

Demand Shocks as Productivity Shocks

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Introduction

- A popular intuitive idea: “demand” matters for business cycles.
 - **Problem:** With decreasing returns to scale in labor counterfactual implications in equilibrium growth models.
 - This paper:
 - Builds a model where demand for goods plays a direct “productive” role:
 - ▶ Demand creates its own supply: demand shocks (today modeled as preference shocks) cause fluctuations in measured productivity.
 - Embeds the simple theory in a standard stochastic growth model.
 - Estimates the contribution of both demand shocks and “true” productivity shocks to aggregate fluctuations.
- Result:** There is essentially no role for productivity shocks.

The context

- In a standard business cycle model, the production function requires that either productivity or inputs change output (The only inputs are capital and labor).

$$Y = z F(K, N)$$

- So either productivity shocks (z) move or inputs (i.e. labor) move.
- Decreasing returns to scale require that labor productivity and wages drop if labor increases.
- This does not happen in the data, the residual z is strongly correlated with output. Hence there have to be TFP shocks.
- We have been looking for them for thirty years with limited success.

The logic

- We postulate that in order to transform produced goods into used goods, both consumers and investors must exert (search) efforts.
- Such efforts are not accounted for in NIPA.
- The economy cannot operate at full capacity.
- Operationally, this works as a search friction in the goods market. Increases in search effort imply increased measured productivity.
- We use competitive search (unique, efficient equilibrium).
- Preference shocks are a stand in for a variety of demand shocks (credit restrictions, animal spirits, terms of trade shocks).

Alternatives in the literature

- 1 Mismeasure inputs: variable capital intensity. (Greenwood, Hercowitz, and Huffman (1988), Basu (1996), and Licandro and Puch (2000), others). Wen (2004): preference shocks cause changes in the measured Solow residual. Labor hoarding (Burnside, Eichenbaum, and Rebelo (1993))/
- 2 Petrosky-Nadeau and Wasmer (2011) model costly search for goods in final goods that interacts with search in the labor market. Lagos (2006), Faig and Jerez (2005), and Alessandria (2005) emphasize the effects of search frictions in shaping TFP but not on business cycles.
- 3 Diamond (1982) and Guerrieri and Lorenzoni (2009) that due to a search friction, the difficulty of coordination of trade can give rise to and exacerbate aggregate fluctuations.
- 4 Demand affecting TFP Fagnart, Licandro, and Portier (1999), monopolistic firms with putty-clay technology are subject to idiosyncratic demand shocks, which causes fluctuations in capacity utilization. Floetotto and Jaimovich (2008) changes in markup rates due to the number of firms changing over the business cycle. Swanson (2006) multisector.

The plan

- 1 We describe the logic in the Lucas tree (output is productivity).
- 2 We move on to a growth model suitable for business cycle analysis.
- 3 We estimate preference shocks from measured Solow residuals.
 - ▶ Contrast estimated model to RBC model.
- 4 We estimate jointly demand shocks and technology shocks and determine the contribution of each.

A Lucas-tree version of the model.

- Continuum of trees, measure $T = 1$. Each yields one fruit per period.
- Search friction: If a shopper finds a tree, then trade at price p ; otherwise the fruit rots.
- So Consumption = Productivity and is endogenous.
- Competitive Search: Agents choose where to search.
- A “market” is characterized by a price and a “market tightness”
 - ① p : Price (numeraire: the value of the tree)
 - ② Q : Market tightness (average available fruits per shopper).

Matching Technology

- Output equal the measure of matches:

$$Y = D^\alpha T^{1-\alpha}$$

- D is the measure of shoppers. α is a parameter.
- Recall: market tightness is $Q \equiv \frac{T}{D}$
- Probability that a tree is randomly matched with a shopper (i.e., number of matches per tree):

$$\psi_T(Q) \equiv \frac{D^\alpha T^{1-\alpha}}{T} = \left(\frac{D}{T}\right)^\alpha = Q^{-\alpha} = \frac{Y}{T} = Y = C = D^\alpha,$$

- Output and productivity depend only on how many shoppers.

Preferences:

- Many identical, infinitely lived, households. Utility is

$$E \sum_t \beta^t U(c_t, d_t, \theta_t),$$

where c_t is fruit consumption, d_t is the measure of shopping units (a search disutility). θ is a Markovian preference shock

- Consumption is \neq shopping units (d) times the probability of a unit finding a fruit (Ψ_D):

$$c = d \cdot \Psi_D(Q) \equiv d Q^{1-\alpha}.$$

- Households own s shares of the trees.
- Aggregate state: θ . Individual state: (θ, s) .

