Calibration, Estimation, and Effects of Technology Shocks

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A Question

- What fraction of the variation in output and hours worked is due to technology shocks?
- This is a long-standing question in business cycle research, see, for instance, Kydland and Prescott (1982) and Fisher (2006).
- We'll focus on hours worked.

Households

- There is a continuum of households solving the following problem

$$\max \quad \boldsymbol{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - B \frac{H_t^{1+1/\nu}}{1+1/\nu} \right) \right] \tag{1}$$

$$\text{s.t.} \quad C_t + P_t^k X_t = W_t H_t + R_t^k P_t^k K_t \tag{2}$$

$$K_{t+1} = (1-\delta) K_t + X_t \tag{3}$$

- C_t is consumption, H_t is hours worked, X_t is investment (physical units), P_t^k is the price of the unit of the investment good (using the consumption good as numeraire), W_t is the wage, and R_t^k the rental rate of capital.

Households

- Labor supply

$$H_t = \left(\frac{1}{B_t} \frac{W_t}{C_t} \right)^{\nu}$$

- Euler Equation

$$1 = \beta \boldsymbol{E}_t \left[\frac{P^k_{t+1}/C_{t+1}}{P_t/C_t} \left((1-\delta) + R^k_{t+1} \right) \right]$$

Firms

- Firms rent capital and labor services from Households and produce consumption and investment goods.
- Technology:

$$C_t + \frac{X_t}{V_t} = A_t K_t^{\alpha} H_t^{1-\alpha}$$

- Profits:

$$\Pi_t = C_t + P_t^k X_t - W_t H_t - R_t^k P_t^k K_t$$

- For the firms to be willing to produce both consumption and investment goods it has to be the case that $P_t^k = 1/V_t$.
- The optimal choice of capital and labor implies

$$W_t = (1-\alpha) A_t K_t^{\alpha} H_t^{-\alpha}, \quad R_t^k P_t^k = \alpha A_t K_t^{\alpha-1} H_t^{1-\alpha}$$

NIPA and Exogenous Processes

- NIPA: investment is measured as $I_t = X_t P_t^k$. Hence,

$$Y_t = C_t + I_t = A_t K_t^{\alpha} H_t^{1-\alpha}.$$

- Neutral technological shocks

$$A_t = \exp\{\gamma_a + \widetilde{a}_t\} A_{t-1}, \quad \widetilde{a}_t = \rho_a \widetilde{a}_{t-1} + \sigma_a \epsilon_{a,t}$$

- Investment-specific technology shocks

$$V_t = \exp\{\gamma_v + \widetilde{v}_t\} V_{t-1}, \quad \widetilde{v}_t = \rho_v \widetilde{v}_{t-1} + \sigma_v \epsilon_{v,t}$$

- To estimate the model, we make the preference shock time-varying

$$\ln(B_t/B) = \rho_b \ln(B_{t-1}/B) + \sigma_b \epsilon_{b,t}.$$

Equilibrium Conditions

- Endogenous variables: $Y_t, C_t, I_t, K_{t+1}, W_t, R_t^k, H_t$.
- The endogenous variables have to satisfy the following set of (nonlinear) rational expectations equations

$$\begin{aligned}
H_t &= \left(\frac{1}{B_t}\frac{W_t}{C_t}\right)^{\nu} \\
1 &= \beta \mathbb{E}_t\left[\frac{C_t V_t}{C_{t+1}V_{t+1}}\left((1-\delta)+R_{t+1}^k\right)\right] \\
K_{t+1} &= (1-\delta)K_t + I_t V_t \\
Y_t &= A_t K_t^{\alpha} H_t^{1-\alpha} \\
Y_t &= C_t + I_t \\
W_t &= (1-\alpha)Y_t/H_t \\
R_t^k &= \alpha Y_t/(P_t^k K_t)
\end{aligned}$$

Detrending

- Along a balanced growth path the following variables are stationary

$$\frac{Y_t}{Q_t}, \quad \frac{C_t}{Q_t}, \quad \frac{I_t}{Q_t}, \quad \frac{K_{t+1}}{Q_t V_t}, \frac{W_t}{Q_t}, \quad R_t^k, \quad H_t.$$

- where

$$Q_t = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}.$$

- We denote a detrended version of X_t by $\hat{X}_t$.

