Demand Shocks as Productivity Shocks

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Introduction

- A popular intuitive idea: “demand” matters for business cycles.
  - **Problem**: With decreasing returns to scale in labor counterfactual implications in equilibrium growth models.

- This paper:
  - Builds a model where demand for goods plays a direct “productive” role:
    - Demand creates its own supply: demand shocks (today modeled as preference shocks) cause fluctuations in measured productivity.
  - Embeds the simple theory in a standard stochastic growth model.
  - Estimates the contribution of both demand shocks and “true” productivity shocks to aggregate fluctuations.

**Result**: There is essentially no role for productivity shocks.
In a standard business cycle model, the production function requires that either productivity or inputs change output (The only inputs are capital and labor).

\[ Y = z \cdot F(K, N) \]

So either productivity shocks \((z)\) move or inputs (i.e. labor) move.

Decreasing returns to scale require that labor productivity and wages drop if labor increases.

This does not happen in the data, the residual \(z\) is strongly correlated with output. Hence there have to be TFP shocks.

We have been looking for them for thirty years with limited success.
The logic

- We postulate that in order to transform produced goods into used goods, both consumers and investors must exert (search) efforts.

- Such efforts are not accounted for in NIPA.

- The economy cannot operate at full capacity.

- Operationally, this works as a search friction in the goods market. Increases in search effort imply increased measured productivity.

- We use competitive search (unique, efficient equilibrium).

- Preference shocks are a stand in for a variety of demand shocks (credit restrictions, animal spirits, terms of trade shocks).
Alternatives in the literature


3. Diamond (1982) and Guerrieri and Lorenzoni (2009) that due to a search friction, the difficulty of coordination of trade can give rise to and exacerbate aggregate fluctuations.

The plan

1. We describe the logic in the Lucas tree (output is productivity).

2. We move on to a growth model suitable for business cycle analysis.

3. We estimate preference shocks from measured Solow residuals.
   - Contrast estimated model to RBC model.

4. We estimate jointly demand shocks and technology shocks and determine the contribution of each.
A Lucas-tree version of the model.

- Continuum of trees, measure $T = 1$. Each yields one fruit per period.

- Search friction: If a shopper finds a tree, then trade at price $p$; otherwise the fruit rots.

- So Consumption = Productivity and is endogenous.

- Competitive Search: Agents choose where to search.

- A “market” is characterized by a price and a “market tightness”
  
  1. $p$: Price (numeraire: the value of the tree)
  2. $Q$: Market tightness (average available fruits per shopper).
Matching Technology

- Output equal the measure of matches:

\[ Y = D^\alpha T^{1-\alpha} \]

- \( D \) is the measure of shoppers. \( \alpha \) is a parameter.

- Recall: market tightness is \( Q \equiv \frac{T}{D} \)

- Probability that a tree is randomly matched with a shopper (i.e., number of matches per tree):

\[ \Psi_T(Q) \equiv \frac{D^\alpha T^{1-\alpha}}{T} = \left( \frac{D}{T} \right)^\alpha = Q^{-\alpha} = \frac{Y}{T} = Y = C = D^\alpha, \]

- Output and productivity depend only on how many shoppers.
Preferences:

- Many identical, infinitely lived, households. Utility is
  \[ E \sum_{t} \beta^{t} U(c_{t}, d_{t}, \theta_{t}), \]
  where \( c_{t} \) is fruit consumption, \( d_{t} \) is the measure of shopping units (a search disutility). \( \theta \) is a Markovian preference shock.

- Consumption is \# shopping units (\( d \)) times the probability of a unit finding a fruit (\( \Psi_{D} \)):
  \[ c = d \cdot \Psi_{D}(Q) \equiv d \cdot Q^{1-\alpha}. \]

- Households own \( s \) shares of the trees.

- Aggregate state: \( \theta \). Individual state: \( (\theta, s) \).
The Values of agents: Households and Firms

Hhold: \[ v(\theta, s) = \max_{c,d,s'} U(c, d, \theta) + \beta E \{ v(\theta', s')|\theta \} \quad \text{s.t.} \]

\[ p(\theta) c + s' = s [1 + R(\theta)] \]
\[ c = d \cdot \Psi_D[Q(\theta)] \]

Firms: \[ 1 + R(\theta) = \varsigma(\theta) + E \left\{ \frac{1 + R(\theta')}{1 + R(\theta')} \right\} = p(\theta) \Psi_T(Q) + 1 \]

