Wealth, Wages, and Employment

Preliminary

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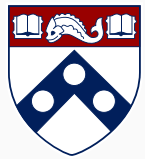


Intro

Introduction

- We want a theory of the joint distribution of employment, wages, and wealth, where
  - Workers are risk averse, so only use self-insurance.
  - Employment and wage risk are endogenous. (More concerned about whether people work than about how long they work.)
  - The economy aggregates into a modern economy (total wealth, labor shares, consumption/investment ratios)
  - Business cycles can be studied. In particular, we want to study employment flows jointly with the other standard objects.
- The most sophisticated version compares well with fluctuations data.

Literature

- The steady state of this economy has as its core Aiyagari (1994) meets Merz (1995), Andolfatto (1996) meets Moen (1997).
- Related Lise (2013), Hornstein et al. (2011), Krusell et al. (2010), Ravn and Sterk (2016, 2017), Den Haan et al. (2015).
- Specially Eeckhout and Sepahsalari (2015), Chaumont and Shi (2017), Griffy (2017).
- Developing empirically sound versions of these ideas compels us to
  - Add extreme value shocks as a form of accommodating quits and on the job search as choices.
  - Use new potent tools to address the study of fluctuations in complicated economies Boppart et al. (2018)

What are the uses?

- The study of Business cycles including gross flows in and out of employment, unemployment and outside the labor force
- Policy analysis where now risk, employment, wealth (including its distribution) and wages are all responsive to policy.
- Get some insights into the extent of wage rigidity
- Life-Cycle versions of these ideas (under construction) will allow us to assess how age dependent policies fare.

Today: Build the Theory Sequentially and discuss & Fluctuations from two types of shocks

1. No Quits: Exogenous Destruction, no Quits. Built on top of Growth Model. (GE version of Eeckhout and Sepahsalari (2015)): Not a lot of wage dispersion. Not a lot of job creation in expansions.
2. Add Endogenous Quits: Higher wage dispersion may arise to keep workers longer (quits via extreme value shocks).
3. On the Job Search workers may get outside offers and take them. (Similar but not the same as in Chaumont and Shi (2017)).
4. Outside of the Labor Force
5. All of the Above
    - Employers commit both to either a wage or a wage schedule $w(z)$ that depends on the aggregate shock.

Key Findings

- If wages are fully fixed and committed (Drastic Wage rigidity)
  - Both endogenous quits and on-the-job yield counter factual procyclical unemployment and massive on the job search.
  - Allowing the wage of an already formed job match to respond some to aggregate shocks corrects this.
  - Getting the right relative volatility of old and new wages and the amount of job-to-job moves and quits provides a way to measure wage rigidity.
- With partial wage rigidity the model fares reasonably well with the data. A few things still to improve. (Excessive Job-to-JOB transitions)
- Similar behavior to that in the Shimer/Hagedorn-Manowski debate. Here we can try to move towards an accommodation of both points of view.

A Brief Look At Data

Relevant Properties in U.S. Data

	Mean Perc	St Dev Relt to Output	Correl w Output	Source
Average Wage	-	0.44-0.84	0.24-0.37	Haefke et al. (2013)
New Wage	-	0.68-1.09	0.79-0.83	Haefke et al. (2013)
Unemployment	4-6	4.84	-0.85	Campolmi and Gnocchi (2016)
Annual Quits	10-40	4.20	0.85	Brown et al. (2017)
Annual Switches	25-35	4.62	0.70	Fujita and Nakajima (2016)
Consumption	75	0.78	0.86	NIPA
Investment	25	4.88	0.90	NIPA

Model 1: No (Endogenous) Quits Model

No Endog Quits: Precautionary Savings, Competitv Search

- Jobs are created by firms (plants). A plant with capital plus a worker produce one (z) unit of the good (z is the aggregate state of the economy).
  - Firms pay flow cost $\bar{c}$ to post a vacancy in market $\{w, \theta\}$.
  - Firms cannot change wage (or wage-schedule) afterwards.
  - Think of a firm as a machine programmed to pay w or $w(z)$
  - Plants (and their capital) are destroyed at rate δ^f.
  - Workers quit exogenously at rate δ^h.
- Households differ in wealth and wages (if working) but not in productivity. There are no state contingent claims, nor borrowing.
  - If employed, workers get w and save.
  - If unemployed, workers produce b and search in some $\{w, \theta\}$.
- General equilibrium: Workers own firms.

Order of Events of No Quits Model

1. Households enter the period with or without a job: $\{e, u\}$.

2. Production & Consumption: Employed produce z on the job. Unemployed produce b at home. They choose savings.

3. Firm Destruction and Exogenous Quits :
   Some Firms are destroyed (rate δ^f) They cannot search this period. Some workers quit their jobs for exogenous reasons δ^h. Total job destruction is δ.

4. Search: Firms and the unemployed choose wage w and tightness θ.

5. Job Matching : $M(V, U)$: Some vacancies meet some unemployed job searchers. A match becomes operational the following period. Job finding and job filling rates $\psi^h(\theta) = \frac{M(V,U)}{U}$, $\psi^f(\theta) = \frac{M(V,U)}{V}$.

No Quits Model: Household Problem

- Individual state: wealth and wage
  - If employed: (a, w)
  - If unemployed: (a)
- Problem of the employed: (Standard)

$$V^e(a, w) = \max_{c, a'} u(c) + \beta \ [(1 - \delta) V^e(a', w) + \delta V^u(a')]$$

$$\text{s.t.} \quad c + a' = a(1 + r) + w, \quad a \geq 0$$

- Problem of the unemployed: Choose which wage to look for

$$V^u(a) = \max_{c, a', w} u(c) + \beta \left\{ \psi^h[\theta(w)] \ V^e(a', w) + [1 - \psi^h[\theta(w)]] \ V^u(a') \right\}$$

$$\text{s.t.} \quad c + a' = a(1 + r) + b, \quad a \geq 0$$

$\theta(w)$ is an equilibrium object

Firms Post vacancies: Choose wages & filling probabilities

- Value of wage-w job: uses constant $\overline{k}$ capital that depreciates at rate δ^k ($\Omega = \overline{k}$)

$$\Omega(w) = z - \overline{k}\delta^k - w + \frac{1-\delta^f}{1+r}\left[(1-\delta^h)\,\Omega(w) + \delta^h\,\Omega\right]$$

- Affine in w: $\Omega(w) = \left[z + \overline{k}\left(\frac{1-\delta^f}{1+r}\delta^h - \delta^k\right) - w\right]\frac{1+r}{r+\delta^f+\delta^h-\delta^f\delta^h}$

Block Recursivity Applies (firms can be ignorant of Eq)

- Value of creating a firm: $\psi^f[\theta(w)]\ \ \Omega(w) + [1-\psi^f[\theta(w)]]\ \Omega$

- Free entry condition requires that for all offered wages

$$\bar{c} + \overline{k} = \psi^f[\theta(w)]\ \ \frac{\Omega(w)}{1+r} + [1-\psi^f[\theta(w)]]\ \ \frac{\Omega}{1+r},$$

No (Endog) Quits Model: Stationary Equilibrium

- Functions $\{V^e, V^u, \Omega, g'^e, g'^u, w^u, \theta\}$, an interest rate r, and a stationary distribution x over (a, w), s.t.

  1. $\{V^e, V^u, g'^e, g'^u, w^u\}$ solve households' problems, $\{\Omega\}$ solves the firm's problem.

  2. Zero profit condition holds for active markets

$$\bar{c} + \overline{k} = \psi^f[\theta(w)] \ \frac{\Omega(w)}{1+r} + [1 - \psi^f[\theta(w)]] \ \frac{\overline{k}(1-\delta-\delta_k)}{1+r}, \quad \forall w \ \text{offered}$$

  3. An interest rate r clears the asset market

$$\int a \ dx = \int \Omega(w) \ dx.$$

Characterization of a worker's decisions

- Standard Euler equation for savings

$$u_c = \beta (1 + r) \; E \{u'_c\}$$

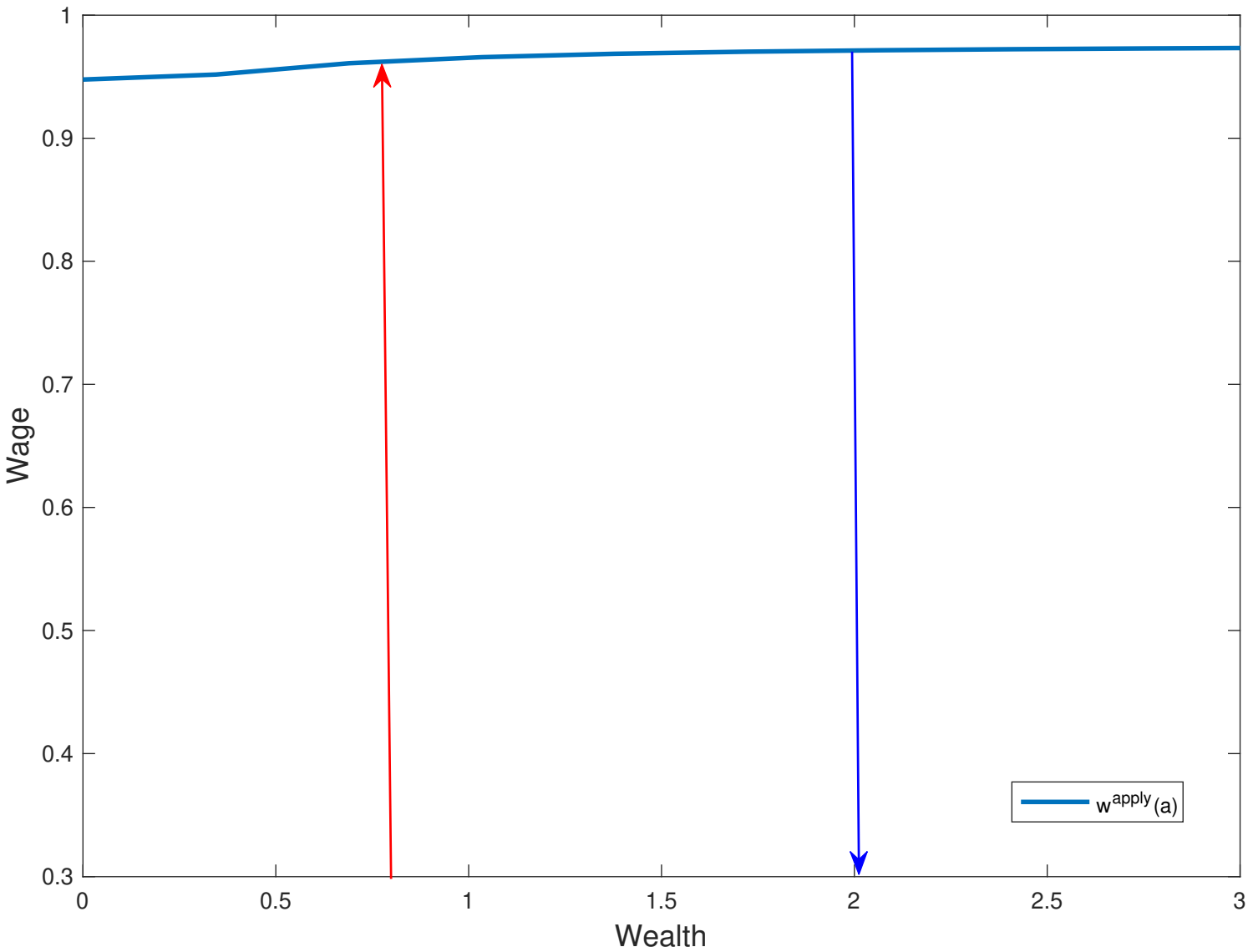
- A F.O.C for wage applicants

$$\psi^h[\theta(w)] \; V^e_w(a', w) = \psi^h_\theta[\theta(w)] \; \theta_w(w) \; [V^u(a') - V^e(a', w)]$$

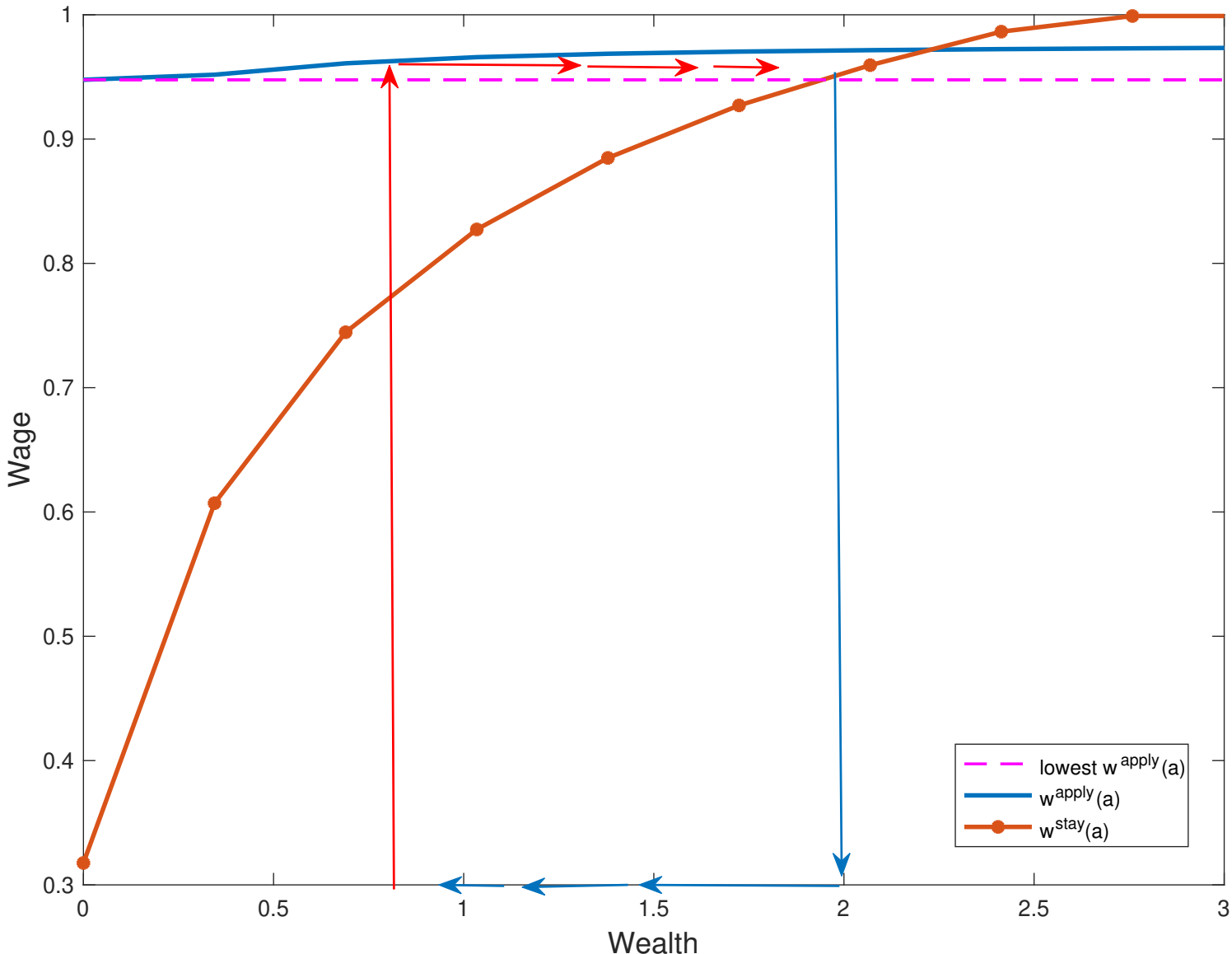
- Households with more wealth are able to insure better against unemployment risk.

- As a result they apply for higher wage jobs and we have dispersion

Worker's wage application decision



Worker's saving decision



Shortcomings of this model

- Silent on Quits and Job-To-Job Movements.
- Low Wage Dispersion
- Small differences in volatility between average and new wages
- Low unemployment volatility

Summary: No (Endog) Quits Model

1. Easy to Compute Steady-State with key Properties
   i. Risk-averse, only partially insured workers, endogenous unemployment
   ii. Can be solved with aggregate shocks too
   iii. Policy such as UI would both have insurance and incentive effects
   iv. Wage dispersion small—wealth doesn't matter too much
   v. ···so almost like two-agent model (employed, unemployed) of Pissarides despite curved utility and savings
2. In the following we examine the implications of a quitting choice

Endogenous Quits

Endogenous Quits: Extreme Value Shocks for finite choice

- Temporary Shocks to the utility of working or not working: Some workers quit. (in addition to any intrinsic taste for leisure)
- Adds a (smoothed) quitting motive so that higher wage workers quit less often: Firms may want to pay high wages to retain workers.
- Conditional on wealth, high wage workers quit less often.
- But Selection (correlation 1 between wage and wealth when hired) makes wealth trump wages and those with higher wages have higher wealth which makes them quite more often: Wage inequality collapses.
- We end up with a model with little wage dispersion but with endogenous quits that respond to the cycle.

Quitting Model: Time-line

1. Workers enter period with or without a job: $\{e, u\}$.

2. Production occurs and consumption/saving choice ensues:

3. Exogenous job/firm destruction happens.

4. Quitting:
   - e draw shocks $\{\epsilon^e, \epsilon^u\}$ and make quitting decision.
     Job losers cannot search this period.
   - u draw shocks $\{\epsilon_1^u, \epsilon_2^u\}$. No decision but same expected means.

