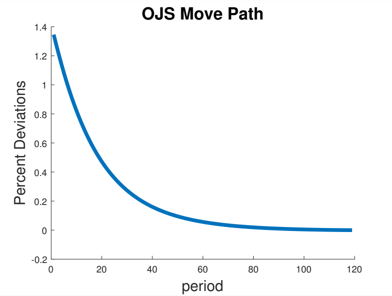


Fig. 8: Job-to-job Moves

- But too much volatility for job-to-job transitions relative to output

Summary, On-the-job Search and Quits

- Pro-cyclical average wages, new wages, and employment, quitting, and job-to-job transitions
- Clear responses of new wages and employment
- Quitting mildly respnds to both shocks
- Job-to-job transitions move too much with both shocks

Assessing Performance in terms of standard hp-filtered 2nd moments

- 1st order data moments are from standard database: CPS, JOLTS, LEHD and NIPA.
- 2nd order data moments are from Haefke, Sonntag, and Van Rens (2013), Campolmi and Gnocchi (2016), Brown et al. (2017) and Fujita and Nakajima (2016).

Productivity Shock: Relative Volatility

- Only Productivity Shock: $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	0.51	0.44-0.84
New Wage	0.95	0.68-1.09
Unemployment	0.35	4.84
Quits + OJS moves	8.94	4.2
OJS moves	10.66	4.62

Table 1: Standard Deviation Relative to Output: Only Productivity Shock

- Unemployment moves too little and Quits and OJS moves too much

Productivity Shock: Correlation

- Only Productivity Shock: $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	1.00	0.24-0.37
New Wage	1.00	0.79-0.83
Unemployment	-0.48	-0.85
Quits + OJS moves	0.99	0.85
OJS moves	0.99	0.70

Table 2: Correlation with Contemprary Output: Only Productivity Shock

- Correlations are on the spot

Delta Shock: Relative Volatility

	Model	Data
Output	1	1
Average Wage	0.09	0.44-0.84
New Wage	2.02	0.68-1.09
Unemployment	4.70	4.84
Quits + OJS moves	41.66	4.2
OJS moves	49.36	4.62

Table 3: Standard Deviation Relative to Output: Only Delta Shock

- Now Unemployment is good but moves are excessive

- Note that relative to output, productivity is very important so employment cannot do that much, but this shock makes employment the only culprit so it has to move a lot

Delta Shock: Correlation Only Delta Shock: $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	0.13	0.24-0.37
New Wage	0.31	0.79-0.83
Unemployment	-0.99	-0.85
Quits + OJS moves	0.40	0.85
OJS moves	0.42	0.70

Table 4: Correlation with Contemprary Output: Only Delta Shock

Both Shocks: Relative Volatility Very correlated (.95)

	Model	Data
Output	1	1
Average Wage	0.49	0.44-0.84
New Wage	1.38	0.68-1.09
Unemployment	3.02	4.84
Quits + OJS moves	25.77	4.2
OJS moves	30.53	4.62

Table 5: Standard Deviation Relative to Output: Both Shocks

- Relative Std of shocks: each shock contributes roughly equal to output volatility

Both Shocks: Correlation 0.95

	Model	Data
Output	1	1
Average Wage	0.77	0.24-0.37
New Wage	0.50	0.79-0.83
Unemployment	-0.37	-0.85
Quits + OJS moves	0.28	0.85
OJS moves	0.29	0.70

Table 6: Correlation with Contemprary Output: Both Shocks

- Relative Std of shocks: each shock contributes roughly equal to output volatility

Both Shocks: Relative Volatility Uncorrelated

	Model	Data
Output	1	1
Average Wage	0.40	0.44-0.84
New Wage	1.35	0.68-1.09
Unemployment	2.59	4.84
Quits + OJS moves	23.98	4.2
OJS moves	28.45	4.62

Table 7: Standard Deviation Relative to Output: Both Shocks

- Relative Std of shocks: each shock contributes roughly equal to output volatility