- Equilibrium objects are 2
  1. Price of consumption (in terms of units of tree): \( p(\theta) \).
  2. Market tightness: \( Q(\theta) \).
     - Consumption: \( C(\theta) = \Psi_T[Q(\theta)] \).
     - Dividends from trees: \( R(\theta) = p(\theta) \Psi_T[Q(\theta)] \).
     - Consumption rate of return: \( 1 + r(\theta') = \frac{p(\theta)[1+R(\theta')]}{p(\theta')} \)
Equilibrium conditions to determine \((p, Q)\)

1. Euler equation:

\[ \frac{\partial U}{\partial c} + \frac{\partial U}{\partial d} \frac{\partial d}{\partial c} = u_c[C(\theta)] - \frac{\theta_d}{\Psi_D(Q)} = p(\theta) M(\theta), \]

where \(M\) is expected discounted marginal utility of saving,

\[ M(\theta) = E \left\{ \frac{[1 + R(\theta')]\beta}{p(\theta')} \left( u_{c'} - \frac{\theta_d'}{\Psi_D(Q')} \right) \mid \theta \right\} \]

2. We need one more equilibrium condition to determine \(Q\).

It comes from competitive search.
Competitive Search in the Market for Goods

- This is the mechanism that determines the additional equilibrium object, market tightness.

- Shoppers choose which market to search in. Those markets are differentiated by $p$ and $Q$.

- Let $\zeta^* = p \Psi_T(Q)$ be the “outside value” for firms of going to the best market to sell their fruit. Shoppers take it as given.

- So shoppers can only open markets where trees get at least $\zeta^*$:

\[ \zeta^* \leq p \Psi_T(Q) \]
The choice of market by the shopper

Let $\theta_d$ be the (sunk) marginal utility cost of an extra shopper. The rewards for the household to send a shopper to a $(p, Q)$ market is

$$\Phi = \max_{p, Q} \left\{-\theta_d + \psi_d(Q) \left( u_c - p M \right)\right\} \quad \text{s.t.}$$

$$s^* \leq p \psi_T(Q),$$

where again

$$M(\theta) = E \left\{ \frac{[1+R(\theta')]}{p(\theta')} \beta \left( u_{c'} - \frac{\theta'_d}{\psi_D(Q')} \right) \bigg| \theta \right\}.$$ 

The FOC is

$$0 = (1 - \alpha) \, Q^{-\alpha} \, u_c - M \, p \, Q^{-\alpha}$$

or

$$p = (1 - \alpha) \, \frac{u_c}{M}$$
Summary of Equilibrium which by the way is Pareto Optimum

- The two conditions that determine the two equilibrium objects \{p, Q\}.

1. The Hold Euler \( U_c - \frac{U_d}{\Psi_D} = p M \) or

\[
\left( \theta_c u_c[C(\theta)] - \frac{\theta_d}{\Psi_D(Q)} \right) = p(\theta) E \left\{ \frac{[1 + R(\theta')]}{p(\theta')} \beta \left( u_{c'}[C(\theta')] - \frac{\theta_{d'}'}{\Psi_D(Q')} \right) \right\} \theta
\]

2. The Search Equilibrium Condition \((1 - \alpha) U_c = p M\) or

\[
(1 - \alpha) u_c[C(\theta)] = p E \left\{ \frac{[1 + R(\theta')]}{p(\theta')} \beta \left( u_{c'}[C(\theta')] - \frac{\theta_{d'}'}{\Psi_D(Q')} \right) \right\} \theta
\]
Example

- Suppose preferences are given by

\[ u(c, d, \theta_c) = \theta_c \log(c) - d, \]

where \( \theta_c \) is i.i.d. with \( E\{\theta_c\} = 1 \)

- Allocations of demand and consumption are

\[ D(\theta_c) = \alpha \theta_c \]
\[ C(\theta_c) = A \alpha^{\alpha} (\theta_c)^{\alpha} \]

- The Solow residual is \( Z(\theta_c) = C(\theta_c) \). Price and interest rate are

\[ P(\theta) = \left(\frac{1}{\beta} - 1\right) \frac{1}{A\alpha^\alpha} \theta_c^{1-\alpha}, \quad 1 + r(\theta) = \frac{\theta_c^{1-\alpha}}{\beta E\{(\theta_c^\alpha)^{1-\alpha}\}}. \]

So a demand shock (\( \theta_c \uparrow \)) implies a partial increase in price and interest rate, and partially increased demand and output.
Putting the model to work: the Growth Version

- We put a growth model with capital investment and labor choice with the shopping structure that we have developed.

- Some important changes.
  1. There is varying capacity or output potential that we denote $F$ and that is the productive capacity.
  2. Both households (when purchasing consumption goods) and firms (when purchasing investment goods) face search frictions.
  3. In this model capital and wealth are NOT the same. The locations have intrinsic value. Extensions will have creation of new locations as a form of investment.
  4. All this generates subtle calibration issues.
Production

- Measure one of firms–locations with installed capital $k$ (depreciates at rate $\delta$). Goods can be used for consumption or investment and capacity is

$$F(Z, k, n) = Z \ k^{\gamma_k} \ n^{\gamma_n}$$

- New capital has to be purchased that requires shoppers $n^k$.