The Values of agents: Households and Firms

$$\text{Hhold: } v(\theta, s) = \max_{c, d, s'} U(c, d, \theta) + \beta E \{v(\theta', s') | \theta\} \quad \text{s.t.}$$

$$p(\theta) c + s' = s [1 + R(\theta)]$$

$$c = d \cdot \Psi_D[Q(\theta)]$$

$$\text{Firms: } 1 + R(\theta) = \varsigma(\theta) + E \left\{ \frac{1 + R(\theta')}{1 + R(\theta')} \right\} = p(\theta) \Psi_T(Q) + 1$$

- Equilibrium objects are 2

① Price of consumption (in terms of units of tree): $p(\theta)$.

② Market tightness: $Q(\theta)$.

▶ Consumption: $C(\theta) = \Psi_T[Q(\theta)]$.

▶ Dividends from trees: $R(\theta) = p(\theta) \Psi_T[Q(\theta)]$.

▶ Consumption rate of return: $1 + r(\theta') = \frac{p(\theta) [1 + R(\theta')]}{p(\theta')}$

Equilibrium conditions to determine (p, Q)

- 1 Euler equation:

$$\frac{\partial U}{\partial c} + \frac{\partial U}{\partial d} \frac{\partial d}{\partial c} = u_c[C(\theta)] - \frac{\theta_d}{\Psi_D(Q)} = p(\theta) M(\theta),$$

where M is expected discounted marginal utility of saving,

$$M(\theta) = E \left\{ \frac{[1 + R(\theta')]}{p(\theta')} \beta \left(u_{c'} - \frac{\theta'_d}{\Psi_D(Q')} \right) \mid \theta \right\}$$

- 2 We need one more equilibrium condition to determine Q .

It comes from competitive search.

Competitive Search in the Market for Goods

- This is the mechanism that determines the additional equilibrium object, market tightness.
- Shoppers choose which market to search in. Those markets are differentiated by p and Q .
- Let $\zeta^* = p \Psi_T(Q)$ be the “outside value” for firms of going to the best market to sell their fruit. Shoppers take it as given.
- So shoppers can only open markets where trees get at least ζ^* :

$$\zeta^* \leq p \Psi_T(Q)$$

The choice of market by the shopper

- Let θ_d be the (sunk) marginal utility cost of an extra shopper. The rewards for the hold to send a shopper to a (p, Q) market is

$$\Phi = \max_{p, Q} \{-\theta_d + \Psi_d(Q) (u_c - p M)\} \quad \text{s.t.}$$

$$\varsigma^* \leq p \Psi_T(Q),$$

where again

$$M(\theta) = E \left\{ \frac{[1+R(\theta')]}{\rho(\theta')} \beta \left(u_{c'} - \frac{\theta'_d}{\Psi_D(Q')} \right) \middle| \theta \right\}.$$

The FOC is

$$0 = (1 - \alpha) Q^{-\alpha} u_c - M p Q^{-\alpha}$$

or

$$p = (1 - \alpha) \frac{u_c}{M}$$

Summary of Equilibrium which by the way is Pareto Optimum

- The two conditions that determine the two equilibrium objects $\{p, Q\}$.

① The Hhold Euler $U_c - \frac{U_d}{\Psi_D} = p M$ or

$$\left(\theta_c u_c[C(\theta)] - \frac{\theta_d}{\Psi_D(Q)} \right) = p(\theta) E \left\{ \frac{[1 + R(\theta')]}{p(\theta')} \beta \left(u_{c'}[C(\theta')] - \frac{\theta'_d}{\Psi_D(Q')} \right) \middle| \theta \right\}$$

② The Search Equilibrium Condition $(1 - \alpha) U_c = p M$ or

$$(1 - \alpha) u_c[C(\theta)] = p E \left\{ \frac{[1 + R(\theta')]}{p(\theta')} \beta \left(u_{c'}[C(\theta')] - \frac{\theta'_d}{\Psi_D(Q')} \right) \middle| \theta \right\}$$

