Sticky Wage Models with Labor Supply Constraint

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2015 Bank of Portugal
The issue: Labor Supply Constraint

- When wage is rigid, how to determine the quantity of labor hired?
  - Dreze (1975): minimum of labor demand $n$ and labor supply $\ell$
    \[
    \text{labor supply constraint: } \hat{n} = \min\{n, \ell\}
    \]
  - New Keynesian models: labor demand determines labor market outcome.

- Can labor demand exceed labor supply?
  - Flexible wage: no, demand is smaller than supply due to wage mark-up
    \[
    \log \ell = \log n + \gamma \left(\log M\right)
    \]
    Frisch elas Mark-up
  - Sticky wage: yes, large demand shifts due to aggregate shocks.
Zhen: It is better to have a movie here
This Paper

1. Investigate whether labor supply constraint is violated popular NK models
   -

2. Solve a simple model: labor supply constraint is respected.

3. Revisit NK models: does labor supply constraint change the estimation?
Quantitative Findings

1. Investigate popular NK models: is labor supply constraint violated?

2. Solve a simple model: labor supply constraint is respected.

3. Revisit NK models: does labor supply constraint change the estimation?
Findings

With the ex-post imposition of the Dreze Equilibrium, we find that about 10% of the workers in the demand-determined economy are working more than desired. Moreover, the variance of the voluntary ex post aggregate labor is around 25% lower than that of the demand-determined quantity of labor, although it varies across specific models, ranging from 12% to 35%. The ex-ante equilibrium in a Taylor economy yields allocations that are very close to the ex-post imposed allocation. So we use the ex-ante...
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   1. The estimates are very different than ignoring the constraint. The role of TFP shocks go from 13% to 70%.
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   2. Persistence is much lower.
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3. We use the approximation to estimate a version of $\tau$ and $\tilde{\tau}$:
   - The estimates are very different than ignoring the constraint. The role of TFP shocks go from 13% to 70%.
   - Persistence is much lower.
   - Other things matter less.
Findings

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4. We use the approximation to estimate a version of $\theta$ and $\gamma$:

   1. The estimates are very different than ignoring the constraint. The role of TFP shocks go from 13% to 70%.
   2. Persistence is much lower.
   3. Other things matter less.
   4. Estimates of wage rigidity are higher.
Sticky Wage Model

- Aggregate labor $n$ across sectors

\[
    n = \left[ \int n_i \frac{\epsilon - 1}{\epsilon} \, di \right]^{\frac{\epsilon}{\epsilon - 1}}
\]

- For each $i$, quantity of labor hired is determined by demand

\[
    n_i = \left( \frac{w_i}{w} \right)^{-\epsilon} n
\]

- Given wage $w_i$, the labor supply $\ell_i$ satisfies

\[
    w_i = \frac{v'(\ell_i)}{u'(c)}
\]

- Wages are reset with probability $\theta$
Construct Voluntary Ex-post Employment

1. Construct cross-sectional wage distribution
   - measure of workers with wage rest $\tau$ periods before: $\mu_\tau = (1 - \theta_w)\theta^\tau_w$
   - the newly set wage every period can be computed by
     \[
     w_t = \left[ \int w_{i,t-1}^{1-\epsilon_w} di \right]^{1-\epsilon_w} = \left[ \theta_w (w_{t-1})^{1-\epsilon_w} + (1 - \theta_w) (w^*_t)^{1-\epsilon_w} \right]^{1-\epsilon_w}
     \]

Solve the model via log-linearization to obtain $n_i, \ell_i$ and construct employment with ex-post labor supply constraint

\[
\tilde{n} = \left[ \int \tilde{n}_i^{\epsilon-1} \epsilon \, di \right]^{\epsilon/(\epsilon-1)} \equiv \left[ \int (\min\{n_i, \ell_i\})^{\epsilon-1} \epsilon \, di \right]^{\epsilon/(\epsilon-1)}
\]

2. How different are $\tilde{n}$ and $\tilde{n}_i$ compared with $n$ and $n_i$?

3. $\tilde{n}$ and $\tilde{n}_i$ are NOT equilibrium allocations, but informative.
The questions

- How different is aggregate employment when the labor supply constraint \( \hat{n}_i = \min\{n_i, \ell_i\} \) is imposed on firms, households and unions ex-ante?

