# TIME-CONSISTENT POLICY

**Preliminary**

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# MOTIVATION

- The following problem is pervasive in economics: a decision maker at time $t$ cares about the future, disagrees with the decision maker at $t+1$, but has no direct influence over him/her.

  As an example, consider the optimal taxation problem without commitment. What does theory predict?
- The early literature (Kydland-Prescott (1977)) focused on finding the Markov equilibrium that is a limit of the corresponding finite-horizon economy.
- Later: focus on using "reputation mechanisms" that are possible in infinite-horizon economies (using Abreu, Pearce, and Stacchetti (1990), as in Chari and Kehoe (1990)).
- What happens when the reputation is "lost", and we are left with the "fundamentals"? What does the Markov equilibrium look like? This is the question we address here, in the context of models with natural state variables such as capital, debt, income distribution, etc.
- The Markov equilibrium gives us information about the results for long-horizon decision problems, and also it focuses on the basic economics dictated by the state variables.

# EARLIER WORK ON MARKOV EQUILIBRIA

- Some linear-quadratic models can be solved explicitly (e.g., Cohen and Michel (1988) and Currie and Levine (1993)).

- Numerical approach: Krusell, Quadrini & Ríos-Rull (1997) and related papers; more recently, e.g., Klein and Ríos-Rull (2001). Problem here: these methods are of the “black-box” type and they did not deliver controlled accuracy.

# CONTRIBUTIONS HERE

- We show how to characterize and solve for the Markov equilibrium:

  1. We derive a "generalized Euler equation"—GEE—allowing us to interpret the incentives facing the key decision maker; this equation does not appear in the existing literature, and it allows qualitative and quantitative interpretations.

  2. We show how to solve this (functional) equation; this is much harder problem than that of solving a standard Euler equation. Reason: to solve for a steady state, one needs to solve jointly for dynamics; to solve for first-order dynamics, one needs to solve for second-order dynamics, and so on. . .

- The methods are, we think, entirely general and applicable to a wide variety of contexts: optimal fiscal and monetary policy, dynamic political economy, dynamic industrial organization issues (e.g., the durable goods monopoly, dynamic oligopoly), models with impure intergenerational altruism, and so on.

# WE ILLUSTRATE WITH AN EXAMPLE ECONOMY: OPTIMAL PUBLIC GOODS PROVISION

Public goods will be financed under a balanced budget (no government debt in this version) through either

- labor income taxes only,
- general income taxes or
- capital income taxes only.

We will describe the general income tax case as the baseline.

We also look at the hyperbolic consumer.

## Households choose

$$\max_{\{c_t,\ell_t,k_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \; u(c_t, 1-\ell_t, g_t)$$

subject to $k_0$ and

$$c_t + k_{t+1} = k_t + (1-\tau_t)\left[w_t\ell_t + (r_t - \delta)k_t\right]$$

The resource constraint is

$$C_t + K_{t+1} + G_t = f(K_t, L_t) + (1-\delta)K_t.$$

The government's period by period balanced budget constraint reads

$$G_t = \tau_t \left[f(K_t, L_t) - \delta K_t\right].$$

# DERIVING THE GEE

In a subgame-perfect equilibrium, the government needs to compare the effects of any current policy choice, $\tau$, on endogenous variables given any current value of the state, $K$. Thus we need to find the three key equilibrium objects:

$$\begin{aligned} L &= \mathcal{L}(K, \tau) \\ K' &= \mathcal{H}(K, \tau) \\ \tau &= \Psi(K). \end{aligned}$$

The functions $\mathcal{L}$, $\mathcal{H}$, and $\Psi$ are unknown to us at this stage.

- Functions $\mathcal{L}$ and $\mathcal{H}$ are determined so as to satisfy the FOC's for the household.
- The equilibrium policy rule $\Psi$ is determined by the GEE: the government's FOC.

