Sticky Wage Models with Labor Supply Constraint

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The issue: Labor Supply Constraint

- When wage is rigid, how to determine the quantity of labor hired?
  - Dreze (1975): minimum of labor demand n and labor supply ℓ

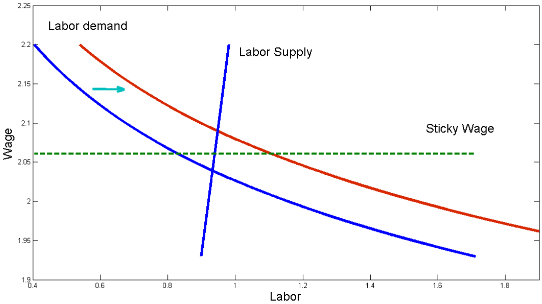
    labor supply constraint: $\widehat{n} = \min\{n, \ell\}$
  - New Keynesian models: labor demand determines labor market outcome.

- Can labor demand exceed labor supply?
  - Flexible wage: no, demand is smaller than supply due to wage mark-up

$$\log \ell = \log n + \underbrace{\gamma}_{\text{Frisch elas}} \underbrace{\log \mathcal{M}}_{\text{Mark-up}}$$

  - Sticky wage: yes, large demand shifts due to aggregate shocks.

Labor Demand and Labor Supply



Zhen: It is better to have a movie here

This Paper

1. Investigate whether labor supply constraint is violated popular NK models
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2. Solve a simple model: labor supply constraint is respected.
3. Revisit NK models: does labor supply constraint change the estimation?

Quantitative Findings

1. Investigate popular NK models: is labor supply constraint violated?
   -
2. Solve a simple model: labor supply constraint is respected.
3. Revisit NK models: does labor supply constraint change the estimation?

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   2. Persistence is much lower.
   3. Other things matter less.
   4. Estimates of wage rigidity are higher.

Sticky Wage Model

- Aggregate labor n across sectors

$$n = \left[\int n_i^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

- For each i, quantity of labor hired is determined by demand

$$n_i = \left(\frac{w_i}{w} \right)^{-\epsilon} n$$

- Given wage w_i, the labor supply ℓ_i satisfies

$$w_i = \frac{v'(\ell_i)}{u'(c)}$$

- Wages are reset with probability θ

Construct Voluntary Ex-post Employment

1. Construct cross-sectional wage distribution
   - measure of workers with wage rest τ periods before: $\mu_\tau = (1-\theta_w)\theta_w^\tau$
   - the newly set wage every period can be computed by

$$w_t = \left[\int w_{i,t}^{1-\epsilon_w} di\right]^{\frac{1}{1-\epsilon_w}} = \left[\theta_w (w_{t-1})^{1-\epsilon_w} + (1-\theta_w)(w_t^*)^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}$$

   Solve the model via log-linearization to obtain n_i, ℓ_i and construct

   employment with ex-post labor supply constraint

$$\widetilde{n} = \left[\int \widetilde{n}_i^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \equiv \left[\int (\min\{n_i, \ell_i\})^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$

2. How different are $\widetilde{n}$ and $\widetilde{n}_i$ compared with n and n_i?

3. $\widetilde{n}$ and $\widetilde{n}_i$ are NOT equilibrium allocations, but informative.

The questions

- How different is aggregate employment when the labor supply constraint $\widehat{n}_i = \min\{n_i, \ell_i\}$ is imposed on firms, households and unions ex-ante?
- **Employment with ex-ante labor supply constraint**

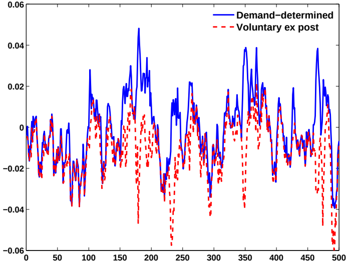
$$\widehat{n} = \left[\int \widehat{n}_i^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \equiv \left[\int \left(\min\{n_i, \ell_i\}\right)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$

- $\widehat{n}_i$ and $\widehat{n}$ are equilibrium allocations.
- Compare n, $\widetilde{n}$ and $\widehat{n}$.