5. Search: New or Idle firms post vacancies. Choose $\{w, \theta\}$.
   Wealth is not observable. (Unlike Chaumont and Shi (2017)).
   Yet it is still Block Recursive

6. Matches occur

Quitting Model: Workers

- Workers receive i.i.d shocks $\{\epsilon^e, \epsilon^u\}$ to the utility of working or not
- Value of the employed right before receiving those shocks:

$$\widehat{V}^e(a', w) = \int \max\{V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u\}\, dF^\epsilon$$

  V^e and V^u are values after quitting decision as described before.

- $\{\epsilon^e, \epsilon^u\} \sim G(\mu, \alpha)$ (Gumbel or Type I Extreme Value), then $\widehat{V}$ has a closed form and the ex-ante quitting probability $q(a, w)$ is

$$q(a, w) = \frac{e^{V^u(a)/\alpha}}{e^{V^e(a,w)/\alpha} + e^{V^u(a)/\alpha}}$$

  - Lower $\alpha \to$ lower chance of quitting (less capricious).
  - The higher the wage the higher the difference bw V^e and V^u, so longer job durations.
  - Firms could pay more to keep workers longer.

Quitting Model: Workers Problem

- Problem of the employed: just change $\widehat{V}^e$ for V^e

$$V^e(a,w) = \max_{c,a'} u(c) + \beta \left[(1-\delta)\widehat{V}^e(a',w) + \delta V^u(a)\right]$$
$$\text{s.t.} \quad c + a' = a(1+r) + w, \quad a \geq 0$$

1. We can add term $E\{\max[\epsilon_1^u, \epsilon_2^u]\}$ to the unemployed to eliminate additional option value to a job.
2. Set μ such that $E\{\max[\epsilon_1^u, \epsilon_2^u]\} = 0$, $\mu = -\alpha\gamma - \ln(2)$.
3. Accept the fact that a job is an option to get utility.

Quitting Model: Value of the firm

- Free entry condition requires that for all offered wages

$$\bar{c} + \overline{k} = \frac{1}{1+r} \left\{ \psi^f [\theta(w)] \, \Omega^0(w) + [1 - \psi^f [\theta(w)]] \, \Omega \right\},$$

$\Omega^j(w)$: Value with with j-tenured worker.

- Probability of retaining a worker with tenure j at wage w is $\ell^j(w)$.

(One to one mapping between wealth and tenure)

$$\ell^j(w) = 1 - q[g^{e,j}(a, w), w]$$

$g^{e,j}(a, w)$ savings rule of a $j-$ *tenured* worker that was hired with wealth a

- Firm's value

$$\Omega^j(w) = z - \overline{k}\delta^k - w + \frac{1-\delta^f}{1+r} \left\{ \ell^j(w) \Omega^{j+1}(w) + [1 - \ell^j(w)] \, \Omega \right\}$$

$$\Omega^0(w) \quad = \quad (z - w - \delta^k k)\, Q^1(w) + (1 - \delta^f - \delta_k) k\, Q^0(w),$$

$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta^f}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta^f}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

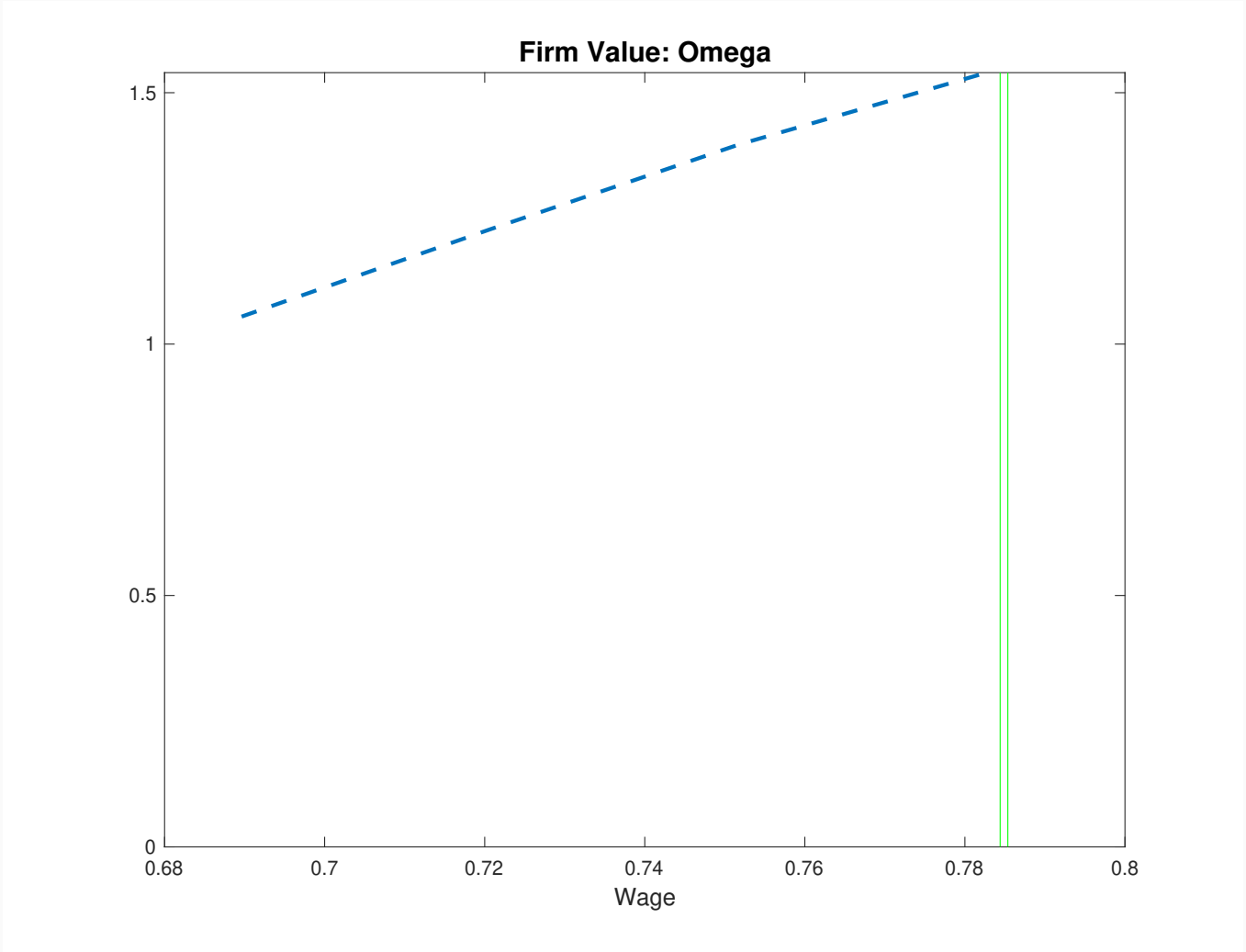
- New equilibrium objects $\{Q^0(w), Q^1(w)\}$. Rest is unchanged.
- It is Block Recursive because wealth can be inferred from w and j. (No need to index contracts by wealth (as in Chaumont and Shi (2017))).

Do we get More Wage Dispersion?

- This Model has the potential to get more wage dispersion
- Conditional on wealth higher wages lead to less quitting.
- So firms are willing to pay more to keep workers longer
- **BUT** we will see a problem

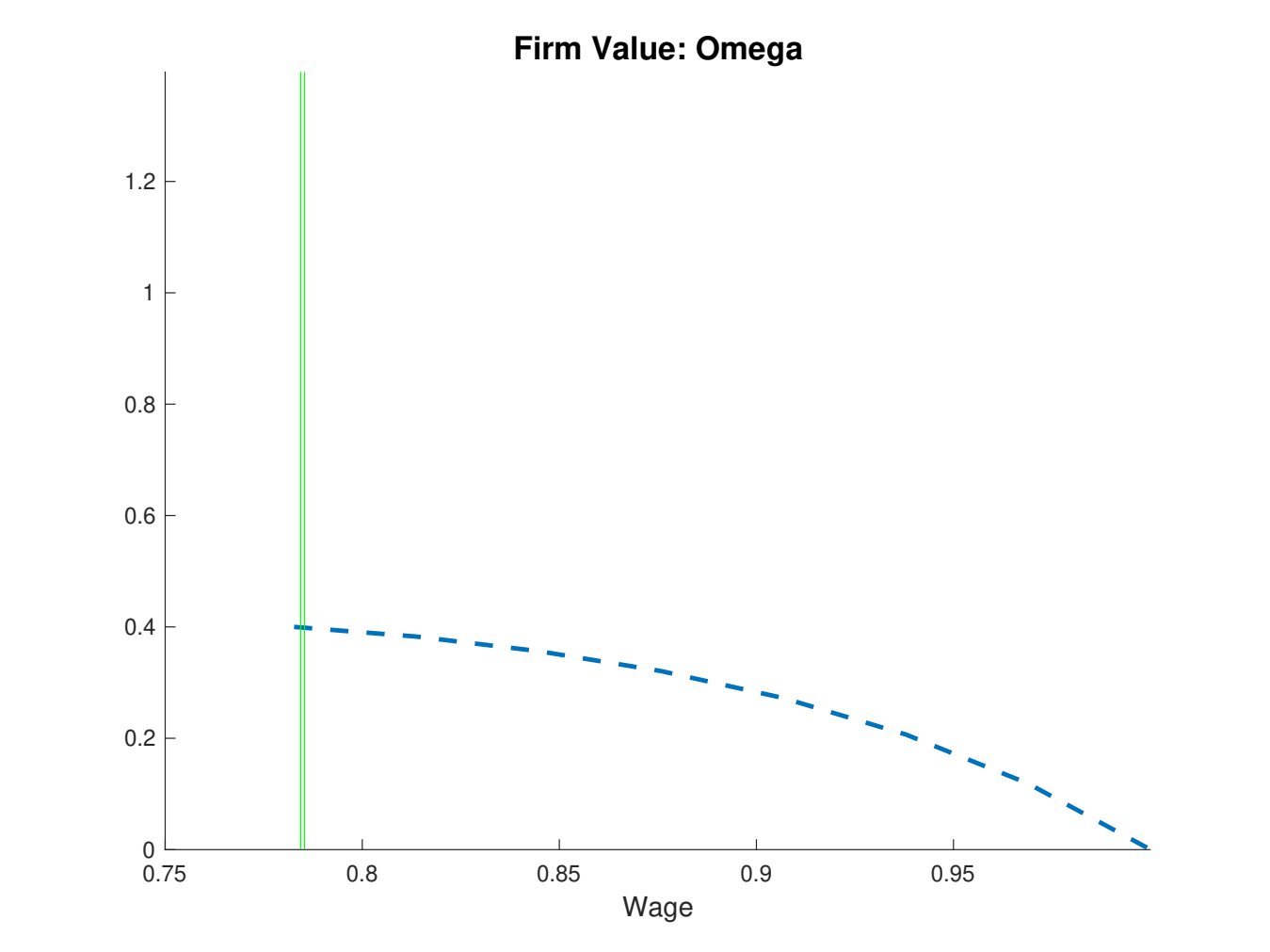
Value of the firm as wage varies: The Poor

- For the poorest, employment duration increases when wage goes up.
- Firms value is increasing in the wage



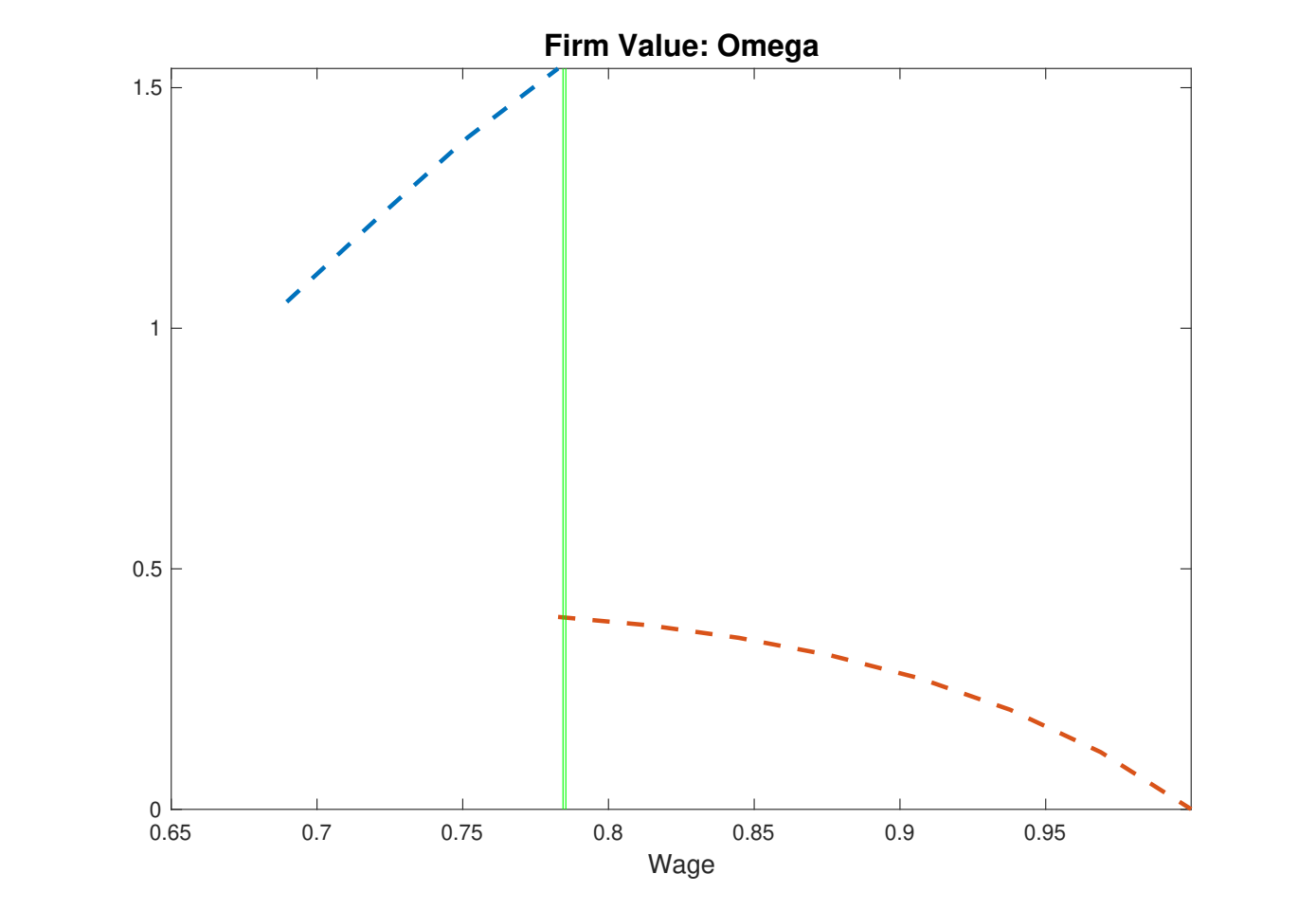
Value of the firm as wage varies: The Rich

- For the richest, employment duration increases but not fast enough.
- Firm value is slowly decreasing in wages (less than static profits).



Value of the firm: Accounting for Worker Selection

- Large drop from below to above equilibrium wages.
- In Equilibrium wage dispersion COLLAPSES due to selection.



- Related to the Diamond dispersion paradox but for very different reasons.

Effect of Quitting: The Mechanism

- Two forces shape the dispersion of wages
  - Agents quit less at higher paid jobs, which enlarge the spectrum of wages that firms are willing to pay (for a given range of vacancy filling probability).
  - However, by paying higher wages, firms attract workers with more wealth.
- Wealthy people quit more often, shrink employment duration.
- In equilibrium, the wage gap is narrow (disappears?) and the effect of wealth dominates. graph

Shortcommings

- Wage Dispersion Collapses
- Silent on Job-To-Job Movements.
- Unemployment Moves little (but more than the previous one) over the cycle
- No difference in volatility between average and new wages
- Correlation 1 between Wealth when starting to work and wage

Need: To Improve the Correlation Between Wealth & Wag

- Pose *aiming* (extreme value) shocks).
- This reduces the correlation between wages and wealth when first hired.
- It will have many uses, we think.

On the Job Search

On the Job Search Model: Time-line

1. Workers enter period with or without a job: V^e, V^u.
2. Production & Consumption:
3. Exogenous Separation
4. Quitting? Searching? Neither?: Employed draw shocks $(\epsilon^e, \epsilon^u, \epsilon^s)$ and make decision to quit, search, or neither. Those who quit become u', those who search join the u, in case of finding a job become $\{e', w'\}$ but in case of no job finding remain e' with the same wage w and those who neither become e' with w. $\widehat{V}^E(a', w)$, is determined with respect to this stage.
5. Search : Potential firms decide whether to enter and if so, the market (w) at which to post a vacancy; u and s assess the value of all wage applying options, receive match specific shocks $\{\epsilon^{w'}\}$ and choose the wage level w' to apply. Those who successfully find jobs become e', otherwise become u'.
6. $\widehat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.
7. Match

- After saving, the unemployed problem is

$$\widehat{V}^u(a') = \int \max_{w'} \left[\psi^h(w') V^e(a', w') + (1 - \psi^h(w')) V^u(a') + \epsilon^{w'} \right] \, dF^\epsilon$$

- After saving, the employed choose whether to quit, search or neither

$$\widehat{V}^e(a', w) = \int \max\{ V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u, V^s(a', w) + \epsilon^s \} dF^\epsilon$$

- The value of searching is

$$V^s(a', w) = \int \max_{w'} \left[\psi^h(w') V^e(a', w') + [1 - \psi^h(w')] V^e(a', w) + \epsilon^{w'} \right] \, dF^\epsilon$$

On the Job Search: Household choices

- The probabilities of quitting and of searching

$$q(a', w) = \frac{1}{1 + \exp(\alpha[V^e(a', w) - V^u(a')]) + \exp(\alpha[V^s(a', w) - V^u(a') + \mu^s])},$$
$$s(a', w) = \frac{1}{1 + \exp(\alpha[V^u(a') - V^s(a', w)]) + \exp(\alpha[V^e(a', w) - V^s(a', w) - \mu^s])}.$$

$\mu^s < 0$ is the mode of the shock ϵ^s which reflects the search cost.