## Auxiliary Functions

There are two other auxiliary functions that are convenient to define:

$$
\begin{aligned}
C &= \mathcal{C}(K, \tau) \\
G &= \mathcal{G}(K, \tau)
\end{aligned}
$$

satisfying

$$
\begin{aligned}
\mathcal{G}(K, \tau) &= \tau \left\{ f[K, \mathcal{L}(K, \tau)] - \delta K \right\} \\
\mathcal{C}(K, \tau) &= f\left[K, \mathcal{L}(K, \tau)\right] + (1 - \delta)K - \mathcal{H}(K, \tau) - \mathcal{G}(K, \tau).
\end{aligned}
$$

## Household Foc's

Functions $\mathcal{H}$ and $\mathcal{L}$ now satisfy, two functional equations: the FOC for labor,

$$u_c\left[\mathcal{C}(K,\tau),1-\mathcal{L}(K,\tau),\mathcal{G}(K,\tau)\right]\cdot f_L\left[K,\mathcal{L}(K,\tau)\right](1-\tau)=$$
$$u_\ell\left[\mathcal{C}(K,\tau),1-\mathcal{L}(K,\tau),\mathcal{G}(K,\tau)\right],$$

for all $(K,\tau)$, and the FOC for saving, also for all $(K,\tau)$.

$$u_c\left[\mathcal{C}(K,\tau),1-\mathcal{L}(K,\tau),\mathcal{G}(K,\tau)\right]=$$
$$\beta u_c\left(\mathcal{C}\{\mathcal{H}(K,\tau),\Psi[\mathcal{H}(K,\tau)]\},1-\mathcal{L}\{\mathcal{H}(K,\tau),\Psi[\mathcal{H}(K,\tau)]\},\mathcal{G}\{\mathcal{H}(K,\tau),\Psi[\mathcal{H}(K,\tau)]\}\right)\cdot$$
$$[1+\{1-\Psi[\mathcal{H}(K,\tau)]\{(f_K\{\mathcal{H}(K,\tau),\mathcal{L}[\mathcal{H}(K,\tau)]\}-\delta)],$$

Notice how $\Psi$ is a determinant of $\mathcal{H}$ and $\mathcal{L}$: the expectations of future government behavior influence how consumers work and save.

## Auxiliary Functions

The government's problem can now be written as

$$\max_{\tau} \quad u\left[\mathcal{C}\left(K,\tau\right),1-\mathcal{L}\left(K,\tau\right),\mathcal{G}\left(K,\tau\right)\right]+\ \beta\, v\left[\mathcal{H}\left(K,\tau\right)\right]$$

where

$$v(K) \equiv u\left(\{\mathcal{C}[K,\Psi(K)],1-\mathcal{L}[K,\Psi(K)],\mathcal{G}[K,\Psi(K)]\}+\ \beta\, v\{\mathcal{H}[K,\Psi(K)]\}.\right.$$

A *subgame-perfect equilibrium* now dictates that $\Psi(K)$ solves the above problem for all $K$.

## Sequential Representation

Notice: in equilibrium

$$v(K) \;=\; \max_{\tau} \;\; u\left[\mathcal{C}(K,\tau), 1-\mathcal{L}(K,\tau), \mathcal{G}(K,\tau)\right] + \;\beta\, v[\mathcal{H}(K,\tau)]$$

This is a **recursive problem**! That is, we can alternatively characterize the optimal policy sequence, $\{\tau_t\}_{t=0}^{\infty}$, as the solution to the following sequential problem

$$\max_{\{k_{t+1},\tau_t\}_{t=0}^{\infty}} \;\; \sum_{t=0}^{\infty} \;\beta^t \;\; u[\mathcal{C}(k_t,\tau_t), 1-\mathcal{L}(k_t,\tau_t), \mathcal{G}(k_t,\tau_t)]$$

subject to

$$k_{t+1} = \mathcal{H}(k_t,\tau_t).$$

Again, this problem is not in terms of primitives: $\mathcal{H}$ and $\mathcal{L}$ (and hence indirectly also $\mathcal{C}$ and $\mathcal{G}$) are endogenous, and depend on the policy rule $\Psi$.