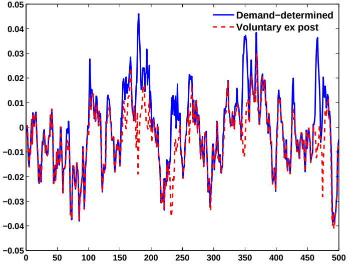
Smets and Wouters (2007)

- Exclude wage mark-up shock
  - The volatility of the wage mark-up shock is too large (CKM, 2010).
  - The implied wage mark-up can be negative, which does not make economic sense.
  - We simulate the model economy without the wage mark-up shock.
- Key parameter: average wage mark-up.
  - We re-estimate the model using wage mark-up from 6% to 18%.
  - With smaller wage mark-up, labor demand is more likely to exceed labor supply.
  - Wage mark-up is 5% in CEE (2005), 11% in CKM (2002).

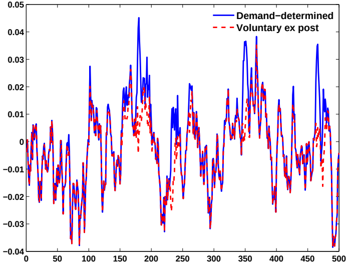
Figure: Demand-Determined and Voluntary Ex Post Labor in the SW (2007) Model



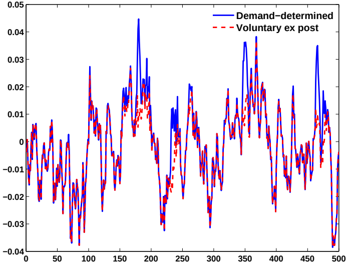
6% Wage Markup



11% Wage Markup



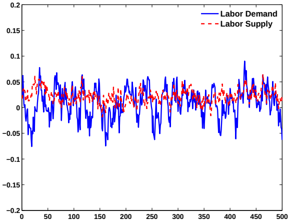
15% Wage Markup



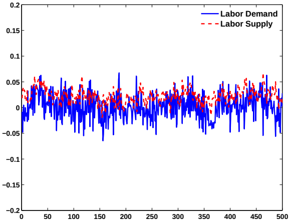
18% Wage Markup

Table: Demand-Determined and Voluntary ex post Aggregate Labor in the Smets and Wouters (2007) Model

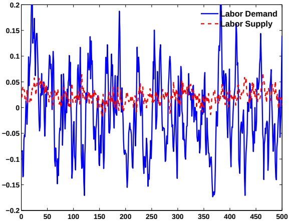
Wage markup: 6%	Mean	Binding Freq.	Var	Corr(N,Y)
Demand-determined labor	—	27.04	1.02	0.80
Voluntary ex post labor	-1.04	—	0.89	0.34
Wage markup: 11%	Mean	Binding Freq.	Var	Corr(N,Y)
Demand-determined labor	—	10.67	0.98	0.79
Voluntary ex post labor	-0.42	—	0.67	0.58
Wage markup: 15%	Mean	Binding Freq.	Var	Corr(N,Y)
Demand-determined labor	—	5.73	0.97	0.79
Voluntary ex post labor	-0.23	—	0.71	0.68
Wage markup: 18%	Mean	Binding Freq.	Var	Corr(N,Y)
Demand-determined labor	—	3.85	0.96	0.79
Voluntary ex post labor	-0.15	—	0.77	0.72



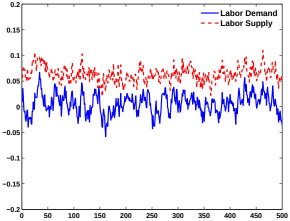
6% Wage Markup: Cohort 1



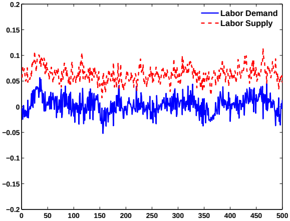
6% Wage Markup: Cohort 2



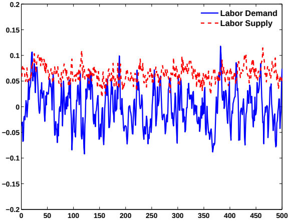
6% Wage Markup: Cohort 4



18% Wage Markup: Cohort 1



18% Wage Markup: Cohort 2



18% Wage Markup: Cohort 4

Findings

- The labor supply constraint is indeed violated.
  - Frequency: the probability of violation ranges from 3.8% to 27.3%.
  - Degree: the reduction in employment ranges from 0.15% to 1.04%.
- The difference in aggregate employment decreases with wage mark-up.
  - With ex-post labor supply constraint, the correlation between output and employment is monotonically increasing in wage mark-up.
  - The employment volatility declines by up to 30%; however, not monotonically in the wage mark-up.
  - The employment with ex-post labor supply constraint is less pro-cyclical or even counter-cyclical.