- Households solve

$$V^e(a, w) = \max_{a' \geq 0} u[a(1 + r) + w - a'] + \beta \left[\delta V^u(a') + (1 - \delta)\widehat{V}^e(a', w)\right]$$

$$V^u(a) = \max_{c, a' \geq 0} u[a(1 + r) + b - a'] + \beta \widehat{V}^u(a')$$

- The value of the firm is again given like in the Quitting Model

$$\Omega^0(w) = (z - w - \delta^k k)\, Q^1(w) + (1 - \delta - \delta_k) k\, Q^0(w),$$

$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

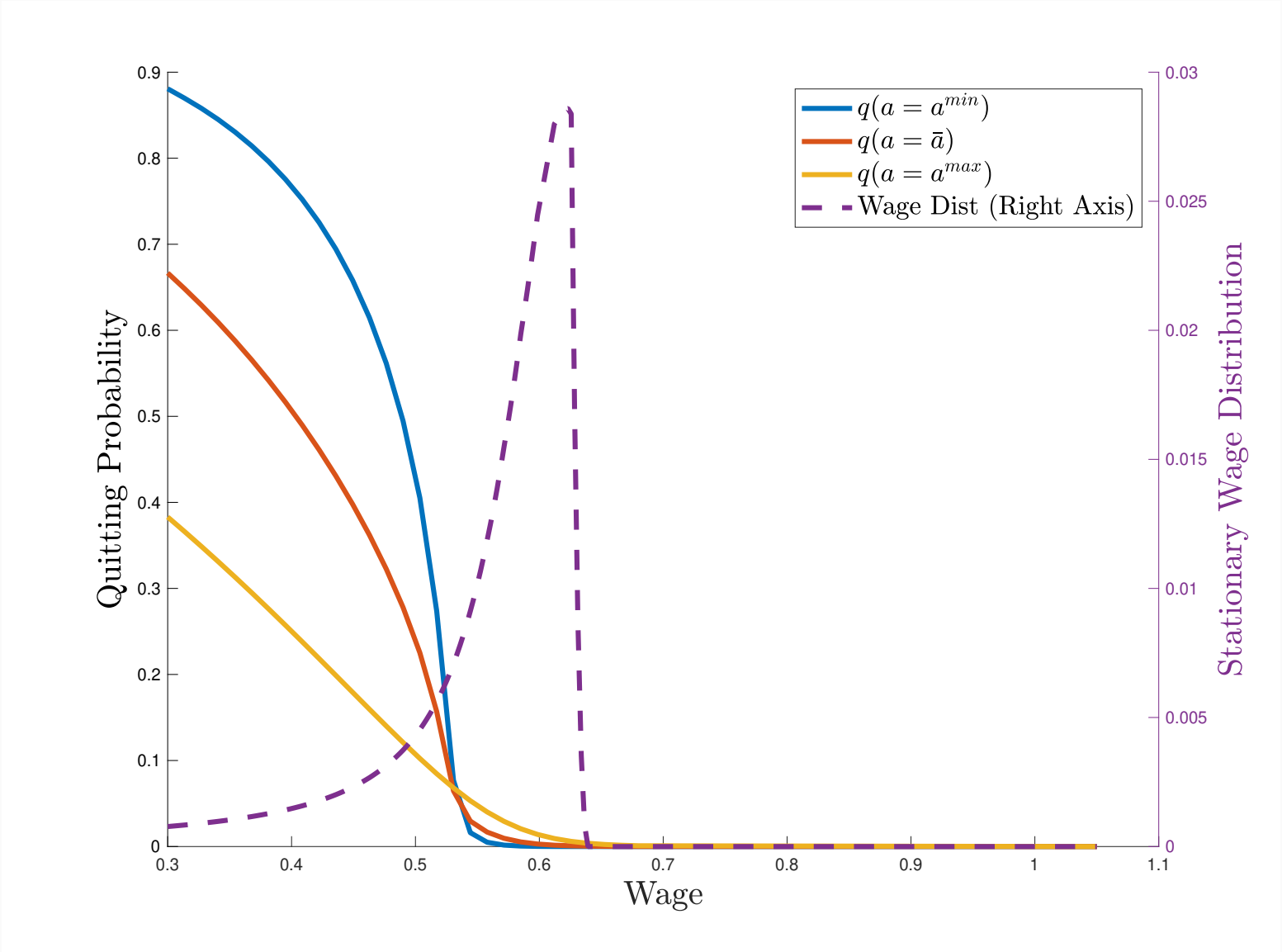
$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- Except that now the probability of keeping a worker after j periods is

$$\ell^j(w) = 1 - \int h(w;a)\, q[g^{e,j}(a,w), w]\, dx^u(a) -$$
$$\int h(w;a)\, s[w; g^{e,j}(a,w)] \left[\int \hat{h}[\widetilde{w}; g^{e,j}(a,w), w] \xi \phi^h(\widetilde{w})\, d(\widetilde{w}) \right] dx^u(a)$$

- Not block recursive but Q^1 and Q^2 are sufficient.

OJS Quitting Probabilities, Various wealths & Wage Density



- The rich pursue often other activities (leisure?)

Outside the Labor Force

Outside the Labor Force Model: Time-line

1. Workers enter period with or without a job: V^e, V^u.
2. In the beginning of the period non Workers get a shock to the utility of either searching or not searching. They then choose whether to sit out and not search or to search. It is an extreme value shock.

   Workers get a utility injection equal to the expected utility of the maximum of those two shocks to get no bias in the value of working versus not.
3. Production & Consumption:
4. Exogenous Separation
5. Quitting? Searching? Neither?:
6. Search
7. $\widehat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.
8. Match

Various Economies with added Life Cycle (live 50 years)

- Provides a mechanism for having poor agents
- Right now we have Four Economies
  1. Only Exogenous Quitting
  2. Endogenous Quitting
  3. Exogenous Quitting with On-the-job Search
  4. Endogenous Quitting and On-the-job Search
  5. ... and some agents do not want to work
- Today we will only look at the Economy with Endogenous quitting and On-the-Job-Search (4)

Quantitative Analysis: Steady States

Parameter Values: Period is half a quarter

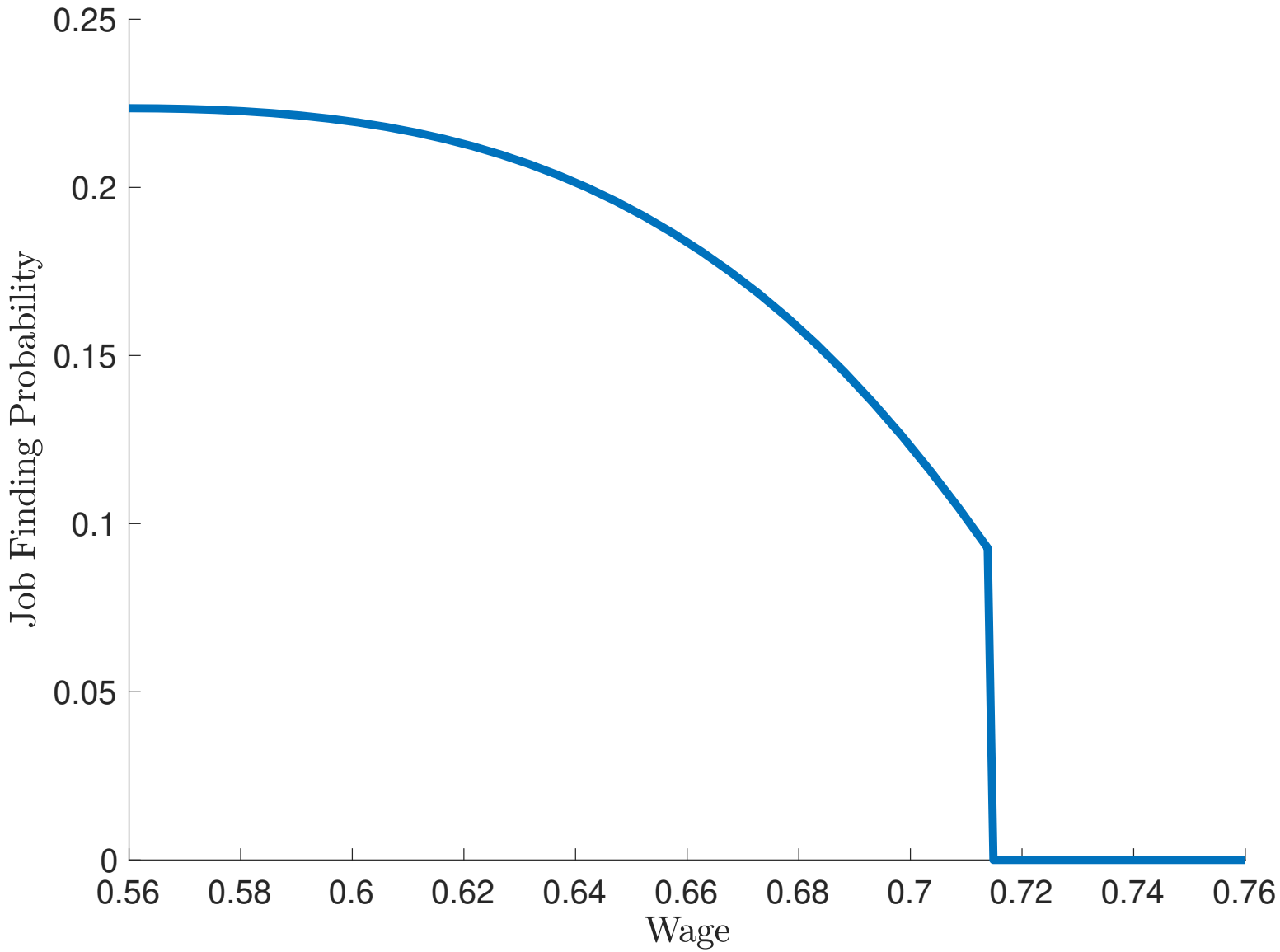
	Definition	Value in Yearly Units
r	interest rate	3%
K	fixed capital required	3
δ^f	firm destruction rate	2.88%
δ^k	capital maintenance rate	6.38%
δ^h	total worker quitting rate	8.56%
c^v	job posting cost	0.03
y	productivity on the job	1
b/w	productivity at home	0.4
σ	risk aversion	2
Matching function	$m = \chi u^\eta v^{1-\eta}$, non-OJS	$\chi = 0.15, \eta = 0.62$
	$m = \chi u^\eta v^{1-\eta}$, OJS	$\chi = 0.3, \eta = 0.5$

• We also explore a lower on the job search economy (high value of leisure economy) $b/w \sim 0.75$

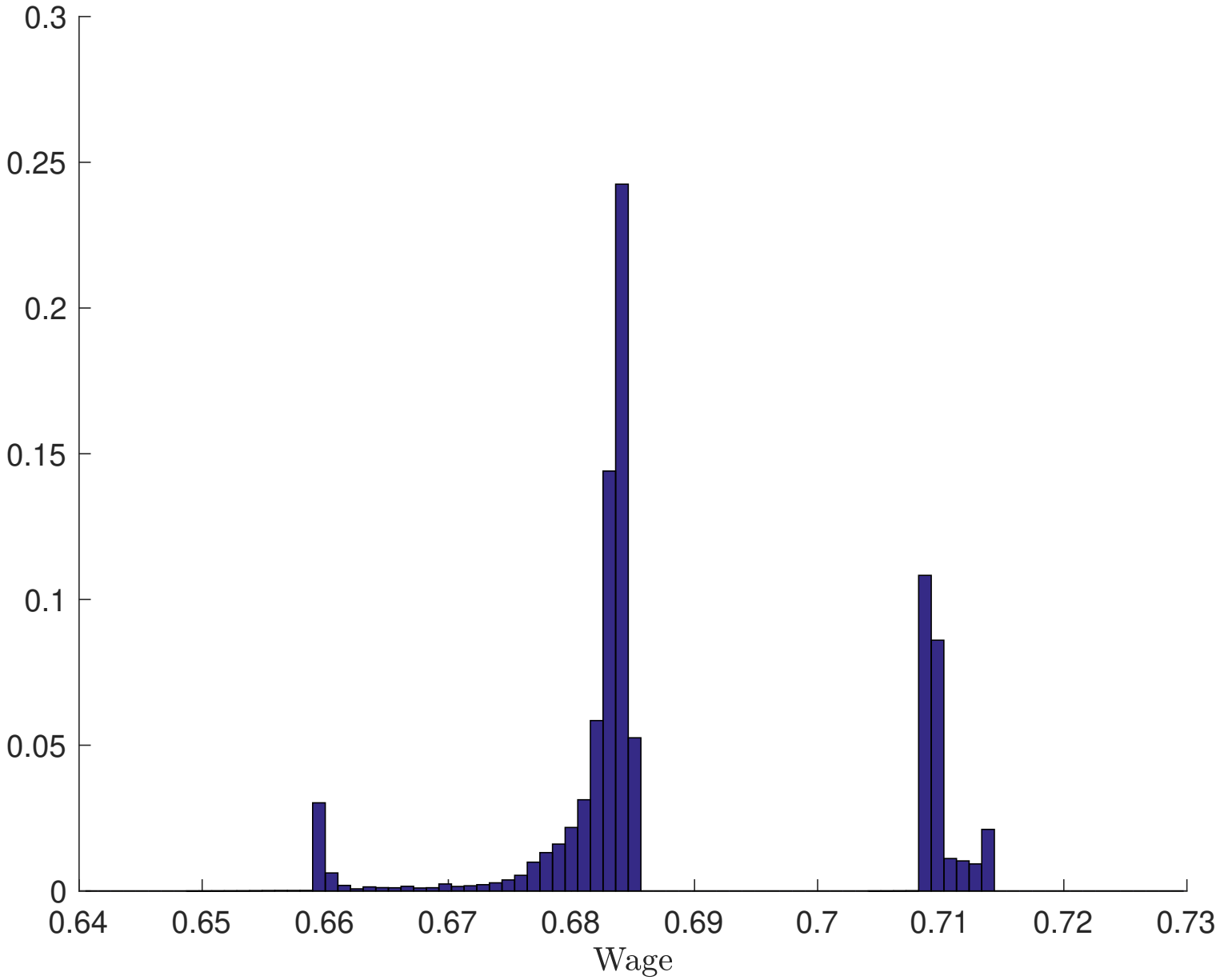
Steady State Allocations in Yearly Units: Endog Quits & C

interest rate	0.030
avg consumption	0.651
avg wage	0.689
avg wealth	3.041
stock market value	2.953
avg labor income	0.654
consumption to wealth ratio	0.225
labor income to wealth ratio	0.215
quit ratio	0.090
unemployment rate	0.097
job losers	0.117
wage of newly hired unemp	0.677
std consumption	0.011
std wage	0.002
std wealth	3.606
mean-min consumption	2.051
mean-min wage	1.058
UE transition	0.125
total vacancy	0.578
avg unemp duration	0.773
avg emp duration	7.228
avg job duration	1.898
OJS move rate	0.395

