International Business Cycles Redux

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Punchline

- Backus-Smith (Backus and Smith (1993)) puzzle: households consume more domestic goods when they are more expensive
  - $corr(RER, cH/cF) > 0$.
  - Yet standard models (e.g. RBC) predict the opposite.
- Literature: demand shocks, low elasticity between home and foreign goods (Corsetti, Dedola, and Leduc (2008)), non-tradable goods (Engel and Wang (2011)), labor wedge from home production (Karabarbounis (2012)).
- We pose an explanation based on demand shocks in the context of environments where expenditures shape productivity (a la Bai, Ríos-Rull, and Storesletten (2011)). We also obtain
  - Countercyclical terms of trade.
  - Volatile net exports.
  - Lower international cons corr than output's.

The two country two good model with shopping

- Two countries $j = \{1, 2\}$ with representative agents in each.
- Build a top of stochastic growth model.
- There are incomplete markets (no insurance for preference shocks).
- There is perfect mobility of capital without impediments to cross country ownership.

Preferences: Current utility of Households in country j

$$u\left[H^c\left(c^{jj}, c^{jj^*}\right), n^j, H^d\left(d^{jj}, d^{jj^*}\right), \theta^j\right]$$

- $H^c(c^{jj}, c^{jj^*})$ is an (Armington) aggregator of the home (c^{jj}) and the foreign (c^{jj^*}) good.
- n^j are hours worked in country j.
- $H^d(d^{jj}, d^{jj^*})$ is an aggregator (maybe linear) of search shopping effort at home (d^{jj}) and abroad (d^{jj^*}).
- θ^j is a Markovian preference shock.

• Households cannot change their country of residence, which makes labor immobile. Like in Bai, Ríos-Rull, and Storesletten (2011) consumption requires that the household finds the goods which in turn requires enough shopping effort to find them.

Production in each country

- Measure one of firms–locations with installed capital k^j (depreciates at rate δ). Goods can be used for consumption or investment and capacity is

$$F(k^j, (n^F)^j) = \bar{z} \;\; \left(k^j\right)^{\gamma_k} \; \left((n^F)^j\right)^{\gamma_n}$$

- New capital consists of an (Armington) aggregator of home and foreign investment goods:

$$k'^j = (1-\delta)k^j + H^i(i^j, i^{j^*})$$

- New capjtal has to be purchased that requires shoppers looking for the home investment good $(n^{j,j})^k$ and for the foreign investment good $(n^{j,j^*})^k$.

- Output in each country can be used by home consumers, foreign consumers, home investors and foreign investors.

- Unmatched capacity rots.

Households

Preferences are

$$u\left[H^c\left(c^{jj}, c^{jj^*}\right), n^j, H^d\left(d^{jj}, d^{jj^*}\right), \theta^j\right]$$

Again both consumptions require that they are shopped so

$$\begin{aligned} c^{jj} &= d^{jj}\ \Psi_d(Q^{cjj})\, F^{cjj} \\ c^{jj^*} &= d^{jj^*}\ \Psi_d(Q^{cjj^*})\, F^{cjj^*} \end{aligned}$$

- $\Psi_d(Q^{cj\ell})$ is the probability that a country j consumption shopper has of matching a consumption firm from country ℓ that catters to j shoppers. $Q^{cj\ell}$ is market tightness in that consumption goods market and $F^{cj\ell}$ is its output capacity.

- Households own shares of a mutual fund that owns all firms.

A few lemmas alleviate notation

1. The state of the economy is $S = \{\theta^1, \theta^2, K^1, K^2, B^1\}$ where K^j is capital installed in country j and B^1 is the share of total wealth held by country 1 households.

2. There are two active markets in consumption goods (one for locals and the other for foreigners) and two markets in investment goods in each country for a total of 8 markets.