Equilibrium Conditions

- The (detrended) endogenous variables have to satisfy the following set of (nonlinear) rational expectations equations

$$
\begin{aligned}
H_t &= \left(\frac{1}{B_t} \frac{\hat{W}_t}{\hat{C}_t} \right)^{\nu} \\
1 &= \beta \mathbb{E}_t \left[\frac{\hat{C}_t Q_t V_t}{\hat{C}_{t+1} Q_{t+1} V_{t+1}} \left((1-\delta) + R^k_{t+1} \right) \right] \\
\hat{K}_{t+1} &= (1-\delta) \hat{K}_t \frac{Q_{t-1} V_{t-1}}{Q_t V_t} + \hat{I}_t \\
\hat{Y}_t &= \hat{K}_t^{\alpha} \left(\frac{Q_{t-1} V_{t-1}}{Q_t V_t} \right)^{\alpha} H_t^{1-\alpha}, \quad \hat{Y}_t = \hat{C}_t + \hat{I}_t \\
\hat{W}_t &= (1-\alpha) \hat{Y}_t / H_t, \quad R^k_t = \alpha \frac{\hat{Y}_t}{\hat{K}_t} \frac{Q_t V_t}{Q_{t-1} V_{t-1}}
\end{aligned}
$$

Equilibrium Conditions

- Recall that

$$\begin{aligned} Q_t/Q_{t-1} &= \exp\left\{ \frac{1}{1-\alpha}(\gamma_a + \widetilde{a}_t) + \frac{\alpha}{1-\alpha}(\gamma_v + \widetilde{v}_t) \right\}, \\ V_t/V_{t-1} &= \exp\{\gamma_v + \widetilde{v}_t\} \end{aligned}$$

- Define $q_t = Q_t/Q_{t-1}$ and $v_t = V_t/V_{t-1}$.

Equilibrium Conditions

$$
\begin{aligned}
H_t &= \left(\frac{1}{B_t} \frac{\hat{W}_t}{\hat{C}_t} \right)^{\nu}, \quad \hat{K}_{t+1} = (1-\delta)\hat{K}_t \frac{1}{q_t v_t} + \hat{I}_t \\
1 &= \beta \boldsymbol{E}_t \left[\frac{\hat{C}_t}{\hat{C}_{t+1} q_{t+1} v_{t+1}} \left((1-\delta) + R^k_{t+1} \right) \right] \\
\hat{Y}_t &= \hat{K}_t^{\alpha} \left(\frac{1}{q_t v_t} \right)^{\alpha} H_t^{1-\alpha}, \quad \hat{Y}_t = \hat{C}_t + \hat{I}_t \\
\hat{W}_t &= (1-\alpha)\hat{Y}_t / H_t, \quad R^k_t = \alpha \frac{\hat{Y}_t}{\hat{K}_t} q_t v_t \\
q_t &= \exp\left\{ \frac{1}{1-\alpha}(\gamma_a + \widetilde{a}_t) + \frac{\alpha}{1-\alpha}(\gamma_v + \widetilde{v}_t) \right\} \\
v_t &= \exp\{\gamma_v + \widetilde{v}_t\}
\end{aligned}
$$

Solving the Model

- We can now calculate a steady state (in terms of the detrended variables), log-linearize the equilibrium conditions around the steady state, and apply a solution technique to solve the system of linear rational expectations difference equations.
- We show subsequently the relevant steady state ratios and log-linearized equations.

Steady States

$$
\begin{aligned}
R^* &= \frac{e^{(\gamma_a+\gamma_v)/(1-\alpha)}}{\beta} - 1 + \delta \\
\frac{K^*}{Y^*} &= \frac{\alpha e^{(\gamma_a+\gamma_v)/(1-\alpha)}}{R^*} \\
\frac{I^*}{Y^*} &= \frac{K^*}{Y^*}\left[1 - (1-\delta)e^{-(\gamma_a+\gamma_v)/(1-\alpha)}\right] \\
\frac{I^*}{K^*} &= 1 - (1-\delta)e^{-(\gamma_a+\gamma_v)/(1-\alpha)}
\end{aligned}
$$

Log-linearizations

$$
\begin{aligned}
\widetilde{H}_t &= \nu(\widetilde{W}_t - \widetilde{C}_t - \widetilde{B}_t) \\
\widetilde{K}_{t+1} &= (1-\delta)e^{-(\gamma_a+\gamma_v)/(1-\alpha)}\left[\widetilde{K}_t - \widetilde{q}_t - \widetilde{v}_t\right] + \frac{I^*}{K^*}\widetilde{I}_t \\
0 &= \mathbb{E}_t\left[\widetilde{C}_t - \widetilde{C}_{t+1} - (\widetilde{q}_{t+1} + \widetilde{v}_{t+1}) + \frac{R^*}{1-\delta+R^*}\widetilde{R}_{t+1}\right] \\
\widetilde{Y}_t &= \alpha\widetilde{K}_t + (1-\alpha)\widetilde{H}_t - \alpha[\widetilde{q}_t + \widetilde{v}_t] \\
\widetilde{Y}_t &= \left(1 - \frac{I^*}{Y^*}\right)\widetilde{C}_t + \frac{I^*}{Y^*}\widetilde{I}_t \\
\widetilde{W}_t &= \widetilde{Y}_t - \widetilde{H}_t \\
\widetilde{R}^k_t &= \widetilde{Y}_t - \widetilde{K}_t + \widetilde{q}_t + \widetilde{v}_t
\end{aligned}
$$