- Employment with ex-ante labor supply constraint

\[
\hat{n} = \left[ \int \hat{n}_i \frac{e-1}{e} \, di \right]^{\frac{e}{e-1}} = \left[ \int \left( \min\{n_i, \ell_i\} \right) \frac{e-1}{e} \, di \right]^{\frac{e}{e-1}}
\]

- \( \hat{n}_i \) and \( \hat{n} \) are equilibrium allocations.

- Compare \( n, \tilde{n} \) and \( \hat{n} \).
Smets and Wouters (2007)

- Exclude wage mark-up shock
  - The volatility of the wage mark-up shock is too large (CKM, 2010).
  - The implied wage mark-up can be negative, which does not make economic sense.
  - We simulate the model economy without the wage mark-up shock.

- Key parameter: average wage mark-up.
  - We re-estimate the model using wage mark-up from 6% to 18%.
  - With smaller wage mark-up, labor demand is more likely to exceed labor supply.
  - Wage mark-up is 5% in CEE (2005), 11% in CKM (2002).
Figure: Demand-Determined and Voluntary Ex Post Labor in the SW (2007) Model

6% Wage Markup

11% Wage Markup

15% Wage Markup

18% Wage Markup
### Table: Demand-Determined and Voluntary ex post Aggregate Labor in the Smets and Wouters (2007) Model

<table>
<thead>
<tr>
<th>Wage markup: 6%</th>
<th>Mean</th>
<th>Binding Freq.</th>
<th>Var</th>
<th>Corr(N,Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand-determined labor</td>
<td>—</td>
<td>27.04</td>
<td>1.02</td>
<td>0.80</td>
</tr>
<tr>
<td>Voluntary ex post labor</td>
<td>-1.04</td>
<td>—</td>
<td>0.89</td>
<td>0.34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wage markup: 11%</th>
<th>Mean</th>
<th>Binding Freq.</th>
<th>Var</th>
<th>Corr(N,Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand-determined labor</td>
<td>—</td>
<td>10.67</td>
<td>0.98</td>
<td>0.79</td>
</tr>
<tr>
<td>Voluntary ex post labor</td>
<td>-0.42</td>
<td>—</td>
<td>0.67</td>
<td>0.58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wage markup: 15%</th>
<th>Mean</th>
<th>Binding Freq.</th>
<th>Var</th>
<th>Corr(N,Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand-determined labor</td>
<td>—</td>
<td>5.73</td>
<td>0.97</td>
<td>0.79</td>
</tr>
<tr>
<td>Voluntary ex post labor</td>
<td>-0.23</td>
<td>—</td>
<td>0.71</td>
<td>0.68</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wage markup: 18%</th>
<th>Mean</th>
<th>Binding Freq.</th>
<th>Var</th>
<th>Corr(N,Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand-determined labor</td>
<td>—</td>
<td>3.85</td>
<td>0.96</td>
<td>0.79</td>
</tr>
<tr>
<td>Voluntary ex post labor</td>
<td>-0.15</td>
<td>—</td>
<td>0.77</td>
<td>0.72</td>
</tr>
</tbody>
</table>
6% Wage Markup: Cohort 1  
6% Wage Markup: Cohort 2  
6% Wage Markup: Cohort 4  
18% Wage Markup: Cohort 1  
18% Wage Markup: Cohort 2  
18% Wage Markup: Cohort 4
Findings

- The labor supply constraint is indeed violated.
  - Frequency: the probability of violation ranges from 3.8% to 27.3%.
  - Degree: the reduction in employment ranges from 0.15% to 1.04%.

- The difference in aggregate employment decreases with wage mark-up.
  - With ex-post labor supply constraint, the correlation between output and employment is monotonically increasing in wage mark-up.
  - The employment volatility declines by up to 30%; however, not monotonically in the wage mark-up.
  - The employment with ex-post labor supply constraint is less pro-cyclical or even counter-cyclical.
Gali, Smets and Wouters (2011)

- Better interpretation of $\min\{n_i, \ell_i\}$. $\ell_i$ is labor force.

- Wage mark-up shock and labor supply shock can be identified.

- The average wage mark-up can be determined by

$$\frac{\epsilon}{\epsilon - 1} \approx \gamma \mu^n$$

- In Gali, Smets and Wouters (2011), $\gamma = 3$ and wage mark-up is 18%.