Both Shocks: Correlation Uncorrelated

	Model	Data
Output	1	1
Average Wage	0.82	0.24-0.37
New Wage	0.62	0.79-0.83
Unemployment	-0.61	-0.85
Quits + OJS moves	0.47	0.85
OJS moves	0.48	0.70

Table 8: Correlation with Contemprary Output: Both Shocks

Clumsy Experiments & Extensions

- Now we move to some experiments/extensions to illustrate/evaluate the business cycle performance of the model
- We look at the following
  1. An Exogenous quitting Economy with higher b that illuminates the Shimer/Hagedorn-Manowski debate.
  2. An Economy with on the job search and quitting with lower ξ (the intensity of on-the-job search) such that J2J is 29% rather than 40% per year.
  3. An Economy with on the job search and quitting and higher wage pegs (from 0.5 to 0.95).
  4. An Economy with on the job search and quitting and different matching functions for UE and EE moves.

1- High-b Economy: (Without quits or OJS only TFP)

	Low-b			High-b		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.70	0.51	1.00	0.74	0.33	0.84
New Wage	0.70	0.73	0.99	0.74	0.38	0.84
Unemp Rate	12.6%	0.28	-0.55	22.2%	0.97	-0.86

Table 9: The High-b Benchmark Economy: M1

- Much higher unemployment volatility due to higher b
  - higher wages and thus lower firm profits in s-s, amplifying the move of job finding probability due to aggregate shocks
- We are moving towards an economy with two types of b and agents occasionally move across types. V: (Not done yet)
  - such that most quits are due to type stwitchers

2- Baseline: and Low Ave J-2-J 1% Productivity Shock ($\rho = .95$)

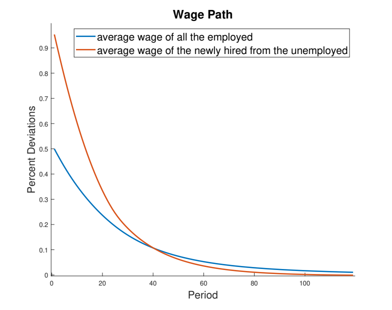


Fig. 9: Wages

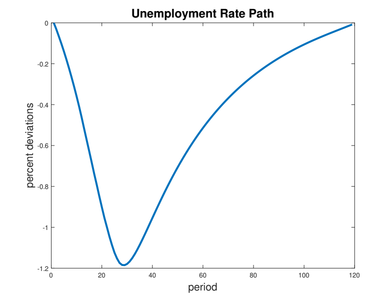


Fig. 10: Unemployment Rate

- Similar Wage Responses
- 70% more unemployment volatility which mainly comes from more responsive quits

2- Baseline and Low Ave J-2-J 1% Productivity Shock ($\rho = .95$)

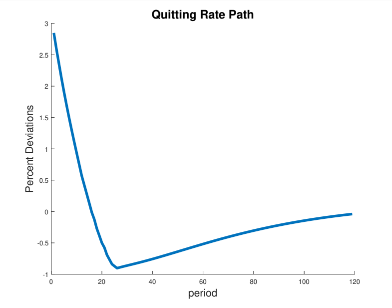


Fig. 11: Quits

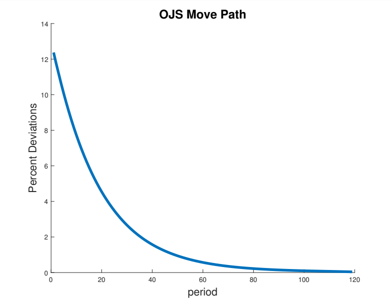


Fig. 12: Job-to-job Moves

- More quitting
- Similar (excessive) J-2-J transitions

2- Baseline: and Low Ave J-2-J 1% Delta Shock ($\rho = .95$)