Log-linearizations

$$
\begin{aligned}
\widetilde{q}_t &= \frac{1}{1-\alpha}\widetilde{a}_t + \frac{\alpha}{1-\alpha}\widetilde{v}_t \\
\widetilde{a}_t &= \rho_a \widetilde{a}_{t-1} + \epsilon_{a,t} \\
\widetilde{v}_t &= \rho_v \widetilde{v}_{t-1} + \epsilon_{v,t} \\
\widetilde{B}_t &= \rho_b \widetilde{B}_{t-1} + \epsilon_{b,t}
\end{aligned}
$$

Answering the Question: Three Approaches

- A Calibration
- Bayesian estimation of the DSGE model
- A structural VAR, loosely based on the model

Data (To be Updated)

- Sample Range: 1955:Q1 to 2004:Q4.
- Relative Price of Investment: $(1/P_t^k)$; Source: Fisher's (2006) interpolation of Violante's series (Equipment and Structures).
- Labor Share: computed as in Cooley and Prescott (1995)
- Population (for conversion into per capita terms): total civilian noninstitutional (thousands, NSA); Source: DRI-Global Insight.
- Hours: Aggregate Hours Index (ID PRS85006033); Source: Bureau of Labor Statistics.

Data (To be Updated)

- According to our model

$$Y_t = C_t + I_t = C_t + X_t P_t^k$$

where output, investment (and consumption) are measured in terms of consumption goods.

- In the data, we start from nominal output, consumption, and investment. Roughly:

$$GDP^{nom} = C^{nom} + I^{nom} + G^{nom} + NetEX^{nom}$$

- We have to take a stand on what to do with G^{nom} and NX^{nom}. How about: treating NX^{nom} as investment, splitting G^{nom} (attributing government expenditures on investment goods to investment and the remainder to consumption). What should we do with consumer durables?

Data (To be Updated)

- After these adjustments we get

$$GDP^{nom} = \widetilde{C}^{nom} + \widetilde{I}^{nom}$$

  which we obtain from the NIPA.
- Using adjustments as above we can computed $\widetilde{C}^{real}$ from NIPA.
- Define a consumption deflator:

$$PCD = \widetilde{C}^{nom} / \widetilde{C}^{real}.$$

- Then we can calculate real investment measured in terms of the consumption good, which is I_t in the model, as $\widetilde{I}^{nom}/PCD$.

Data (To be Updated)

- We can obtain X_t in the model as $\widetilde{I}^{nom}/(PCD * P^k)$.
- What remains to do:
- Capital Stock: Real capital in 1955; Source: Bureau of Economic Analysis, Fixed Asset Tables. Do we treat the real NIPA value as physical units (K_0 in our model)? Does it matter?
- Decide how to treat depreciation: depreciation rates versus: real consumption of fixed capital (from Bureau of Economic Analysis (NIPA); we would need to convert this into consumption units).
- We need a discount factor β: compute averages of real interest rates to choose β.

Calibration: Investment-specific Technology

- According to our model, the investment-specific technology shock corresponds to the relative price of investment goods, which we can measure in the data. Hence, we treat $V_t = 1/P_t^k$ as observed.

Calibration: Total Factor Productivity

- We compute the total factor productivity

$$A_t = \frac{Y_t}{K_t^{\alpha} H_t^{1-\alpha}},$$

which requires α and K_t.

- We can average data on the labor share $W_t H_t / Y_t$ to obtain an estimate of α.
- Capital stock in period $t = 0$ is assumed to be in steady state. We then calculate a capital stock series recursively:

$$K_{t+1} = (1-\delta)K_t + I_t / P_t^k,$$

- Investment (I_t, valued in terms of consumption goods) is observed.
- We are using an average depreciation rate based on the Cummins-Violante depreciation series.