- With more reasonable Frisch elasticity $\gamma = 1.5$, wage mark-up is 9%.
Figure: Demand-Determined and Voluntary Ex Post Labor in the GSW (2011) Model
<table>
<thead>
<tr>
<th>Wage markup: 6%</th>
<th>Mean</th>
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<th>Var</th>
<th>Corr(N,Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Determined Employment</td>
<td>—</td>
<td>9.68</td>
<td>0.86</td>
<td>0.84</td>
</tr>
<tr>
<td>Voluntary ex post Employment</td>
<td>-0.49</td>
<td>—</td>
<td>0.93</td>
<td>0.68</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wage markup: 11%</th>
<th>Mean</th>
<th>Binding Freq</th>
<th>Var</th>
<th>Corr(N,Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Determined Employment</td>
<td>—</td>
<td>5.05</td>
<td>0.61</td>
<td>0.79</td>
</tr>
<tr>
<td>Voluntary ex post Employment</td>
<td>-0.21</td>
<td>—</td>
<td>0.48</td>
<td>0.71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wage markup: 15%</th>
<th>Mean</th>
<th>Binding Freq</th>
<th>Var</th>
<th>Corr(N,Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Determined Employment</td>
<td>—</td>
<td>4.20</td>
<td>0.54</td>
<td>0.76</td>
</tr>
<tr>
<td>Voluntary ex post Employment</td>
<td>-0.15</td>
<td>—</td>
<td>0.41</td>
<td>0.70</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wage markup: 18%</th>
<th>Mean</th>
<th>Binding Freq</th>
<th>Var</th>
<th>Corr(N,Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Determined Employment</td>
<td>—</td>
<td>3.86</td>
<td>0.51</td>
<td>0.75</td>
</tr>
<tr>
<td>Voluntary ex post Employment</td>
<td>-0.13</td>
<td>—</td>
<td>0.39</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Findings

- The labor supply constraint is indeed violated.
  - Frequency: the probability of violation ranges from 3.8% to 28.1%.
  - Degree: the average employment reduces from 0.13% to 0.77%.

- The difference in aggregate employment decreases with wage mark-up.
  - With ex-post labor supply constraint, the correlation between output and employment is monotonically increasing in wage mark-up.
  - The employment volatility declines by up to 30%; however, not monotonically in the wage mark-up.
  - The employment with ex-post labor supply constraint is less pro-cyclical or even counter-cyclical.
**Monetary policy shock: Smets and Wouters (2007)**

**Table**

![Graphs showing monetary policy shock effects](chart.png)

- **6% Wage Markup**: 1st. d. shock
- **6% Wage Markup**: 2nd. d. shock
- **6% Wage Markup**: 3rd. d. shock
- **11% Wage Markup**: 1st. d. shock
- **11% Wage Markup**: 2nd. d. shock
- **11% Wage Markup**: 3rd. d. shock
Monetary policy shock: Gali, Smets and Wouters (2011)
Findings

With ex-post labor supply constraint

- Employment increases less or even decreases after an expansionary monetary policy shock.

- The response of employment is non-linear in the size of the shock.

- The effect of an expansionary monetary policy shock on employment increases with wage mark-up.
A model with ex-ante labor supply constraint

- Households consist of one consumer (who saves and consumes) and a continuum of workers.

- Firms, households, unions take the labor supply constraint into account ex-ante.

- Nominal wage is fixed for four periods (one year).

- No adjustment costs, no price stickiness in order to simplify the computation.

- We consider two kinds of shocks: TFP shock and monetary policy shock.
Firm

Firm’s problem:

\[
\max_{k_t, n_t, n_{i,t}} z_t k_t^\alpha n_t^{1-\alpha} - r_t^k k_t - \int w_{i,t} n_{i,t} di
\]

subject to

\[
n_t = \left[ \int n_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}
\]

\[
n_{i,t} \leq \Phi(w_{i,t}) = \left( \frac{u'(c_t) w_{i,t}}{\phi} \right)^{\frac{1}{\gamma}}
\]
Labor union

The union chooses the nominal wage $w$ for $T^w$ periods:

$$\max_w E_t \sum_{k=0}^{T^w-1} \beta^k \left\{ u'(c_{t+k}) \frac{w}{p_{t+k}} n_{i,t+k} - \frac{n_{i,t+k}^{1+\gamma}}{1 + \gamma} \right\}$$

subject to:

$$n_{i,t+k} = \min \left\{ \left( \frac{u'(c_{t+k})w}{\phi p_{t+k}} \right)^{\frac{1}{\gamma}}, \Psi_{t+k} \left( \frac{w}{p_{t+k}} \right) \right\}$$
TFP Shock
Monetary policy shock