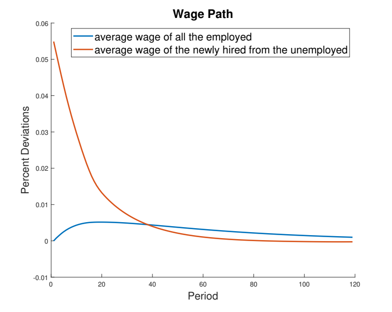


Fig. 13: Wages

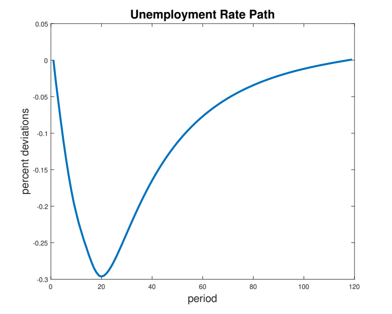


Fig. 14: Unemployment Rate

- Similar Wage Response
- 16% more unemployment response
- Note wage is not pegged to the delta shock

2- Baseline: with Low Ave J-2-J 1% Delta Shock ($\rho = .95$)

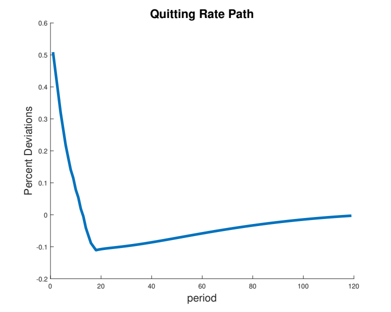


Fig. 15: Quits

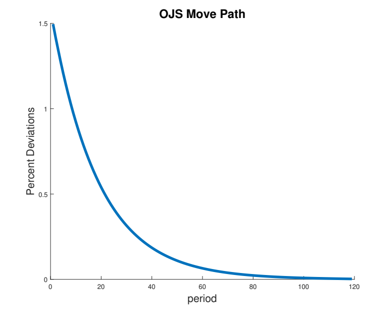


Fig. 16: Job-to-job Moves

- More Quits
- Similar (excessive) volatility for job-to-job transitions

2- Baseline: with Low Ave J-2-J: Business Cycle Statistics

- Two ways to aggregate shocks

	shock corr = 0.95		shock corr = 0	
	Std	corr	Std	corr
output	1.00	1.00	1.00	1.00
avg wage	0.41	0.93	0.41	0.90
new wage	1.69	0.76	1.38	0.52
unemployment	2.59	-0.73	2.80	-0.63
quits + j2j movers	29.85	0.77	26.72	0.38
J2J movers	36.30	0.79	32.51	0.41

- Not too successful in reducing volatility of quits and J2J movers.
- Need to look for alternatives.

3- Baseline with Higher Wage Peg (.8): 1% TFP ($\rho = .95$)

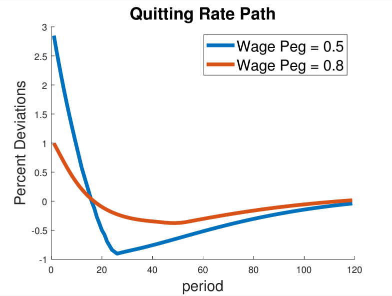


Fig. 17: Quits

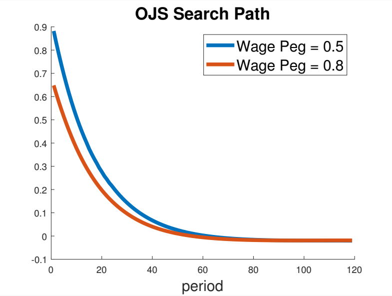


Fig. 18: OJS Searchers

- Higher wage peg lowers the reponse of on-the-job search and quit.
  - Workers find it less so attractive to move/quit as existing wages now comove more with the productivity shock

3- Baseline with Higher Wage Peg (.8): 1% TFP ($\rho = .95$)