Calibration: Shock Processes

- Now that we have constructed estimates of A_t and V_t we can fit autoregressive processes.
- Using data from 1955 to 2006 we obtain the following point estimates

$$\begin{aligned}\Delta \ln A_t &= \Delta \ln A_{t-1} + 0.007\widetilde{\epsilon}_{A,t} \\ \Delta \ln V_t &= (1-0.8)\cdot 0.007 + 0.8\Delta \ln V_{t-1} + 0.003\widetilde{\epsilon}_{V,t}\end{aligned}$$

- We do not utilize the preference shock: $B_t = B$.

Calibration

- We can calibrate β based on observations on real interest rates.
- Traditional approach: link labor supply elasticity to steady state relationship. Suppose preferences are of the form

  $$\ln C_t + \ln(1 - H_t)$$

  Then Frisch elasticity is given by $(1 - H^*)/H^*$. If households work $1/3$ of their time then Frisch elasticity is 2.
- We choose three values for ν: 0.2, 2, and 100.

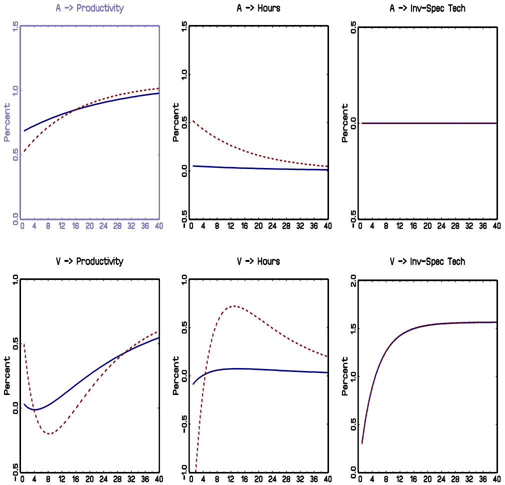
Calibration

- Parameter uncertainty: we have posteriors for the coefficients of the shock process, we can interpret the sample averages that were used to calculate α and β as posterior means and compute posterior standard deviations.
- Treat all parameter blocks as independent, generate parameter draws, for each parameter draw simulate the DSGE model for 200 periods using
  - only neutral technology shocks A_t;
  - only investment-specific technology shocks V_t;
  - both technology shock
- Compute the ratio of the variance of hours based on actual and model generated data.

Three Calibrations

	Calibration 1		Calibration 2		Calibration 3	
	Mean	90% Cred. Intv	Mean	90% Cred Intv	Mean	90% Cred Intv
α	0.340		0.340		0.340	
β	0.990		0.990		0.990	
δ	0.013		0.013		0.013	
ν	0.200		2.000		100.0	
γ_V	0.007	[0.005, 0.009]	0.007	[0.005, 0.009]	0.007	[0.005, 0.009]
ρ_V	0.799	[0.737, 0.868]	0.800	[0.737, 0.865]	0.800	[0.733, 0.865]
σ_A	0.007	[0.006, 0.008]	0.007	[0.006, 0.008]	0.007	[0.006, 0.008]
σ_V	0.003	[0.003, 0.003]	0.003	[0.003, 0.003]	0.003	[0.003, 0.003]

Impulse Response Functions for $\nu = 0.2$ and $\nu = 100$



Sample Variance Ratios for Hours: Model / Data

	Calibration 1		Calibration 2		Calibration 3	
Shock	Mean	90% Intv	Mean	90% Intv	Mean	90% Intv
A	.002	[.001, .003]	0.05	[0.02, 0.07]	0.14	[0.07, 0.21]
V	.010	[.002, .017]	0.23	[0.06, 0.42]	0.83	[0.25, 1.45]
A, V	.012	[.003, .020]	0.28	[0.09, 0.47]	0.97	[0.33, 1.61]

DSGE Model Estimation

- Alternatively we can estimate the DSGE model directly.
- To make the DSGE model estimation comparable to the VAR estimation (see below) we will by using the following three series: growth rate of investment price ($\Delta \ln P_t^k$), labor productivity growth ($\Delta \ln Y_t/H_t$), and hours worked H_t.
- Notice: so far we have three observables and two shocks, which means that the likelihood function is degenerate.
- To overcome this degeneracy, we introduce a preference shock, that is we let B_t evolve according to

$$\ln(B_t/B) = \rho_b \ln(B_{t-1}/B) + \sigma_b \epsilon_{b,t}.$$

Priors

- Details to be added...

Posteriors

- We use MCMC methods reviewed in An and Schorfheide (2007) to obtain draws from the posterior of the DSGE model parameters.
- We use the Kalman smoother to obtain an estimate of total factor productivity $\ln A_t$. We compare this estimate to the estimate obtained with the "calibration" approach. Notice that we have employed different information sets.