![Graph showing monetary policy shock](image)

- Blue line: Demand determined
- Red line: Approximated Dreze
- Green line: Dreze equilibrium

The graph illustrates the impact of monetary policy shocks on economic variables, comparing demand-determined outcomes with approximated Dreze equilibrium and standard Dreze equilibrium scenarios.
### Compare Dreze Equilibrium with other Solution Methods: Employment

**Table:** Properties of Employment in the Simple Model for Various Solutions

<table>
<thead>
<tr>
<th>Solution Method</th>
<th>Mean</th>
<th>Var</th>
<th>Corr(N,Y)</th>
<th>Prob of binding</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP Shock</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dreze equilibrium (global solution)</td>
<td>-0.10</td>
<td>2.67</td>
<td>0.93</td>
<td>—</td>
</tr>
<tr>
<td>Approximation to Dreze equilibrium</td>
<td>-0.29</td>
<td>2.53</td>
<td>0.94</td>
<td>—</td>
</tr>
<tr>
<td>Voluntary ex post employment</td>
<td>-0.33</td>
<td>2.60</td>
<td>0.88</td>
<td>—</td>
</tr>
<tr>
<td>Demand-determined employment</td>
<td>—</td>
<td>3.79</td>
<td>0.96</td>
<td>0.11</td>
</tr>
<tr>
<td>Monetary Policy Shock</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dreze Equilibrium (global solution)</td>
<td>-0.52</td>
<td>1.60</td>
<td>0.94</td>
<td>—</td>
</tr>
<tr>
<td>Approximation to Dreze equilibrium</td>
<td>-0.41</td>
<td>1.29</td>
<td>0.99</td>
<td>—</td>
</tr>
<tr>
<td>Voluntary ex post employment</td>
<td>-0.45</td>
<td>1.38</td>
<td>0.77</td>
<td>—</td>
</tr>
<tr>
<td>Demand-determined employment</td>
<td>—</td>
<td>2.26</td>
<td>1.00</td>
<td>0.12</td>
</tr>
</tbody>
</table>
Compare Dreze Equilibrium with other Solution Methods: Business Cycles

Table: Business Cycle Statistics for the Simple Model with Various Solutions

<table>
<thead>
<tr>
<th></th>
<th>TFP Shock</th>
<th>Monetary Policy Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dreze Equilibrium</td>
<td>Approximated Dreze Equil.</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>3.12</td>
<td>2.95</td>
</tr>
<tr>
<td>Employment</td>
<td>2.67</td>
<td>2.53</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>Investment</td>
<td>42.82</td>
<td>38.10</td>
</tr>
<tr>
<td>Correlation with output</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.61</td>
<td>0.57</td>
</tr>
<tr>
<td>Investment</td>
<td>1.00</td>
<td>0.99</td>
</tr>
</tbody>
</table>

1 Demand-determined solution is much more volatile than Dreze equilibrium.

2 Approximated Dreze provides a good approximation to Dreze equilibrium.
Application: Re-estimate CEE (2005) and ACEL(2011) Model

- A medium-scale DSGE model with three shocks
  - neutral technology, investment technology, and monetary policy shocks

- Estimation method: generalized GMM
  - matching the impulse response functions recovered from SVAR model

- Re-estimate the model with approximated Dreze equilibrium
  - both estimated parameters and the quantitative properties change
## Estimation Results