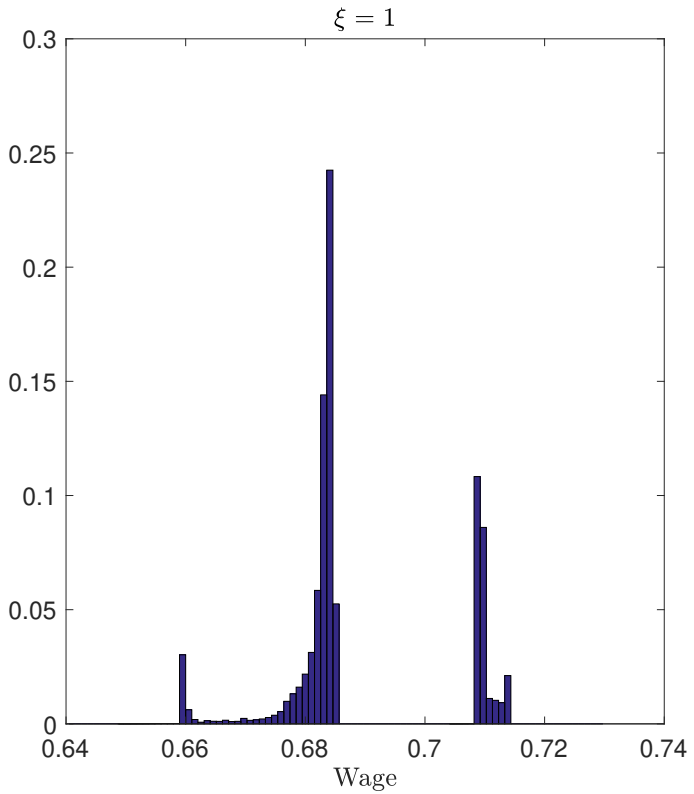
Job Finding Probability Curves

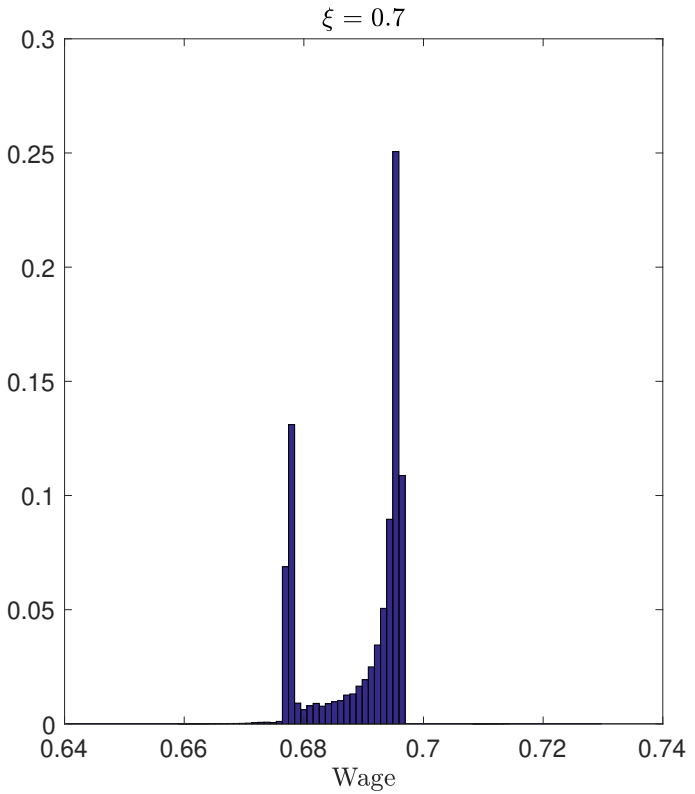


Wage Distributions: Baseline

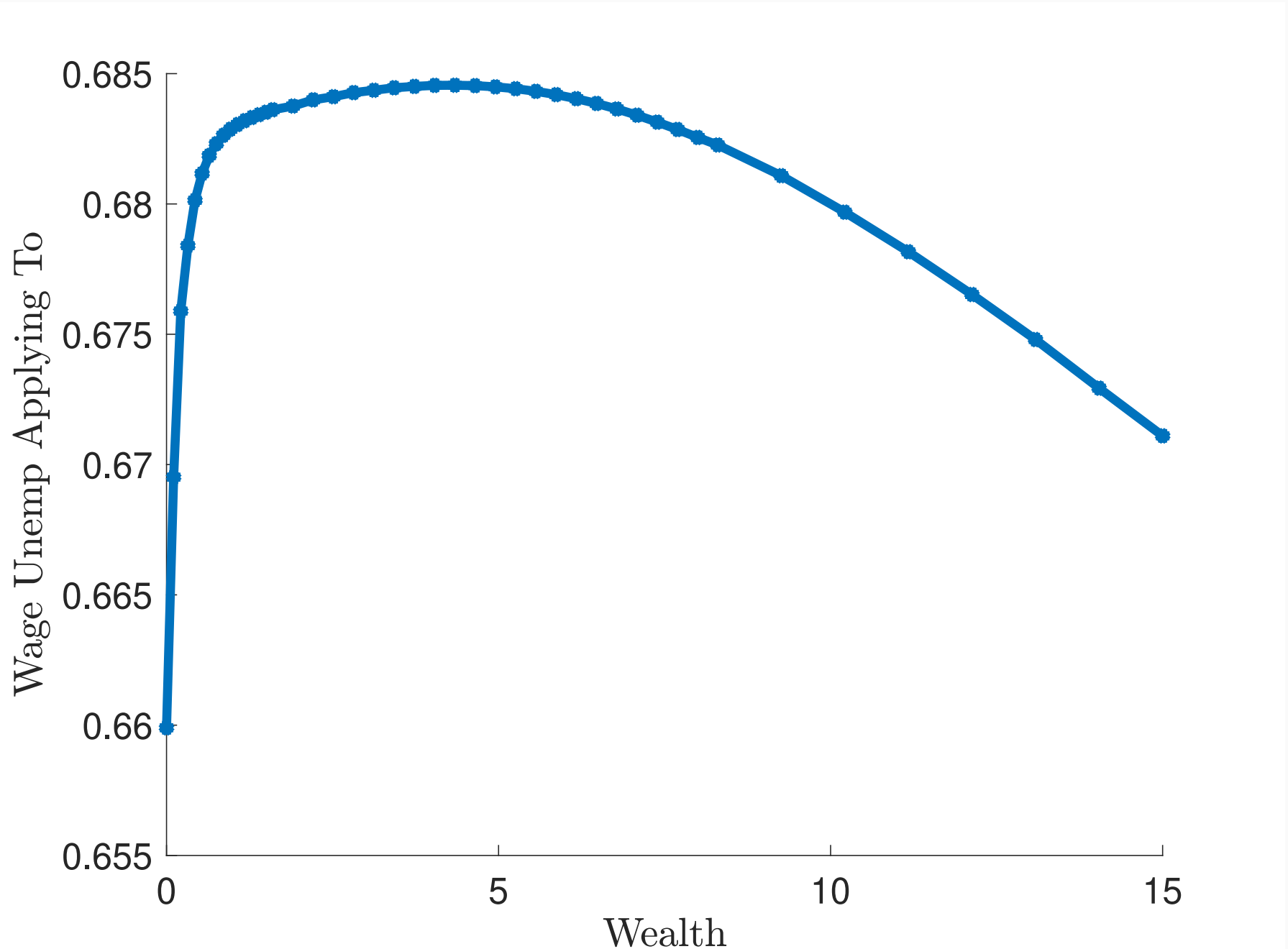


Wage Distributions: Comparing with lower OJS

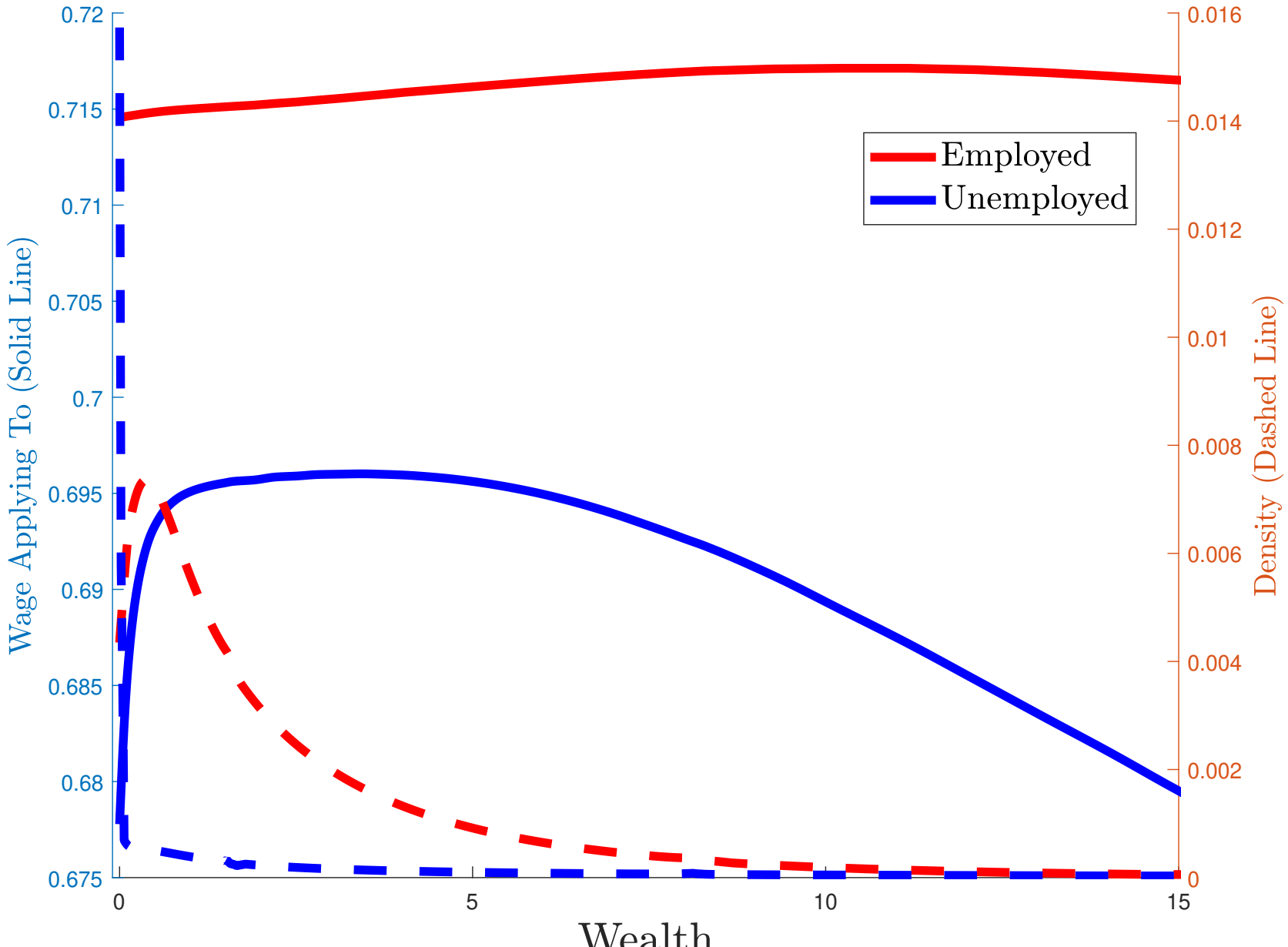




Wage Applications of the Unemployed by Wealth



Wage Applications of U and $\overline{w}$ and densities of all



Summary of Steady States

- Without on-the-job search the wage dispersion due to competitive search is not much (wage mean-min ratio = 1.021).
- With on-the-job we can get 1.108, which is about half the data (≈ 1.2).
- Endogenous quitting, leads to complete collapse of wage dispersion.
- On the job search and Quitting allow firms to tolerate more wage dispersion (they require smaller changes in worker finding probabilities).
- Can Match many properties of employment inclduing gross flows
- Have to be aware of implicit option values of extreme value shocks.

Aggregate Fluctuations

Introduce Aggregate Shocks

- We examine the model responses to two type of shocks
  1. Productivity shocks z_t: Output = EmpRate $\times (1 + z_t)$
  2. Firm destruction shocks d_t: Firm Destruction Rate = $\delta^f \times (1 - d_t)$
- We introduce a wage peg assumption:
  - To allow the wage of an already formed job match to respond to z_t shocks directly (by 50%) (but not to d_t shocks)
  - If wages were completely rigid there would be massive quits: counterfactual.

Baseline: IRF to z shock

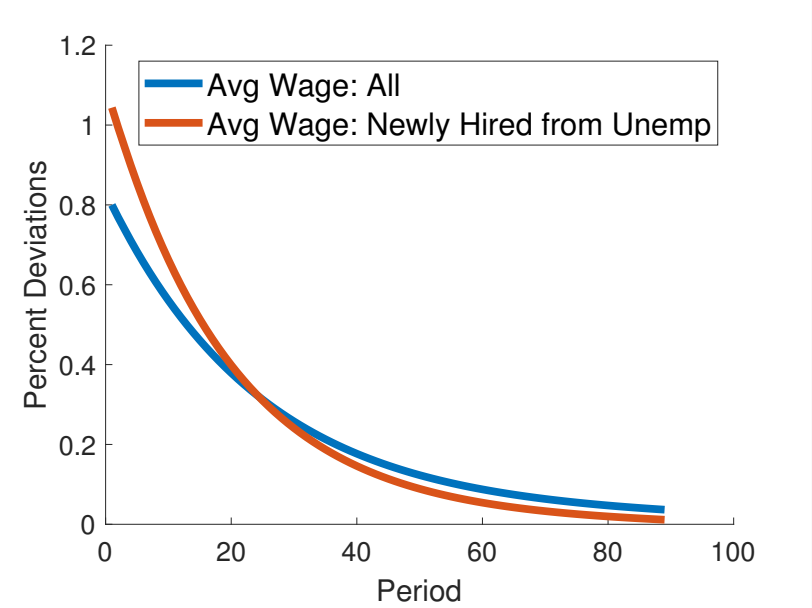


Figure 1: Wages

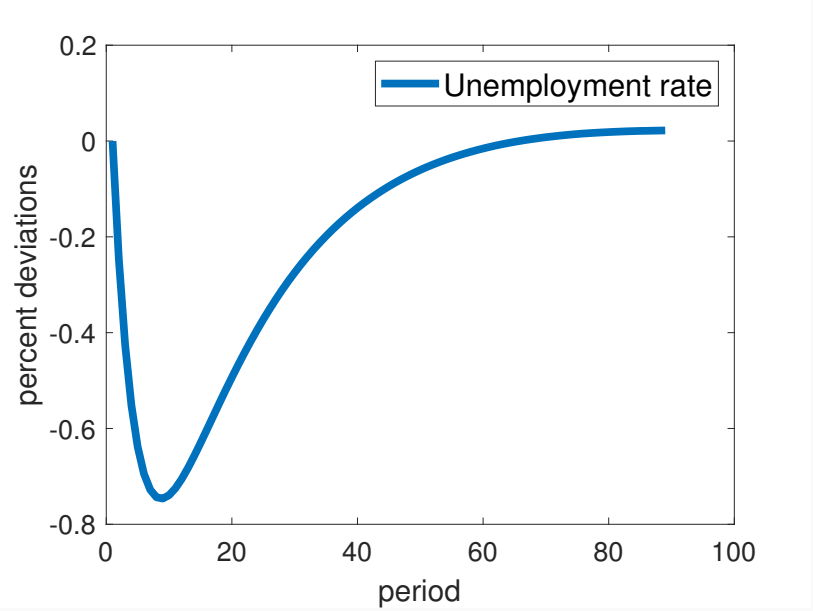


Figure 2: Unemployment Rate

- Responsive new wage (directed search) and average wage (wage peg)
- Non-trivial response of unemployment

Baseline: IRF to z shock

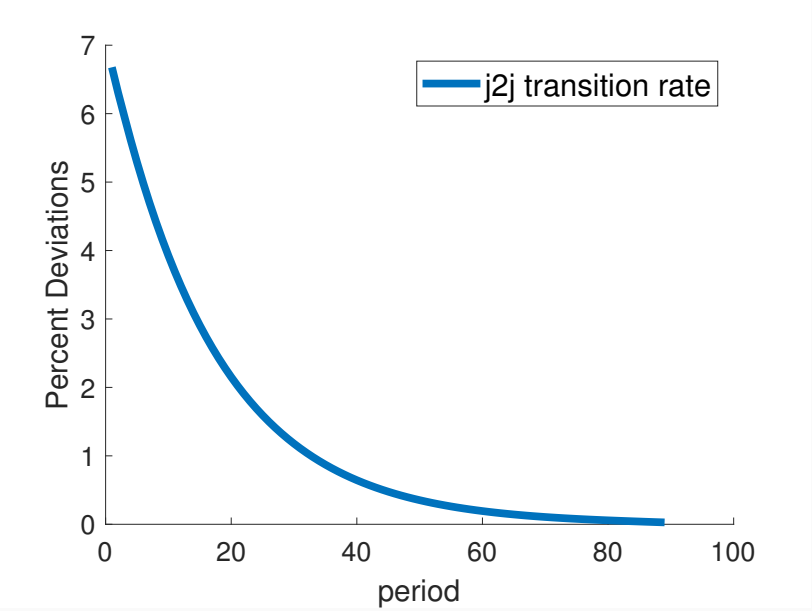


Figure 3: J2J transitions

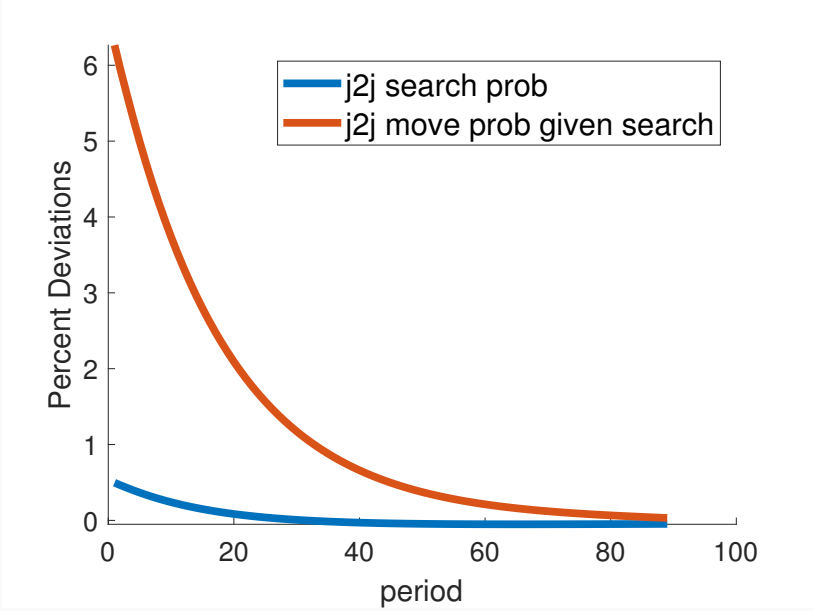


Figure 4: J2J search & JFP

- Too much responsive j2j transitions
- Due to improved job finding probability, not more searchers