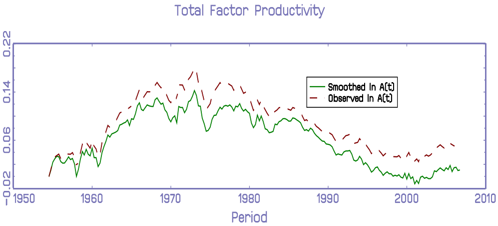
Calibration versus Estimation

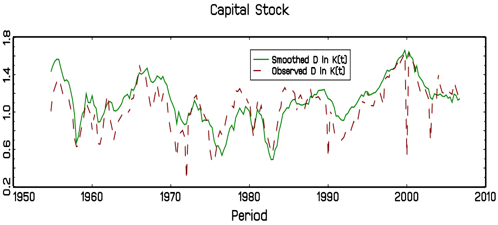
	Calibration 1		Estimation	
Name	Mean	90% Cred. Intv	Mean	90% Cred Intv
α	0.340		0.354	[0.322, 0.387]
β	0.990		0.990	
δ	0.013		0.013	
ν	0.200		0.229	[0.056, 0.388]
$\ln H^*$			-0.037	[-0.084, 0.018]
γ_A	0.000		0.000	[-0.001, 0.001]
γ_V	0.007	[0.005, 0.009]	0.007	[0.005, 0.008]
ρ_V	0.799	[0.737, 0.868]	0.732	[0.651, 0.807]
ρ_B	0.000	[0.000, 0.000]	0.973	[0.954, 0.995]
σ_A	0.007	[0.006, 0.008]	0.007	[0.007, 0.008]
σ_V	0.003	[0.003, 0.003]	0.003	[0.003, 0.004]
σ_B	0.000		0.010	[0.008, 0.011]

Sample Variance Ratios for Hours: Model / Data

	Calibration 1		Estimation	
Name	Mean	90% Cred. Intv	Mean	90% Cred Intv
A	.002	[.001, .003]	.004	[.000, .008]
V	.010	[.002, .017]	.008	[.000, .017]
A, V	.012	[.003, .020]	.012	[.000, .025]

Estimation: Capital Growth and Total Factor Productivity





VAR Analysis

- Finally, we will use a structural VAR to tackle our substantive question.
- Let y_t be composed of the *growth rates* of the investment goods price and labor productivity, and the *log level* of hours worked .
- Here is a (structural) VAR:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + \Phi_\epsilon \epsilon_t$$

- Our interpretation: the vector ϵ_t is composed of the two technology shock innovations as well as the innovation to a third shock.
- One can think of the third shock as preference shock, but we don't have to take a stand. The innovations are normalized to have unit variance.

VAR Analysis

- Define reduced-form innovation $u_t = \Phi_\epsilon \epsilon_t$. Denote covariance matrix of u_t by Σ_u.
- Write VAR in matrix form as linear regression model:

$$Y = X\Phi + U$$

where $T \times n$, X is $T \times k$.

VAR Approximation of DSGE Model

- We can link the DSGE model and the VAR by assuming that we estimate a VAR based on infinitely many observations generated from the DSGE model, conditional on structural parameters θ.
- Let $\boldsymbol{E}^D_\theta[\cdot]$ be the expectation under DSGE model and define the autocovariance matrices

$$\Gamma_{XX}(\theta) = \boldsymbol{E}^D_\theta[x_t x_t'], \quad \Gamma_{XY}(\theta) = \boldsymbol{E}^D_\theta[x_t y_t'].$$

- Then we can define a VAR approximation of the DSGE model by population least squares:

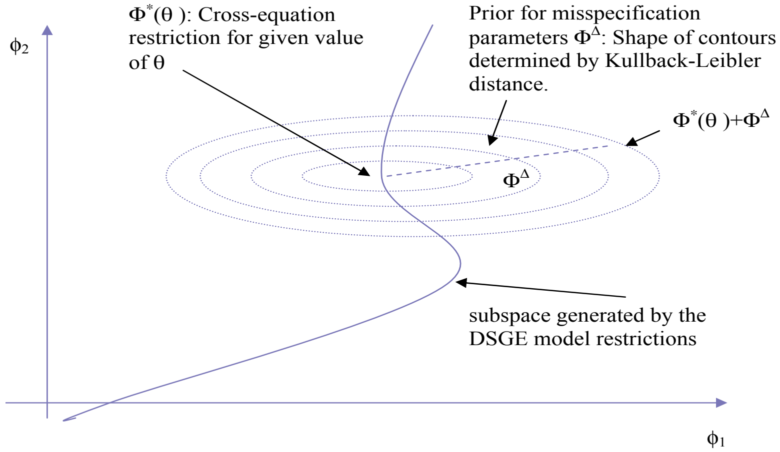
$$\Phi^*(\theta) = \Gamma^{-1}_{XX}(\theta)\Gamma_{XY}(\theta), \quad \Sigma^*(\theta) = \Gamma_{YY}(\theta) - \Gamma_{YX}(\theta)\Gamma^{-1}_{XX}(\theta)\Gamma_{XY}(\theta). \tag{4}$$