3. Firms that produce consumption and investment, for local buyers and to export choose the same inputs $F^{cjj} = F^{i\ell j} = .. = F(K^j, N^{yj}) = F^j(S)$. We use $n(k, F)$ to denote the inverse $f(k, n)$.

4. All firms in each country get the same expected revenue (but not necessarily the same price and market tightness).

The household problem

$$v^j(S,b) = \max_{c^{\cdot\cdot}, d^{\cdot\cdot}, n, b'} u\left[H^c\Big(c^{jj}, c^{jj^*}\Big), n^j, H^d\Big(d^{jj}, d^{jj^*}\Big), \theta^j\right] + \beta E\Big\{v^j(S', b')$$

$$b' + \sum_{\ell=j}^{j^*} p^{cj\ell}(S)\; c^{j\ell} = b\,[1 + R(S)] + n^j\; w^j(S)$$

$$c^{j\ell} = d^{j\ell}\;\; \Psi_d\left(Q^{cj\ell}(S)\right)\; F^{cj\ell}(S) \qquad \ell = j, j^*,$$

$$S' = G(S)$$

The household problem — First Order Conditions

Hholds' FOC (and RA): for $\ell = j, j^*$ and $m = j, j^*$

$$
\begin{aligned}
u_c^j H_\ell^{cj} - \frac{u_d^j H_\ell^{dj}}{\Psi_d^{cj\ell} F^{cj\ell}} &= \beta E \left\{ \frac{p^{cj\ell}(1+R')}{p^{cjm\prime}} \left[u_c^{j\prime} H_m^{cj\prime} - \frac{u_d^{j\prime} H_m^{dj\prime}}{\Psi_d^{cjm\prime} F^{ckm\prime}} \right] |\theta \right\} \\
u_c^j H_\ell^{c} - \frac{u_d^j H_\ell^{d}}{\Psi_d^{cj\ell} F^{cj\ell}} &= u_n \frac{p^{cj\ell}}{w^j} \\
\frac{1}{p^{cj\ell}} \left[u_c^j H_\ell^{cj} - \frac{u_d^j H_\ell^{dj}}{\Psi_d^{cj\ell} F^{cj\ell}} \right] &= \frac{1}{p^{cjm}} \left[u_c^j H_m^{cj} - \frac{u_d^j H_m^{dj}}{\Psi_d^{cjm} F^{cjm}} \right]
\end{aligned}
$$

Define marginal utility of savings $M(S)$:

$$
M(S) = \beta E \left\{ \frac{(1+R')}{p^{cjm\prime}} \left[u_c^{j\prime} H_m^{cj\prime} - \frac{u_d^{j\prime} H_m^{dj\prime}}{\Psi_d^{cjm\prime} F^{ckm\prime}} \right] |\theta \right\}
$$

Consumption (or invt) firms in a $(p^{cj\ell}, F^{cj\ell}, Q^{cj\ell})$ submarket

$$\begin{aligned}
\Omega^j(S,k) \quad = \quad & \max_{n^{k\cdots}, k', i^{jj}, i^{jj*}} \Psi_T(Q^{cj\ell})\, p^{cj\ell}\, F^{cj\ell} - w^j(S)\, \left[n(k, F^{cj\ell}) + n^{kjj} + n^{kjj^*}\right] \\
& \qquad -p^{ijj}(S)\, i^{jj} - p^{ijj^*}(S)\, i^{jj^*} + E\left\{ \left. \frac{\Omega^j(S',k')}{1+R(S')} \right| \theta \right\}
\end{aligned}$$

$$\begin{aligned}
\text{s.t.} \qquad & i^{j\ell} = (n^{kj\ell}\,\zeta)\;\; \Psi_d[Q^{ij\ell}(S)]\;\; F^{\ell}(S) \qquad \text{for } \ell = j, j^* \\
& k' = (1-\delta)k + H^i\left(i^{jj}, i^{jj^*}\right) \\
& S' = G(S)
\end{aligned}$$

with FOC (and RA condition) for $\ell = j, j^*$

$$H^{ij}_{\ell} E\left\{ \left. \frac{\Omega^j_k\left(S', K^{j\prime}\right)}{1+R(S')} \right| \theta \right\} = \frac{w^j(S)}{\zeta\, \Psi_d[Q^{ij\ell}(S)]\, F^{\ell}(S)} + p^{ij\ell}(S)$$

