Banking Dynamics and Capital Regulation in General Equilibrium

José-Víctor Ríos-Rull   Tamon Takamura   Yaz Terajima

Penn and UCL   *Bank of Canada*   *Bank of Canada*

April 29, 2019 Econ 712 Penn

A Growth Model around a Banking Industry

- There is a Rep hhold
    - It owns a Mutual Fund that yields dividends
    - It gets utility from deposits
    - It holds bonds (risk free in St St, not necessarily so outside)
    - Some of its members work
- Many *Putty Clay* firms
    - Start up with bank loans. Become equity firms after Calvo shock.
    - All proceeds go to Mutual Funds
- A Banking Industry.
    - Individual Banks make Loans to firms with maturity λ
    - Borrow and issue deposits
    - Startup costs paid by Mutual Funds with difficulty (via func u^b)
- Mutual Funds
    - Manage Loan firms
    - Own Equity firms
    - Open and own banks with transfer difficulties

1 Steady State

Prices, Aggregate Variables, and Other Objects

- Prices
    - Interest rate q for bonds: Safe
    - Interest rate r^ℓ for loans: Unsafe
    - Interest rate for deposits q^D Safe because insured by Gov.
    - Wage function $w(k, C)$ (I am using a guess and verify based on logs)
- Quantities
    - Employment, and Number of Firms/Plants N
    - Capital per Plant K
    - Output, Cons, Inv, $C + \delta NK = Y = NAK^\alpha -$ Intermediate Inputs
    - Loans $L = (1 - \lambda)NK$ V: (Double check, but similar formula)
    - Deposits D
    - Bonds B
    - Taxes, Banks Loses T
- Other Elements
    - A Banking Industry with a measure of banks x, new entrants m^E, and dividends C^b
    - Mutual funds that manage/own all firms

Bank's Problem

$$V^i(a,\ell) = \max\left\{0, W^i(a,\ell)\right\}$$

$$W^i(a,\ell) = \max_{\ell^n \geq 0, c \geq 0, b',} \left\{u^b(c^b) + \beta \sum_{i'} \Gamma_{i,i'} \sum_{\delta'} \pi(\delta')\, V^{i'}[a'(\delta'), \ell'(\delta')]\right\} \text{ s.t.}$$

$$(TL) \qquad \ell' = (1-\lambda)\,(1-\delta')\,\ell + (1-\overline{\delta})\ell^n$$

$$(TA) \qquad a' = \left(\lambda + r^\ell\right)(1-\delta')\ell + r^\ell(1-\overline{\delta})\ell^n - \xi^{i,d} - b'$$

$$(BC) \qquad c^b + \ell^n + \xi^{i,n}(\ell^n) + \xi^{i,b}(b') \leq a + q^{i,b}(\ell,\ell^n,b')b' + q^d\xi^{i,d}$$

$$(KR) \qquad \frac{\ell^n + \ell - q^d\xi^{i,d} - q^{i,b}(\ell,\ell^n,b')b'}{\omega^r(n+\ell) + \omega^s\, 1_{b'<0} b' q^{i,b}(\ell,\ell^n,b')} \geq \theta$$

Entry and exit of banks

- Some banks go bankrupt when they cannot roll over debt. Let the default set be $M^i(A, L)$

- There is entry of new banks, (m^E is the measure of entrants), occurs as long as the free-entry condition is satisfied:

$$W^E(a^E, \ell^E) = u^b(\kappa^{Eb})$$

  - a^E, ℓ^E is the prespecified values of new entrants.
  - Function $u^b(.)$ translates units of the good into units of the objective function of banks
  - $\kappa^{E,b}$ is the opening cost of a new bank.

Industry Equilibria

- The definition is exactly like the one in the other paper. But for our purposes we need to link it with the rest of the model.
- We proceed by specifying what are inputs to the banks
- Given safe interest rate, $1/q$, deposit rate $1/q^d$, loan rate r^ℓ and cost of entry κ^{Eb}, it yields
    - A measure of Banks over their states x, including entrants m^E, and fraction of loans in hands of failing banks d^B.
    - Total Quantity of Bonds B
    - Total Quantity of Deposits D
    - Total Dividends C^b
    - Total Loses T to be covered by government
    - Total resources needed by new entrants $m^E \kappa^{Eb}$

Investment and firms: Putty-Clay

- Under Free Entry, One-Worker Putty-Clay Plants arise: $y = A\ k^{\alpha}$.
- Firms get destroyed with probability δ. From the point of view of banks $\delta \sim \gamma_{\delta}$, with mean δ_1.
- Financed with Bank loans of stochastic maturity λ. Upon arrival of Maturity, becomes Equity firm. Mutual Fund pays loan
- All cash flows of firms end up in Mutual Funds.
- Extensive margin: There are N^n new firms each period.
- Intensive margin: Each period firms invest k units.
- Total amount of new loans is $L^n = k\ N^n$.
- Employment or the number of plants is

$$N' = (1 - \delta_1)N + N^n.$$

- Output is

$$Y' = (1 - \delta_1)Y + N^n\ A\ k^{\alpha}.$$

Investment and firms: Financing

- Firms must borrow 100% of their investment k from a bank.