- Shoppers and sellers trade in decentralized markets at prices (in terms of shares of the economy’s wealth) $p^i$ if investment and $p^c$ if consumption.

- Unmatched capacity rots.
Preferences are

\[ E \left\{ \sum_t \beta^t U(c, n, d, \theta) \right\}, \quad \theta, \text{ Markovian} \]

Again consumption requires that it is shopped so

\[ c = d \Psi_d(Q^c) F^c \]

- \( \Psi_d(Q^c) \) is the probability of matching a consumption firm, \( Q^c \) is market tightness in the consumption good market and \( F^c \) is output capacity in a consumption location.

- Households own the firms.
A few lemmas alleviate notation

1. The state of the economy is the pair \( \{ \theta, Z, K \} \).

2. There is only one active market in consumption goods and another in investment goods.

3. Firms that produce consumption and investment choose the same inputs \( F^i = F^c = F(Z, K, N^F) \). Use \( n^F(k, Z, F) \) to denote the inverse fn.

4. Consumption and investment firms get the same expected revenue (but not necessarily the same price and market tightness).
Consumption (or invt) firms in a \((p^c, F, Q^c)\) submarket

\[
\Omega(\theta, Z, K, k) = \max_{n^k, k', i} \Psi_T(Q^c) F p^F - w(\theta, K) \left[ n^F(k, Z, F) + n^k \right]
\]

\[
- p^i(\theta, Z, K) i + E \left\{ \frac{\Omega(\theta', Z', K', k')}{1 + R(\theta', Z', K')} \bigg| \theta \right\}
\]

s.t.

\[
i = (n^k \zeta) \Psi_d[Q^i(\theta, Z, K)] \quad F^i[\theta, Z, K]
\]

\[
k' = i + (1 - \delta)k
\]

\[
K' = G(\theta, Z, K)
\]

with FOC (and RA condition)

\[
E \left\{ \frac{\Omega_3(\theta', Z', K', K')}{1 + R(\theta', Z', K')} \bigg| \theta \right\} = \frac{w(\theta, Z, K)}{\zeta \Psi_d[Q^i(\theta, K)] F^i(Z, K, N)} + p^i(\theta, Z, K).
\]
The household problem

\[ v(\theta, Z, K, s) = \max_{c,d,n,s'} U(c, d, n, \theta) + \beta E \left\{ v(\theta', K', s') | \theta \right\} \quad \text{s.t.} \]

\[ p^F(\theta, Z, K) c + s' = s [1 + R(\theta, Z, K)] + n w(\theta, Z, K) \]

\[ c = d \psi_d [Q^c(\theta, Z, K)] F[K, N^F(\theta, Z, K)] \]

\[ K' = G(\theta, Z, K) \]

- Hholds’ FOC (and RA)

\[ U_c - \frac{U_d}{\psi_d F} = \beta E \left\{ \frac{p^F (1 + R')}{p^{c'}} \left[ U'_c - \frac{U'_d}{\psi_d F'} \right] | \theta \right\} , \]

\[ U_c - \frac{U_d}{\psi_d F} = U_n \frac{p^F}{w}. \]
Competitive Search in Markets

- Markets are now indexed by quantity, price, and market tightness (the latter two will be different for consumption and investment).

- We get additional conditions from the FOC of shoppers given expected revenue for sellers.

- The equilibrium objects are functions of $(\theta, Z, K)$ for

$$\left\{ Q^c, Q^i, N^F, N^k, N, p^c, p^i, R, G, T^c \right\}.$$
1. *Households and firms* solve their problems (3).

2. *Competitive Search Conditions.* (3).

3. *Representative Agent Conditions*

4. *Equal Profit Condition:* \( p^i \psi_T(Q^i) = p^c \psi_T(Q^c) \).

5. *Market Clearing Conditions:*

   \[
   N = N^F + N^k = N,
   \]

   \[
   C = T^c \psi_T(Q^C) F(Z, K, N^F).
   \]

6. Value of the firms is 1.
Putting the model to work

Assume separable utility with constant Frisch elasticity and Cobb-Douglas technology. Allow shocks to preferences and to the investment shopping technology.