The household problem

$$v^j(S,b) = \max_{c^{\cdot\cdot},d^{\cdot\cdot},n,b'} u\left[H^c\Big(c^{jj},c^{jj^*}\Big),n^j,H^d\Big(d^{jj},d^{jj^*}\Big),\theta^j\right] + \beta E\Big\{v^j(S',b')$$

$$b' + \sum_{\ell=j}^{j^*} p^{cj\ell}(S)\ c^{j\ell} = b\,[1+R(S)] + n^j\ w^j(S)$$

$$c^{j\ell} = d^{j\ell}\ \ \Psi_d\left(Q^{cj\ell}(S)\right)\ F^{cj\ell}(S) \qquad \ell = j, j^*,$$

$$S' = G(S)$$

The household problem — First Order Conditions

Hholds' FOC (and RA): for $\ell = j, j^*$ and $m = j, j^*$

$$
\begin{aligned}
u_c^j\, H_\ell^{cj} - \frac{u_d^j\, H_\ell^{dj}}{\Psi_d^{cj\ell}\, F^{cj\ell}} &= \beta E\left\{ \frac{p^{cj\ell}\,(1+R')}{p^{cjm\prime}} \left[u_c^{j\prime}\, H_m^{cj\prime} - \frac{u_d^{j\prime}\, H_m^{dj\prime}}{\Psi_d^{cjm\prime} F^{ckm\prime}} \right] |\theta \right\} \\
u_c^j\, H_\ell^{c} - \frac{u_d^j\, H_\ell^{d}}{\Psi_d^{cj\ell}\, F^{cj\ell}} &= u_n\ \frac{p^{cj\ell}}{w^j} \\
\frac{1}{p^{cj\ell}} \left[u_c^j\, H_\ell^{cj} - \frac{u_d^j\, H_\ell^{dj}}{\Psi_d^{cj\ell}\, F^{cj\ell}} \right] &= \frac{1}{p^{cjm}} \left[u_c^j\, H_m^{cj} - \frac{u_d^j\, H_m^{dj}}{\Psi_d^{cjm}\, F^{cjm}} \right]
\end{aligned}
$$

We define marginal utility of savings $M(S)$ as

$$
M(S) = \beta E\left\{ \frac{(1+R')}{p^{cjm\prime}} \left[u_c^{j\prime}\, H_m^{cj\prime} - \frac{u_d^{j\prime}\, H_m^{dj\prime}}{\Psi_d^{cjm\prime} F^{ckm\prime}} \right] |\theta \right\}
$$

Competitive Search in Goods Markets

- Markets are now indexed by good type (country of production), quantity, price, and market tightness.

- There are four possible purchasers of goods (home and foreign, consumption and investment. A total of 8 markets.

- We get additional conditions from the FOC of shoppers given expected revenue for sellers.

- The equilibrium objects (44) are functions of (S) for

$$\left\{Q^{cj\ell}, Q^{ij\ell}, N^{kj\ell}, C^{j\ell}, I^{j\ell}, p^{cj\ell}, p^{ij\ell}, T^{cj\ell}, T^{ij\ell}, w^{j}, N^{yj}, N^{j}, B', R\right\}_{j\in\{1,2\},\ell\in\{}$$