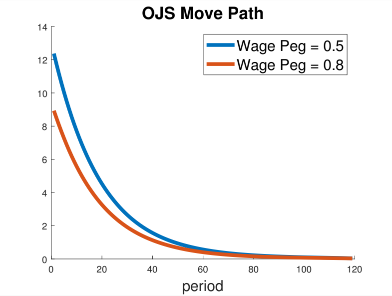


Fig. 19: Job-to-job transitions

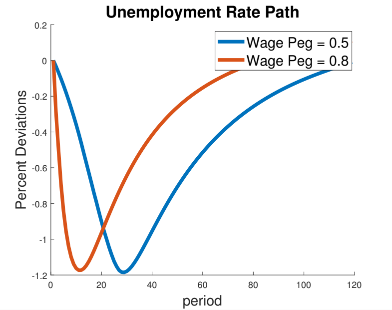


Fig. 20: Unemployment

- Job-to-job transition rate also lowers: from 12% to 9%. This is from
  - less search on the job (see Fig 18)
  - less improvement of job finding rate due to smaller s-s firm profits
- Also less persistence of the unemployment response (less turnover).
- However j2j transition rate still moves much more than unemployment

3- Baseline with Higher Wage Peg (.8): 1% TFP ($\rho = .95$)

	Wage Peg = 0.5			Wage Peg = 0.8		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.690	0.76	0.99
New Wage	0.689	0.95	1.00	0.689	1.04	0.99
Unemp Rate	10.6%	0.35	-0.48	10.6%	0.42	-0.64
Quits+J2J moves	38.4%	8.94	0.99	38.4%	6.65	-0.99
J2J moves	29.2%	10.66	0.99	29.2%	8.50	-0.99

Table 10: M4 Compare Wage Pegs: Productivity Shock ($\rho = 0.95$)

- Lowers j2j transition volatility while raises unemployment volatility
- Still, j2j transition volatility is much higher than unemployment volatility
- In the next several pages we take a closer look at this problem

A Fundamental Tension

- In all of the above exercises we find that the volatility of j2j transition rate is a magnitude larger than unemployment rate
- However, in the data unemployment rate is as volatile as (or even more volatile than) the j2j transition rate.
- Difficult to deliver this in the model from aggregate shocks affecting jobs at all wage levels
    - The percentage changes of firm value, vacancy filling probability and job finding probability are similar at all wage levels
    - Thus as a stock, the response of unemployment would thus be a magnitude smaller than the j2j transition rate (a flow)

A Fundamental Tension: the Fix

- Two potential fixes
  - Make the value of the firm at high wages be more volatile $\Rightarrow$ hard since high-wage matches feature low profits
  - Make the job finding probability of the employed less responsive to the same percentage change in the firm value $\Rightarrow$ curvature in the matching function controls this
- Motivated by this, we will allow η in the matching function $m = \chi u^{\eta} v^{1-\eta}$ to be low in UE moves but high in EE moves
  - $\psi^h(w) = \chi(\frac{\chi}{\psi^f(w)})^{\frac{1-\eta}{\eta}} \Rightarrow \ln \psi^h(w) = \frac{1}{\eta} \ln \chi - \frac{1-\eta}{\eta} \ln \psi^f(w)$
  - Higher $\eta \Rightarrow$ smaller response of $\psi^h(w)$ to $\psi^f(w)$
- Lower η^u from 0.5 to 0.35 and raise η^e from 0.5 to 0.75

4- Baseline plus Different Matching Functions for UE & EE

	$\eta^e = \eta^u = 0.5$			$\eta^e = 0.75, \eta^u = 0.35$		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1.00
Avg Wage	0.690	0.51	1.00	0.688	0.53	1.00
New Wage	0.689	0.95	1.00	0.654	0.92	1.00
Unemp Rate	10.6%	0.35	-0.48	7.7%	0.78	-0.84
Quits+J2J moves	38.4%	8.94	0.99	34.9%	1.42	1.00
J2J moves	29.2%	10.66	0.99	26.9%	1.98	1.00

Table 11: Baseline with Different Matching Functions: TFP Shocks

- It greatly reduces the volatility gap between unemployment and j2j transitions
- But they both show insufficient volatility compared to output, in response to the productivity shock