Relaxing Restrictions

- A concern when estimating a DSGE model is that we are imposing invalid cross-coefficient restrictions on the data.
- VARs are in general less restrictive and try to let the data speak.
- To relax the cross-coefficient restrictions, we can use a prior distribution that has a lot of mass *near* the restrictions but does not dogmatically impose them:

$$\begin{aligned} \Sigma|\theta &\sim \mathcal{IW}\Big(\lambda T\Sigma^*(\theta), \lambda T - k, n\Big) \qquad (5) \\ \Phi|\Sigma,\theta &\sim \mathcal{N}\left(\Phi^*(\theta), \frac{1}{\lambda T}\Big[\Sigma^{-1}\otimes\Gamma_{XX}(\theta)\Big]^{-1}\right). \end{aligned}$$

- The larger λ, the more tightly the prior contours are concentrated.


$\Phi^*(\theta)$: Cross-equation restriction for given value of θ
Prior for misspecification parameters Φ^Δ: Shape of contours determined by Kullback-Leibler distance.
$\Phi^*(\theta)+\Phi^\Delta$
Φ^Δ
subspace generated by the DSGE model restrictions
ϕ_2
ϕ_1

Identification

- To answer our substantive questions we need to identify the technology shocks, that is, we need to parameterize the VAR in terms of Φ_ϵ instead of Σ.
- Let Σ_{tr} be the Cholesky factor of Σ and Ω an orthonormal matrix. Then

$$\Phi_\epsilon = \Sigma_{tr}\Omega$$

- Our prior for Σ induces a prior for Σ_{tr}. We only need to add a prior for Ω.

Identification

- In the DSGE model we can calculate:

$$\left(\frac{\partial y_t}{\partial \epsilon_t'}\right)_{DSGE} = A(\theta),$$

  say. Then use QR decomposition of $A(\theta)$ to decompose $A(\theta)$ into a lower triangular matrix and an orthonormal matrix $\Omega^*(\theta)$.
- For the VAR analysis we can now use:

$$\Phi_\epsilon = \Sigma_{tr}\Omega^*(\theta)$$

- Hence, along the restriction function the VAR impulse responses to structural shocks will closely resemble the DSGE model impulse responses, at least in the short run.

The DSGE-VAR

- We now have the following hierarchical model:
  - Likelihood function: $p(Y|\Phi, \Sigma)$
  - Prior for DSGE model parameters: $p(\theta)$
  - Prior for VAR parameters: $p(\Phi, \Sigma, \Omega|\theta, \lambda)$
- Joint distribution (conditional on λ):

$$p(Y|\Phi, \Sigma)p(\Phi, \Sigma|\theta, \lambda)p(\Omega|\theta)p(\theta)$$

- Use MCMC methods described in Del Negro and Schorfheide (2004) to generate draws from the joint posterior distribution.

Choosing the Hyperparameter λ

- We can study the fit of the DSGE model and determine by how much the cross-coefficient restrictions need to be relaxed by examining the marginal likelihood function of the hyperparameter λ:

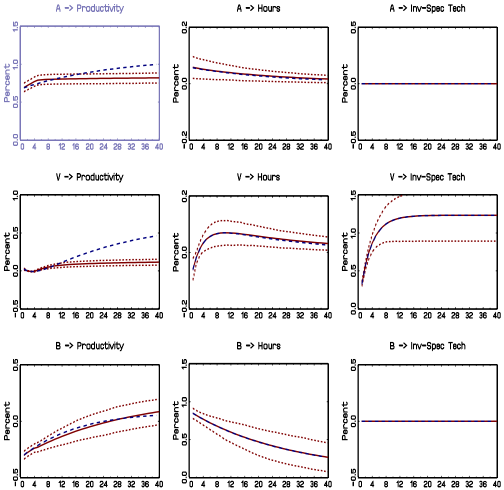
$$p(Y|\lambda) = \int p(Y|\Phi, \Sigma_u,)p(\Phi, \Sigma, \Omega, \theta|\lambda)d(\theta, \Phi, \Sigma, \Omega). \tag{6}$$

- The marginal likelihood penalizes the in-sample-fit of the estimated VAR by a measure of complexity. The larger λ, the more restricted the prior, the smaller the model complexity, and the smaller the penalty.