- If the Bank does not fail (prob $1-d^B$), then with probability $1-\lambda$, the firm continues to be debt-financed and pays interest kr^ℓ; with probability λ, a loan terminates. With probability γ, the firm chooses refinancing by banks. Otherwise, the mutual fund pays $(1+r^\ell)k$ at the beginning of next period, and the firm becomes an Equity firm.

- If the bank fails (prob d^B), we assume that the loan also terminates with prob γ and the Mutual pays the government $k(1+r^\ell+\zeta^F)$. V: What happens with prob $(1-\lambda)$?
- d^B is the endogenous fraction of loans held by defaulting banks:

$$d^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell)\in D_i} \ell \; dm_i(a,\ell)}{\sum_{i=1}^{N_\xi} \int \ell \; dm_i(a,\ell)}$$

Value of firms: There are measures $m^0(k, r^\ell)$ and $m^1(k)$ of them

- Given capital k, the maintenance cost δ_2, interest rate r^ℓ, wage $w(k)$, and the repayment cost ζ^F when banks default, the value of a loan firm is

$$\Pi^0(k, r^\ell) = Ak^\alpha - w(k) - (r^\ell + \delta_2)k + (1-d^B)(1-\lambda)q(1-\delta_1)\Pi^0(k, r^\ell)$$
$$+q\left(1-\delta_1\right)\left\{\lambda(1-d^B)+d^B\right\}(1-\gamma)\Pi^0(k)$$
$$+q(1-\delta_1)\left[\lambda(1-d^B)+d^B\right]\gamma\left[-k+\Pi^1(k)\right] - q(1-\delta_1)d^B\gamma\zeta^F k$$

- The value of an equity firm is

$$\Pi^1(k) = Ak^\alpha - w(k) - \delta_2 k + q(1-\delta_1)\Pi^1(k)$$

- Letting $R(k) = Ak^\alpha - w(k)$, $\Pi^0 < \Pi^1$ due to loan repayment costs:

$$\Pi^1(k) = \frac{R(k) - \delta_2 k}{1 - q(1-\delta_1)}$$
$$\Pi^0(k, r^\ell) = \frac{R(k) - \delta_2 k}{1 - q(1-\delta_1)} - \frac{r^\ell + q(1-\delta_1)\gamma\left[\lambda(1-d^B) + d^B + d^B\zeta^f\right]}{1 - q(1-\delta_1)\left[1 - \gamma\left\{\lambda(1-d^B) + d^B\right\}\right]}k$$

- Given the expected value, a firm chooses the size of capital:

$$k^* = \arg\max_k \left\{ q\, \Pi^0(k, r^\ell) - \kappa^{Ef} \right\}$$

- With FOC

$$k^* = \left\{ \frac{(1-\mu)\alpha A}{\frac{r^\ell + q(1-\delta_1)\gamma[\lambda(1-d^B)+d^B+d^B\zeta^f][1-q(1-\delta_1)]}{1-q(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]} + \delta_2} \right\}^{\frac{1}{1-\alpha}}$$

- Firms enter until profits are zero:

$$\kappa^{E,f} = q\Pi^0(k^*; r^\ell)$$

Outcome of Investment Decisions

- Given $r^{\ell}, q, d^{B}, L^{n}$, δ_1 and wage function $w(k)$
- Pose parameters of firm problem: δ_2, A, α, μ, $\bar{b}$
- Yields k, w, N, new firms $\delta_1 N$, that satisfy
  1. Wage equation
  2. FOC of firms
  3. Zero Profit Condition
  4. Feasibility: $Y = A\ N\ k^{\alpha} = C + I +$ costs of starting firms and operating banks
  5. $I = (\delta_1 + \delta_2)kN$

Mutual Funds

- Households own Mutual Funds which in turn own firms and banks, but do not trade its shares, just passively receive its dividends.
- Mutual Funds create banks and receive its dividends. Even though, banks assess the dividends according to function $u^b()$. Its cash flow is

$$\pi^b = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D_i} c^{i,b}(a,\ell) dm^i(a,\ell) + (c^{E,b} - \kappa^{E,b}) m^E$$

- Mutual Funds manage Loan-firms and own Equity Firms:

$$\begin{aligned} \pi^f &= Y - \mu Y - (1-\mu)\bar{b}N - r^\ell K^0 \\ &\quad -(1-d^B)\lambda K^0 - d^B(1+\zeta^F)K^0 - \kappa^{E,f} N^n \\ &= \int_{k,r^\ell} \left[R^0(k,r^\ell) - kr^\ell - (1-d^B)\lambda k - d^B(1+\zeta^F)k \right] dm^0(k,r^\ell) \\ &\quad + \int_k R^1(k) dm^1(k) - \kappa^{E,f} N^n \end{aligned}$$

