Equity Issuance in Non-Traded Firms:

A Tale of Time Inconsistency

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November 12, 2019

Wharton Macro Lunch

VERY PRELIMINARY

- Many firms are not publicly traded.
  - Manager Compensation cannot depend on *Firm Value*. Not traded.
  - It depends on dividends.
  - Manager captures value
- Trade-off between investment, dividends and new equity issuance.
- We pose an environment where there is a firm featuring a manager who makes the decisions
  1. Issues outside equity and dividends (*time inconsistency* problem);
  2. Is *impatient*; and faces *moral hazard* through limited liability.

Main Results

- Under-capitalization due to time-inconsistency problem. Time inconsistency problems exist because of:
  - Reoptimization of dividend payment, and dilution of existing equity.
  - When interpreting as banks with limited liability additional moral hazard problem
- Some regulation may fix it.

Outline

1. Simple model to highlight the time-inconsistency issue
   - i Time-consistent solution in Markov Perfect Equilibrium
   - ii Commitment solution
   - iii Insufficient Investment
2. A Model with loan, deposit and default (firms are banks)
   - i Default and equity valuation
   - ii Higher leverage together with under-capitalization

- Consider a manager with preferences $u(c)$ and discount rate χ that issues equity to fund investment.
- Issuance is costly.
- The manager itself has no equity.
- Its compensation is linked to dividends
- It has no commitment to the amount of new equity issuance

$$V(n) = \max_{\{c,z,y,e,m,n'\}} \left\{ u(c) + \chi \, V\left(n'\right) \right\} \qquad \text{s.t.}$$

$$c + z + y = n + \alpha m, \qquad \left\{ \begin{array}{lr} n & \text{liquid assets} \\ c & \text{manager compensation} \\ z & \text{dividends} \\ y & \text{investment} \\ \alpha m & \text{new equity} \quad \alpha < 1 \end{array} \right.$$

$$n' = f(y), \qquad \text{productive investment}$$

$$m = e \; R^{-1} \; \Omega\left(n'\right) \qquad \text{equity rewards: } e \text{ share of the firm}$$

$$c \leq \psi z \qquad \text{comp linked to dividends} \; =$$

$$e \leq \frac{m}{(n - \gamma c - z) + m} \qquad \text{anti-dilution protection} =$$

γ?

- Value for shareholders of the firm that takes as given manager's behavior

$$\Omega\left(n\right)=z\left(n\right)+R^{-1}\left[1-e\left(n\right)\right]\ \Omega\left(n'\right).$$

- Substituting we get

$$R^{-1}\ \Omega(n')=n+m-(1+\gamma\ \psi)\ z \qquad \text{(Accounting Rule)}$$

$$\Omega(n)=n-\psi\ \gamma\ z(n) \qquad \text{(Value of firm)}$$

$$z=Q(n,y)=\frac{\xi}{\psi}\left\{(1-\alpha)\,n-\alpha R^{-1}\gamma\psi\ z'[f(y)]+\alpha R^{-1}f\left(y\right)-y\right\}$$

(Budg Constr)

- Where $\xi\equiv\frac{\psi}{1+\psi-\alpha(1+\psi\gamma)}$
- Tomorrow dividends are written as a direct choice of tomorrow's manager. This is where the time inconsistency is.

$$V(n) = \max_{y} \quad u\left[\psi Q(n,y)\right] + \chi \ V\left[f(y)\right]$$

- Guessing (and verifying later) for an interior solution The FOC is

$$\xi \ u_c \left[\alpha R^{-1} f_y \left(1 - \gamma \ \psi \ z_n'\right) - 1\right] = \chi \ f_y(y) \ V_n'(y)$$