Example

- Suppose preferences are given by

$$u(c, d, \theta_c) = \theta_c \log(c) - d,$$

where θ_c is i.i.d. with $E\{\theta_c\} = 1$

- Allocations of demand and consumption are

$$\begin{aligned} D(\theta_c) &= \alpha \theta_c \\ C(\theta_c) &= A \alpha^\alpha (\theta_c)^\alpha \end{aligned}$$

- The Solow residual is $Z(\theta_c) = C(\theta_c)$. Price and interest rate are

$$P(\theta) = \left(\frac{1}{\beta} - 1 \right) \frac{1}{A \alpha^\alpha} \theta_c^{1-\alpha}, \quad 1 + r(\theta) = \frac{\theta_c^{1-\alpha}}{\beta E\{(\theta'_c)^{1-\alpha}\}}.$$

So a demand shock ($\theta_c \uparrow$) implies a partial increase in price and interest rate, and partially increased demand and output

Putting the model to work: the Growth Version

- We put a growth model with capital investment and labor choice with the shopping structure that we have developed.
- Some important changes.
 - ① There is varying capacity or output potential that we denote F and that is the productive capacity.
 - ② Both households (when purchasing consumption goods) and firms (when purchasing investment goods) face search frictions.
 - ③ In this model capital and wealth are NOT the same. The locations have intrinsic value. Extensions will have creation of new locations as a form of investment.
 - ④ All this generates subtle calibration issues.

- Measure one of firms–locations with installed capital k (depreciates at rate δ). Goods can be used for consumption or investment and capacity is

$$F(Z, k, n) = Z k^{\gamma_k} n^{\gamma_n}$$

- New capital has to be purchased that requires shoppers n^k .
- Shoppers and sellers trade in decentralized markets at prices (in terms of shares of the economy's wealth) p^i if investment and p^c if consumption.
- Unmatched capacity rots.

Preferences are

$$E \left\{ \sum_t \beta^t U(c, n, d, \theta) \right\}, \quad \theta, \text{ Markovian}$$

Again consumption requires that it is shopped so

$$c = d \Psi_d(Q^c) F^c$$

- $\Psi_d(Q^c)$ is the probability of matching a consumption firm, Q^c is market tightness in the consumption good market and F^c is output capacity in a consumption location.
- Households own the firms.

A few lemmas alleviate notation

- 1 The state of the economy is the pair $\{\theta, Z, K\}$.
- 2 There is only one active market in consumption goods and another in investment goods.
- 3 Firms that produce consumption and investment choose the same inputs $F^i = F^c = F(Z, K, N^F)$. Use $n^F(k, Z, F)$ to denote the inverse fn.
- 4 Consumption and investment firms get the same expected revenue (but not necessarily the same price and market tightness).

Consumption (or invt) firms in a (p^c, F, Q^c) submarket

$$\Omega(\theta, Z, K, k) = \max_{n^k, k', i} \Psi_T(Q^c) F p^F - w(\theta, K) [n^F(k, Z, F) + n^k] \\ - p^i(\theta, Z, K) i + E \left\{ \frac{\Omega(\theta', Z', K', k')}{1 + R(\theta', Z', K')} \middle| \theta \right\}$$

$$\text{s.t.} \quad i = (n^k \zeta) \Psi_d[Q^i(\theta, Z, K)] F^i[\theta, Z, K] \\ k' = i + (1 - \delta)k \\ K' = G(\theta, Z, K)$$

with FOC (and RA condition)

$$E \left\{ \frac{\Omega_3(\theta', Z', K', k')}{1 + R(\theta', Z', K')} \middle| \theta \right\} = \frac{w(\theta, Z, K)}{\zeta \Psi_d[Q^i(\theta, K)] F^i(Z, K, N)} + p^i(\theta, Z, K).$$

The household problem

$$v(\theta, Z, K, s) = \max_{c, d, n, s'} U(c, d, n, \theta) + \beta E \{v(\theta', K', s') | \theta\} \quad \text{s.t.}$$

$$p^F(\theta, Z, K) c + s' = s [1 + R(\theta, Z, K)] + n w(\theta, Z, K)$$

$$c = d \Psi_d [Q^c(\theta, Z, K)] F[K, N^F(\theta, Z, K)]$$

$$K' = G(\theta, Z, K)$$

- Holds' FOC (and RA)

$$U_c - \frac{U_d}{\Psi_d F} = \beta E \left\{ \frac{p^F (1 + R')}{p^{c'}} \left[U'_c - \frac{U'_d}{\Psi_d F'} \right] | \theta \right\},$$