Baseline: IRF to d shock

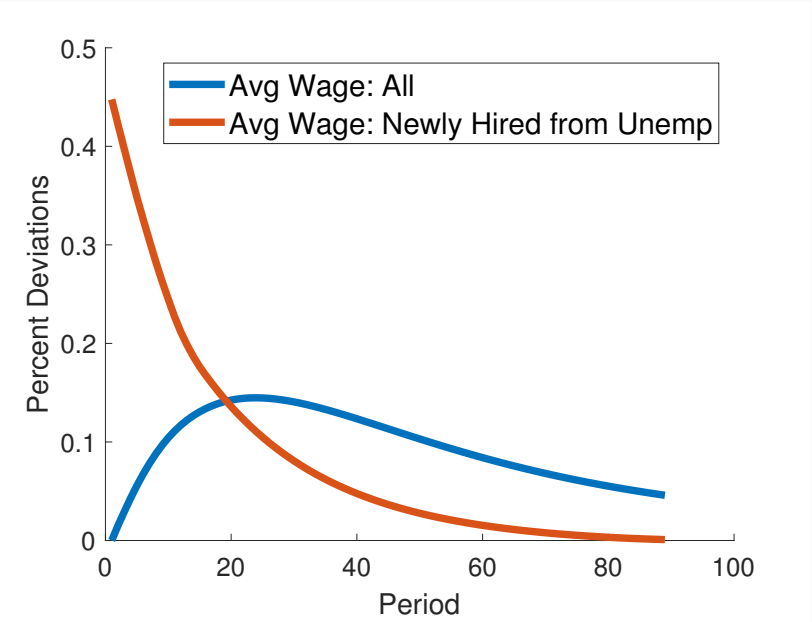


Figure 5: Wages

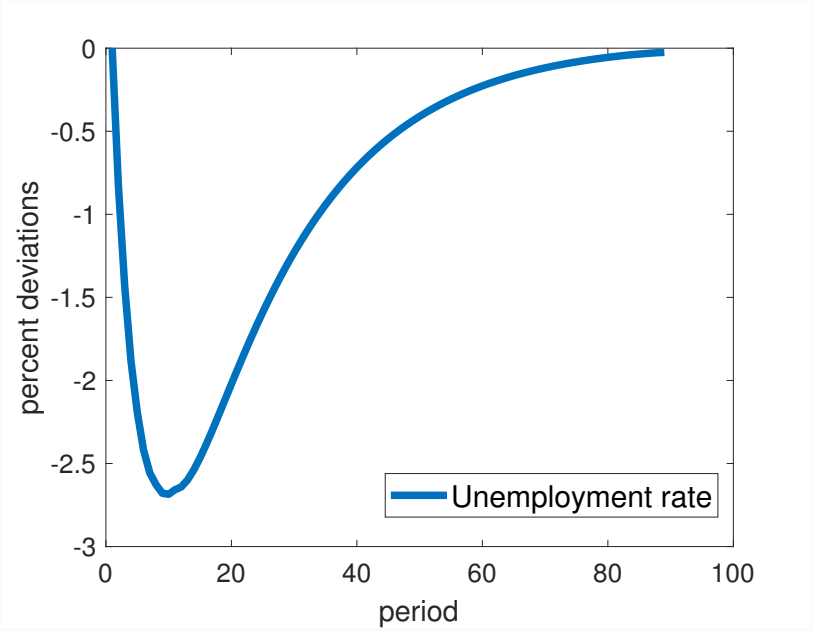


Figure 6: Unemployment Rate

- 1% delta shock = 0.36 base points
- Large response of wage and unemployment to the delta shock
- Note wage is not pegged to the delta shock

Baseline: IRF to d shock

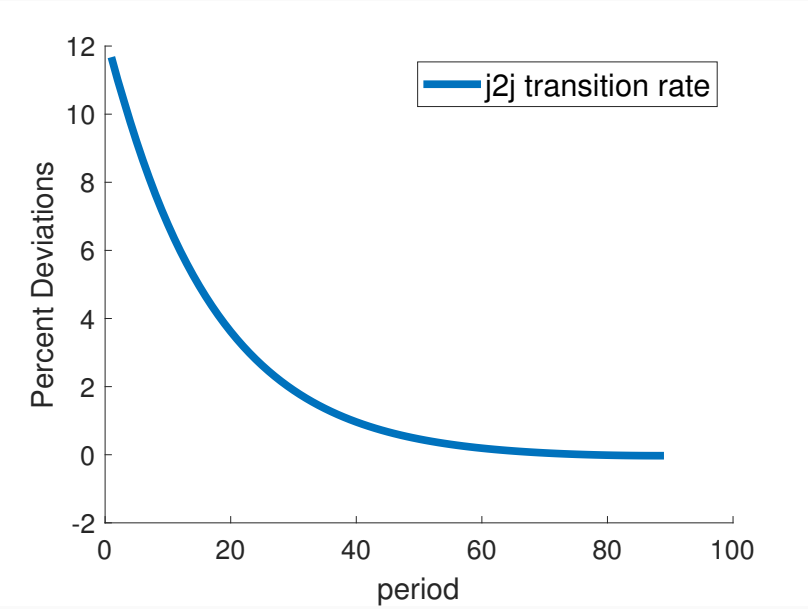


Figure 7: J2J transitions

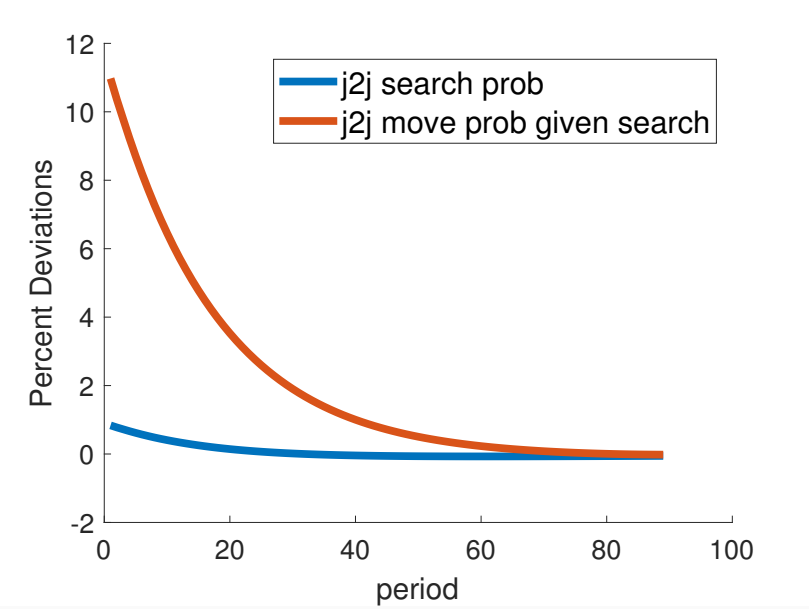


Figure 8: J2J search & JFP

- But too much volatility for job-to-job transitions

Summary, On-the-job Search and Quits

- Pro-cyclical average wages, new wages, and employment, qutting, and job-to-job transitions
- Clear responses of new wages and employment
- Quitting mildly respnds to both shocks
- Job-to-job transitions move too much with both shocks

Assessing Performance in terms of standard HP-filtered 2ND moments

- 1st order data moments are from standard database: CPS, JOLTS, LEHD and NIPA.

- 2nd order data moments are from Haefke et al. (2013), Campolmi and Gnocchi (2016), Brown et al. (2017) and Fujita and Nakajima (2016).

Productivity Shock: Relative Volatility

- Only Productivity Shock: $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	0.51	0.44-0.84
New Wage	0.95	0.68-1.09
Unemployment	0.35	4.84
Quits + OJS moves	8.94	4.2
OJS moves	10.66	4.62

Table 1: Standard Deviation Relative to Output: Only Productivity Shock

- Unemployment moves too little and Quits and OJS move too much

Productivity Shock: Correlation

- Only Productivity Shock: $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	1.00	0.24-0.37
New Wage	1.00	0.79-0.83
Unemployment	-0.48	-0.85
Quits + OJS moves	0.99	0.85
OJS moves	0.99	0.70

Table 2: Correlation with Contemprary Output: Only Productivity Shock

- Correlations are on the spot

Delta Shock: Relative Volatility

	Model	Data
Output	1	1
Average Wage	0.09	0.44-0.84
New Wage	2.02	0.68-1.09
Unemployment	4.70	4.84
Quits + OJS moves	41.66	4.2
OJS moves	49.36	4.62

Table 3: Standard Deviation Relative to Output: Only Delta Shock

- Now Unemployment is good but moves are excessive
- Note that relative to output, productivity is very important so employment cannot do that much, but this shock makes employment the only culprit so it has to move a lot

Delta Shock: Correlation

- Only Delta Shock: $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	0.13	0.24-0.37
New Wage	0.31	0.79-0.83
Unemployment	-0.99	-0.85
Quits + OJS moves	0.40	0.85
OJS moves	0.42	0.70

Table 4: Correlation with Contemprary Output: Only Delta Shock

Both Shocks: Relative Volatility Very correlated

- Interact productivity shock and delta shock
  - High Correlation of shocks = 0.95
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.49	0.44-0.84
New Wage	1.38	0.68-1.09
Unemployment	3.02	4.84
Quits + OJS moves	25.77	4.2
OJS moves	30.53	4.62

Table 5: Standard Deviation Relative to Output: Both Shocks

Both Shocks: Correlation

- Interact productivity shock and delta shock
  - High Correlation of shocks = 0.95
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.77	0.24-0.37
New Wage	0.50	0.79-0.83
Unemployment	-0.37	-0.85
Quits + OJS moves	0.28	0.85
OJS moves	0.29	0.70

Table 6: Correlation with Contemprary Output: Both Shocks

Both Shocks: Relative Volatility Uncorrelated

- Interact productivity shock and delta shock
  - Low Correlation of shocks = 0
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.40	0.44-0.84
New Wage	1.35	0.68-1.09
Unemployment	2.59	4.84
Quits + OJS moves	23.98	4.2
OJS moves	28.45	4.62

Table 7: Standard Deviation Relative to Output: Both Shocks

Both Shocks: Correlation Uncorrelated

- Interact productivity shock and delta shock
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.82	0.24-0.37
New Wage	0.62	0.79-0.83
Unemployment	-0.61	-0.85
Quits + OJS moves	0.47	0.85
OJS moves	0.48	0.70

Table 8: Correlation with Contemprary Output: Both Shocks

Clumsy Experiments & Extensions

Several Experiments/Extensions

- Now we move to some experiments/extensions to evaluate the business cycle performance of the model

- We look at the following
  - An extension to allow for different matching function elasticities for UE and EE moves ($\eta^u \neq \eta^e$).
  - On top of that an economy with higher b (from 0.3 to 0.5-0.6) that illuminates the Shimer/Hagedorn-Manowski debate.

Hetero-η Economy: Motivation

- For all the above exercises we find that the volatility of j2j transition rate is a magnitude larger than unemployment rate
- In the data unemployment rate is as volatile as (or even more volatile than) the j2j transition rate.
- Difficult to deliver this in the model from aggregate shocks affecting jobs at all wage levels
  - The percentage changes of firm value, vacancy filling probability and job finding probability are similar at all wage levels
  - Thus as a stock, the response of unemployment would thus be a magnitude smaller than the j2j transition rate (a flow)

Hetero-η Economy: Motivation

- Two potential fix
  - Make the firm value at high wages more volatile $\Rightarrow$ hard since high-wage matches feature low profits
  - Make the job finding probability of the employed less responsive to the same percentage change in the firm value $\Rightarrow$ curvature in the matching function controls this

- Motivated by this, we will allow η in the matching function $m = \chi u^{\eta} v^{1-\eta}$ to be low in UE moves but high in EE moves
  - $\psi^h(w) = \chi(\frac{\chi}{\psi^f(w)})^{\frac{1-\eta}{\eta}} \Rightarrow \ln \psi^h(w) = \frac{1}{\eta} \ln \chi - \frac{1-\eta}{\eta} \ln \psi^f(w)$
  - Higher $\eta \Rightarrow$ smaller response of $\psi^h(w)$ to $\psi^f(w)$

- Lower η^u from 0.5 to 0.35 and raise η^e from 0.5 to 0.75

Hetero-η Economy: IRF to z shock

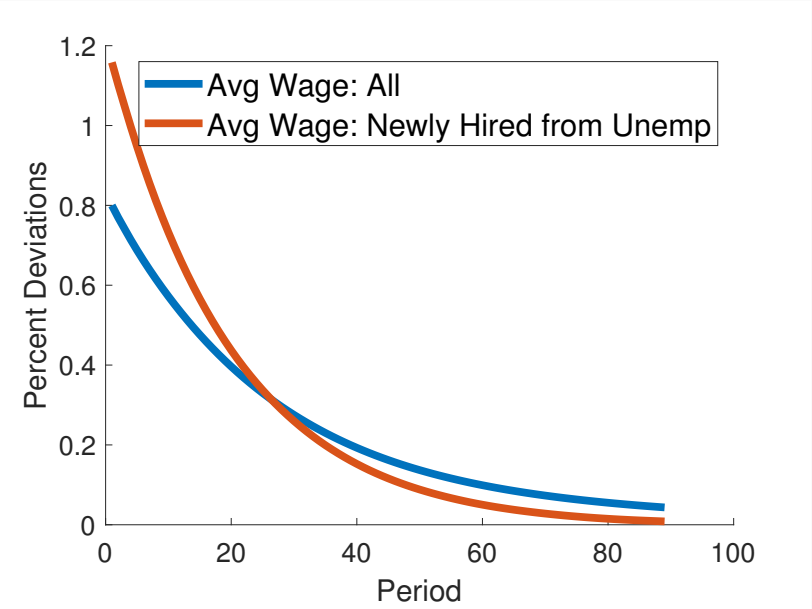


Figure 9: Wages

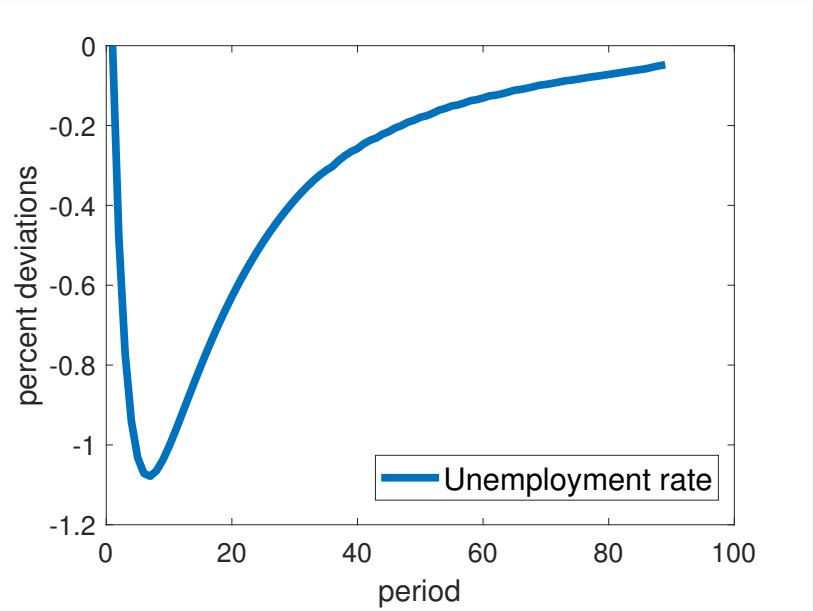


Figure 10: Unemployment Rate

- Similar wage response
- More responsive unemployment (still not enough)

Hetero-η Economy: IRF to z shock

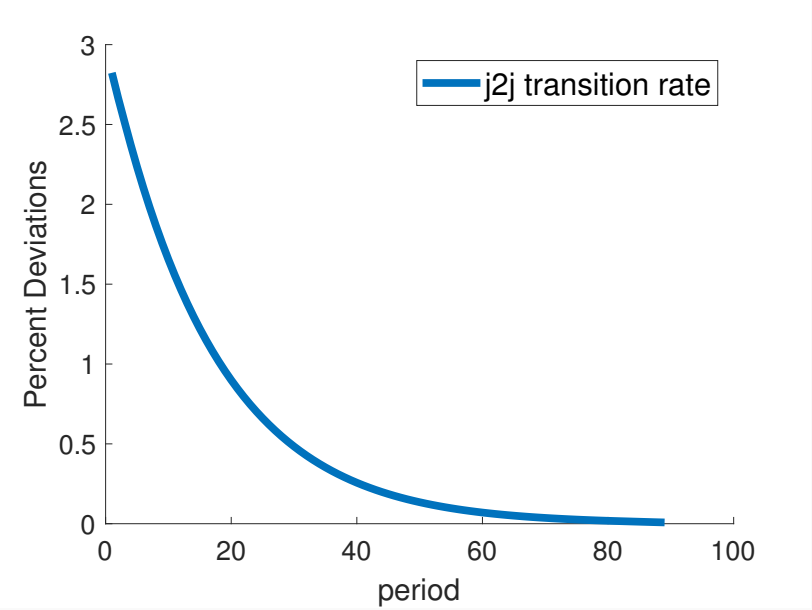


Figure 11: J2J transitions

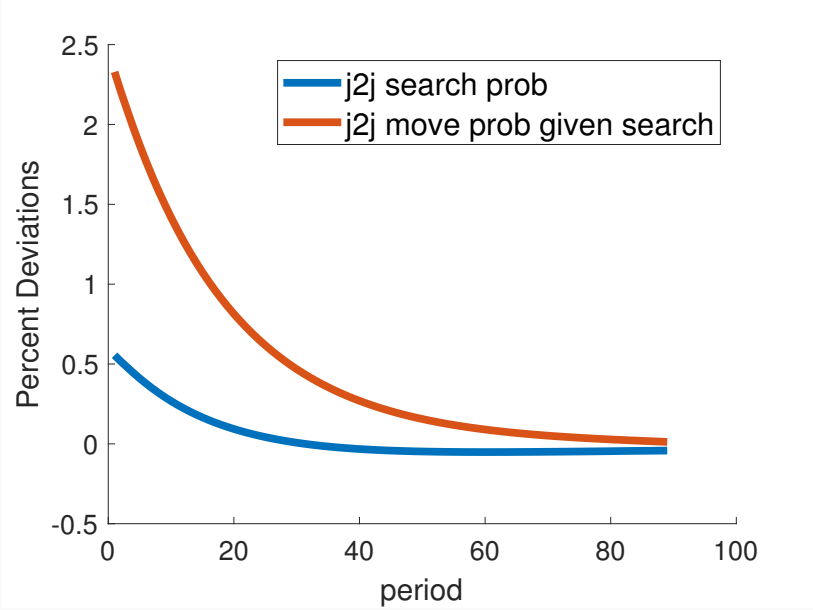


Figure 12: J2J search & JFP

- Response of J2J transition is mitigated
- Due to less responsive job finding probability for the employed workers