- and envelope condition (the time inconsistency is not here)

$$V_n = \xi \left(1 - \alpha\right) u_c,$$

- together yields the GEE (in compact form)

$$u_c = \frac{\chi \left(1 - \alpha\right)}{1 - \alpha \ R^{-1} \ f_y (1 - \gamma \ \psi \ z_n')} \ f_y \ u_c'.$$

$$u_c = \frac{\chi\,(1-\alpha)}{1-\alpha\ R^{-1}\ f_y(1-\gamma\ \psi\ z'_n)}\ f_y\ u'_c.$$

- Today's manager takes it into account tomorrow's manager recklessness via $z'_n \equiv \frac{\partial z'}{\partial n'}$.
- The GEE collapses to a usual Euler equation when $\alpha = 0$: $u_c = \chi f_y u'_c$.
- The manager considers the "cost" of increasing y through

$$\Omega\,[f\,(y)] = -\psi\ \gamma\ z\,[f\,(y)] + f\,(y)\,.$$

- To see how much of an issue this is (when $\alpha > 0$) we pose the problem under commitment.
- We proceed by posing the model as of time zero and looking at the implied Euler equation after in a generic future period.
- The manager still optimally chooses how much equity Ω_t to emit.

$$\max_{\{c_t, z_t, y_t, m_t, e_t, n_{t+1}, \Omega_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \chi^t u(c_t) \qquad \text{s.t.}$$

$$\begin{aligned}
c_t + z_t + y_t &= n_t + \alpha\, m_t, \\
m_t &= R^{-1} e_t \Omega_{t+1}, \\
e_t &\leq \frac{m_t}{m_t + n_t - \gamma c_t - z_t}, \\
c_t &\leq \psi\ z_t \\
\Omega_t &= z_t + R^{-1}(1 - e_t)\ \Omega_{t+1}, \\
n_{t+1} &= f(y_t).
\end{aligned}$$

Euler Equation: (note the accumulated effect over time)

$$u_{c,t} + \sum_{j=1}^{t} \left(-\alpha\ R^{-1}\ \chi^{-1} \widetilde{\psi}\gamma\right)^j u_{c,t-j} = \frac{\chi(1-\alpha) f_{y,t}}{1 + (1-\alpha)\ \alpha\ R^{-1}\ \gamma\ \widetilde{\psi}\ f_{y,t} - \alpha\ R^{-1}\ f_{y,t}} u_{c,t+1}.$$

- The problem for the manager under commitment can be recursively written by separating the first period problem (with value function W_0) and the rest of the periods (with value function W).

- The first period problem is

$$W_0(n) = \max_{c,z,y,e,m,\Omega,\Omega'} \left\{ u\left(c\right) + \chi \;\; W\left[f(y), \Omega'\right] \right\}.$$

s.t. all relevant constraints.

- Substituting the constraints into the problem
- The commitment is to a value of the shares

$$W(n,\Omega) = \max_{\Omega'} \quad u\left(\frac{n-\Omega+\gamma\underline{c}}{\gamma}\right) + \chi$$
$$W\left\{ f\left[\left(1-\alpha-\frac{1+\psi-\alpha(1+\psi\gamma)}{\psi\,\gamma}\right) n + \alpha\, R^{-1}\Omega' \right.\right.$$
$$+ \frac{1+\psi-\alpha(1+\psi\,\gamma)}{\psi\gamma}\Omega -$$
$$\left.\left.\left(\alpha+\frac{1+\psi-\alpha(1+\psi\,\gamma)}{\psi\,\gamma}\right)\gamma\underline{c}\right], \Omega' \right\}$$

- FOC

$$\alpha \ R^{-1} W n' f_y + W'_\Omega = 0$$

- Envelope conditions

$$\begin{aligned} W_n &= \frac{1}{\gamma} u_c + \chi \left(1 - \alpha - \frac{1 + \psi - \alpha \left(1 + \psi \gamma\right)}{\psi \gamma} \right) W'_n f_y \\ W_\Omega &= -\frac{1}{\gamma} u_c + \chi \frac{1 + \psi - \alpha \left(1 + \psi \gamma\right)}{\psi \gamma} W'_n f_y \end{aligned}$$