Competitive Search in Consumption Market $cj\ell$

The rewards for the hhold to send a shopper to a $(p^{cj\ell}, F^{cj\ell}, Q^{cj\ell})$ market is

$$\Phi = \max_{Q^{cj\ell}, p^{cj\ell}, F^{cj\ell}} -u_d^j(S)H_\ell^{dj}(S) + \Psi_d(Q^{cj\ell})F^{cj\ell}\left(u_c^j(S)H_\ell^{cj}(S) - p^{cj\ell}M(S)\right)$$

$$\varsigma^{cj\ell} \leq p^{cj\ell}\frac{\Psi_d(Q^{cj\ell})}{Q^{cj\ell}}F^{cj\ell} - w^j(S)n(k, F^{cj\ell}) \tag{1}$$

Solving $p^{cj\ell}$ from equation (1) and plugging it into the objective function, we have (and ignore the sunk shopping cost)

$$\max_{Q^{cj\ell}, p^{cj\ell}, F^{cj\ell}} \Psi_d(Q^{cj\ell})F^{cj\ell}\left(u_c^j(S)H_\ell^{cj}(S) - \frac{\varsigma^{cj\ell} + w^j(S)n(k, F^{cj\ell})}{\Psi_d(Q^{cj\ell})F^{cj\ell}/Q^{cj\ell}}M(S)\right)$$

Competitive Search in Consumption Market $cj\ell$

First order condition over $Q^{cj\ell}$:

$$(1-\alpha)A(Q^{cj\ell})^{-\alpha}F^{cj\ell}u_c^j(S)H_\ell^{cj}(S) - \left[\varsigma^{cj\ell} + w^j(S)n(k, F^{cj\ell})\right]M(S) = 0$$

First order condition over $F^{cj\ell}$:

$$A(Q^{cj\ell})^{1-\alpha}u_c^j(S)H_\ell^{cj}(S) - w^j(S)\frac{dn(k, F^{cj\ell})}{dF^{cj\ell}}Q^{cj\ell}M(S) = 0$$

Thus, two equations characterize the equilibrium in market $cj\ell$,

$$\begin{aligned} p^{cj\ell} &= (1-\alpha)\frac{u_c^j(S)H_\ell^{cj}(S)}{M(S)} \\ \frac{w^j(S)}{p^{cj\ell}} &= \frac{1}{1-\alpha}\frac{\Psi_d(Q^{cj\ell})}{Q^{cj\ell}}f_n(S) \end{aligned}$$

Competitive Search in Investment Market $ij\ell$

The rewards for a firm to send a shopper to a $(p^{ij\ell}, F^{ij\ell}, Q^{ij\ell})$ market is

$$\Phi_F = \max_{Q^{ij\ell}, p^{ij\ell}, F^{ij\ell}} -w^j(S) + \zeta \Psi_d(Q^{ij\ell}) F^{ij\ell} \left[H_\ell^{ij}(S) E\left\{ \Omega(S', k') \Pi(S, S') \right\} - p^{ij\ell} \right]$$

$$\varsigma^{ij\ell} \leq p^{ij\ell} \frac{\Psi_d(Q^{ij\ell})}{Q^{ij\ell}} F^{ij\ell} - w^j(S) n(k, F^{ij\ell}) \tag{2}$$

Solving $p^{ij\ell}$ from equation (2) and plugging it into the objective function, we have (and ignore the sunk wage cost)

$$\max_{Q^{ij\ell}, p^{ij\ell}, F^{ij\ell}} \zeta \Psi_d(Q^{ij\ell}) F^{ij\ell} \left[H_\ell^{ij}(S) E\left\{ \Omega(S', k') \Pi(S, S') \right\} - \frac{\zeta^{ij\ell} + w^j(S) n(k, F^{ij\ell})}{\Psi_d(Q^{ij\ell}) F^{ij\ell} / Q^{ij\ell}} \right]$$

Competitive Search in Investment Market $ij\ell$

First order condition over $Q_{ij\ell}$:

$$(1-\alpha)A(Q^{ij\ell})^{-\alpha}F^{ij\ell}H_{\ell}^{ij}(S)E\left\{\Omega(S',k')\Pi(S,S')\right\}-\left[\varsigma^{ij\ell}+w^{j}(S)n(k,F^{ij\ell})\right]=0$$