$$U_c - \frac{U_d}{\Psi_d F} = U_n \frac{p^F}{w}.$$

Competitive Search in Markets

- Markets are now indexed by quantity, price, and market tightness (the latter two will be different for consumption and investment).
- We get additional conditions from the FOC of shoppers given expected revenue for sellers.
- The equilibrium objects are functions of (θ, Z, K) for

$$\left\{ Q^c, Q^i, N^F, N^k, N, p^c, p^i, R, G, T^c \right\}.$$

Recursive Equilibrium

- 1 *Households and firms solve their problems (3).*
- 2 *Competitive Search Conditions. (3).*
- 3 *Representative Agent Conditions*
- 4 *Equal Profit Condition: $p^i \Psi_T(Q^i) = p^c \Psi_T(Q^c)$.*
- 5 *Market Clearing Conditions:*

$$\begin{aligned}N &= N^F + N^k = N, \\C &= T^c \Psi_T(Q^c) F(Z, K, N^F).\end{aligned}$$

- 6 *Value of the firms is 1.*

Putting the model to work

Assume separable utility with constant Frisch elasticity and Cobb-Douglas technology. Allow shocks to preferences and to the investment shopping technology.

- Preferences

$$\tilde{u}(c, n, d, \theta) = \theta_c \frac{c^{1-\sigma}}{1-\sigma} - \theta_n \chi \frac{n^{1+\psi}}{1+\psi} - \theta_d d$$

- Production function

$$F(Z, k, n^F) = Z k^{\gamma_k} (n^F)^{\gamma_n}$$

- Shocks

$$\begin{aligned} X &\sim [\log(\theta_{c,t}), \log(\theta_{d,t}), \log(\zeta), \log(Z_t)] \\ X_t &= \rho_X X_{t-1} + v_t, \quad v_t \sim N(0, \Sigma^2) \end{aligned}$$

and $\log(\theta_{n,t})$ follows an AR(2) process

A couple of asides

- The Equilibrium is Optimal (when triple indexing markets).

The (measured) Solow Residual

- Potential output is

$$F(K, N^F) = \bar{z} K^{\gamma_k} (N^F)^{\gamma_n}$$

- The measured Solow residual is obtained from

$$Y_t = \bar{Z}_t K_t^{1-\bar{\gamma}} N_t^{\bar{\gamma}}$$

(where $\bar{\gamma} = \frac{1}{1-\alpha} \gamma_n + \frac{\alpha\delta}{(1-\alpha)(1/\beta-1+\delta)} \gamma_k$: measured labor share).

- The expression for the measured Solow residual \bar{Z}_t includes

$$\bar{Z} = A\bar{z} \underbrace{[(D^c)^\alpha (T^c)^{1-\alpha} + \bar{p}^i (D^i)^\alpha (T^i)^{1-\alpha}]}_{\text{Demand Effect 1.04}}$$

Demand Effect 1.04

$$\underbrace{\left(\frac{N^F}{N} \right)^{\gamma_n}}_{\text{Effective Work -.00}}$$

Effective Work -.00

$$\underbrace{K^{\gamma_k - (1-\bar{\gamma})} N^{\gamma_n - \bar{\gamma}}}_{\text{Share's Error -.00}}$$

Share's Error -.00

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$$\tilde{u}(c, n, d, \theta) = \theta_c \frac{c^{1-\sigma}}{1-\sigma} - \theta_n \chi \frac{n^{1+\psi}}{1+\psi} - \theta_d d$$

- Production function

$$F(Z, k, n^F) = Z k^{\gamma_k} (n^F)^{\gamma_n}$$

- Shocks

$$\begin{aligned} X &\sim [\log(\theta_{c,t}), \log(\theta_{d,t}), \log(\zeta), \log(Z_t)] \\ X_t &= \rho_X X_{t-1} + v_t, \quad v_t \sim N(0, \Sigma^2) \end{aligned}$$

and $\log(\theta_{n,t})$ follows an AR(2) process

Calibration

- There are 11 parameters.
 - Preferences: $\{\beta, \sigma, \chi, \psi\}$
 - Production Technology: $\{\bar{z}, \gamma_k, \gamma_n, \delta\}$.
 - Matching technologies: $\{A, \alpha, \bar{\zeta}\}$.
- Some moments/specific choices are Standard.