- From the envelope conditions:

$$W_n + W_\Omega = \chi \left(1 - \alpha\right) W'_n f_y$$

- Assuming $W_n = W'_n$ and $W_\Omega = W'_\Omega$ in the steady state (same as before):

$$1 - \alpha \ R^{-1} f_y = \chi \left(1 - \alpha\right) f_y.$$

A Comparison of MPE and Commitment Solutions: Under-Capitalization in MPE (Steady State)

- Markov Perfect Equilibrium:

$$f_y^{ME} = \frac{1}{\chi\,(1-\alpha) + \alpha\ R^{-1}\,(-\gamma\ z_n' + 1)}$$

- Commitment:

$$f_y^{CM} = \frac{1}{\chi\,(1-\alpha) + \alpha\ R^{-1}}$$

- Social Planner

$$f_y^{SP} = \frac{1}{R^{-1}}$$

- Insufficient capitalization if $z_n' > 0$.

$$y^{SP} > y^{CM} > y^{ME}.$$

- Functional forms: $u(c) = \log(c)$, $f(y) = y^{\nu}$.
- Parameter values:

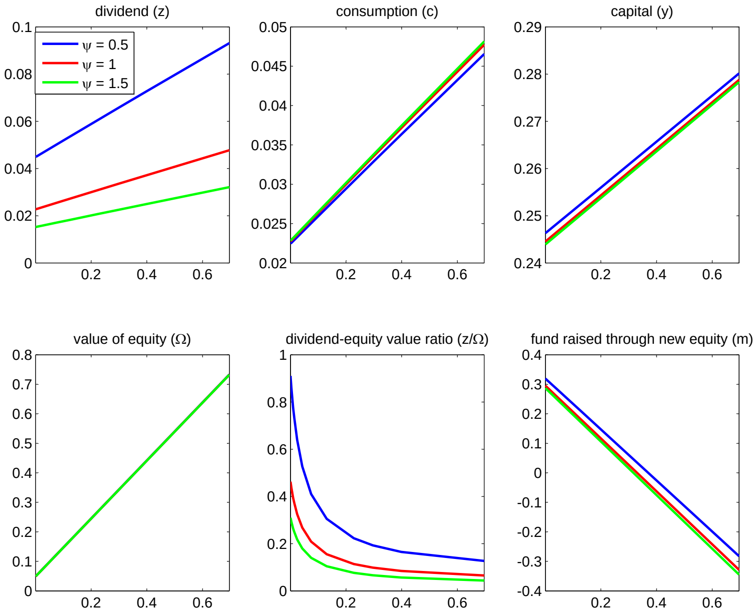
α	R^{-1}	γ	χ	ψ	ν
0.98	0.99	0.5	0.9	1.0	0.9

- Results: $z_n' = 0.036 > 0$. Thus, $y^{CM} > y^{ME}$.

Commitment Equilibrium vs Markov Perfect Equilibrium

	y	z	Ω	z/Ω	m/Ω
Commitment	0.31	0.035	0.33	0.10	0.09
Markov Perfect	0.26	0.034	0.28	0.12	0.11

Global Solution – Markov Perfect Equilibrium



An Extension with Default, Borrowing and Investment

Firms are Banks Not Normal Firms

- Here investment is in risky loans
- Crucially there is Borrowing
  - Unsecured debt
  - **Government Insured** (i.e. deposits) (se we do not have to worry about interest rate penalties for excessive risk). These deposits may be increasing in costs of acquisition $\zeta(d)$
- Brings an obvious concern which implies regulation

$$V(n;\Omega) = \max_{\{c,z,y,\ell,d,e,m\}} \Big\{ u(c) + \chi \underbrace{\int_{\eta^*(d,y)} V\left[f\left(d,y,\eta'\right);\Omega\right] G\left(d\eta'\right)}_{\text{continuation value of survival}} + \underbrace{\chi\, \underline{V}\, G\left(\eta^{*'}\left(d,y\right)\right)}_{\text{outside option}} \Big\} \qquad \text{s.t}$$