- **Preferences**

  \[ \tilde{u}(c, n, d, \theta) = \theta_c \frac{c^{1-\sigma}}{1 - \sigma} - \theta_n \frac{n^{1+\psi}}{1 + \psi} - \theta_d \ d \]

- **Production function**

  \[ F(Z, k, n^F) = Z \ k^{\gamma_k} \ (n^F)^{\gamma_n} \]

- **Shocks**

  \[ X \sim [\log(\theta_{c,t}), \log(\theta_{d,t}), \log(\zeta), \log(Z_t)] \]

  \[ X_t = \rho X_{t-1} + v_t, \quad v_t \sim N(0, \Sigma^2) \]

  and \( \log(\theta_{n,t}) \) follows an AR(2) process
A couple of asides

- The Equilibrium is Optimal (when triple indexing markets).
The (measured) Solow Residual

- Potential output is
  \[ F(K, N^F) = \bar{z} K^{\gamma_k} (N^F)^{\gamma_n} \]

- The measures Solow residual is obtained from
  \[ Y_t = \bar{Z}_t K_t^{1-\bar{\gamma}} N_t^{\bar{\gamma}} \]
  (where \( \bar{\gamma} = \frac{1}{1-\alpha} \gamma_n + \frac{\alpha \delta}{(1-\alpha)(1/\beta-1+\delta)} \gamma_k \): measured labor share).

- The expression for the measured Solow residual \( \bar{Z}_t \) includes
  \[
  \bar{Z} = A \bar{z} \left[ (D^c)^{\alpha} (T^c)^{1-\alpha} + \bar{p}^i (D^i)^{\alpha} (T^i)^{1-\alpha} \right] \\
  \text{Demand Effect 1.04} \\
  \left( \frac{N^F}{N} \right)^{\gamma_n} \cdot K^{\gamma_k - (1-\bar{\gamma})} N^{\gamma_n - \bar{\gamma}} \\
  \text{Effective Work -.00} \\
  \text{Share’s Error -.00}
  \]
Putting the model to work

Assume separable utility with constant Frisch elasticity and Cobb-Douglas technology. Allow shocks to preferences and to the investment shopping technology.

- Preferences

\[ \tilde{u}(c, n, d, \theta) = \theta_c \frac{c^{1-\sigma}}{1 - \sigma} - \theta_n \frac{n^{1+\psi}}{1 + \psi} - \theta_d \, d \]

- Production function

\[ F(Z, k, n^F) = Z \, k^{\gamma_k} \, (n^F)^{\gamma_n} \]

- Shocks

\[ X \sim [\log(\theta_{c,t}), \log(\theta_{d,t}), \log(\zeta), \log(Z_t)] \]

\[ X_t = \rho_X \, X_{t-1} + \nu_t, \quad \nu_t \sim N(0, \Sigma^2) \]

and \( \log(\theta_{n,t}) \) follows an AR(2) process.
Calibration

- There are 11 parameters.
  - Preferences: \( \{ \beta, \sigma, \chi, \psi \} \)
  - Production Technology: \( \{ \bar{z}, \gamma_k, \gamma_n, \delta \} \).
  - Matching technologies: \( \{ A, \alpha, \bar{\zeta} \} \).

- Some moments/specific choices are Standard.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of return</td>
<td>.04</td>
</tr>
<tr>
<td>Coefficient of Risk Aversion</td>
<td>2.</td>
</tr>
<tr>
<td>Frisch Elasticity of Labor</td>
<td>.7</td>
</tr>
<tr>
<td>Time spent working</td>
<td>.3</td>
</tr>
<tr>
<td>Labor Share</td>
<td>.67</td>
</tr>
<tr>
<td>Investment to Output Ratio</td>
<td>.20</td>
</tr>
<tr>
<td>Physical Capital to Output Ratio</td>
<td>2.75</td>
</tr>
</tbody>
</table>
Calibration II

- The other moments are specific to this economy

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Tobin’s average Q</td>
<td>1.21</td>
</tr>
<tr>
<td>Capacity in Consumption</td>
<td>.81</td>
</tr>
<tr>
<td>Industries</td>
<td></td>
</tr>
<tr>
<td>Capacity in Investment</td>
<td>.81</td>
</tr>
<tr>
<td>Industries</td>
<td></td>
</tr>
</tbody>
</table>

- This calibration uniquely specifies the model economy.

- Other implications are

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<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Wealth to Output Ratio</td>
<td>3.33</td>
</tr>
<tr>
<td>Relative price of consumption</td>
<td>1</td>
</tr>
<tr>
<td>and investment</td>
<td></td>
</tr>
<tr>
<td>Percentage of Cost of New Capital that is internal</td>
<td>9</td>
</tr>
<tr>
<td>Share of production workers</td>
<td>0.97</td>
</tr>
</tbody>
</table>
Quantitative analysis

- Use Bayesian methods to estimate processes for various shocks, targeting the measured Solow residual.