Gali, Smets and Wouters (2011)

- Better interpretation of min$\{n_i, \ell_i\}$. ℓ_i is labor force. Detail
- Wage mark-up shock and labor supply shock can be identified.
- The average wage mark-up can be determined by

$$\frac{\epsilon}{\epsilon - 1} \approx \gamma u^n$$

- In Gali, Smets and Wouters (2011), $\gamma = 3$ and wage mark-up is 18%.
- With more reasonable Frisch elasticity $\gamma = 1.5$, wage mark-up is 9%.

Figure: Demand-Determined and Voluntary Ex Post Labor in the GSW (2011) Model

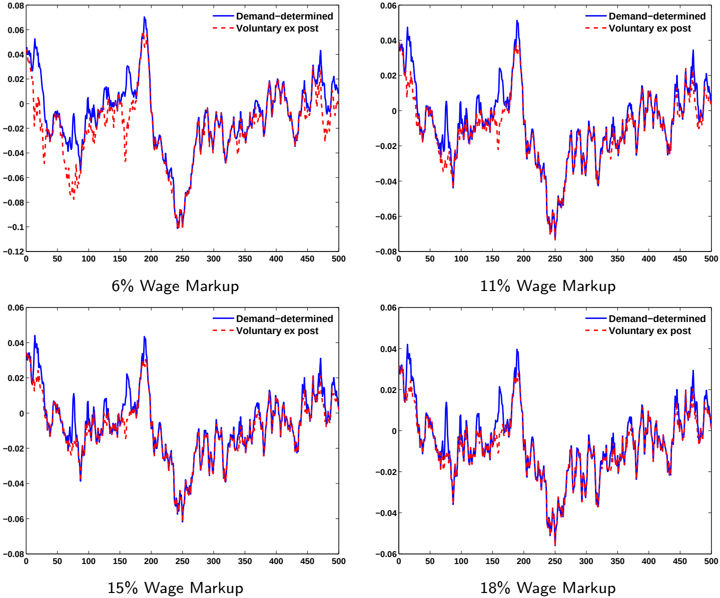


Table: Demand determined and Voluntary ex post Employment in the Gali, Smets and Wouters (2011) Model

Wage markup: 6%	Mean	Binding Freq	Var	Corr(N,Y)
Demand Determined Employment	—	9.68	0.86	0.84
Voluntary ex post Employment	-0.49	—	0.93	0.68
Wage markup: 11%	**Mean**	**Binding Freq**	**Var**	**Corr(N,Y)**
Demand Determined Employment	—	5.05	0.61	0.79
Voluntary ex post Employment	-0.21	—	0.48	0.71
Wage markup: 15%	**Mean**	**Binding Freq**	**Var**	**Corr(N,Y)**
Demand Determined Employment	—	4.20	0.54	0.76
Voluntary ex post Employment	-0.15	—	0.41	0.70
Wage markup: 18%	**Mean**	**Binding Freq**	**Var**	**Corr(N,Y)**
Demand Determined Employment	—	3.86	0.51	0.75
Voluntary ex post Employment	-0.13	—	0.39	0.70

Findings

- The labor supply constraint is indeed violated.
  - Frequency: the probability of violation ranges from 3.8% to 28.1%.
  - Degree: the average employment reduces from 0.13% to 0.77%.
- The difference in aggregate employment decreases with wage mark-up.
  - With ex-post labor supply constraint, the correlation between output and employment is monotonically increasing in wage mark-up.
  - The employment volatility declines by up to 30%; however, not monotonically in the wage mark-up.
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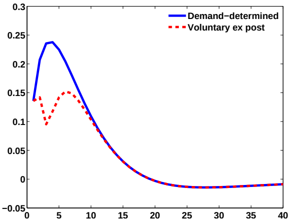
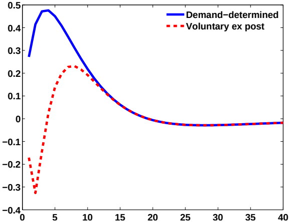
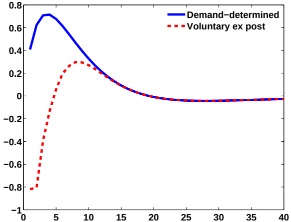
Monetary policy shock: Smets and Wouters (2007) Table