$$c + z + y = n + \alpha m \qquad \text{Budget Constraint}$$

$$\ell = d + y \qquad \text{Balance Sheet}$$

$$m = e\,R^{-1} \int_{\eta^*(d,y)} \Omega\left[f\left(d,y,\eta'\right)\right]\, dG\left(\eta'\right) \qquad \text{Equity Issuance}$$

$$c \leq \psi z \qquad \text{Compensation Constraint}$$

$$e \leq \frac{m}{(n - \gamma c - z) + m} \qquad \text{Dilution Constraint}$$

$$f(\ell, y, \eta') = R^{\ell} \cdot \ell^{1-\nu} \eta' - \left[R^{d} + \zeta(\ell - y)\right] \cdot (\ell - y) \qquad \text{Loan Returns}$$

$\eta^*(\ell, y)$ given by $f(\ell, y, \eta' = \eta^*) = \underline{A}$ — Default Threshold set by regulator

Excessive Leverage due to Moral Hazard

- We can separate the problem in stages
- Without loss of generality
- Imagine the choice of loans conditional on equity issuance.
- It displays excessive lending (relative to what the regulator wishes due to the moral hazard of not paying for deposits)

- Define auxiliary variables for cash-flow f, dividends Q, and default threshold η^*.

$$f\left(\ell, y, \eta'\right) = \max\left\{\left(R\ell^{-\gamma}\eta' - \left(R_d + \zeta\left(\ell - y\right)\right)\right)\ell + \left(R_d + \zeta\left(\ell - y\right)\right)y, \underline{n}\right\}$$

$$Q\left(\ell, y, n\right) = \frac{\xi}{\psi}\Bigg[\left(1-\alpha\right)n +$$

$$\alpha R^{-1}\int_{\eta'^*(\ell,y)}\left[f\left(\ell, y, \eta'\right) - \psi\gamma\, z'(f(\ell, y, \eta'))\right] dG\left(\eta'\right) - y\Bigg]$$

$$\eta'^*\left(\ell, y\right) = \frac{\underline{A} + \left[R^d + \zeta(\ell - y)\right]\cdot(\ell - y)}{R^\ell \cdot \ell^{1-\nu}}$$

The problem of the manager then simplifies to...

$$V(n) = \max_{\ell,y} \left\{ u\left[\psi\ Q\left(\ell, y, n\right)\right] + \chi \int_{\eta'^{*}(\ell,y)} V\left[f\left(\ell, y, \eta'\right)\right] G(d\eta') + \chi\ G\left[\eta'^{*}\left(\ell, y\right)\right] \underline{V} \right\}$$

- With Optimality conditions (again under interiority)

$$
\begin{array}{lrcl}
[y] & \psi\ u_c\ Q_y & = & -\chi \int_{\eta^*(\ell,y)} V_n'\ f_y\ G(d\eta') \\
[\ell] & \psi\ u_c\ Q_\ell & = & -\chi \int_{\eta^*(\ell,y)} V_n'\ f_\ell\ G(d\eta') \\
[\text{envelope}] & V_n & = & u_c\ \psi\ Q_n
\end{array}
$$

$[y]$

$$u_c\left[\alpha R^{-1}\left(\int_{\eta'^*(\ell,y)}\left[f_y\left(\ell,y,\eta'\right)\;\left(1-\psi\gamma\; z_n'(f(\ell,y,\eta'))\right)\right]dG\left(\eta'\right)+\eta_y^*\gamma\psi z(\underline{A})g(\eta^*)\right)\right.$$

$$\left.-1\right]=\chi\left\{(\alpha-1)\int_{\eta^*}u_c'\;f_y\;G(d\eta')+g(\eta^*)\;\eta_y^*(\ell,y)\;\frac{1}{\xi}\left[V(\underline{A})-\underline{V})\right]\right\}$$