- Use model to address two questions:
  1. How does each shock work (on its own) in terms of business cycle behavior?
  2. Estimate full model with all shocks, including a technology shock. Which ones account for the main aggregate variables?
1. Univariate shock versions of economy

- Start with main business cycle moments: U.S. and Standard RBC Model

<table>
<thead>
<tr>
<th></th>
<th>U.S. Data</th>
<th></th>
<th></th>
<th>Standard RBC</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variance</td>
<td>Cor w Y</td>
<td>Autocor</td>
<td>Variance</td>
<td>Cor w Y</td>
<td>Autocor</td>
<td></td>
</tr>
<tr>
<td>$Z$</td>
<td>3.19</td>
<td>0.43</td>
<td>0.94</td>
<td>3.45</td>
<td>0.99</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>2.38</td>
<td>1.00</td>
<td>0.86</td>
<td>0.82</td>
<td>1.00</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>2.50</td>
<td>0.87</td>
<td>0.91</td>
<td>0.04</td>
<td>0.96</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>1.55</td>
<td>0.87</td>
<td>0.87</td>
<td>0.05</td>
<td>0.95</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>34.15</td>
<td>0.92</td>
<td>0.80</td>
<td>13.74</td>
<td>0.99</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>cor($C, I$)</td>
<td><strong>0.74</strong></td>
<td></td>
<td></td>
<td><strong>0.93</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Business Cycle Moments: Ec with Demand Shocks

<table>
<thead>
<tr>
<th></th>
<th>(a) Shop Disut $\theta_d$</th>
<th>(b) Labor Disut $\theta_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variance</td>
<td>Cor w Y</td>
</tr>
<tr>
<td>$Z$</td>
<td>3.44</td>
<td>1.00</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.37</td>
<td>1.00</td>
</tr>
<tr>
<td>$N$</td>
<td>0.07</td>
<td>-1.00</td>
</tr>
<tr>
<td>$C$</td>
<td>0.51</td>
<td>1.00</td>
</tr>
<tr>
<td>$I$</td>
<td>0.03</td>
<td>0.99</td>
</tr>
<tr>
<td>$\text{cor}(C, I)$</td>
<td>0.99</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(c) Firms’ Shopping Tech $\zeta$</th>
<th>(d) Technology Shock $z$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variance</td>
<td>Cor w Y</td>
</tr>
<tr>
<td>$Z$</td>
<td>2.08</td>
<td>0.73</td>
</tr>
<tr>
<td>$Y$</td>
<td>1.73</td>
<td>1.00</td>
</tr>
<tr>
<td>$N$</td>
<td>0.54</td>
<td>0.78</td>
</tr>
<tr>
<td>$C$</td>
<td>0.45</td>
<td>-0.53</td>
</tr>
<tr>
<td>$I$</td>
<td>69.22</td>
<td>0.96</td>
</tr>
<tr>
<td>$\text{cor}(C, I)$</td>
<td>-0.74</td>
<td></td>
</tr>
</tbody>
</table>

All variables except the Solow residual are HP-filtered.
Impulse response for a 1% technology shock

- Labor
- Consumption
- Investment
- Rate of Return

RBC model
Shopping model
Assessment of univariate shock models

1. The Standard RBC has little endogenous movements (recall the low Frisch elasticity) but the right comovements.

2. Demand shocks alone have bad implications for hours (it is like a positive wealth effect). It may require a combination of shocks to both consumption ($\theta_d$) and investment ($\zeta$) to generate the right comovements.

3. Other shocks to preferences by themselves ($\theta_n$, $\theta_c$ have also very bad properties to be responsible for the movements in TFP.

4. In particular, TFP shocks in the shopping model generates countercyclical hours.
2. Full (multivariate shock) shopping economy

1. Run a horse race between demand shocks and productivity shocks in the context of the shopping model using the Solow residual, output, hours, and consumption.

2. Estimate the processes for 4 shocks using Bayesian methods:
   1. Consumption demand shocks $\theta_d$
   2. Investment demand shocks $\zeta$
   3. Direct TFP shock $z$
   4. Shock to the MRS $\theta_n$