4- Baseline plus Different Matching Functions for UE & EE

	$\eta^e = \eta^u = 0.5$			$\eta^e = 0.75, \eta^u = 0.35$		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.15	0.13	0.688	0.45	0.47
New Wage	0.689	2.02	0.31	0.654	2.40	0.73
Unemp Rate	10.6%	4.55	-0.99	7.7%	9.37	-0.99
Quits+J2J moves	38.4%	42.41	0.40	34.9%	11.65	0.70
J2J moves	29.2%	49.40	0.42	26.9%	15.55	0.70

Table 12: M4 Different Matching Functions: Delta Shock ($\rho = 0.95$)

- It reduces the volatility gap between unemployment and j2j transitions
- Unemployment is much more volatile compared to output in response to the delta shock, because the delta shock only affects total output through employment

4- Baseline plus Different Matching Functions for UE & EE

- Two ways to aggregate shocks

	shock corr = 0		shock corr = 0.95	
	Std	corr	Std	corr
output	1.00	1.00	1.00	1.00
avg wage	0.48	0.91	0.41	0.94
new wage	1.20	0.80	1.34	0.96
unemployment	3.70	-0.52	3.30	-0.91
quits + j2j movers	4.88	0.60	5.01	0.94
J2J movers	6.50	0.62	6.68	0.96

Table 13: M4 Both Shocks ($\eta^e = 0.75, \eta^u = 0.35, \rho = 0.95$)

- By allowing for two types of shocks, and different matching functions for UE and EE moves, the model delivers a pretty good match to the data

Conclusions I

- Develop tools to get a joint theory of wages, employment and wealth that marry the two main branches of modern macro:
  1. Aiyagari models (output, consumption, investment, interest rates)
  2. Labor search models with job creation, turnover, wage determination, flows between employment, unemployment and outside the labor force.
  3. Add tools from Empirical Micro to generate quits
- Useful for business cycle analysis: We are getting procyclical
  - Quits
  - Employment
  - Investment and Consumption
  - Wages
- On the Job Search are quite important to understand employment volatility and give us a good sense of how (upward) flexible wages are.

Conclusions II

- Exciting set of continuation projects:
  1. Incorporate the movements outside of the labor force.
  2. Endogenous Search intensity on the part of firms
  3. Aiming Shocks to soften correlation between wages and wealth
  4. Efficiency Wages: Endogenous Productivity (firms use different technologies with different costs of idleness)
  5. Move towards more sophisticated household structures (more life cycle movements, multiperson households).

Firms choose Search Intensity

- The number of vacancies posted is chosen by firms
- Easy to implement
- Slightly Different steady state

- Let $\upsilon(\overline{c})$ be a technology to post vacancies where $\overline{c}$ is the cost paid.

- Then the free entry condition requires that for all offered wages

$$0 = \max_{\overline{c}} \left\{ \upsilon(\overline{c})\ \psi^f[\theta(w)]\ \ \frac{\Omega(w)}{1+r} + \left[1 - \upsilon(\overline{c})\ \psi^f[\theta(w)]\right]\ \ \frac{\overline{k}(1-\delta_k)}{1+r} - \overline{c} - \overline{k} \right\},$$

- With FOC given by

$$v_{\overline{c}}(\overline{c})\ \left\{ \psi^f[\theta(w)]\ \ \left[\frac{\Omega(w)}{1+r} - \ \ \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

How to make it consistent with the current steady state

- If $v(\overline{c}) = \frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_{2\ \overline{c}}$, we have

$$(\upsilon_1 \overline{c} + \upsilon_2) \left\{ \psi^f[\theta(w)] \left[\frac{\Omega(w)}{1+r} - \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

- By Choosing υ so that for the numbers that have now

$$\left\{ \left[\frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_2 \overline{c} \right] \psi^f[\theta(w)] \frac{\Omega(w)}{1+r} + \left[1 - \frac{\upsilon_1 \overline{c}^2}{2} - \upsilon_2 \overline{c} \right] \psi^f[\theta(w)] \frac{\overline{k}(1-\delta_k)}{1+r} \right\} = \overline{c} + \overline{k}$$