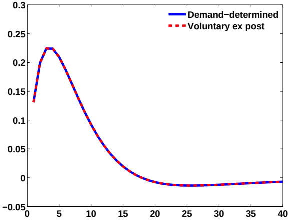
6% Wage Markup: 1 st.d shock



6% Wage Markup: 2 st.d shock



6% Wage Markup: 3 st.d shock



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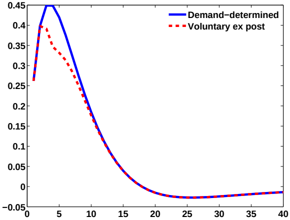

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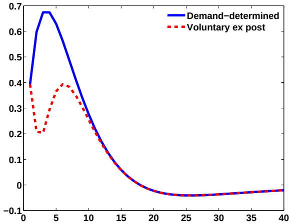

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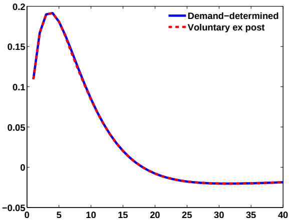
Monetary policy shock: Gali, Smets and Wouters (2011) Table



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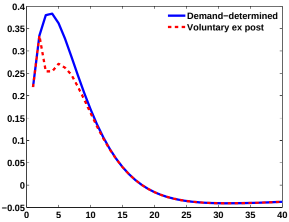

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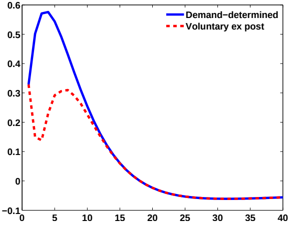

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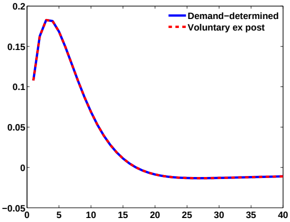

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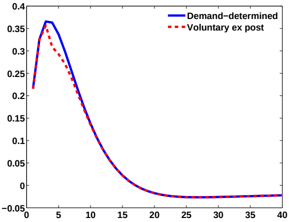

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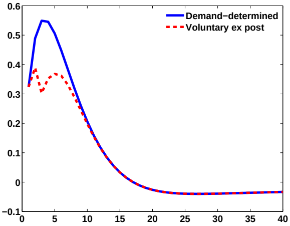


11% Wage Markup: 3 st.d shock

Findings

With ex-post labor supply constraint

- Employment increases less or even decreases after an expansionary monetary policy shock.
- The response of employment is non-linear in the size of the shock.
- The effect of an expansionary monetary policy shock on employment increases with wage mark-up.

A model with ex-ante labor supply constraint

- Households consist of one consumer (who saves and consumes) and a continuum of workers.
- Firms, households, unions take the labor supply constraint into account ex-ante.
- Nominal wage is fixed for four periods (one year).
- No adjustment costs, no price stickiness in order to simplify the computation.
- We consider two kinds of shocks: TFP shock and monetary policy shock.

Firm

- Firm's problem:

$$\max_{k_t, n_t, n_{i,t}} z_t k_t^{\alpha} n_t^{1-\alpha} - r_t^k k_t - \int w_{i,t} n_{i,t} di$$

subject to

$$n_t = \left[\int n_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$n_{i,t} \leq \Phi(w_{i,t}) = \left(\frac{u'(c_t) w_{i,t}}{\phi} \right)^{\frac{1}{\gamma}}$$

More

Labor union

- The union chooses the nominal wage w for T^w periods:

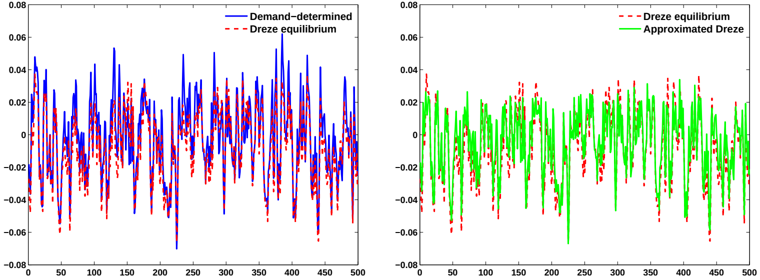
$$\max_w E_t \sum_{k=0}^{T^w-1} \beta^k \left\{ u'(c_{t+k}) \frac{w}{p_{t+k}} n_{i,t+k} - \frac{n_{i,t+k}^{1+\gamma}}{1+\gamma} \right\}$$

subject to:

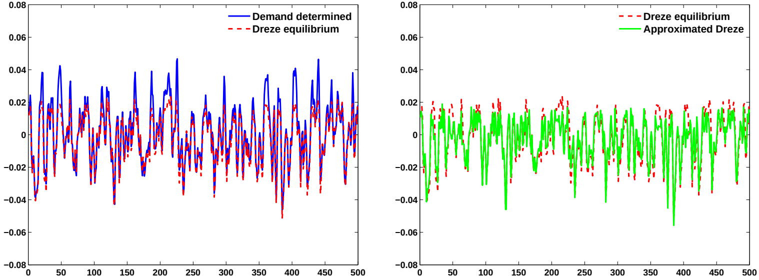
$$n_{i,t+k} = \min \left\{ \left(\frac{u'(c_{t+k})w}{\phi p_{t+k}} \right)^{\frac{1}{\gamma}}, \ \Psi_{t+k} \left(\frac{w}{p_{t+k}} \right) \right\}$$

Detail

TFP Shock


Demand−determined
Dreze equilibrium
Approximated Dreze
0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
−0.08
0
50
100
150
200
250
300
350
400
450
500

Monetary policy shock



Compare Dreze Equilibrium with other Solution Methods: Employment

Table: Properties of Employment in the Simple Model for Various Solutions

TFP Shock				
Solution Method	Mean	Var	Corr(N,Y)	Prob of binding
Dreze equilibrium (global solution)	-0.10	2.67	0.93	—
Approximation to Dreze equilibrium	-0.29	2.53	0.94	—
Voluntary ex post employment	-0.33	2.60	0.88	—
Demand-determined employment	—	3.79	0.96	0.11
Monetary Policy Shock				
Solution Method	Mean	Var	Corr(N,Y)	Prob of binding
Dreze Equilibrium (global solution)	-0.52	1.60	0.94	—
Approximation to Dreze equilibrium	-0.41	1.29	0.99	—
Voluntary ex post employment	-0.45	1.38	0.77	—
Demand-determined employment	—	2.26	1.00	0.12

Compare Dreze Equilibrium with other Solution Methods: Business Cycles

Table: Business Cycle Statistics for the Simple Model with Various Solutions

	TFP Shock			Monetary Policy Shock		
	Dreze Equilibrium	Approximated Dreze Equil.	Demand Determined	Dreze Equilibrium	Approximated Dreze Equil.	Demand Determined
		Variance			*Variance*	
Output	3.12	2.95	3.90	0.65	0.52	0.92
Employment	2.67	2.53	3.79	1.60	1.29	2.26
Consumption	0.17	0.17	0.19	0.01	0.01	0.02
Investment	42.82	38.10	52.11	10.12	7.96	13.95
		Correlation with output			*Correlation with output*	
Employment	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.61	0.57	0.62	0.61	0.57	0.62
Investment	1.00	0.99	1.00	1.00	0.99	1.00

1. Demand-determined solution is much more volatile than Dreze equilibrium.
2. Approximated Dreze provides a good approximation to Dreze equilibrium

Application: Re-estimate CEE (2005) and ACEL(2011) Model

- A medium-scale DSGE model with three shocks
  - neutral technology, investment technology, and monetary policy shocks
- Estimation method: generalized GMM
  - matching the impulse response functions recovered from SVAR model
- Re-estimate the model with **approximated Dreze equilibrium**
  - both estimated parameters and the quantitative properties change

Estimation Results

Table: Subset of Estimated Parameter Values with 5% Wage Markup Using 1 St.d IRF

	Demand determined	Approximated Dreze
Std of neutral technology shock, $\sigma_{\mu z}$	0.068 (0.046)	0.140 (0.089)
Autocor neutral technology shock, $\rho_{\mu z}$	0.902 (0.102)	0.697 (0.240)
Price rigidity, γ	0.040 (0.029)	0.054 (0.039)
Variable capital utilization, σ_a	1.995 (2.222)	4.564 (7.070)
Investment adjustment cost, S''	3.281 (2.038)	4.752 (2.378)
Interest semi-elasticity of money demand, ϵ	0.808 (0.208)	0.779 (0.193)
Habit formation, b	0.706 (0.045)	0.698 (0.058)
Effects of neutral technology shock on policy, ρ_{xz}	0.343 (0.266)	0.195 (0.480)
Scaling factor of neutral technology shock, c_z	2.997 (2.310)	1.027 (0.749)
Scaling factor of neutral technology shock, c_z^p	1.327 (1.381)	0.665 (0.650)