Rate of return	.04
Coefficient of Risk Aversion	2.
Frisch Elasticity of Labor	.7
Time spent working	.3
Labor Share	.67
Investment to Output Ratio	.20
Physical Capital to Output Ratio	2.75

Calibration II

- The other moments are specific to this economy

Tobin's average Q	1.21
Capacity in Consumption Industries	.81
Capacity in Investment Industries	.81

- This calibration uniquely specifies the model economy.
- Other implications are

Wealth to Output Ratio	3.33
Relative price of consumption and investment	1
Percentage of Cost of New Capital that is internal	9
Share of production workers	0.97

Quantitative analysis

- Use Bayesian methods to estimate processes for various shocks, targeting the measured Solow residual.
- Use model to address two questions:
 - ① How does each shock work (on its own) in terms of business cycle behavior?
 - ② Estimate full model with all shocks, including a technology shock. Which ones account for the main aggregate variables?

1. Univariate shock versions of economy

- Start with main business cycle moments: U.S. and Standard RBC Model

	U.S. Data			Standard RBC		
	Variance	Cor w Y	Autocor	Variance	Cor w Y	Autocor
Z	3.19	0.43	0.94	3.45	0.99	0.95
Y	2.38	1.00	0.86	0.82	1.00	0.71
N	2.50	0.87	0.91	0.04	0.96	0.72
C	1.55	0.87	0.87	0.05	0.95	0.76
I	34.15	0.92	0.80	13.74	0.99	0.71
cor(C, I)	0.74			0.93		

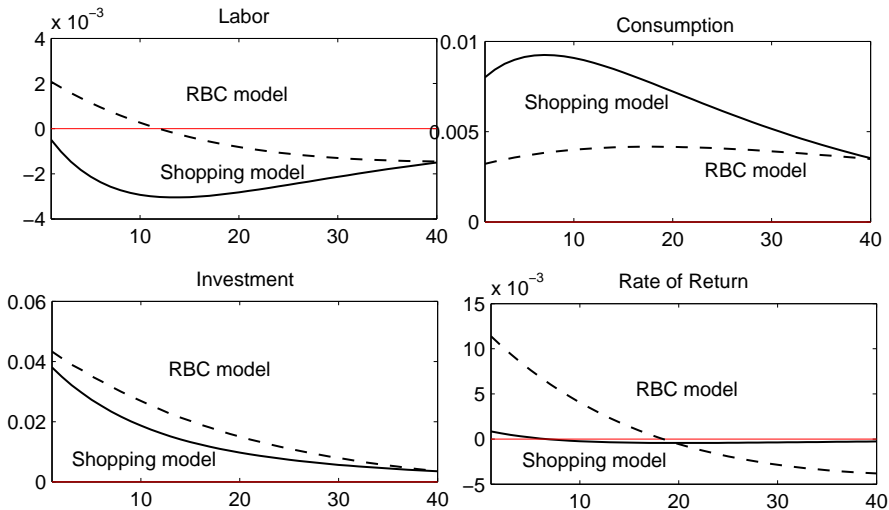
Business Cycle Moments: Ec with Demand Shocks

(a) Shop Disut θ_d				(b) Labor Disut θ_n		
	Variance	Cor w Y	Autocor	Variance	Cor w Y	Autocor
Z	3.44	1.00	0.94	3.02	-0.85	0.94
Y	0.37	1.00	0.71	35.49	1.00	0.58
N	0.07	-1.00	0.72	97.13	0.99	0.55
C	0.51	1.00	0.72	2.90	0.76	0.84
I	0.03	0.99	0.69	626.37	0.98	0.55
cor(C, I)	0.99			0.63		

(c) Firms' Shopping Tech ζ				(d) Technology Shock z		
	Variance	Cor w Y	Autocor	Variance	Cor w Y	Autocor
Z	2.08	0.73	0.91	3.45	0.99	0.95
Y	1.73	1.00	0.75	0.59	1.00	0.73
N	0.54	0.78	0.69	0.01	-0.52	0.96
C	0.45	-0.53	0.74	0.22	0.98	0.78
I	69.22	0.96	0.70	4.08	0.98	0.70
cor(C, I)	-0.74			0.92		

All variables except the Solow residual are HP-filtered.