- Solving for $\{\upsilon_1, \upsilon_2\}$ that satisfy both equations given our choice of $\overline{c}$ we are done

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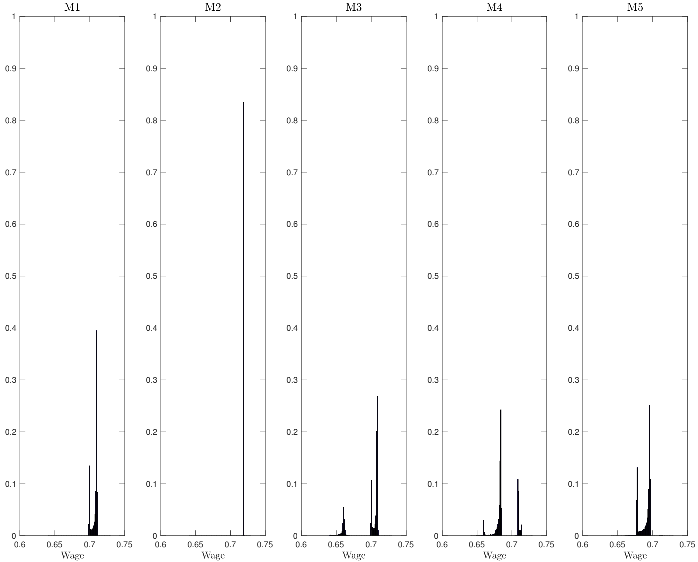
Steady-States

	m1	m2	m3	m4	m4 (low xi)
β	0.975	0.972	0.975	0.976	0.976
interest rate	0.030	0.030	0.030	0.030	0.030
avg consumption	0.686	0.682	0.691	0.684	0.680
avg wage	0.707	0.719	0.696	0.689	0.690
avg wealth	2.789	2.763	2.361	3.041	2.919
stock market value	2.971	2.692	3.065	2.953	2.931
avg labor income	0.659	0.655	0.668	0.654	0.652
consumption to wealth ratio	0.246	0.247	0.293	0.225	0.233
labor income to wealth ratio	0.236	0.237	0.283	0.215	0.223
quit ratio	0.090	0.088	0.090	0.090	0.092
unemployment rate	0.129	0.165	0.076	0.097	0.106
job losers	0.117	0.115	0.117	0.117	0.119
wage of newly hired unemployed	0.707	0.719	0.656	0.677	0.689
std consumption	0.013	0.010	0.011	0.011	0.011
std wage	0.000	0.000	0.003	0.002	0.001
std wealth	2.989	2.715	2.624	3.606	3.677
mean-min consumption	2.057	2.045	2.072	2.051	2.039
mean-min wage	1.012	1.001	1.094	1.058	1.042
UE transition	0.121	0.114	0.128	0.125	0.126
total vacancy	0.544	0.308	0.704	0.578	0.707
avg unemp duration	1.062	1.449	0.589	0.773	0.745
avg emp duration	7.228	7.335	7.228	7.228	7.131
OJS move rate	0.000	0.000	0.420	0.395	0.292
avg job duration	7.228	7.335	1.814	1.898	2.342

Wage Distributions



Derive the Idle Value

- Value of an idle firm is

$$\Omega^0 = -\delta^k k + \frac{1-\delta^f}{1+r}\left[-c^v + \psi^f \Omega + (1-\psi^f)\Omega^0\right]$$

- Free entry

$$k = \frac{1}{1+r}\left[-c^v + \psi^f \Omega + (1-\psi^f)\Omega^0\right]$$

  - Newly entered firms do not receive the destruction shock immediately
  - Vacancy posting cost is paid immediately before searching

- Combine the above

$$\Omega^0 = (1-\delta^f-\delta^k)k$$