Comparison between Demand-Determined and Approximated Dreze

Table: Effects on Labor of the Shocks of Each Set of Parameters over Each Solution Concept

	Estimated with Demand Determined			Estimated with Approximated Dreze: 1 std shock		
Neutral technology shock	Mean	Var	Corr(N,Y)	Mean	Var	Corr(N,Y)
Demand determined	—	**0.18**	**0.87**	—	0.24	0.97
Approximated Dreze	-1.57	1.16	0.96	**-2.59**	**1.41**	**0.95**
Investment technology shock	Mean	Var	Corr(N,Y)	Mean	Var	Corr(N,Y)
Demand determined	–	**0.67**	**0.99**	–	0.52	0.99
Approximated Dreze	-0.42	0.34	0.98	**-0.55**	**0.32**	**0.99**
Monetary shock	Mean	Var	Corr(N,Y)	Mean	Var	Corr(N,Y)
Demand determined	—	**0.46**	**1.00**	—	0.33	1.00
Approximated Dreze	-0.07	0.33	0.99	**-0.01**	**0.30**	**1.00**
All shocks	Mean	Var	Corr(N,Y)	Mean	Var	Corr(N,Y)
Demand determined	—	**1.38**	**0.96**	—	1.15	0.95
Approximated Dreze	-2.28	2.06	0.98	**-3.41**	**1.99**	**0.96**

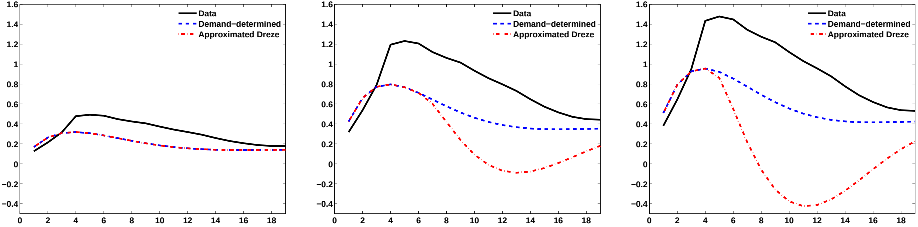
Findings

1. The neutral technology shock is both much more volatile and less persistent
   - more persistent shocks make labor supply constraint be violated more often
2. There is more wage and price rigidity.
   - labor demand become more responsive to overcome labor supply constraint
3. Contributions of shocks change in the approximated Dreze estimation
   - contribution of neutral technology shock becomes much larger
   - contribution of the other shocks is smaller
4. Variance of employment is much larger in approximated Dreze equilibrium.
   - larger movement is needed to overcome the labor supply constraint in the IRF

Robustness

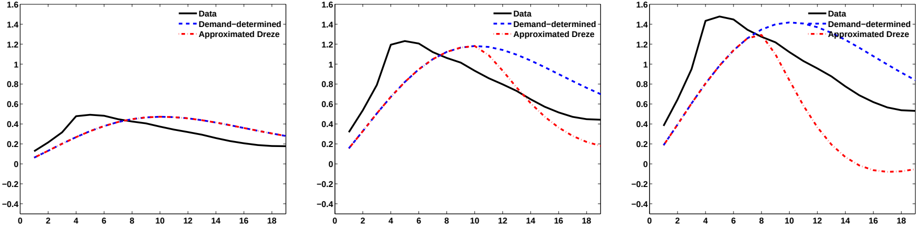
- Our results are robust under the following variations
- Various wage markup
  - CEE(2005) and ACEL(2011) use 5% wage markup
  - We also experiment with larger wage markups
- Various sizes of shocks in the IRF
  - With approximated Dreze solution, the model is nonlinear
  - We experiment with 1 to 3 std shocks in IRF

Figure: IRF of Labor to Neutral Tech. Shock: Estimation with Demand-Determined Model, 9% Markup



1 std shock 2.5 std shock 3 std shock

Figure: IRF of Labor to Neutral Tech. Shock: Estimation with Approximated Dreze Equilibrium, 9% Markup



1 std shock 2.5 std shock 3 std shock

Conclusion

- In popular New Keynesian models with ex-post labor supply constraint
  - Employment becomes less pro-cyclical or even counter-cyclical.
  - Employment increases less or even decreases after an expansionary monetary policy shock.
- In a simple model with ex-ante labor supply constraint
  - Employment becomes less pro-cyclical with smaller volatility.
  - The monetary policy is less effective.
  - Employment behaves similar with ex-post and ex-ante labor supply.