Impulse response for a 1% technology shock



Assessment of univariate shock models

- 1 The Standard RBC has little endogenous movements (recall the low Frisch elasticity) but the right comovements.
- 2 Demand shocks alone have bad implications for hours (it is like a positive wealth effect). It may require a combination of shocks to both consumption (θ_d) and investment (ζ) to generate the right comovements.
- 3 Other shocks to preferences by themselves (θ_n, θ_c have also very bad properties to be responsible for the movements in TFP.
- 4 In particular, TFP shocks in the shopping model generates countercyclical hours.

2. Full (multivariate shock) shopping economy

- 1 Run a horse race between demand shocks and productivity shocks in the context of the shopping model using the Solow residual, output, hours, and consumption.
- 2 Estimate the processes for 4 shocks using Bayesian methods:
 - 1 Consumption demand shocks θ_d
 - 2 Investment demand shocks ζ
 - 3 Direct TFP shock z
 - 4 Shock to the MRS θ_n
- 3 Focus on variance decomposition

Full Estimation of the Shopping Model with all shocks

Priors and Posteriors for the Shock Parameters (Likelihood = 2244.29)

Parameter	Density	Para(1)	Para(2)	Mean	90% Intv.
ρ_d	Beta	0.915	0.05	0.956	[0.930, 0.981]
σ_d	Inverse Gamma	0.059	0.50	0.074	[0.063, 0.084]
ρ_ζ	Beta	0.928	0.05	0.932	[0.896, 0.969]
σ_ζ	Inverse Gamma	0.126	0.50	0.123	[0.106, 0.142]
ρ_z	Beta	0.907	0.05	0.899	[0.805, 0.974]
σ_z	Inverse Gamma	0.002	0.50	0.002	[0.0004, 0.004]
$\rho_{n,1}$	Beta	0.580	0.05	0.830	[0.807, 0.850]
$\rho_{n,2}$	Beta	0.147	0.05	0.162	[0.139, 0.188]
σ_n	Inverse Gamma	0.022	0.50	0.024	[0.022, 0.026]
$\text{Cor}(\theta_d, \zeta)$	Normal	0.001	0.40	0.013	[-0.227, 0.192]

Full Estimation of the Shopping Model with all shocks

	Variance Decomposition (%)				Business Cycle Statistics	
	θ_d	ζ	z	θ_n	Variance	Cor w Y
<i>Y</i>	31.14	31.00	0.77	37.09	1.08	1.00
Solow	83.17	13.61	1.22	1.99	5.85	0.63
<i>N</i>	3.25	11.89	0.01	84.85	1.35	0.62
<i>C</i>	54.82	16.98	0.34	27.85	0.71	0.59
<i>I</i>	1.00	85.97	0.41	12.62	17.93	0.76

- Productivity Shocks play a minor role.

Where does identification come from?

- The search friction make firms a fixed factor in the economy. This reduces capital share in the production function and the high rate of return of savings when we invest too much. Same for the wage.
- Additional factor: Investment requires some labor input (for investment shopping). A higher TFP increases the marginal product of labor. Thus, the cost of investment increases slightly. This works in the same way as a negative investment shock.
- If sum of capital and labor share is set to e.g. 0.995, the impulse response to a technology shock mimics that of RCB economy

A bonus: More predictions of the shopping economies

	Variance			Correlation with Y		
	Data	θ_d, ζ	θ_d, ζ z, θ_n	Data	θ_d, ζ	θ_d, ζ z, θ_n
p_i/p_c	0.47	0.98	2.61	-0.23	-1.00	-0.27
Stock Market (S&P 500)	42.64	0.26	1.97	0.41	0.26	0.36
Capacity Utilization	10.02	0.68	0.63	0.89	0.99	0.74
Output	2.38	0.84	1.08	1	1	1

- We have a theory of the relative price of investment.
- The value of locations moves some the price of equity.
- Capacity utilization is also endogenous.

Conclusions

- 1 We built a theory where demand shocks where
 - ▶ demand shocks have real effects (move productivity procyclically)
 - ▶ prices are flexible
- 2 It compares positively with the standard RBC model with TFP shocks.
- 3 It has (yet to be explored) implications for equity prices and for the relative price between cons. and inv. (non-technological).
- 4 It is very easy to use (dynare code will be on the web).
- 5 Next step is to go beyond simple preference shocks and generate demand fluctuations coming from financial frictions, terms of trade, monetary and fiscal policy, etc.