$[\ell]$

$$u_c\left[\alpha R^{-1}\left(\int_{\eta'^*(\ell,y)}\left[f_\ell\left(\ell,y,\eta'\right)\;\left(1-\psi\gamma\; z_n'(f(\ell,y,\eta'))\right)\right]dG\left(\eta'\right)+g\eta_\ell^*\gamma\psi z(\underline{A})g(\eta^*)\right)\right]$$

$$=\chi\left\{(\alpha-1)\int_{\eta^*}u_c'\;f_\ell\;G(d\eta')+g(\eta^*)\;\eta_\ell^*(\ell,y)\;\frac{1}{\xi}\left[V(\underline{A})-\underline{V})\right]\right\}$$

$[y]$

$$0 = \psi\, u_c \cdot \frac{1}{1+\psi-\alpha(1+\gamma\psi)} \Big\{$$

-1 direct loss in consumption from y↑

$+\alpha R^{-1} \int_{\eta_*(d,y)} f_y\, G(d\eta')$ gain in equity valuation from more loans

$-\alpha R^{-1} \int_{\eta_*(d,y)} \gamma\psi z_n' f_y\, G(d\eta')$ loss in equity valuation from higher z'

$-\alpha R^{-1} \gamma\psi z(\underline{A}) g(\eta_*)(-\eta_{*\,y})\Big\}$ loss in equity valuation from lower default

$+\chi \int_{\eta_*(d,y)} \frac{\psi(1-\alpha)}{1+\psi-\alpha(1+\gamma\psi)} u_c' f_y\, G(d\eta')$ gain in future (consumption) from more loans

$+\frac{\chi}{\xi} [V(\underline{A}) - \underline{V}]\, g(\eta_*)(-\eta_{*\,y})$ gain in continuation value from lower default

- Suppose $\eta' \in \{0, 1\}$ and $\zeta(d) = \kappa \cdot d$. Let $p_1 = \Pr(\eta' = 1)$.
- Long-surviving bankers' leverage and capital can be orderd as

$$\text{leverage}^{ME} > \text{leverage}^{CM} > \text{leverage}^{SP},$$

$$y^{ME} < y^{CM} < y^{SP}.$$

Proof (Step 1)

- Compare a Markovian manager and a committed manager.
- Regardless of com. tech., $\ell(y)$ is the same under moral hazard.

$$D_\ell f(\ell(y), y, 1) = 0.$$

- DRS implies $d\ell(y)/dy < 1$ and **leverage is decreasing in** y.
- Again, $y^{ME} < y^{CM}$ due to time inconsistency. Hence,

$$\text{leverage}^{ME} > \text{leverage}^{CM}.$$

Proof (Step 2)

- Compare a committed banker and the social planner.
- **Only the banker is protected by limited liability.**

- Solutions to d and ℓ satisfy $d^{CM} > d^{SP}$ and $\ell^{CM} < \ell^{SP}$:

$$d^{CM} = \frac{p_1^{-1}\left[\chi\left(1-\alpha\right)+\alpha R^{-1}\right]^{-1} - R_d}{2\kappa} > \frac{R^{-1} - R_d}{2\kappa} = d^{SP},$$

$$\ell^{CM} = \left[\left[\chi\left(1-\alpha\right)+\alpha R^{-1}\right] p_1\left(1-\gamma\right)R\right]^{1/\gamma} < \left[R^{-1} p_1\left(1-\gamma\right)R\right]^{1/\gamma} = \ell^{SP}$$

- This implies that $y^{CM} < y^{SP}$. As a result,

$$\text{leverage}^{CM} > \text{leverage}^{SP},$$

- Time inconsistency problem arises in firms where manager's compensation is linked to profits not value (effective problem in non-traded firms).
- It generates excessive dividends and equity issuance
- When added to environments with moral hazard such as banking or other ones where there is an external cost in bankruptcy the problem leads to too much borrowing and thus excessive leverage of banks.
- It may also lead to insufficient investment, although this depends on the details.