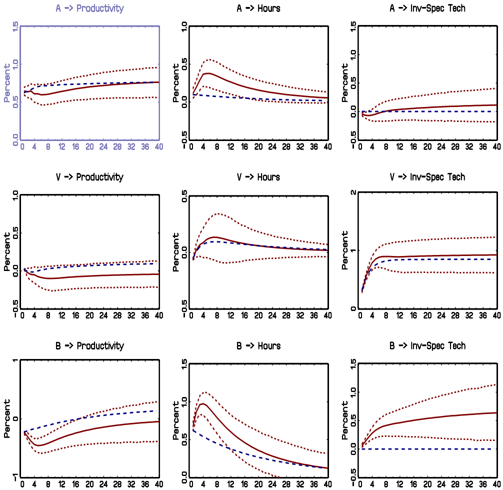
DSGE-VAR Estimates

	DSGE-VAR($\lambda = \infty$)		DSGE-VAR($\lambda = 1$)	
Name	Mean	90% Cred. Intv	Mean	90% Cred Intv
α	0.353	[0.322, 0.386]	0.360	[0.327, 0.395]
β	0.990		0.990	
δ	0.013		0.013	
ν	0.229	[0.056, 0.395]	0.484	[0.151, 0.815]
$\ln H^*$	-0.029	[-0.064, 0.007]	-0.031	[-0.070, 0.004]
γ_A	0.000	[-0.001, 0.001]	0.000	[-0.001, 0.001]
γ_V	0.007	[0.005, 0.008]	0.007	[0.005, 0.009]
ρ_A	0.000	[0.000, 0.000]	0.000	[0.000, 0.000]
ρ_V	0.727	[0.652, 0.800]	0.615	[0.506, 0.725]
ρ_B	0.970	[0.952, 0.989]	0.958	[0.931, 0.985]
σ_A	0.007	[0.007, 0.008]	0.007	[0.006, 0.007]
σ_V	0.003	[0.003, 0.004]	0.003	[0.003, 0.003]
σ_B	0.010	[0.008, 0.011]	0.008	[0.006, 0.009]
$\ln p(Y\|\lambda)$	2278.14		2322.83	

DSGE versus DSGE-VAR($\lambda = \infty$)



DSGE-VAR($\lambda = 1$) versus DSGE-VAR($\lambda = \infty$)



Sample Variance Ratios for Hours: Model / Data

	DSGE		DSGE-VAR($\lambda = \infty$)		DSGE-VAR($\lambda = 1$)	
Name	Mean	90% Intv	Mean	90% Intv	Mean	90% Intv
A	.004	[.000, .008]	.004	[.000, .009]	.128	[.004, .249]
V	.008	[.000, .017]	.010	[.000, .021]	.030	[.001, .070]
A, V	.012	[.000, .025]	.014	[.000, .029]	.158	[.001, .298]

Population Variance Decomposition

	DSGE		DSGE-VAR($\lambda = \infty$)		DSGE-VAR($\lambda = 1$)	
Name	Mean	90% Intv	Mean	90% Intv	Mean	90% Intv
A	.004	[.000, .010]	.005	[.000, .011]	.140	[.023, .253]
V	.011	[.000, .023]	.012	[.000, .024]	.033	[.000, .081]
A, V	.015	[.000, .033]	.017	[.000, .035]	.173	[.023, .334]

Deterministic Trends

- We repeat the calibration and the estimation of the DSGE model for a version of the model with deterministic trends in the two technology processes:

$$\begin{aligned}
(\ln A_t - \ln A_0 - \gamma_a t) &= \rho_{a,1}(\ln A_{t-1} - \ln A_0 - \gamma_a t) \\
&\quad + \rho_{a,2}(\ln A_{t-2} - \ln A_0 - \gamma_a t) + \sigma_a \epsilon_{a,t} \\
(\ln V_t - \ln V_0 - \gamma_v t) &= \rho_{v,1}(\ln V_{t-1} - \ln V_0 - \gamma_v t) \\
&\quad + \rho_{v,2}(\ln V_{t-2} - \ln V_0 - \gamma_v t) + \sigma_v \epsilon_{v,t}.
\end{aligned}$$

Deterministic Trends

- The point estimates for the sample 1955:I to 2006:IV are given by

$$\begin{aligned}&(\ln A_t - 4.841)\\ &\quad = \quad 1.028(\ln A_{t-1} - 4.841)\\ &\qquad\qquad -0.055\rho_{a,2}(\ln A_{t-2} - 4.841) + 0.007\epsilon_{a,t}\\ &(\ln V_t + 0.320 - 0.008t)\\ &\quad = \quad 1.766(\ln V_{t-1} + 0.320 - 0.008t)\\ &\qquad\qquad -0.773(\ln V_{t-2} + 0.320 - 0.008t) + 0.003\epsilon_{v,t}.\end{aligned}$$

- If the sum of the AR coefficients is 1, the model reduces to the stochastic trend specification.
- We re-parameterize the exogenous shocks in terms of partial autocorrelations: $\rho_1 = \psi_1(1-\psi_2)$; $\rho_2 = \psi_2$.