## Now we derive the GEE from the Sequential Representation

To derive the GEE is now easy: just differentiate. This yields, after simplification and using primes instead of $t$'s, the functional equation

$$\mathcal{L}_\tau \left[u_c f_L - u_\ell\right] + \mathcal{G}_\tau \left[u_g - u_c\right] + \beta \mathcal{H}_\tau \cdot$$

$$\mathcal{L}'_K \;\; u'_c f'_L - u_\ell \;\; + \mathcal{G}'_\tau \left[u'_g - u'_c\right] - \frac{\mathcal{H}'_K}{\mathcal{H}'_\tau} \;\; \mathcal{L}'_\tau \;\; u'_c f'_L - u'_\ell \;\; + \mathcal{G}'_\tau \left[u'_g - u'_c\right] \qquad +$$

$$\mathcal{H}_\tau \quad -u_c + \beta u'_c (1 + f'_K - \delta) \quad = 0.$$

that has to hold for all $K$ (this argument is suppressed for readability). This equation, together with the two first-order conditions, represent three functional equations in three unknown functions.

## Deriving the GEE from the Recursive Representation

Alternatively, we can take the slightly more cumbersome route of using the recursive form of the government's problem to derive the GEE by

1. Take first order conditions with respect to $\tau$.
2. Differentiate the expression for $v(K)$ to get an envelope condition.
3. Use the first order condition to solve for $v_K(\mathcal{H}(K, \Psi(K)))$ and substitute this into the envelope condition.
4. Evaluate the expression for $v(K)$ thus obtained at $\mathcal{H}(K, \Psi(K))$ and substitute into the first order condition. We now have the GEE.

Side remark: The approach has to be modified if the number of states is not equal to the number of government controls.

## About the GEE:

$$\mathcal{L}_\tau \left[u_c f_L - u_\ell\right] + \mathcal{G}_\tau \left[u_g - u_c\right] + \beta \mathcal{H}_\tau \cdot$$

$$\mathcal{L}'_K \quad u'_c f'_L - u_\ell \quad + \mathcal{G}'_\tau \left[u'_g - u'_c\right] - \frac{\mathcal{H}'_K}{\mathcal{H}'_\tau} \quad \mathcal{L}'_\tau \quad u'_c f'_L - u'_\ell \quad + \mathcal{G}'_\tau \left[u'_g - u'_c\right] \qquad +$$

$$\mathcal{H}_\tau \quad -u_c + \beta u'_c (1 + f'_K - \delta) \quad = 0.$$

- It is a linear combination of wedges.
- In the case of a labor income tax and a capital income tax, some of the terms disappear.
- In the case of lump sum taxes, a solution satisfies $u_g = u_c$.
- The Markov equilibrium is not necessarily equal to the Ramsey solution, even in the labor tax only case. Reason: the Ramsey policy maker takes into account the fact that a tax hike at $t$ not only lowers labor supply at $t$ but raises it at $t-1$. A Markov policy-maker treats the latter as a bygone.

# EXISTENCE, COMPUTATION, ETC.

To see some issues that arise with solving this type of GEE, let's look at the simplest example that we can imagine. That implied by the decision problem of an agent with multiple selves and quasi–geometric discounting (Laibson and so on).

$$u'[f(k) - g(k)] = \beta\ \delta\ u'\{f[g(k)] - g[g(k)]\} \qquad f'[g(k)] - g'[g(k)](1 - 1/\beta)$$

for all $k$. Here we have one functional equation in one unknown function: $g(k)$. Issues:

- To find a steady state: 1 equation and two unknowns.
- Computational solution: Perturbation methods.
  - Assume $g'(k) = 0$. Solve for St-St $k_0^* = g(k_0^*)$.
  - Assume $g''(k) = 0$. Solve for St-St $k_1^* = g(k_1^*)$ and $g'(k_1^*)$.
  - Keep going
  - Assume $g^n(k) = 0$. Solve for St-St $k_n^* = g(k_n^*)$, and $g'(k_n^*)$, up to $g^n(k_n^*)$.
  - Hope $k_n^*$ converges (so far it has).
- Existence.