Hetero-η Economy: Model Statistics

	$\eta^e = \eta^u = 0.5$			$\eta^e = 0.75, \eta^u = 0.35$		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1.00
Avg Wage	0.690	0.51	1.00	0.688	0.53	1.00
New Wage	0.689	0.95	1.00	0.654	0.92	1.00
Unemp Rate	10.6%	0.35	-0.48	7.7%	0.78	-0.84
Quits+J2J moves	38.4%	8.94	0.99	34.9%	1.42	1.00
J2J moves	29.2%	10.66	0.99	26.9%	1.98	1.00

Table 9: Productivity Shock ($\rho = 0.95$)

- Allowing for different matching functions for UE and EE moves greatly reduce the gap of volatility between unemployment and j2j transitions
- But they both show insufficient volatility compared to output, in response to the productivity shock

Hetero-η Economy: Model Statistics

	$\eta^e = \eta^u = 0.5$			$\eta^e = 0.75, \eta^u = 0.35$		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.15	0.13	0.688	0.45	0.47
New Wage	0.689	2.02	0.31	0.654	2.40	0.73
Unemp Rate	10.6%	4.55	-0.99	7.7%	9.37	-0.99
Quits+J2J moves	38.4%	42.41	0.40	34.9%	11.65	0.70
J2J moves	29.2%	49.40	0.42	26.9%	15.55	0.70

Table 10: Delta Shock ($\rho = 0.95$)

- Allowing for different matching functions for UE and EE moves has similar effect on reduce volatility gap between unemployment and j2j transitions
- Unemployment is much more volatile compared to output in response to the delta shock, because the delta shock only affects total output through employment

Hetero-η Economy: Model Statistics

- Two ways to aggregate shocks

	shock corr = 0		shock corr = 0.95	
	Std	corr	Std	corr
output	1.00	1.00	1.00	1.00
avg wage	0.48	0.91	0.41	0.94
new wage	1.20	0.80	1.34	0.96
unemployment	3.70	-0.52	3.30	-0.91
quits + j2j movers	4.88	0.60	5.01	0.94
J2J movers	6.50	0.62	6.68	0.96

Table 11: Both Shocks ($\eta^e = 0.75, \eta^u = 0.35, \rho = 0.95$)

- By allowing for two types of shocks, and different matching functions for UE and EE moves, the model delivers a pretty good match to the data

High-b & Hetero-η Economy: Motivation

- The non-market value b is well recognized to be a key driver of the unemployment volatility (Hagedorn and Manovski, 2008).

- We now raise b from 0.3 (Shimer, 2005) to 0.5-0.6 (near the upper limit of our model) in the hetero-η economy.

High-b (0.5) & Hetero-η Economy: IRF to z shock

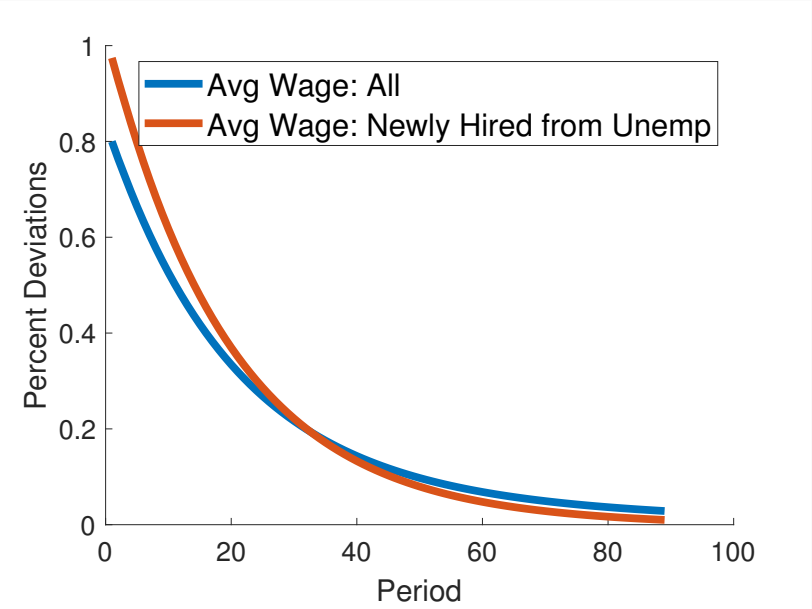


Figure 13: Wage

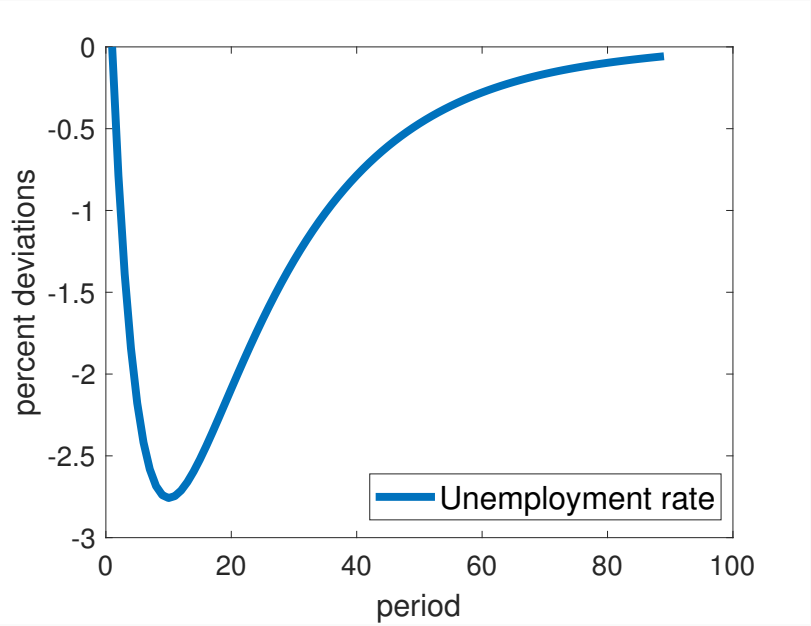


Figure 14: Unemployment Rate

- New wages are a bit less reponsive
- Unemployment drops up to 2.7% for 1% increase in productivity

High-b (0.5) & Hetero-η Economy: IRF to z shock

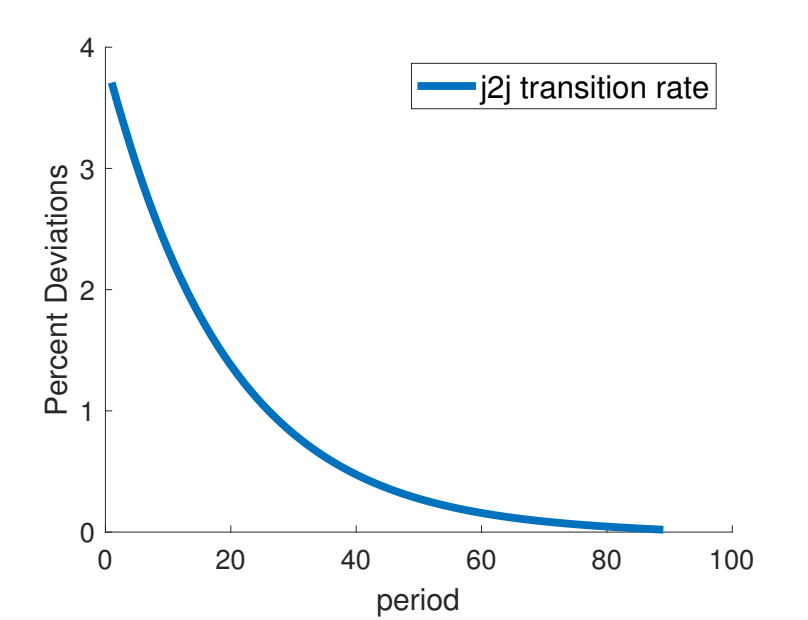


Figure 15: J2J transitions

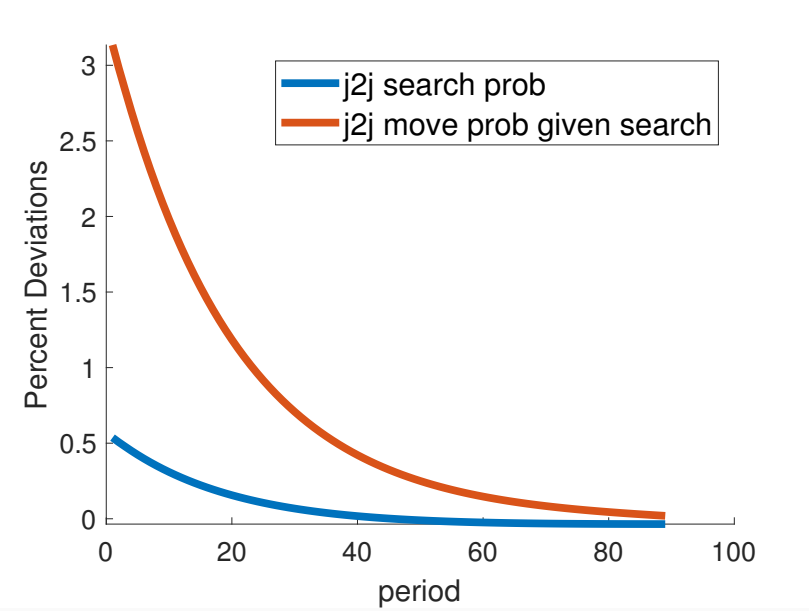


Figure 16: J2J search & JFP

- Response of J2J transition is further mitigated

High-b (0.5) & Hetero-η Economy: Model Statistics

	Benchmark			New		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.665	0.66	0.98
New Wage	0.689	0.95	1.00	0.656	0.80	0.97
Unemp Rate	10.6%	0.35	-0.48	9.4%	1.24	-0.83
Unemp Rate (normalized)	10.6%	0.35	-0.48	10.6%	1.21	-0.82
Quits+J2J moves	38.4%	8.94	0.99	37.7%	2.32	0.98
J2J moves	29.2%	10.66	0.99	28.7%	3.05	0.98

Table 12: High-b & Hetero-η: Productivity Shock ($\rho = 0.95$)

- New: $b = 0.5$, $\eta^u = 0.35, \eta^e = 0.75$. Benchmark: $b = 0.3$, $\eta^u = \eta^e = 0.5$
- All together, these extensions lead to 3.5 times more unemployment volatility, and shrink OJS move volatility to less than 30% the

High-b (0.6) & Hetero-η Economy: IRF to z shock

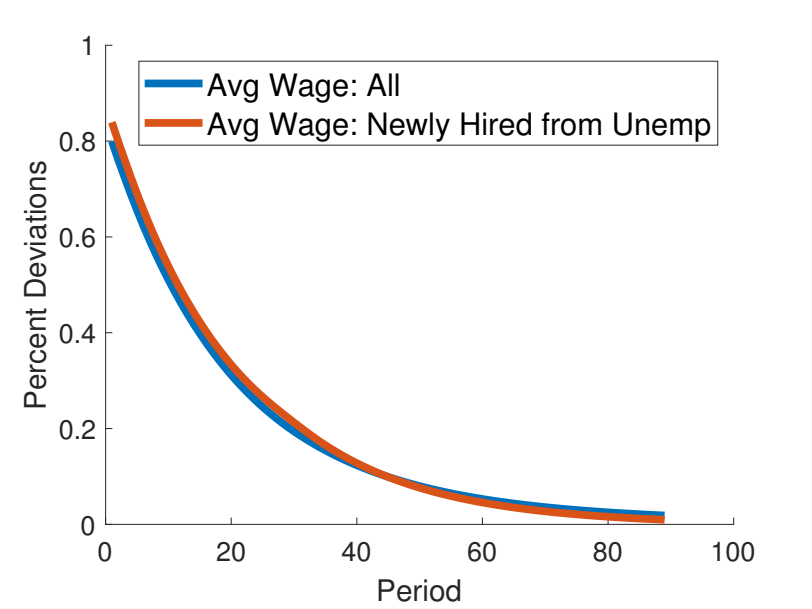


Figure 17: Wage

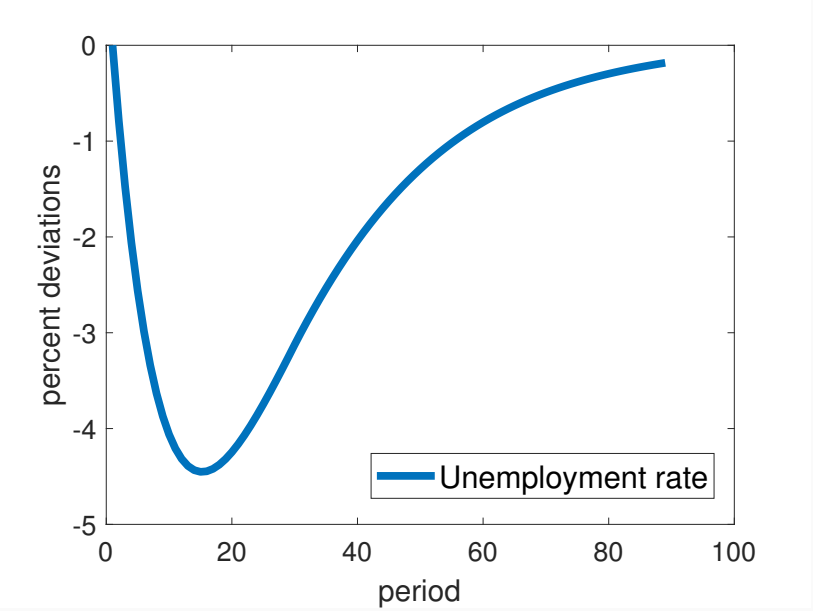


Figure 18: Unemployment Rate

- b to the limit: new wage only slightly more responsive than the average
- Unemployment drops up to 4.5% for 1% increase in productivity

High-b (0.6) & Hetero-η Economy: IRF to z shock

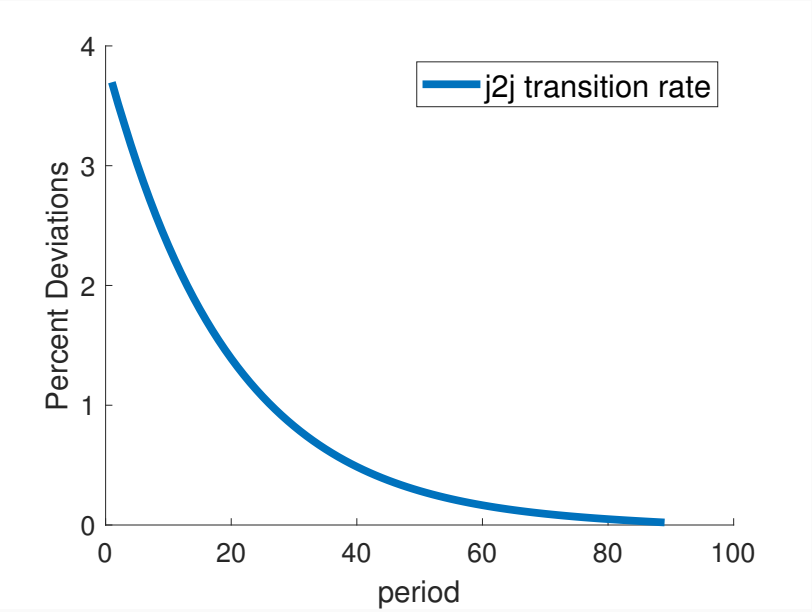


Figure 19: J2J transitions

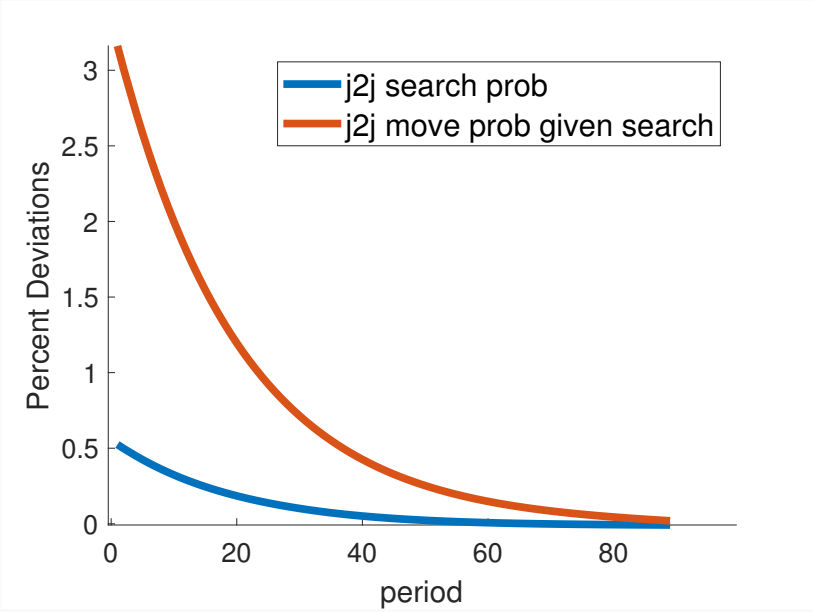


Figure 20: J2J search & JFP

- Response of J2J transition is the same magnitude as unemployment (like in the data)

High-b (0.6) & Hetero-η Economy: Model Statistics

	Benchmark			New		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.665	0.44	0.98
New Wage	0.689	0.95	1.00	0.656	0.47	0.97
Unemp Rate	10.6%	0.35	-0.48	20.9%	1.25	-0.83
Unemp Rate (normalized)	10.6%	0.35	-0.48	10.6%	2.46	-0.84
Quits+J2J Moves	38.4%	8.94	0.99	37.7%	1.54	0.98
J2J Moves	29.2%	10.66	0.99	28.7%	2.06	0.98

Table 13: High-b & Hetero-η Economy: Productivity Shock ($\rho = 0.95$)

-

Conclusions I

- Develop tools to get a joint theory of wages, employment and wealth that marry the two main branches of modern macro:
  1. Aiyagari models (output, consumption, investment, interest rates)
  2. Labor search models with job creation, turnover, wage determination, flows between employment, unemployment and outside the labor force.
  3. Add tools from Empirical Micro to generate quits
- Useful for business cycle analysis: We are getting procyclical
  - Quits
  - Employment
  - Investment and Consumption
  - Wages
- On the Job Search seems to Magnify Fluctuation a lot

Conclusions II

- Exciting set of continuation projects:
  1. Incorporate the movements outside of the labor force.
  2. Endogenous Search intensity on the part of firms
  3. Aiming Shocks to soften correlation between wages and wealth
  4. Efficiency Wages: Endogenous Productivity (firms use different technologies with different costs of idleness)
  5. Move towards more sophisticated household structures (more life cycle movements, multiperson households).