Three Calibrations

	Calibration 1		Calibration 2		Calibration 3	
	Mean	90% Cred. Intv	Mean	90% Cred Intv	Mean	90% Cred Int
β	0.990		0.990		0.990	
δ	0.013		0.013		0.013	
ν	0.200		2.000		100.0	
γ_V	0.007	[0.005, 0.009]	0.007	[0.005, 0.009]	0.007	[0.005, 0.009
$\psi_{1,A}$	0.980		0.980		0.980	
$\psi_{2,A}$	-0.049	[-0.161, 0.067]	-0.050	[-0.163, 0.066]	-0.050	[-0.169, 0.059
$\psi_{1,V}$	0.980		0.980		0.980	
$\psi_{2,V}$	-0.770	[-0.832, -0.701]	-0.770	[-0.839, -0.705]	-0.770	[-0.836, -0.70
σ_A	0.007	[0.006, 0.008]	0.007	[0.006, 0.008]	0.007	[0.006, 0.008
σ_V	0.003	[0.003, 0.003]	0.003	[0.003, 0.003]	0.003	[0.003, 0.003

Sample Variance Ratios for Hours: Model / Data

	Calibration 1		Calibration 2		Calibration 3	
Shock	Mean	90% Intv	Mean	90% Intv	Mean	90% Intv
A	.003	[.001, .005]	0.09	[0.04, 0.13]	0.31	[0.14, 0.45]
V	.007	[.003, .010]	0.22	[0.09, 0.34]	0.98	[0.34, 1.54]
A, V	.010	[.005, .015]	0.31	[0.15, 0.46]	1.29	[0.56, 1.91]

Bayesian Estimation

- We now estimate the deterministic trend model using the following series: labor productivity (log level); hours worked (log level); investment-specific technology (log level)
- We also re-estimate the stochastic growth version of the DSGE model, using log levels (instead of growth rates) of labor productivity and investment-specific technology. The likelihood is constructed as in Chang, Doh, and Schorfheide (2007).
- For the log-level estimation we parameterize the DSGE model in terms of $\ln Y_0$ rather than $\ln A_0$.
- Posterior odds in favor of stochastic trend are 20 to 1.

Posterior Estimates

	Deterministic Trend		Stochastic Trend	
	Mean	90% Cred. Intv	Mean	90% Cred Intv
β	0.990		0.990	
δ	0.013		0.013	
ν	0.670	[0.296, 1.038]	0.302	[0.050, 0.533]
γ_A	-0.001	[-0.002, -0.001]	-0.001	[-0.002, 0.001]
γ_V	0.007	[0.007, 0.008]	0.007	[0.005, 0.008]
$\psi_{1,A}$	0.975	[0.962, 0.990]	1.000	
$\psi_{2,A}$	-0.087	[-0.202, 0.041]	0.121	[0.038, 0.207]
$\psi_{1,V}$	0.990	[0.988, 0.994]	1.000	
$\psi_{2,V}$	-0.728	[-0.807, -0.646]	0.714	[0.636, 0.794]
ρ_B	0.970	[0.952, 0.990]	0.972	[0.955, 0.993]
σ_A	0.007	[0.007, 0.008]	0.007	[0.007, 0.008]
σ_V	0.003	[0.003, 0.004]	0.003	[0.003, 0.004]
σ_B	0.011	[0.010, 0.013]	0.010	[0.009, 0.011]
$\ln p(Y)$	2264.74		2267.60	

Sample Variance Ratios for Hours: Model / Data

	Deterministic Trend		Stochastic Trend	
Shock	Mean	90% Intv	Mean	90% Intv
A	0.03	[0.01, 0.06]	0.01	[0.00, 0.02]
V	0.06	[0.01, 0.10]	0.01	[0.00, 0.03]
A, V	0.10	[0.02, 0.18]	0.02	[0.00, 0.05]

Impulse Response Functions for Deterministic and Stochastic Trend Version