# OUTLINE OF AN ALGORITHM FOR THE TAXING PROBLEM

- The first order conditions for the household form two functional equations. The GEE is a third functional equation. Meanwhile, there are three unknown functions: $\mathcal{L}$, $\mathcal{H}$, and $\Psi$.
- Fundamental difference between this problem and a finding a RCE or a Ramsey probl: The steady state is **not** the solution to a finite-dimensional system of nonlinear equations.
- To see this, notice that the GEE features unknown *derivatives* of the private decision rules. Thus the unknowns are $\bar{L}$, $\bar{K}$, $\bar{g}$, $\bar{\mathcal{H}}_\tau$, $\bar{\mathcal{H}}_K$, $\bar{\mathcal{L}}_\tau$, and $\bar{\mathcal{L}}_K$. Thus there are seven unknowns but just three equations.
- So, differentiate the private focs with respect to $\tau$ and $K$ and the GEE with respect to $K$. This gives us five new equations. It also gives us several more unknowns, including $\bar{\Psi}_K$ as well as *second* derivatives of $\mathcal{H}$ and $\mathcal{L}$ evaluated at the ST ST.
- This never stops. There are always more unknowns than equations.
- Then: set all derivatives of degree $n$ and greater equal to zero. Then increase $n$ until further increases make a negligible difference.
- The simplest case: $n = 2$. Then we have eight unknowns and eight equations. With $n = 3$ we have 15 equations and 15 unknowns. And so on.

# CALIBRATED EXAMPLES

We specify the period utility function as

$$u(c, \ell, g) = \frac{\left[\alpha_p \left(\alpha_c c^\rho + (1-\alpha_c)\ell^\rho\right)^{\psi/\rho} + (1-\alpha_p) g^\psi\right]^{\frac{1-\sigma}{\psi}} - 1}{1-\sigma}$$

When $\sigma \to 1$, $\rho \to 0$ and $\psi \to 0$, this reduces to

$$u(c, \ell, g) = \alpha_p \alpha_c \ln c + \alpha_p (1-\alpha_c) \ln \ell + \alpha_p \ln g$$

Meanwhile, the production function is

$$f(K, L) = A \cdot K^\theta L^{1-\theta}.$$

| Labor taxes, endogenous g | | | |
|---|---|---|---|
| | $\theta = 0$ | $\alpha_c = 1$ | Baseline |
| Markov | 0.3328 | 0.0994 | 0.1900 |
| Pareto | 0.3328 | 0.0994 | 0.2540 |
| Ramsey | 0.3328 | 0.0994 | 0.2540 |

Table 1: $\bar{g}/\bar{y}$, *baseline case*

The Pareto and Ramsey equilibria are not the same. Labor supply is smaller under Ramsey than under Pareto. But their $\bar{g}/\bar{y}$ are the same because because it turns out that one of the Ramsey optimality conditions says $u_g = u_c$. It does because cross derivatives of the period utility function are zero.

The Markov $\bar{g}/\bar{y}$ is smaller than the other for, I think, the following reason. An uncommitted policy maker does not take into account that today's taxes increase yesterday's incentives to work. We have talked about this before.

| Labor taxes, endogenous g | | | | |
|---|---|---|---|---|
| | $\rho = 0.5$ | $\rho = 0.1$ | $\rho = -0.1$ | $\rho = -5$ |
| Markov | 0.1356 | 0.1818 | 0.1973 | 0.2990 |
| Pareto | 0.3487 | 0.2632 | 0.2468 | 0.1967 |

Table 2: $\bar{g}/\bar{y}$, *higher and lower consumption/leisure substitutability*

I don't report any Ramsey equilibria here. It turns out that, even after weeks of debugging, the Ramsey $\bar{g}/\bar{y}$ depends on what total factor productivity is.