First order condition over $F_{ij\ell}$:

$$A(Q^{ij\ell})^{1-\alpha}H_{\ell}^{ij}(S)E\left\{\Omega(S',k')\Pi(S,S')\right\}-w^{j}(S)\frac{dn(k,F^{ij\ell})}{dF^{ij\ell}}Q^{ij\ell}=0$$

Thus, two equations characterize the equilibrium in market $ij\ell$,

$$\begin{aligned}
p^{ij\ell} &= (1-\alpha)H_{\ell}^{ij}(S)E\left\{\Omega(S',k')\Pi(S,S')\right\} \\
\frac{w^{j}(S)}{p^{ij\ell}} &= \frac{1}{1-\alpha}\frac{\Psi_{d}(Q^{ij\ell})}{Q^{ij\ell}}f_{n}(S)
\end{aligned}$$

Lemma: All firms with $k = K^j$ in country j choose the same labor

The revenue of a firm in country j to produce m goods ($x = c, i$) for country $\ell = j, j^*$ is given by

$$\begin{aligned} \varsigma^{xj\ell} &= p^{xj\ell} A[Q^{xj\ell}(S)]^{-\alpha} F^{xj\ell} - w^j(S) n(K^j, F^{xj\ell}) \\ &= w^j(S) \left[\frac{p^{xj\ell} A[Q^{xj\ell}(S)]^{-\alpha}}{w^j(S)} F^{xj\ell} - n(K^j, F^{xj\ell}) \right] \end{aligned}$$

Using the equilibrium condition from competitive search, $\frac{w^j(S)}{p^{xj\ell}} = \frac{1}{1-\alpha} A[Q^{xj\ell}(S)]^{-\alpha} f_n(S)$, we can rewrite firm's revenue

$$\varsigma^{xj\ell} = w^j(S) \left[(1-\alpha) \frac{F^{xj\ell}}{f_n(K^j, n(K^j, F^{xj\ell}))} - n(K^j, F^{xj\ell}) \right].$$

In equilibrium, all firms in country j have the same revenue, $\varsigma^{cjj} = \varsigma^{cjj^*} = \varsigma^{ijj} = \varsigma^{ijj^*}$, and thus have the same output and the same labor.

Recursive Equilibrium

1. *Households and firms* solve their problems (12 in households, 4 in firms).

2. *Competitive Search Conditions.* (16).

3. *Representative Agent Conditions*

4. *Market Clearing Conditions for j: (12)*

$$
\begin{aligned}
N^j &= N^{yj} + N^{kjj} + N^{kjj^*}, \\
X^{j\ell} &= T^{xj\ell}\, \Psi_T(Q^{xj\ell})\, F(K^j, N^{yj}) \qquad \text{for } X = \{C, I\}, \ell = \{j, j^*\} \\
1 &= \sum_{\ell=j}^{j^*} \left(T^{cj\ell} + T^{ij\ell} \right) \qquad \text{for } j \in \{1, 2\}
\end{aligned}
$$

5. Value of all firms is 1.

Putting the model to work

- We want a clear version of this model. So separable utility with constant Frisch elasticiy and Cobb-Douglas technology. We will place shocks on preferences and on the investment shopping technology.
  - Preferences
  $$u(H^c, n, d^j, d^{j^*}, \theta) = \theta \ \frac{(H^c)^{1-\sigma}}{1-\sigma} - \ \chi \frac{(n^j)^{1+\psi}}{1+\psi} - \ (d^j + d^{j^*})$$
  - Aggregator
  $$H^c(c^j, c^{j^*}) = \left[\mu (c^j)^\eta + (1-\mu)(c^{j^*})^\eta\right]^{\frac{1}{\eta}}$$
  - Production function
  $$F(k, n^y) = \overline{z} \ k^{\gamma_k} \ (n^y)^{\gamma_n}$$
  - Shocks
  $$\log(\theta_t) = \rho_\theta \ \log(\theta_{t-1}) + v_t, \quad v_t \sim N(0, \Sigma^2)$$