References

Gali, Smets and Wouters (2011)

- Each household has a continuum of workers represented by (i, j).
  - i labor sector
  - j disutility from labor
- Preference

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \phi \int_i \int_0^{n_{i,t}} j^\gamma \, dj \, di \right)$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \phi \int_i \frac{n_{i,t}^{1+\gamma}}{1+\gamma} di \right)$$

Gali, Smets and Wouters (2011)

- Workers only choose whether or not to work. They choose to work if:

$$w_i u'(c) > j^\gamma$$

- The labor force l_i solves:

$$w_i = \frac{(n_i)^\gamma}{u'(c)} = \frac{v'(l_i)}{u'(c)}$$

- The labor demand n_i^d:

$$n_i = \left(\frac{w_i}{w}\right)^{-\epsilon} n$$

Return

Firm

- Solution to firm's problem:

$$r_t^k = \alpha z_t k_t^{\alpha-1} n_t^{1-\alpha}$$

$$n_{i,t} = \min\left\{ \left[\frac{w_{i,t}}{(1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha}} \right]^{-\epsilon} n_t, \Phi(w_{i,t}) \right\}$$

$$n_t = \left[\int n_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

- $\Psi_t(w_{i,t})$ to denote the desired labor demand:

$$\Psi_t(w_{i,t}) = \left[\frac{w_{i,t}}{(1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha}} \right]^{-\epsilon} n_t$$

Return

Household

- Preference

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \phi \int_i \int_0^{n_{i,t}} j^\gamma dj di \right)$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \phi \int_i \frac{n_{i,t}^{1+\gamma}}{1+\gamma} di \right)$$

- Budget constraint

$$p_t[c_t + k_{t+1} - (1-\delta)k_t] + \frac{1}{R_t} b_{t+1} = r_t^k k_t + \int_i w_{i,t} n_{i,t} di + b_t + \Pi_t$$

Return

Monetary policy

Taylor type monetary policy rule

$$\log R_t = \log \frac{1}{\beta} + \phi_\pi \pi_t + \phi_y \log \frac{y_t}{y^*} + \eta_t$$

The disturbance term η_t follows an AR(1) process

$$\eta_t = \rho_m \eta_{t-1} + \zeta_t^m, \quad \zeta_t^m \sim \mathsf{N}(0, \sigma_m)$$

Aggregate labor

- Firm side

$$n_{i,t} = \min\left\{ \left[\frac{w_{i,t}}{(1-\alpha) z_t k_t^\alpha n_t^{-\alpha}} \right]^{-\epsilon} n_t, \Phi(w_{i,t}) \right\}$$

- Union side

$$n_{i,t} = \min\left\{ \left(\frac{u'(c_t) w_{i,t}}{\phi p_t} \right)^{\frac{1}{\gamma}}, \Psi_t(w_{i,t}) \right\}$$

- In equilibrium

$$n_{i,t} = \min\left\{ \left[\frac{w_{i,t}}{(1-\alpha) z_t k_t^\alpha n_t^{-\alpha}} \right]^{-\epsilon} n_t, \left(\frac{u'(c_t) w_{i,t}}{\phi p_t} \right)^{\frac{1}{\gamma}} \right\}$$

$$n_t = \left[\int \left(\min\{n_{i,t}, l_{i,t}\} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

Parameter values

Return

- Utility function is $E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \phi \int_i \frac{n_{i,t}^{1+\gamma}}{1+\gamma} di \right)$
- Production function is $F(k_t, n_t) = z_t k_t^{\alpha} n_t^{1-\alpha}$

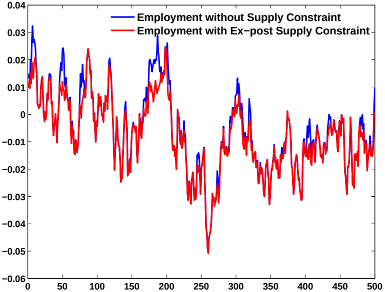
$$\log z_t = \rho_z \log z_{t-1} + \zeta_t^z \quad \zeta_t^m \sim \mathrm{N}(0, \sigma_z)$$

- Monetary policy rule $\log R_t = \log \frac{1}{\beta} + \phi_\pi \pi_t + \phi_y \log \frac{y_t}{y^*} + \eta_t$