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Digression Standard Lucas-tree model: Eq object is just $p(\theta)$

- 1 Market tightness: $Q(\theta) = \infty$ or $\Psi_T(\infty) = 1$.
- 2 Consumption $C(\theta) = \Psi_T[Q(\theta)] = 1$.
- 3 Dividends from trees: $R(\theta) = p(\theta) \Psi_T[Q(\theta)] = p(\theta)$,

$$v(\theta, s) = \max_{c, d, s'} U(c, d, \theta) + \beta E \{ v(\theta', s') | \theta \} \quad \text{s.t.}$$

$$p(\theta) c + s' = s [1 + R(\theta)]$$

$$d = 0$$

$$\text{Firms:} \quad 1 + R(\theta) = p(\theta) + E \left\{ \frac{1 + R(\theta')}{1 + R(\theta')} \right\} = p(\theta) + 1$$

- Equilibrium derives from FOC: $\frac{1}{p(\theta)} U_c(\theta) = \beta E \left\{ \frac{1+p(\theta')}{p(\theta')} U_c(\theta') \right\}$

Calibration Targets, Implied Aggregates and (quarterly) Parameter values

Targets	Value	Parameter	Value
First Group: Parameters are set autonomously			
Risk aversion	2.	σ	2.
Real interest rate	4.%	β	0.99
Frisch elasticity	0.72	$\frac{1}{\psi}$	0.72
Second Group: Standard Targets			
Fraction of time spent working	30%	χ	16.81
Physical Capital to Output Ratio	2.75	δ	0.07
Consumption Share of Output	0.80	γ_k	0.23
Labor Share of income	0.67	γ_n	0.59
Units of output	1.	z	2.03
Third Group: Targets specific to this economy			
Share of production workers	97.%	ζ	3.16
Capacity Utilization of Consumption Sector	0.81	A	0.97
Capacity Utilization of Investment Sector	0.81	α	0.09
Fourth Group: Aggregate Variables Implied			
% of GDP payable to Shoppers	2.%	From hhold budget constraint Equal capacity utilizations	
% of Cost of New Capital that is internal	9%		
Wealth to output ratio	3.33		
Price of investment relative to consumption	1.		

A few more things: Starting with an example

- Let $u(c) = \log c$ and θ be i.i.d., with $E\{\theta_c\} = E\{\theta_d\} = 1$ and $\theta_d/\theta_c \leq \alpha$. Then,

$$D(\theta) = \alpha \frac{\theta_c}{\theta_d}$$

$$C(\theta) = (D(\theta))^\alpha = \left(\alpha \frac{\theta_c}{\theta_d}\right)^\alpha$$

$$p(\theta) = \left(\frac{1}{\beta} - 1\right) \left(\frac{\theta_d}{\alpha}\right)^\alpha \theta_c^{1-\alpha}$$

$$R(\theta) = \left(\frac{1}{\beta} - 1\right) \theta_c$$

- Note: “TFP” can be defined as $Y = TFP \cdot T = (\alpha \theta_c/\theta_d)^\alpha$, so TFP is driven by demand shocks

Intuition: express stuff in units of consumption

- Price of the tree in terms of consumption units:

$$\frac{1}{p(\theta)} = \frac{\beta}{1-\beta} \frac{C(\theta)}{\theta_c} = \frac{\beta}{1-\beta} \frac{1}{\theta_c u_c}.$$

(Lucas model has the same price of the tree in terms of c)

- Dividends in terms of consumption units:

$$\frac{R(\theta)}{p(\theta)} = C(\theta)$$

(... as in the Lucas model)

- The interest rate (in terms of consumption) is

$$1 + r(\theta) = \frac{\theta_d^\alpha \theta_c^{1-\alpha}}{\beta E \{ \theta_d'^\alpha \theta_c'^{1-\alpha} \}}$$

$\Rightarrow r(\theta)$ is increasing in θ_c , with elasticity $1 - \alpha$.