**Table:** Subset of Estimated Parameter Values with 5% Wage Markup Using 1 St.d IRF

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Demand determined</th>
<th>Approximated Dreze</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std of neutral technology shock, $\sigma_{\mu z}$</td>
<td>0.068</td>
<td>0.140</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.089)</td>
</tr>
<tr>
<td>Autocor neutral technology shock, $\rho_{\mu z}$</td>
<td>0.902</td>
<td>0.697</td>
</tr>
<tr>
<td></td>
<td>(0.102)</td>
<td>(0.240)</td>
</tr>
<tr>
<td>Price rigidity, $\gamma$</td>
<td>0.040</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.039)</td>
</tr>
<tr>
<td>Variable capital utilization, $\sigma_a$</td>
<td>1.995</td>
<td>4.564</td>
</tr>
<tr>
<td></td>
<td>(2.222)</td>
<td>(7.070)</td>
</tr>
<tr>
<td>Investment adjustment cost, $S''$</td>
<td>3.281</td>
<td>4.752</td>
</tr>
<tr>
<td></td>
<td>(2.038)</td>
<td>(2.378)</td>
</tr>
<tr>
<td>Interest semi-elasticity of money demand, $\epsilon$</td>
<td>0.808</td>
<td>0.779</td>
</tr>
<tr>
<td></td>
<td>(0.208)</td>
<td>(0.193)</td>
</tr>
<tr>
<td>Habit formation, $b$</td>
<td>0.706</td>
<td>0.698</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>Effects of neutral technology shock on policy, $\rho_{xz}$</td>
<td>0.343</td>
<td>0.195</td>
</tr>
<tr>
<td></td>
<td>(0.266)</td>
<td>(0.480)</td>
</tr>
<tr>
<td>Scaling factor of neutral technology shock, $c_z$</td>
<td>2.997</td>
<td>1.027</td>
</tr>
<tr>
<td></td>
<td>(2.310)</td>
<td>(0.749)</td>
</tr>
<tr>
<td>Scaling factor of neutral technology shock, $c^p_z$</td>
<td>1.327</td>
<td>0.665</td>
</tr>
<tr>
<td></td>
<td>(1.381)</td>
<td>(0.650)</td>
</tr>
</tbody>
</table>
Comparison between Demand-Determined and Approximated Dreze

Table: Effects on Labor of the Shocks of Each Set of Parameters over Each Solution Concept

<table>
<thead>
<tr>
<th></th>
<th>Estimated with Demand Determined</th>
<th>Estimated with Approximated Dreze: 1 std shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
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Findings

1. The neutral technology shock is both much more volatile and less persistent
   - more persistent shocks make labor supply constraint be violated more often

2. There is more wage and price rigidity.
   - labor demand become more responsive to overcome labor supply constraint

3. Contributions of shocks change in the approximated Dreze estimation
   - contribution of neutral technology shock becomes much larger
   - contribution of the other shocks is smaller

4. Variance of employment is much larger in approximated Dreze equilibrium.
   - larger movement is needed to overcome the labor supply constraint in the IRF
Robustness

- Our results are robust under the following variations

  - Various wage markup
    - CEE(2005) and ACEL(2011) use 5% wage markup
    - We also experiment with larger wage markups

  - Various sizes of shocks in the IRF
    - With approximated Dreze solution, the model is nonlinear
    - We experiment with 1 to 3 std shocks in IRF
**Figure:** IRF of Labor to Neutral Tech. Shock: Estimation with Demand-Determined Model, 9% Markup

![Data, Demand-determined, Approximated Dreze for 1 std shock, 2.5 std shock, 3 std shock](image)

**Figure:** IRF of Labor to Neutral Tech. Shock: Estimation with Approximated Dreze Equilibrium, 9% Markup

![Data, Demand-determined, Approximated Dreze for 1 std shock, 2.5 std shock, 3 std shock](image)
Conclusion

In popular New Keynesian models with ex-post labor supply constraint

- Employment becomes less pro-cyclical or even counter-cyclical.
- Employment increases less or even decreases after an expansionary monetary policy shock.

In a simple model with ex-ante labor supply constraint

- Employment becomes less pro-cyclical with smaller volatility.
- The monetary policy is less effective.
- Employment behaves similar with ex-post and ex-ante labor supply.
References
Each household has a continuum of workers represented by \((i,j)\).

- \(i\) labor sector
- \(j\) disutility from labor

Preference

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( u(c_t) - \phi \int_i \int_0^{n_{i,t}} j^\gamma \, dj \, di \right)
\]

\[
= E_0 \sum_{t=0}^{\infty} \beta^t \left( u(c_t) - \phi \int_i \frac{n_{i,t}^{1+\gamma}}{1+\gamma} \, di \right)
\]
Workers only choose whether or not to work. They choose to work if:

\[ w_i u'(c) > j^\gamma \]

The labor force \( l_i \) solves:

\[ w_i = \frac{(n_i)^\gamma}{u'(c)} = \frac{v'(l_i)}{u'(c)} \]