3. Focus on variance decomposition
### Priors and Posteriors for the Shock Parameters (Likelihood = 2244.29)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Density</th>
<th>Para(1)</th>
<th>Para(2)</th>
<th>Mean</th>
<th>90% Intv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_d$</td>
<td>Beta</td>
<td>0.915</td>
<td>0.05</td>
<td>0.956</td>
<td>[0.930, 0.981]</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>Inverse Gamma</td>
<td>0.059</td>
<td>0.50</td>
<td>0.074</td>
<td>[0.063, 0.084]</td>
</tr>
<tr>
<td>$\rho_\zeta$</td>
<td>Beta</td>
<td>0.928</td>
<td>0.05</td>
<td>0.932</td>
<td>[0.896, 0.969]</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
<td>Inverse Gamma</td>
<td>0.126</td>
<td>0.50</td>
<td>0.123</td>
<td>[0.106, 0.142]</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Beta</td>
<td>0.907</td>
<td>0.05</td>
<td>0.899</td>
<td>[0.805, 0.974]</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Inverse Gamma</td>
<td>0.002</td>
<td>0.50</td>
<td>0.002</td>
<td>[0.0004, 0.004]</td>
</tr>
<tr>
<td>$\rho_{n,1}$</td>
<td>Beta</td>
<td>0.580</td>
<td>0.05</td>
<td>0.830</td>
<td>[0.807, 0.850]</td>
</tr>
<tr>
<td>$\rho_{n,2}$</td>
<td>Beta</td>
<td>0.147</td>
<td>0.05</td>
<td>0.162</td>
<td>[0.139, 0.188]</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>Inverse Gamma</td>
<td>0.022</td>
<td>0.50</td>
<td>0.024</td>
<td>[0.022, 0.026]</td>
</tr>
<tr>
<td>Cor($\theta_d, \zeta$)</td>
<td>Normal</td>
<td>0.001</td>
<td>0.40</td>
<td>0.013</td>
<td>[-0.227, 0.192]</td>
</tr>
</tbody>
</table>
### Variance Decomposition (%)

<table>
<thead>
<tr>
<th></th>
<th>$\theta_d$</th>
<th>$\zeta$</th>
<th>$z$</th>
<th>$\theta_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>31.14</td>
<td>31.00</td>
<td>0.77</td>
<td>37.09</td>
</tr>
<tr>
<td>Solow</td>
<td>83.17</td>
<td>13.61</td>
<td>1.22</td>
<td>1.99</td>
</tr>
<tr>
<td>$N$</td>
<td>3.25</td>
<td>11.89</td>
<td>0.01</td>
<td>84.85</td>
</tr>
<tr>
<td>$C$</td>
<td>54.82</td>
<td>16.98</td>
<td>0.34</td>
<td>27.85</td>
</tr>
<tr>
<td>$I$</td>
<td>1.00</td>
<td>85.97</td>
<td>0.41</td>
<td>12.62</td>
</tr>
</tbody>
</table>

### Business Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>Variance</th>
<th>Cor w $Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>1.08</td>
<td>1.00</td>
</tr>
<tr>
<td>Solow</td>
<td>5.85</td>
<td>0.63</td>
</tr>
<tr>
<td>$N$</td>
<td>1.35</td>
<td>0.62</td>
</tr>
<tr>
<td>$C$</td>
<td>0.71</td>
<td>0.59</td>
</tr>
<tr>
<td>$I$</td>
<td>17.93</td>
<td>0.76</td>
</tr>
</tbody>
</table>

- Productivity Shocks play a minor role.
Where does identification come from?

- The search friction make firms a fixed factor in the economy. This reduces capital share in the production function and the high rate of return of savings when we invest too much. Same for the wage.

- Additional factor: Investment requires some labor input (for investment shopping). A higher TFP increases the marginal product of labor. Thus, the cost of investment increases slightly. This works in the same way as a negative investment shock.

- If sum of capital and labor share is set to e.g. 0.995, the impulse response to a technology shock mimics that of RCB economy.
A bonus: More predictions of the shopping economies

<table>
<thead>
<tr>
<th></th>
<th>Variance</th>
<th>Correlation with Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>$\theta_d, \zeta$</td>
</tr>
<tr>
<td></td>
<td>$z, \theta_n$</td>
<td></td>
</tr>
<tr>
<td>$p_i/p_c$</td>
<td>0.47</td>
<td>0.98</td>
</tr>
<tr>
<td>Stock Market (S&amp;P 500)</td>
<td>42.64</td>
<td>0.26</td>
</tr>
<tr>
<td>Capacity Utilization</td>
<td>10.02</td>
<td>0.68</td>
</tr>
<tr>
<td>Output</td>
<td>2.38</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.27</td>
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<tr>
<td></td>
<td></td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
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<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

- We have a theory of the relative price of investment.
- The value of locations moves some the price of equity.
- Capacity utilization is also endogenous.
We built a theory where demand shocks have real effects (move productivity procyclically) and prices are flexible.

It compares positively with the standard RBC model with TFP shocks.

It has (yet to be explored) implications for equity prices and for the relative price between cons. and inv. (non-technological).

It is very easy to use (dynare code will be on the web).