Calibration

Targets	Value	Parameter	Value
First Group: Parameters Set Exogenously			
Risk aversion	2.	σ	2.
Real interest rate	4%	β	0.99
Frisch elasticity	0.72	$\frac{1}{\nu}$	0.72
Armington elasticity	1.1	η	0.09
		α	0.25
Second Group: Standard Targets			
Fraction of time spent working	30%	χ	2.38
Physical capital to output ratio	2.75	δ	0.07
Consumption share of output	0.80	γ_k	0.31
Labor share of income	0.67	γ_n	0.45
Steady-state output	1	$\bar{z}$	1.10
Import share	0.10	μ	0.88
Third Group: Targets Specific to This Economy			
Capacity utilization of consumption sector	0.81	A	1.66
Capacity utilization of investment sector	0.81	ζ	0.38

Implications over Other Aggregate Variables	
Share of production workers	0.90

Results

Data: for US and EU15							
Quantities	Data	IRBC		Shopping Model			
		Y	Z	Y^c	Y^{uc}	Z^c	Z^{uc}
A. Variance (US)							
Output	1.76	1.76	0.57	1.75	1.76	2.16	2.05
Consumption	0.98	0.06	0.02	1.52	1.77	2.48	2.06
Investment	13.74	37.43	12.43	21.88	26.02	37.32	30.25
Employment	1.16	0.14	0.05	1.44	1.52	1.97	1.76
Net exports	0.16	0.01	0.006	0.44	0.62	0.98	0.72
TFP	0.63	1.17	0.38	0.29	0.28	0.31	0.32
Terms of trade	4.98	0.15	0.06	6.39	8.79	13.84	10.22
B. International comovement							
Output	0.40	0.40	0.22	0.40	0.16	-0.10	0.16
Consumption	0.25	0.38	0.20	0.15	-0.11	-0.35	-0.11
Investment	0.23	0.26	0.08	-0.65	-0.80	-0.89	-0.80
Employment	0.21	0.45	0.28	0.15	-0.12	-0.37	-0.12
TFP	0.30	0.39	0.21	0.73	0.57	0.36	0.57
C. Co-movement within a country							
Net exports, Output	-0.49	-0.50	-0.56	-0.51	-0.61	-0.71	-0.62
Terms of trade, Output	-0.33	0.46	0.52	-0.51	-0.61	-0.71	-0.61
RER, cH/cF	-0.71	0.99	0.99	-0.96	-0.96	-0.96	-0.96

Sensitivity analysis

Data: for US and EU15

Quantities	Data	Shopping Model	
		Benchmark	Low Armington (0.55)
A. Variance (US)			
Output	1.76	1.75	1.76
Consumption	0.98	1.52	1.14
Investment	13.74	21.88	16.76
Employment	1.16	1.44	1.40
Net exports	0.16	0.44	0.16
TFP	0.63	0.29	0.30
Terms of trade	4.98	6.39	5.37
B. International comovement			
Output	0.40	0.40	0.41
Consumption	0.25	0.15	0.28
Investment	0.23	-0.65	-0.35
Employment	0.21	0.15	0.23
TFP	0.30	0.73	0.65
C. Co-movement within a country			
Net exports, Output	-0.49	-0.51	-0.52
Terms of trade, Output	-0.33	-0.51	-0.51
RER, cH/cF	-0.71	-0.96	-0.95

Summary

With demand shocks only, our shopping model can account for puzzles in the international economics:

- Backus-Smith puzzle: $corr(RER, cH/cF) < 0$
- Countercyclical terms of trade
- Volatile net exports
- International consumption correlation smaller than output correlation

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