The labor demand \( n^d_i \):

\[ n_i = \left( \frac{w_i}{w} \right)^{-\epsilon} n \]
**Firm**

- Solution to firm’s problem:

\[ r_t^k = \alpha z_t k_t^{\alpha - 1} n_t^{1-\alpha} \]

\[ n_{i,t} = \min \left\{ \left[ \frac{w_{i,t}}{(1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha}} \right]^{-\epsilon} n_t, \Phi(w_{i,t}) \right\} \]

\[ n_t = \left[ \int n_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \]

- \( \Psi_t(w_{i,t}) \) to denote the desired labor demand:

\[ \Psi_t(w_{i,t}) = \left[ \frac{w_{i,t}}{(1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha}} \right]^{-\epsilon} n_t \]
Household

- **Preference**

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( u(c_t) - \phi \int_i \int_0^{n_{i,t}} j^\gamma dj di \right)
\]

\[
= E_0 \sum_{t=0}^{\infty} \beta^t \left( u(c_t) - \phi \int_i \frac{n_{i,t}^{1+\gamma}}{1+\gamma} di \right)
\]

- **Budget constraint**

\[
p_t [c_t + k_{t+1} - (1 - \delta)k_t] + \frac{1}{R_t} b_{t+1} = r_t^k k_t + \int_i w_{i,t} n_{i,t} di + b_t + \Pi_t
\]

Return
Monetary policy

Taylor type monetary policy rule

$$\log R_t = \log \frac{1}{\beta} + \phi_\pi \pi_t + \phi_y \log \frac{y_t}{y^*} + \eta_t$$

The disturbance term $\eta_t$ follows an AR(1) process

$$\eta_t = \rho_m \eta_{t-1} + \zeta^m_t, \quad \zeta^m_t \sim N(0, \sigma_m)$$
Aggregate labor

- Firm side

\[ n_{i,t} = \min \left\{ \left[ \frac{w_{i,t}}{(1 - \alpha)z_t k_t^{\alpha} n_t^{-\alpha}} \right]^{-\epsilon} n_t, \Phi(w_{i,t}) \right\} \]

- Union side

\[ n_{i,t} = \min \left\{ \left( \frac{u'(c_t)w_{i,t}}{\phi p_t} \right)^{\frac{1}{\gamma}}, \Psi_t(w_{i,t}) \right\} \]

- In equilibrium

\[ n_{i,t} = \min \left\{ \left[ \frac{w_{i,t}}{(1 - \alpha)z_t k_t^{\alpha} n_t^{-\alpha}} \right]^{-\epsilon} n_t, \left( \frac{u'(c_t)w_{i,t}}{\phi p_t} \right)^{\frac{1}{\gamma}} \right\} \]

\[ n_t = \left[ \int \left( \min\{n_{i,t}, l_{i,t}\} \right)^{\frac{\epsilon - 1}{\epsilon}} di \right]^\frac{\epsilon}{\epsilon - 1} \]
Utility function is \( E_0 \sum_{t=0}^{\infty} \beta^t \left( \log c_t - \phi \int_i \frac{n_{i,t}^{1+\gamma}}{1+\gamma} di \right) \)

Production function is \( F(k_t, n_t) = z_t k_t^\alpha n_t^{1-\alpha} \)

\[
\log z_t = \rho_z \log z_{t-1} + \zeta_t^z \\
\zeta_t^m \sim \text{N}(0, \sigma_z)
\]

Monetary policy rule \( \log R_t = \log \frac{1}{\beta} + \phi_\pi \pi_t + \phi_y \log \frac{y_t}{y^*} + \eta_t \)

\[
\eta_t = \rho_m \eta_{t-1} + \zeta_t^m, \quad \zeta_t^m \sim \text{N}(0, \sigma_m)
\]

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<th>( \phi_\pi )</th>
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GSW(2011) with wage mark-up shock

St. d of Wage Markup: 0.0

St. d of Wage Markup: 0.015
**GSW(2011) with wage mark-up shock**

**Table:** Gali, Smets and Wouters (2011) Model with Ex-post Labor Supply Constraint

<table>
<thead>
<tr>
<th>St.d of Wage Markup Shock</th>
<th>Mean</th>
<th>Var</th>
<th>Corr(N,Y)</th>
<th>Binding Prob</th>
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### Table: Smets and Wouters (2007) Model with Different Wage Markups

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