Next step is to go beyond simple preference shocks and generate demand fluctuations coming from financial frictions, terms of trade, monetary and fiscal policy, etc.
References


Digression Standard Lucas-tree model: Eq object is just $p(\theta)$

1. Market tightness: $Q(\theta) = \infty$ or $\Psi_T(\infty) = 1$.

2. Consumption $C(\theta) = \Psi_T[Q(\theta)] = 1$.

3. Dividends from trees: $R(\theta) = p(\theta) \Psi_T[Q(\theta)] = p(\theta)$,

$$v(\theta, s) = \max_{c,d,s'} U(c, d, \theta) + \beta E \{v(\theta', s')|\theta\} \quad \text{s.t.}$$

$$p(\theta) c + s' = s \left[1 + R(\theta)\right]$$

$$d = 0$$

Firms: $1 + R(\theta) = p(\theta) + E \left\{\frac{1 + R(\theta')}{1 + R(\theta')}\right\} = p(\theta) + 1$

- Equilibrium derives from FOC: $\frac{1}{p(\theta)} U_c(\theta) = \beta E \left\{\frac{1 + p(\theta')}{p(\theta')} U_c(\theta')\right\}$
### Calibration Targets, Implied Aggregates and (quarterly) Parameter values

<table>
<thead>
<tr>
<th>Targets</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Group: Parameters are set autonomously</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk aversion</td>
<td>2.</td>
<td>$\sigma$</td>
<td>2.</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>4. %</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Frisch elasticity</td>
<td>0.72</td>
<td>$\frac{1}{\psi}$</td>
<td>0.72</td>
</tr>
<tr>
<td><strong>Second Group: Standard Targets</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of time spent working</td>
<td>30%</td>
<td>$\chi$</td>
<td>16.81</td>
</tr>
<tr>
<td>Physical Capital to Output Ratio</td>
<td>2.75</td>
<td>$\delta$</td>
<td>0.07</td>
</tr>
<tr>
<td>Consumption Share of Output</td>
<td>0.80</td>
<td>$\gamma_k$</td>
<td>0.23</td>
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<tr>
<td>Labor Share of income</td>
<td>0.67</td>
<td>$\gamma_n$</td>
<td>0.59</td>
</tr>
<tr>
<td>Units of output</td>
<td>1</td>
<td>$z$</td>
<td>2.03</td>
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<tr>
<td><strong>Third Group: Targets specific to this economy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of production workers</td>
<td>97. %</td>
<td>$\zeta$</td>
<td>3.16</td>
</tr>
<tr>
<td>Capacity Utilization of Consumption Sector</td>
<td>0.81</td>
<td>$A$</td>
<td>0.97</td>
</tr>
<tr>
<td>Capacity Utilization of Investment Sector</td>
<td>0.81</td>
<td>$\alpha$</td>
<td>0.09</td>
</tr>
<tr>
<td><strong>Fourth Group: Aggregate Variables Implied</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% of GDP payable to Shoppers</td>
<td>2. %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% of Cost of New Capital that is internal</td>
<td>9%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wealth to output ratio</td>
<td>3.33</td>
<td>From hhold budget constraint</td>
<td></td>
</tr>
<tr>
<td>Price of investment relative to consumption</td>
<td>1.</td>
<td>Equal capacity utilizations</td>
<td></td>
</tr>
</tbody>
</table>

---

Bai, Ríos-Rull, and Storesletten

Demand Shocks as Productivity Shocks
A few more things: Starting with an example

- Let \( u(c) = \log c \) and \( \theta \) be i.i.d., with \( E \{ \theta_c \} = E \{ \theta_d \} = 1 \) and \( \theta_d / \theta_c \leq \alpha \). Then,

\[
D(\theta) = \alpha \frac{\theta_c}{\theta_d}
\]

\[
C(\theta) = (D(\theta))^\alpha = \left( \alpha \frac{\theta_c}{\theta_d} \right)^\alpha
\]

\[
p(\theta) = \left( \frac{1}{\beta} - 1 \right) \left( \frac{\theta_d}{\alpha} \right)^\alpha \theta_c^{1-\alpha}
\]

\[
R(\theta) = \left( \frac{1}{\beta} - 1 \right) \theta_c
\]

- Note: “TFP” can be defined as \( Y = TFP \cdot T = (\alpha \theta_c / \theta_d)^\alpha \), so TFP is driven by demand shocks.
Intuition: express stuff in units of consumption

- Price of the tree in terms of consumption units:
  \[
  \frac{1}{p(\theta)} = \frac{\beta}{1 - \beta} \frac{C(\theta)}{\theta_c} = \frac{\beta}{1 - \beta} \frac{1}{\theta_c u_c}.
  \]
  (Lucas model has the same price of the tree in terms of $c$)