It is not surprising that the Markov $\bar{g}/\bar{y}$ is higher, the less substitutable leisure and consumption are. The incentive effects of taxation are smaller.

However, it is surprising that if $\rho$ is small enough, the $\bar{g}/\bar{y}$ is higher under Markov than under Ramsey. This I don't understand.

| Labor taxes, endogenous g | | | | |
|---|---|---|---|---|
| | $\psi = 0.5$ | $\psi = 0.1$ | $\psi = -0.1$ | $\psi = -5$ |
| Markov | 0.1563 | 0.1852 | 0.1942 | 0.2374 |
| Pareto | 0.1020 | 0.2324 | 0.2724 | 0.4352 |

Table 3: $\bar{g}/\bar{y}$, *higher and lower public/private consumption substitutability*

| Labor taxes, endogenous g | | | | |
|---|---|---|---|---|
| | $\sigma = 2$ | $\sigma = 3$ | $\sigma = 5$ | $\sigma = 50$ |
| Markov | 0.1995 | 0.2055 | 0.2128 | 0.2342 |
| Pareto | 0.2540 | 0.2540 | 0.2540 | 0.2540 |

Table 4: $\bar{g}/\bar{y}$, *Changing the intertemporal elasticity of substitution*

As intertemporal substitutability falls, the Markov solution converges to the Pareto solution. Recall the story we told about Ramsey differing from Markov. It depends on there being intertemporal substitutability. Without it, we approach the Ramsey solution which in this respect ($\bar{g}/\bar{y}$) coincides with Pareto.

| Income taxes, endogenous g | | | | | |
|---|---|---|---|---|---|
| | $\rho = 0.5$ | $\rho = 0.1$ | $\rho = 0$ | $\rho = -0.1$ | $\rho = -5$ |
| Markov | 0.1681 | 0.1962 | 0.2006 | 0.2045 | 0.2436 |
| Pareto | 0.3487 | 0.2632 | 0.2540 | 0.2468 | 0.1967 |

Table 5: $\bar{g}/\bar{y}$, *higher and lower consumption/leisure substitutability*

| Income taxes, endogenous g | | | | | |
|---|---|---|---|---|---|
| | $\psi = 0.5$ | $\psi = 0.1$ | $\psi = 0$ | $\psi = -0.1$ | $\psi = -5$ |
| Markov | 0.1309 | 0.1905 | 0.2006 | 0.2095 | 0.2971 |
| Pareto | 0.1020 | 0.2324 | 0.2540 | 0.2724 | 0.4352 |

Table 6: $\bar{g}/\bar{y}$, *higher and lower public/private consumption substitutability*

| Income taxes, endogenous g | | | | |
|---|---|---|---|---|
| | $\sigma = 2$ | $\sigma = 3$ | $\sigma = 5$ | $\sigma = 50$ |
| Markov | 0.2128 | 0.2206 | 0.2303 | 0.2590 |
| Pareto | 0.2540 | 0.2540 | 0.2540 | 0.2540 |

Table 7: $\bar{g}/\bar{y}$, *Changing the intertemporal elasticity of substitution*

| Capital taxes, endogenous g | | | | | |
|---|---|---|---|---|---|
| | $\rho = 0.5$ | $\rho = 0.1$ | $\rho = 0$ | $\rho = -0.1$ | $\rho = -5$ |
| Markov | 0.2031 | 0.2138 | 0.2148 | 0.2155 | 0.2189 |
| Pareto | 0.3487 | 0.2632 | 0.2540 | 0.2468 | 0.1967 |