Firms choose Search Intensity

- The number of vacancies posted is chosen by firms
- Easy to implement
- Slightly Different steady state

Free entry with variable recruiting intensity

- Let $\upsilon(\overline{c})$ be a technology to post vacancies where $\overline{c}$ is the cost paid.
- Then the free entry condition requires that for all offered wages

$$0 = \max_{\overline{c}} \left\{ \upsilon(\overline{c})\, \psi^f[\theta(w)] \;\; \frac{\Omega(w)}{1+r} + \left[1 - \upsilon(\overline{c})\, \psi^f[\theta(w)]\right] \;\; \frac{\overline{k}(1-\delta_k)}{1+r} - \overline{c} - \overline{k} \right\},$$

- With FOC given by

$$v_{\overline{c}}(\overline{c}) \;\; \left\{ \psi^f[\theta(w)] \;\; \left[\frac{\Omega(w)}{1+r} - \;\; \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

How to make it consistent with the current steady state

- If $v(\overline{c}) = \frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_2 \overline{c}$, we have

$$(\upsilon_1 \overline{c} + \upsilon_2) \left\{ \psi^f[\theta(w)] \left[\frac{\Omega(w)}{1+r} - \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

- By Choosing υ so that for the numbers that have now

$$\left\{ \left[\frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_2 \overline{c} \right] \psi^f[\theta(w)] \frac{\Omega(w)}{1+r} + \left[1 - \frac{\upsilon_1 \overline{c}^2}{2} - \upsilon_2 \overline{c} \right] \psi^f[\theta(w)] \frac{\overline{k}(1-\delta_k)}{1+r} \right\} = \overline{c} + \overline{k}$$

- Solving for $\{\upsilon_1, \upsilon_2\}$ that satisfy both equations given our choice of $\overline{c}$ we are done

References

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659–684.

Andolfatto, D. (1996): "Business Cycles and Labor-Market Search," *American Economic Review*, 86(1), 112–132.

Boppart, T., P. Krusell, and K. Mitman (2018): "Exploiting MIT shocks in heterogeneous-agent economies: the impulse response as a numerical derivative," *Journal of Economic Dynamics and Control*, 89, 68–92.

Brown, A. J., B. Kohlbrecher, C. Merkl, and D. J. Snower (2017): "The effects of productivity and benefits on unemployment: Breaking the link," Tech. rep., GLO Discussion Paper.

Campolmi, A. and S. Gnocchi (2016): "Labor market participation, unemployment and monetary policy," *Journal of Monetary Economics*, 79, 17–29.

Chaumont, G. and S. Shi (2017): "Wealth Accumulation, On the Job Search and Inequality," Https://ideas.repec.org/p/red/sed017/128.html.

Den Haan, W., P. Rendahl, and M. Riegler (2015): "Unemployment (Fears) and Deflationary Spirals," CEPR Discussion Papers 10814, C.E.P.R. Discussion Papers.

Eeckhout, J. and A. Sepahsalari (2015): "Unemployment Risk and the Distribution of Assets," Unpublished Manuscript, UCL.

Fujita, S. and M. Nakajima (2016): "Worker flows and job flows: A quantitative investigation," *Review of Economic Dynamics*, 22, 1–20.

Griffy, B. (2017): "Borrowing Constraints, Search, and Life-Cycle Inequality," Unpublished Manuscript, UC Santa Barbara.

Haefke, C., M. Sonntag, and T. Van Rens (2013): "Wage rigidity and job creation," *Journal of Monetary Economics*, 60, 887–899.

Hornstein, A., P. Krusell, and G. Violante (2011): "Frictional Wage Dispersion in Search Models: A Quantitative Assessment," *American Economic Review*, 101, 2873–2898.

Krusell, P., T. Mukoyama, and A. Şahin (2010): "Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations," *Review of Economic Studies*, 77, 1477–1507.

Lise, J. (2013): "On-the-Job Search and Precautionary Savings," *The Review of Economic Studies*, 80, 1086–1113.

Merz, M. (1995): "Search in the Labor Market and the Real Business Cycle," *Journal of Monetary Economics*, 36, 269–300.

Moen, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385–411.

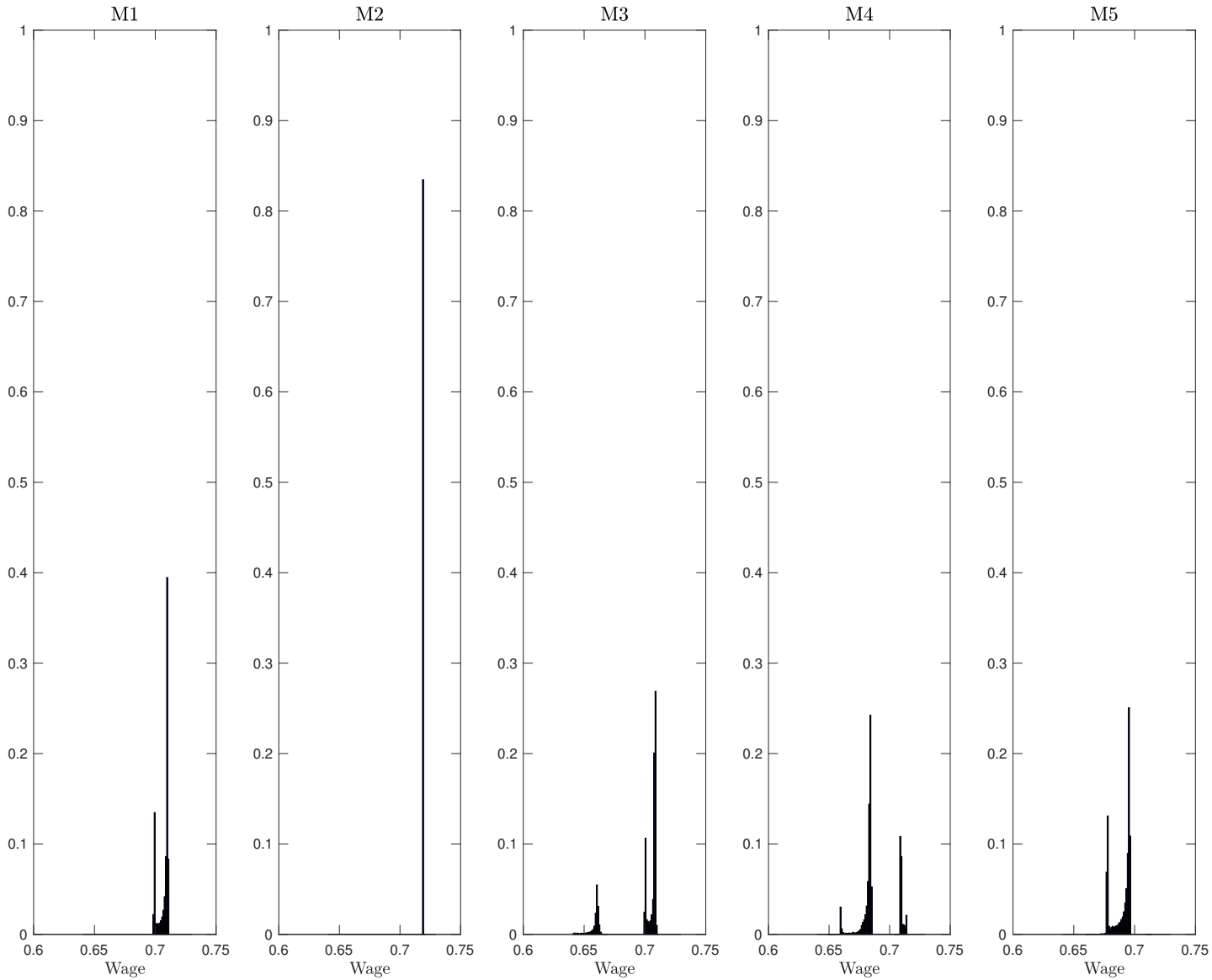
Ravn, M. O. and V. Sterk (2016): "Macroeconomic Fluctuations with HANK & SAM: An Analytical Approach," Discussion Papers 1633, Centre for Macroeconomics (CFM).

——— (2017): "Job uncertainty and deep recessions," *Journal of Monetary Economics*, 90, 125–141.

Steady-States

	m1	m2	m3	m4	m4 (low xi)
β	0.975	0.972	0.975	0.976	0.976
interest rate	0.030	0.030	0.030	0.030	0.030
avg consumption	0.686	0.682	0.691	0.684	0.680
avg wage	0.707	0.719	0.696	0.689	0.690
avg wealth	2.789	2.763	2.361	3.041	2.919
stock market value	2.971	2.692	3.065	2.953	2.931
avg labor income	0.659	0.655	0.668	0.654	0.652
consumption to wealth ratio	0.246	0.247	0.293	0.225	0.233
labor income to wealth ratio	0.236	0.237	0.283	0.215	0.223
quit ratio	0.090	0.088	0.090	0.090	0.092
unemployment rate	0.129	0.165	0.076	0.097	0.106
job losers	0.117	0.115	0.117	0.117	0.119
wage of newly hired unemployed	0.707	0.719	0.656	0.677	0.689
std consumption	0.013	0.010	0.011	0.011	0.011
std wage	0.000	0.000	0.003	0.002	0.001
std wealth	2.989	2.715	2.624	3.606	3.677
mean-min consumption	2.057	2.045	2.072	2.051	2.039
mean-min wage	1.012	1.001	1.094	1.058	1.042
UE transition	0.121	0.114	0.128	0.125	0.126
total vacancy	0.544	0.308	0.704	0.578	0.707
avg unemp duration	1.062	1.449	0.589	0.773	0.745
avg emp duration	7.228	7.335	7.228	7.228	7.131
OJS move rate	0.000	0.000	0.420	0.395	0.292
avg job duration	7.228	7.335	1.814	1.898	2.342

Wage Distributions



Derive the Idle Value

- Value of an idle firm is

$$\Omega^0 = -\delta^k k + \frac{1-\delta^f}{1+r}\left[-c^v + \psi^f \Omega + (1-\psi^f)\Omega^0\right]$$

- Free entry

$$k = \frac{1}{1+r}\left[-c^v + \psi^f \Omega + (1-\psi^f)\Omega^0\right]$$

  - Newly entered firms do not receive the destruction shock immediately
  - Vacancy posting cost is paid immediately before searching

- Combine the above

$$\Omega^0 = (1-\delta^f-\delta^k)k$$

M4 Low Ave J-2-J 1% Productivity Shock ($\rho = .95$) [IRF]

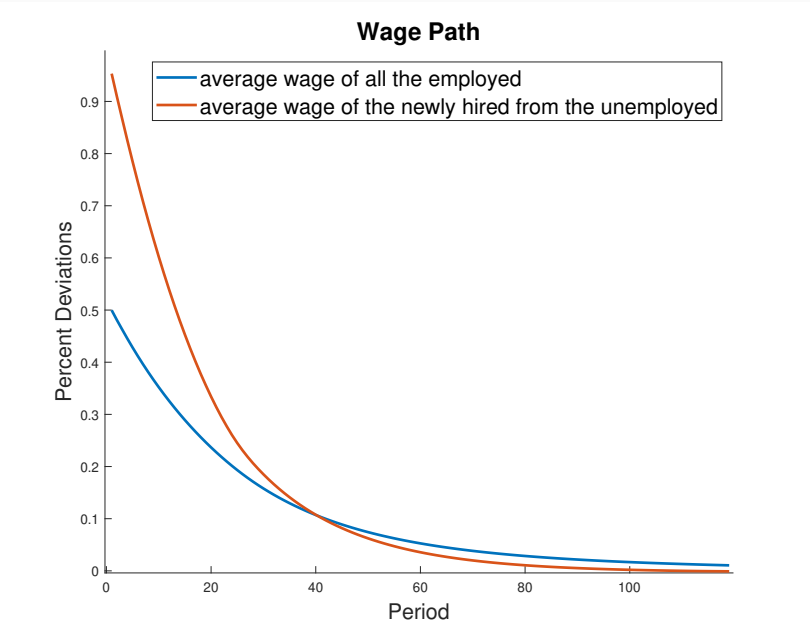


Figure 21: Wages

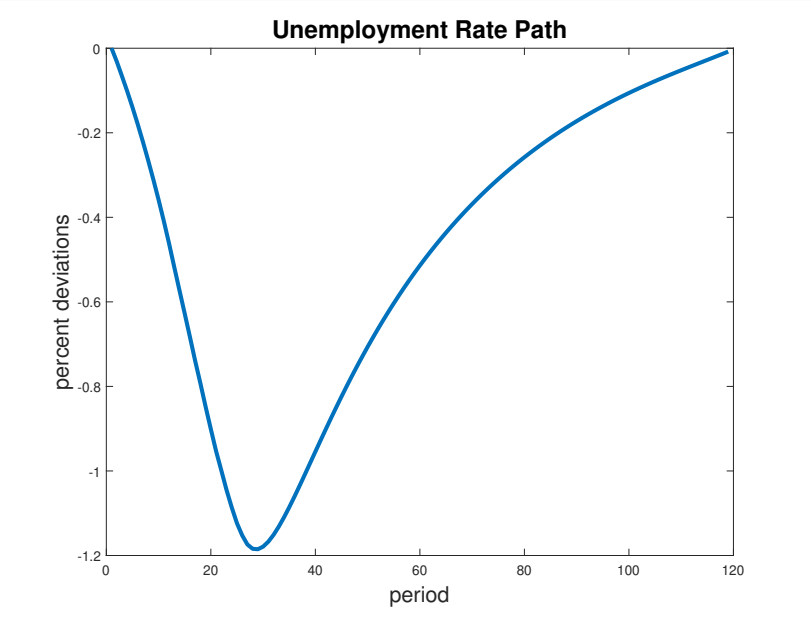


Figure 22: Unemployment Rate

- Similar Wage Responses
- 70% more unemployment volatility: J: mainly comes from more responsive quits

M4 Low Ave J-2-J 1% Productivity Shock ($\rho = .95$) IRF

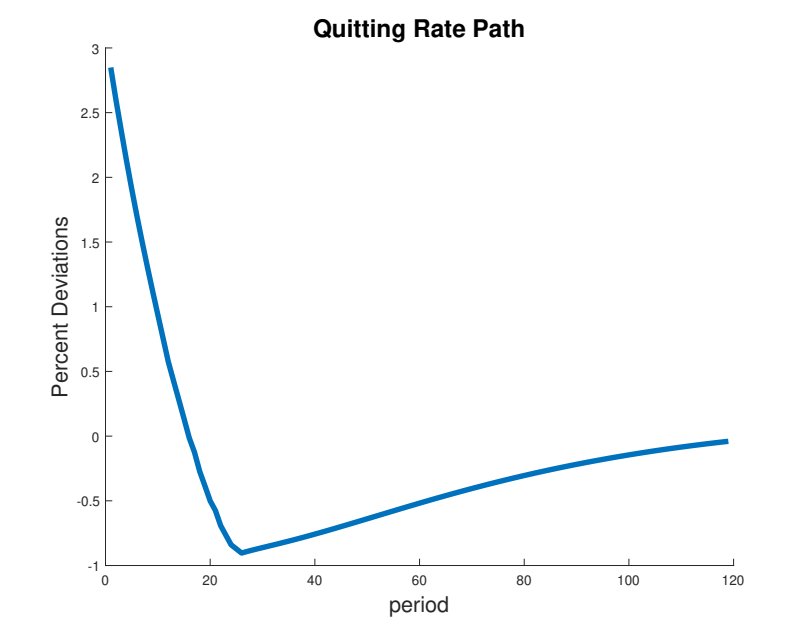


Figure 23: Quits

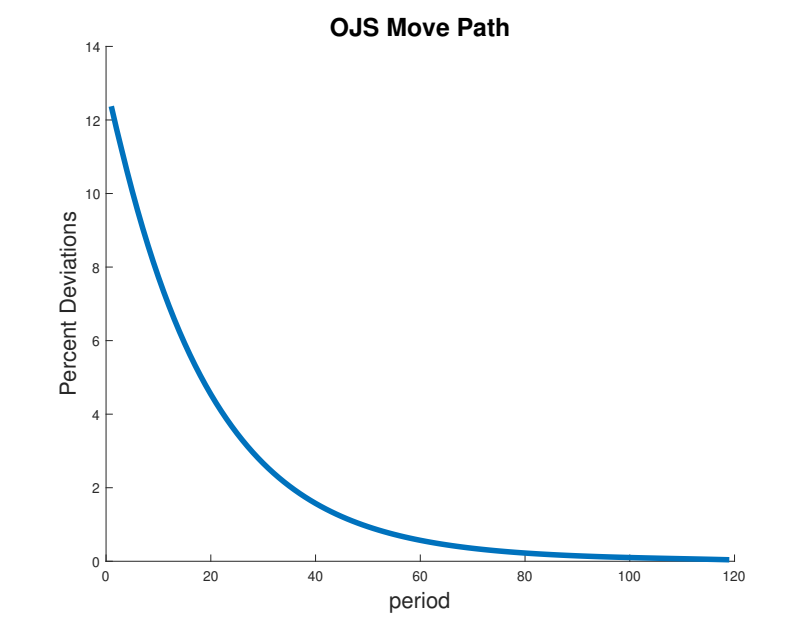


Figure 24: Job-to-job Moves

- More quitting
- Similar (excessive) J-2-J transitions

M4 Low Ave J-2-J 1% Delta Shock ($\rho = .95$)