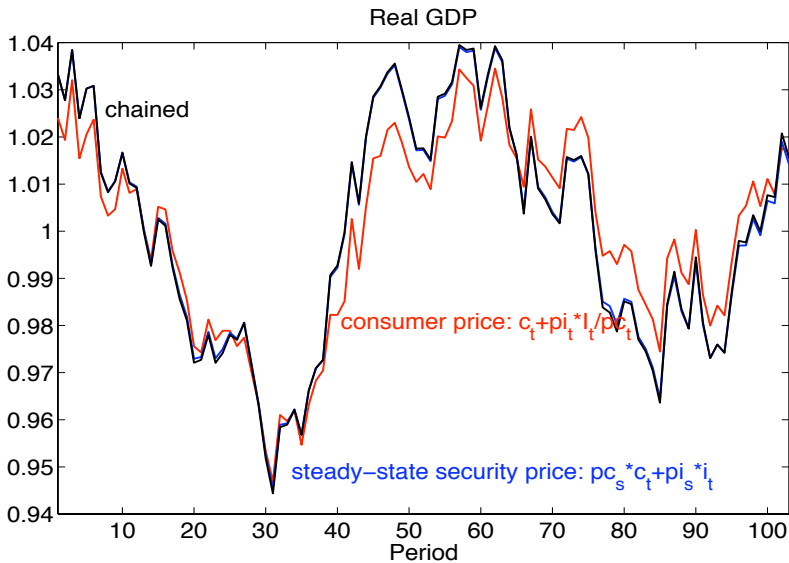
Comparison with the standard Lucas tree model

- Lucas model: Lucas tree model $\alpha \rightarrow 0 \Rightarrow Y = C, D = 0$, and

$$\begin{aligned}p(\theta) &= \left(\frac{1}{\beta} - 1\right) \theta_c \\R(\theta) &= p(\theta) Y \\1 + r(\theta) &= \frac{\theta_c}{\beta E\{(\theta'_c)\}} = \frac{\theta_c}{\beta}.\end{aligned}$$

- Aggregate consumption is invariant to the demand shock (so the elasticity is zero)
- All the adjustment to θ_c takes place through the prices:
 - The elasticity of $1 + r$ and p to θ_c is unity
 - In the shopping model, the elasticity is $1 - \alpha$

- The exact manner in which we measure output does not really matter that much: Ways to measure output
 - ① Consumption Goods units. $C_t + \frac{p_t^i}{p_t^c} I_t$.
 - ② Base year prices (Old NIPA) $GDP_t = C_t p_0^c + I_t p_0^i$.
 - ③ Chained-Indexed prices (New NIPA)
- We use base year prices. It is just easier (in Dynare).



Equilibrium

- Decisions: $c(\theta, s)$, $d(\theta, s)$, $s'(\theta, s)$; aggregates $D(\theta)$ $Q(\theta) = \frac{1}{D(\theta)}$, $C(\theta)$, $p(\theta)$ and $R(\theta)$ such that

- 1 Households solve their problem.
- 2 Representative Agent Condition:

$$D(\theta) = d(\theta, 1)$$

$$C(\theta) = c(\theta, 1) = D(\theta)^\alpha$$

- 3 Equilibrium in the asset market

$$s'(\theta, 1) = 1$$

- 4 Equilibrium in the good markets (via competitive search)

$$p(\theta) = (1 - \alpha) \frac{\theta_c u_c(C(\theta))}{E \left(\frac{[1+R(\theta')]}{p(\theta')} \beta (\theta'_c u_{c'} - \theta'_d D'^\alpha) \mid \theta \right)}$$

$$R(\theta) = p(\theta) D^\alpha.$$

Let's reestimate the full system with the high Frisch's (1.1)

- Estimate by CI ML 4 variables (detrended output, labor, Solow residual (TFP) and investment) and four uncorrelated shocks (θ_d , θ_c , z ζ).

Table: Variance Decomposition in percentages: 1960.Q1–2006.Q4

	θ_d	θ_c	z	ζ
TFP	72.9	0.2	20.1	6.7
N	1.6	95.3	0.3	2.7
Y	4.5	75.0	4.1	16.4
I	0.0	14.9	6.6	78.4

- Same findings with a bit less extreme.
- This may be exaggerated by trends. In terms of HP filtered variance decomposition, shocks to TFP account for 32% of Solow, and θ_c plays a smaller role.

Optimal Fiscal Policy

- Imagine there is a public good G , with

$$u(c, d, G, \theta) = \theta_c \log c - \theta_d d + G$$

- Payable with lump sum taxes in terms of consumption.

- How should a benevolent government set G ?

(in this simply environment there are no commitment problems).

- The answer is that it depends.

- 1 If a recession is caused by low θ_c is like low aggregate demand. Then completely stabilize to make output constant.
- 2 If a recession is caused by high θ_d is like a supply shock, then reduce G like you would do with a technology shock.