Table 8: $\bar{g}/\bar{y}$, *higher and lower consumption/leisure substitutability*

| Capital taxes, endogenous g | | | | | |
|---|---|---|---|---|---|
| | $\psi = 0.5$ | $\psi = 0.1$ | $\psi = 0$ | $\psi = -0.1$ | $\psi = -5$ |
| Markov | 0.1837 | 0.2113 | 0.2148 | 0.2174 | 0.1857 |
| Pareto | 0.1020 | 0.2324 | 0.2540 | 0.2724 | 0.4352 |

Table 9: $\bar{g}/\bar{y}$, *higher and lower public/private consumption substitutability*

| Capital taxes, endogenous g | | | | |
|---|---|---|---|---|
| | $\sigma = 2$ | $\sigma = 3$ | $\sigma = 5$ | $\sigma = 25$ |
| Markov | 0.2344 | 0.2484 | 0.2669 | 0.3091 |
| Pareto | 0.2540 | 0.2540 | 0.2540 | 0.2540 |

Table 10: $\bar{g}/\bar{y}$, *Changing the intertemporal elasticity of substitution*

| Capital taxes, endogenous g | | | | |
|---|---|---|---|---|
| | $\sigma = 2$ | $\sigma = 3$ | $\sigma = 5$ | $\sigma = 25$ |
| Markov | 0.3433 | 0.3662 | 0.3973 | 0.4702 |
| Pareto | 0.4989 | 0.4989 | 0.4989 | 0.4989 |

Table 11: $\bar{g}/\bar{c}$, *Changing the intertemporal elasticity of substitution*

# CONCLUSION

- We have introduced a new, powerful approach to characterizing and calculating time consistent policy equilibria.
- It remains to apply the approach to more cases, including government debt policy.

# CALIBRATED EXAMPLES

We specify the period utility function as

$$u(c,\ell,g) = \frac{\left[\alpha_p\left(\alpha_c c^\rho + (1-\alpha_c)\ell^\rho\right)^{\psi/\rho} + (1-\alpha_p)g^\psi\right]^{\frac{1-\sigma}{\psi}} - 1}{1-\sigma}$$

When $\sigma \longrightarrow 1$, $\rho \longrightarrow 0$ and $\psi \longrightarrow 0$, this reduces to

$$u(c,\ell,g) = \alpha_p\alpha_c \ln c + \alpha_p(1-\alpha_c)\ln\ell + \alpha_p \ln g$$

Meanwhile, the production function is

$$f(K,L) = A \cdot K^\theta L^{1-\theta}.$$

Labor taxes, endogenous g

| | $\theta = 0$ | $\alpha_c = 1$ | Baseline |
|---|---|---|---|
| Markov | 0.3328 | 0.0994 | 0.1900 |
| Pareto | 0.3328 | 0.0994 | 0.2540 |

Table 12: $\bar{g}/\bar{y}$, *baseline case*

| Labor taxes, endogenous g | | | | |
|---|---|---|---|---|
| | $\rho = 0.5$ | $\rho = 0.1$ | $\rho = -0.1$ | $\rho = -5$ |
| Markov | 0.1356 | 0.1818 | 0.1973 | 0.2990 |
| Pareto | 0.3487 | 0.2632 | 0.2468 | 0.1967 |

Table 13: $\bar{g}/\bar{y}$, *higher and lower public/private consumption substitutability*

| Labor taxes, endogenous g | | | | |
|---|---|---|---|---|
| | $\psi = 0.5$ | $\psi = 0.1$ | $\psi = -0.1$ | $\psi = -5$ |
| Markov | 0.1563 | 0.1852 | 0.1942 | 0.2374 |
| Pareto | 0.1020 | 0.2324 | 0.2724 | 0.4352 |

Table 14: $\bar{g}/\bar{y}$, *higher and lower public/private consumption substitutability*

Labor taxes, endogenous g

| | $\sigma = 2$ | $\sigma = 3$ | $\sigma = 5$ | $\sigma = 50$ |
|---|---|---|---|---|
| Markov | 0.1995 | 0.2055 | 0.2128 | 0.2342 |
| Pareto | 0.2540 | 0.2540 | 0.2540 | 0.2540 |