- Dividends in terms of consumption units:
  \[
  \frac{R(\theta)}{p(\theta)} = C(\theta)
  \]
  (... as in the Lucas model)

- The interest rate (in terms of consumption) is
  \[
  1 + r(\theta) = \frac{\theta_d^\alpha \theta_c^{1-\alpha}}{\beta E \left\{ \theta_d^{\alpha} \theta_c^{1-\alpha} \right\}}
  \]
  \[
  \Rightarrow r(\theta) \text{ is increasing in } \theta_c, \text{ with elasticity } 1 - \alpha.
  \]
Comparison with the standard Lucas tree model

- Lucas model: Lucas tree model $\alpha \to 0 \Rightarrow Y = C, D = 0$, and
  \[
  p(\theta) = \left(\frac{1}{\beta} - 1\right) \theta_c \\
  R(\theta) = p(\theta) Y \\
  1 + r(\theta) = \frac{\theta_c}{\beta E \{((\theta'_c)\}} = \frac{\theta_c}{\beta}.
  \]

- Aggregate consumption is invariant to the demand shock (so the elasticity is zero)

- All the adjustment to $\theta_c$ takes place through the prices:
  - The elasticity of $1 + r$ and $p$ to $\theta_c$ is unity
  - In the shopping model, the elasticity is $1 - \alpha$
The exact manner in which we measure output does not really matter that much: Ways to measure output

1. Consumption Goods units. \( C_t + \frac{p_t^i}{p_t^c} l_t \).

2. Base year prices (Old NIPA) \( GDP_t = C_t \ p_t^c + l_t \ p_t^i \).

3. Chained-Indexed prices (New NIPA)

We use base year prices. It is just easier (in Dynare).
consumer price: \( c_t + \pi_t^* I_t / pc_t \)

steady-state security price: \( pc_s^* c_t + \pi_s^* i_t \)

Period
Real GDP
Equilibrium

- Decisions: \( c(\theta, s), d(\theta, s), s'(\theta, s); \) aggregates \( D(\theta) \) \( Q(\theta) = \frac{1}{D(\theta)}, \)
  \( C(\theta), p(\theta) \) and \( R(\theta) \) such that

1. Households solve their problem.
2. Representative Agent Condition:
   \[
   D(\theta) = d(\theta, 1) \\
   C(\theta) = c(\theta, 1) = D(\theta)^\alpha
   \]
3. Equilibrium in the asset market
   \[s'(\theta, 1) = 1\]
4. Equilibrium in the good markets (via competitive search)
   \[
p(\theta) = (1 - \alpha) \frac{\theta_c \ u_c(C(\theta))}{E \left( \frac{[1+R(\theta')]}{p(\theta')} \beta \left( \theta_c' u_{c'} - \theta_d' D'^\alpha \right) \mid \theta \right)} \\
   R(\theta) = p(\theta) \ D^\alpha.
   \]
Let’s reestimate the full system with the high Frisch’s (1.1)

- Estimate by CI ML 4 variables (detrended output, labor, Solow residual (TFP) and investment) and four uncorrelated shocks ($\theta_d$, $\theta_c$, $z$, $\zeta$).

**Table:** Variance Decomposition in percentages: 1960.Q1–2006.Q4

<table>
<thead>
<tr>
<th></th>
<th>$\theta_d$</th>
<th>$\theta_c$</th>
<th>$z$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>72.9</td>
<td>0.2</td>
<td>20.1</td>
<td>6.7</td>
</tr>
<tr>
<td>$N$</td>
<td>1.6</td>
<td>95.3</td>
<td>0.3</td>
<td>2.7</td>
</tr>
<tr>
<td>$Y$</td>
<td>4.5</td>
<td>75.0</td>
<td>4.1</td>
<td>16.4</td>
</tr>
<tr>
<td>$I$</td>
<td>0.0</td>
<td>14.9</td>
<td>6.6</td>
<td>78.4</td>
</tr>
</tbody>
</table>

- Same findings with a bit less extreme.

- This may be exaggerated by trends. In terms of HP filtered variance decomposition, shocks to TFP account for 32% of Solow, and $\theta_c$ plays a smaller role.
Imagine there is a public good $G$, with

$$u(c, d, G, \theta) = \theta_c \log c - \theta_d d + G$$

Payable with lump sum taxes in terms of consumption.

How should a benevolent government set $G$?

(in this simply environment there are no commitment problems).

The answer is that it depends.

1. If a recession is caused by low $\theta_c$ is like low aggregate demand. Then completely stabilize to make output constant.

2. If a recession is caused by high $\theta_d$ is like a supply shock, then reduce $G$ like you would do with a technology shock.