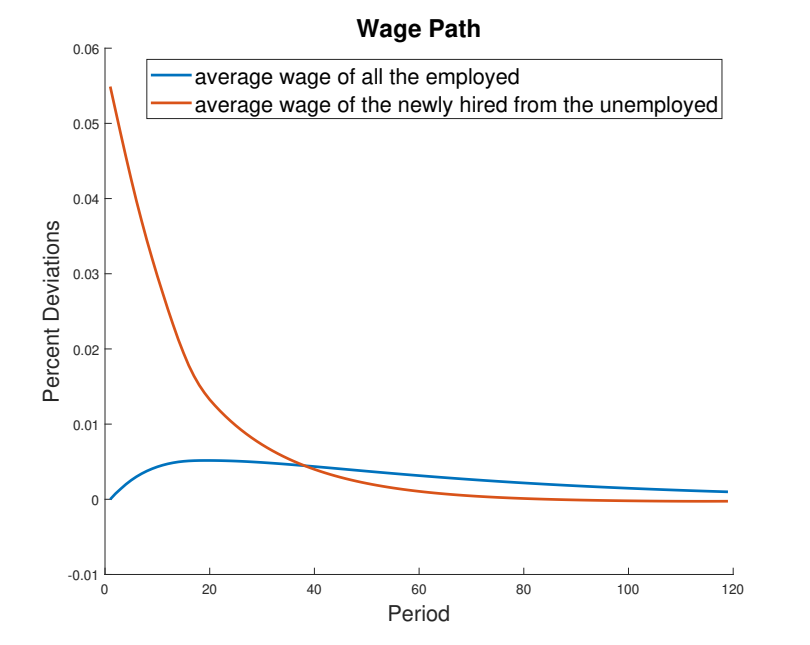


Figure 25: Wages

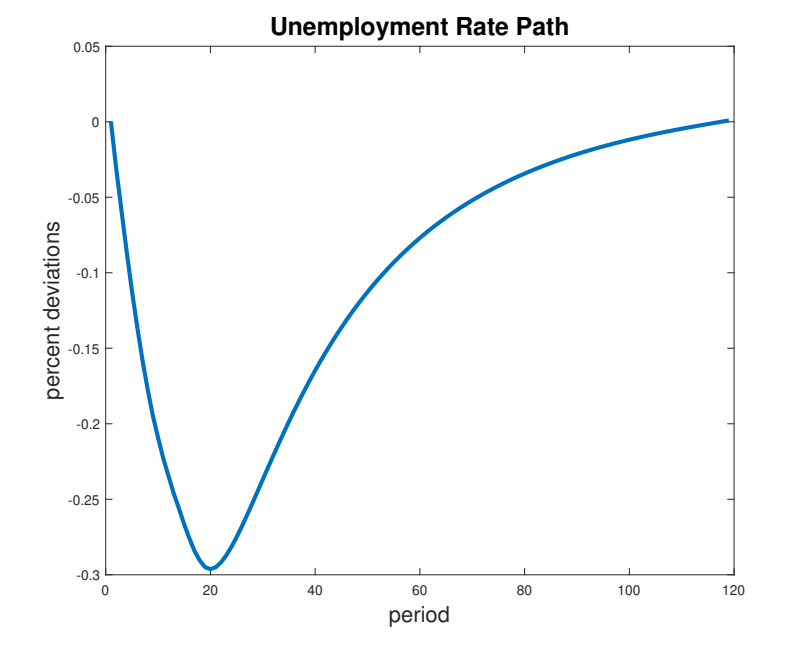


Figure 26: Unemployment Rate

- Similar Wage Response
- 16% more unemployment response
- Note wage is not pegged to the delta shock

M4 Low Ave J-2-J 1% Delta Shock ($\rho = .95$)

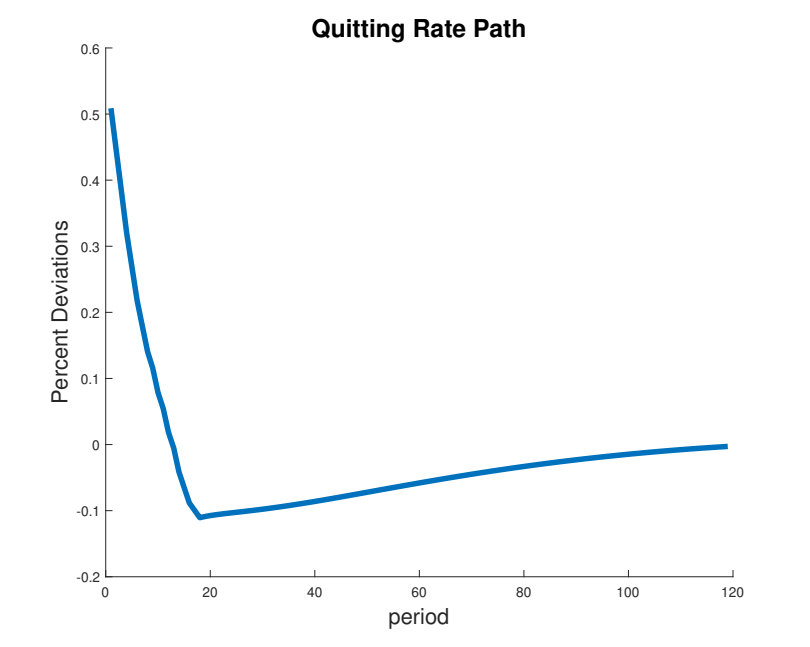


Figure 27: Quits

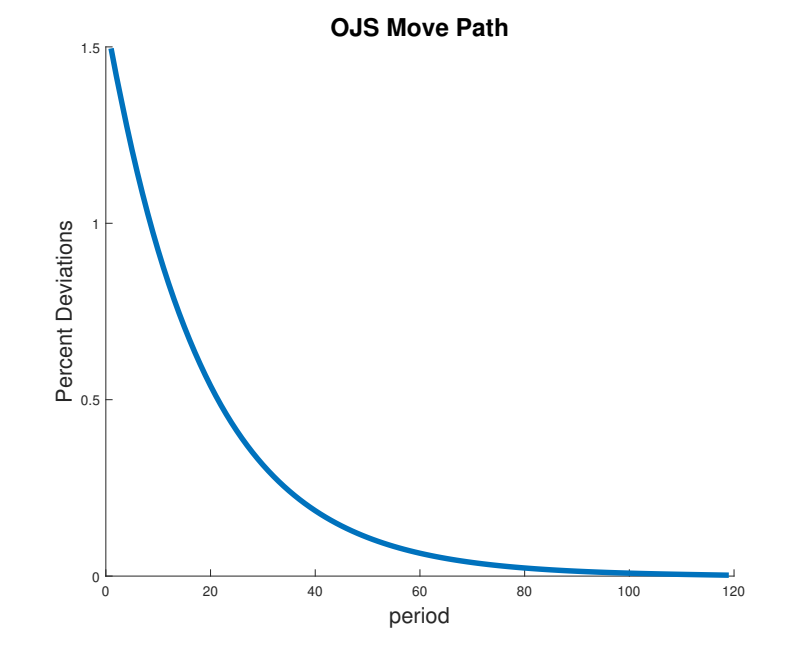


Figure 28: Job-to-job Moves

- More Quit similar (excessive) volatility for job-to-job transitions

M4 Low Ave J-2-J: Business Cycle Statistics

- Two ways to aggregate shocks

	shock corr = 0.95		shock corr = 0	
	Std	corr	Std	corr
output	1.00	1.00	1.00	1.00
avg wage	0.41	0.93	0.41	0.90
new wage	1.69	0.76	1.38	0.52
unemployment	2.59	-0.73	2.80	-0.63
quits + j2j movers	29.85	0.77	26.72	0.38
J2J movers	36.30	0.79	32.51	0.41

- Not too successful in reducing volatility of quits and J2J movers.
- Need to look for alternatives.

M4 Higher Wage Peg: 1% Productivity Shock ($\rho = .95$)

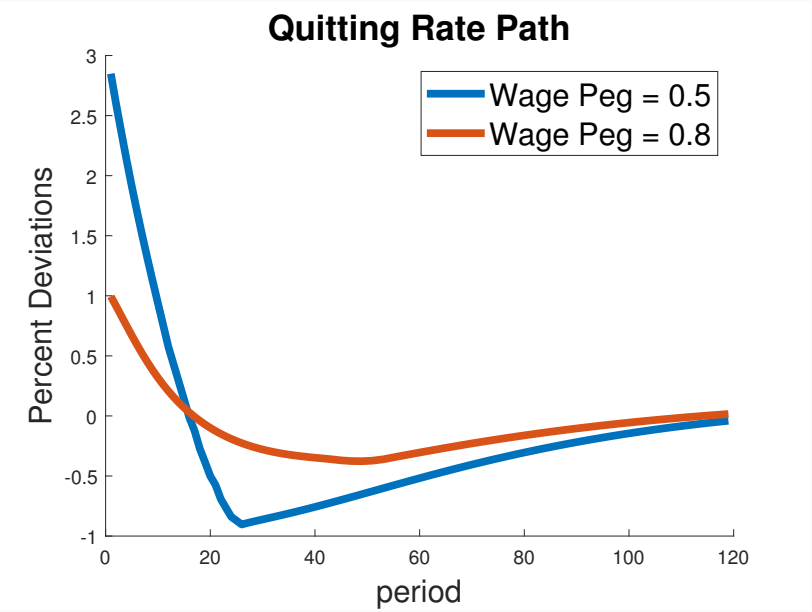


Figure 29: Quits

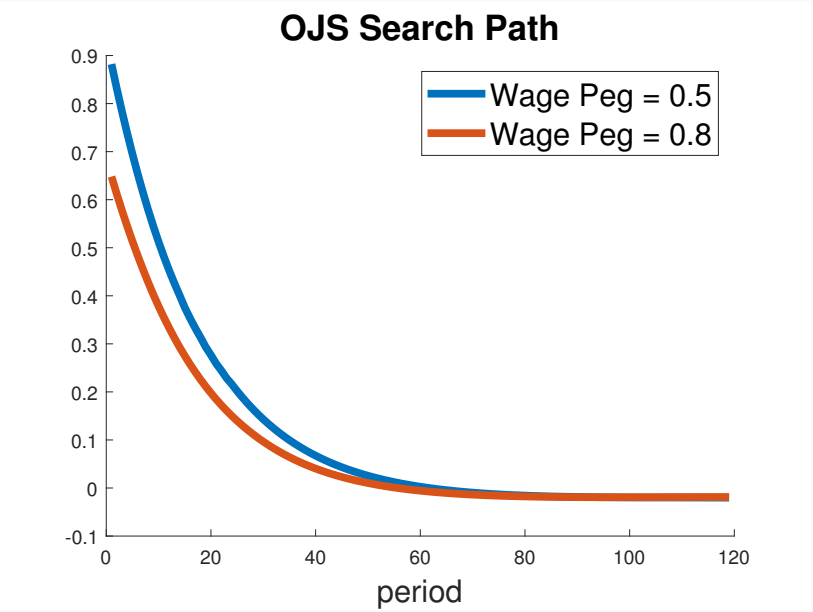


Figure 30: OJS Searchers

- Higher wage peg lowers the reponse of on-the-job search and quit.
  - Workers find it less so attractive to move/quit as existing wages now comove more with the productivity shock

M4 Higher Wage Peg: 1% Productivity Shock ($\rho = .95$)

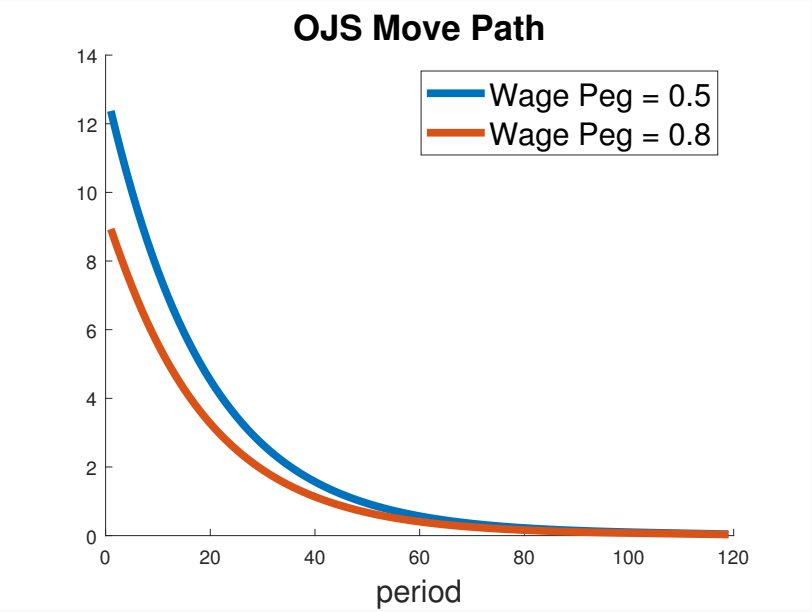


Figure 31: Job-to-job transitions

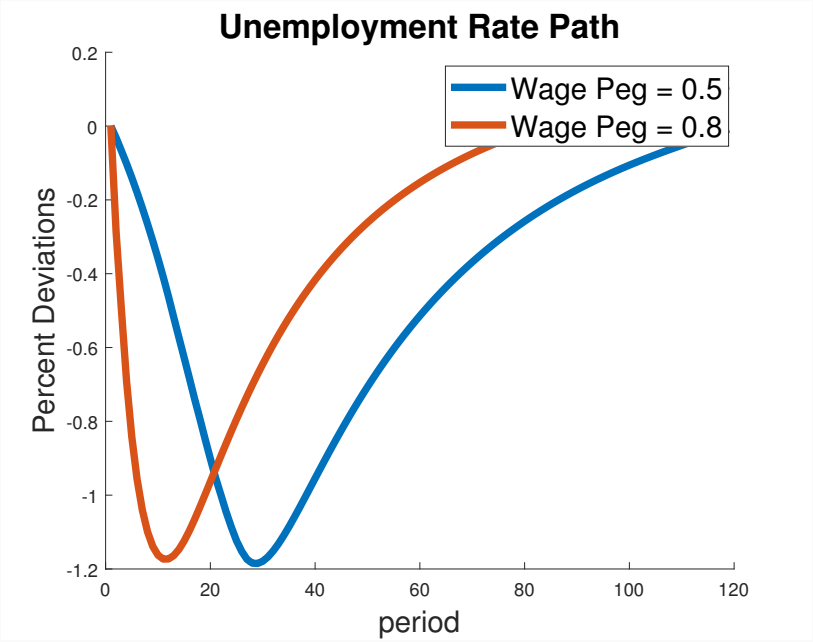


Figure 32: Unemployment

- Job-to-job transition rate also lowers: from 12% to 9%. This is from
  - less search on the job (see Fig 30)
  - less improvement of job finding rate due to smaller s-s firm profits
- Also less persistence of the unemployment response (less turnover).
- However the j2j transition rate is still far more responsive than the unemployment

M4 Higher Wage Peg: Business Cycle Statistics

	Wage Peg = 0.5			Wage Peg = 0.8		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.690	0.76	0.99
New Wage	0.689	0.95	1.00	0.689	1.04	0.99
Unemp Rate	10.6%	0.35	-0.48	10.6%	0.42	-0.64
Quits+J2J moves	38.4%	8.94	0.99	38.4%	6.65	-0.99
J2J moves	29.2%	10.66	0.99	29.2%	8.50	-0.99

Table 14: M4 Compare Wage Pegs: Productivity Shock ($\rho = 0.95$)

- Higher wage pegs lower the j2j transition volatility while raise the unemployment volatility
- However even we make the existing wages comove with productivity closely, the j2j transition volatility is still much higher than the unemployment volatility
- In the next several pages we take a closer look at this problem

Quitting Makes a Big Difference

- Job finding Rates back