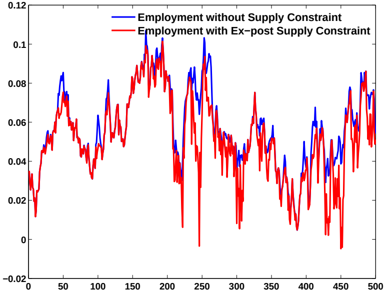
$$\eta_t = \rho_m \eta_{t-1} + \zeta_t^m, \quad \zeta_t^m \sim \mathrm{N}(0, \sigma_m)$$

β	γ	ϕ_π	ϕ_y	α	δ	ϵ_w	ρ_z	σ_z	ρ_m	σ_m
0.99	1.5	1.5	0.0	0.36	0.02	11.0	0.95	0.006	0.50	0.004

GSW(2011) with wage mark-up shock



St.d of Wage Markup: 0.0



St.d of Wage Markup: 0.015

GSW(2011) with wage mark-up shock

Table: Gali, Smets and Wouters (2011) Model with Ex-post Labor Supply Constraint

St.d of Wage Markup Shock: 0.000	Mean	Var	Corr(N,Y)	Binding Prob
Employment w/o constraint	—	0.51	0.75	3.79
Employment w/ ex-post labor supply constraint	-0.13	0.39	0.70	—
St.d of Wage Markup Shock: 0.005	**Mean**	**Var**	**Corr(N,Y)**	**Binding Prob**
Employment w/o constraint	—	0.51	0.75	3.90
Employment w/ ex-post labor supply constraint	-0.13	0.39	0.70	—
St.d of Wage Markup Shock: 0.010	**Mean**	**Var**	**Corr(N,Y)**	**Binding Prob**
Employment w/o constraint	—	0.51	0.75	4.94
Employment w/ ex-post labor supply constraint	-0.14	0.41	0.69	—
St.d of Wage Markup Shock: 0.015	**Mean**	**Var**	**Corr(N,Y)**	**Binding Prob**
Employment w/o constraint	—	0.51	0.75	7.21
Employment w/ ex-post labor supply constraint	-0.20	0.51	0.65	—

Monetary policy shock: Smets and Wouters (2007) ◂ Return

Table: Smets and Wouters (2007) Model with Different Wage Markups

Wage Markup: 6%	Mean	Var	Corr(N,Y)	Binding Prob
Employment w/o constraint	—	0.12	1.00	12.31
Employment w/ ex-post labor supply constraint	-0.32	0.10	0.31	—
Wage Markup: 11%	**Mean**	**Var**	**Corr(N,Y)**	**Binding Prob**
Employment w/o constraint	—	0.11	1.00	3.75
Employment w/ ex-post labor supply constraint	-0.09	0.07	0.87	—
Wage Markup: 15%	**Mean**	**Var**	**Corr(N,Y)**	**Binding Prob**
Employment w/o constraint	—	0.11	1.00	1.56
Employment w/ ex-post labor supply constraint	-0.03	0.09	0.97	—
Wage Markup: 18%	**Mean**	**Var**	**Corr(N,Y)**	**Binding Prob**
Employment w/o constraint	—	0.11	1.00	0.77
Employment w/ ex-post labor supply constraint	-0.02	0.09	0.99	—

Monetary policy shock: Gali, Smets and Wouters (2011) ◂ Return

Table: Gali, Smets and Wouters (2011) Model with Different Wage Markups

Wage Markup: 6%	Mean	Var	Corr(N,Y)	Binding Prob
Employment w/o constraint	—	0.08	1.00	16.25
Employment w/ ex-post labor supply constraint	-0.31	0.08	0.33	—
Wage Markup: 11%	**Mean**	**Var**	**Corr(N,Y)**	**Binding Prob**
Employment w/o constraint	—	0.07	1.00	4.38
Employment w/ ex-post labor supply constraint	-0.11	0.04	0.79	—
Wage Markup: 15%	**Mean**	**Var**	**Corr(N,Y)**	**Binding Prob**
Employment w/o constraint	—	0.07	1.00	2.29
Employment w/ ex-post labor supply constraint	-0.05	0.05	0.93	—
Wage Markup: 18%	**Mean**	**Var**	**Corr(N,Y)**	**Binding Prob**
Employment w/o constraint	—	0.07	1.00	1.39
Employment w/ ex-post labor supply constraint	-0.03	0.05	0.97	—