Table 15: $\bar{g}/\bar{y}$, *Changing the intertemporal elasticity of substitution*

Income taxes, endogenous g

| | $\rho = 0.5$ | $\rho = 0.1$ | $\rho = 0$ | $\rho = -0.1$ | $\rho = -5$ |
|---|---|---|---|---|---|
| Markov | 0.1681 | 0.1962 | 0.2006 | 0.2045 | 0.2436 |
| Pareto | 0.3487 | 0.2632 | 0.2540 | 0.2468 | 0.1967 |

Table 16: $\bar{g}/\bar{y}$, *higher and lower consumption/leisure substitutability*

Income taxes, endogenous g

| | $\psi = 0.5$ | $\psi = 0.1$ | $\psi = 0$ | $\psi = -0.1$ | $\psi = -5$ |
|---|---|---|---|---|---|
| Markov | 0.1309 | 0.1905 | 0.2006 | 0.2095 | 0.2971 |
| Pareto | 0.1020 | 0.2324 | 0.2540 | 0.2724 | 0.4352 |

Table 17: $\bar{g}/\bar{y}$, *higher and lower public/private consumption substitutability*

| Income taxes, endogenous g | | | | |
|---|---|---|---|---|
| | $\sigma = 2$ | $\sigma = 3$ | $\sigma = 5$ | $\sigma = 50$ |
| Markov | 0.2128 | 0.2206 | 0.2303 | 0.2590 |
| Pareto | 0.2540 | 0.2540 | 0.2540 | 0.2540 |

Table 18: $\bar{g}/\bar{y}$, *Changing the intertemporal elasticity of substitution*

Capital taxes, endogenous g

| | $\rho = 0.5$ | $\rho = 0.1$ | $\rho = 0$ | $\rho = -0.1$ | $\rho = -5$ |
|---|---|---|---|---|---|
| Markov | 0.2031 | 0.2138 | 0.2148 | 0.2155 | 0.2189 |
| Pareto | 0.3487 | 0.2632 | 0.2540 | 0.2468 | 0.1967 |

Table 19: $\bar{g}/\bar{y}$, *higher and lower consumption/leisure substitutability*

Capital taxes, endogenous g

| | $\psi = 0.5$ | $\psi = 0.1$ | $\psi = 0$ | $\psi = -0.1$ | $\psi = -5$ |
|---|---|---|---|---|---|
| Markov | 0.1837 | 0.2113 | 0.2148 | 0.2174 | 0.1857 |
| Pareto | 0.1020 | 0.2324 | 0.2540 | 0.2724 | 0.4352 |

Table 20: $\bar{g}/\bar{y}$, *higher and lower public/private consumption substitutability*

| Capital taxes, endogenous g | | | | |
|---|---|---|---|---|
| | $\sigma = 2$ | $\sigma = 3$ | $\sigma = 5$ | $\sigma = 25$ |
| Markov | 0.2344 | 0.2484 | 0.2669 | 0.3091 |
| Pareto | 0.2540 | 0.2540 | 0.2540 | 0.2540 |

Table 21: $\bar{g}/\bar{y}$, *Changing the intertemporal elasticity of substitution*

| Capital taxes, endogenous g | | | | |
|---|---|---|---|---|
| | $\sigma = 2$ | $\sigma = 3$ | $\sigma = 5$ | $\sigma = 25$ |
| Markov | 0.3433 | 0.3662 | 0.3973 | 0.4702 |
| Pareto | 0.4989 | 0.4989 | 0.4989 | 0.4989 |

Table 22: $\bar{g}/\bar{c}$, *Changing the intertemporal elasticity of substitution*

# CONCLUSION

- We develop what we believe are very general methods for solving for “the” Markov equilibrium.
- The class of problems for which these methods are relevant seems vast.
- We apply our methods here to analyze, quantitatively, the choice of optimal public goods provision.