Outcome of Mutual Funds

- By Aggregation we get Profits to be Distributed to Households. It needs
  1. New Banks Creation
  2. Profits and loses from Banks C^b
  3. Cash Flow net of Interest from Loan firms (not zero because of fixed costs)
  4. Loan Repayment
  5. Profits from Equity Firms

Wage Determination

- A bargaining process between the firm and the worker. V: (We may change this to get more wage rigidity and avoid the Shymer puzzle)

- The bargaining process is repeated every period and if unsuccesfull neither firm nor worker can partner with anybody else within a period. We assume that the financial obligations to the bank by the firm do not disappear. Let μ be the bargaining weight of the worker and $\overline{b}$ is workers' outside option. Then, we have

$$w^0(k) = w^1(k) = \mu\, A\, k^\alpha + (1-\mu)\overline{b}$$

- Total (per capita) Labor Income paid in the Economy are

$$W\ N = N \int \left[\mu\, A\, k^\alpha + (1-\mu)\overline{b}\right]\ di = \mu Y + (1-\mu)\overline{b}N$$

Household

$$v(a) = \max_{c,b',d'} \quad u(c, d') + \beta v(a') \qquad \text{s.t.}$$

$$\begin{aligned} c + q^d d' + qb' &= a + W\,N + (1-N)\overline{b} + \pi^f + \pi^B - T \\ a' &= d' + b' \end{aligned}$$

where T is the taxes needed to pay for bank losses. FOCs:

$$\begin{aligned} u_c &= \frac{\beta}{q} u'_c \\ u_d &= q^d u_c - \beta\, u'_c \end{aligned}$$

Taxes

The cost of deposit insurance is the amount of deposits that defaulting banks owe minus liquidated capital.

$$T = \sum_{i=1}^{N_\xi} \xi^{i,d} \int_{(a,\ell)\in D} dm^i(a,\ell) - K^0 d^B (1-\zeta^B)$$

where ζ^B is the fraction that the government is unable to recover during the liquidation process.

Output of Household Problem

- Given safe interest rate, $1/q$, deposit rate $1/q^d$, Taxes T, wages W, Profits Π, and Bonds B, Employment N we obtain

  1. Consumption C
  2. Deposits D

Deposits

$$D' = \sum_{i=1}^{N_\xi} \xi_i^d \int_{(a,\ell)\notin D_i} dm^{bi}(a,\ell) + \xi^{dE}\, m^E$$

Bonds

$$qB' = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D_i} q^{ib}\Big(\ell, \ell^{in}(a,\ell), b^{i'}(a,\ell)\Big)\; b^{i'}(a,\ell) dm^i(a,\ell) + q^{Eb} b'^E m^E$$

Market clearing (continued): V: How does NIPA treat F? Intermediate goods?

New loans

$$k^* N^n + (1-\gamma)\left\{\lambda(1-d^B)+d^B\right\}K^0 = \sum_{i=1}^{N_\xi}\int_{(a,\ell)\notin D_i} \ell_i^n(a,\ell)dm_i(a,\ell) + \ell_E^n m_E$$

Goods

$$
\begin{aligned}
Y = {} & C + kN^n + \delta_2 kN + \\
& + \sum_{i=1}^{N_\xi}\int_{(a,\ell)\notin D_i} \xi_i^n\left(\ell_i^n(a,\ell)\right)dm_i(a,\ell) + \xi_E^n\left(\ell_E^n\right) && \text{(Bank loan issuance costs)}\\
& + \sum_{i=1}^{N_\xi}\int_{(a,\ell)\notin D_i} \xi_b\left(b_i'(a,\ell)\right)dm_i(a,\ell) + \xi_E^b\left(b_E'\right) && \text{(Bank bond issuance costs)}\\
& + \kappa_E^b m_E + \kappa_E^f N^n && \text{(Entry costs)}\\
& + d^B(\zeta^B + \zeta^F)K^0 && \text{(Bank default costs)}
\end{aligned}
$$

Households: $u(C, D, N) = log(C) + \eta^D log(D)$,

$$q = \beta \tag{1}$$

$$\frac{\eta^D C}{D} = q^d - \beta \tag{2}$$

Firms:

$$k^* = \left\{ \frac{(1-\mu)\alpha A}{\frac{[r^\ell + q(1-\delta_1)\gamma\{\lambda(1-d^B)+d^B+d^B\zeta^f\}][1-q(1-\delta_1)]}{1-q(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]} + \delta_2} \right\}^{\frac{1}{1-\alpha}} \tag{3}$$

$$\kappa^{Ef} = q\,\Pi^0 \tag{4}$$

$$\Pi^0 = \frac{(1-\mu)\left(A(k^*)^\alpha - \overline{b}\right) - \delta_2 k^*}{1-q(1-\delta_1)} - \frac{r^\ell + q(1-\delta_1)\gamma\left[\lambda(1-d^B) + d^B + d^B\zeta^f\right]}{1-q(1-\delta_1)\left[1-\gamma\left\{\lambda(1-d^B)+d^B\right\}\right]} k^* \tag{5}$$

Steady state conditions (2)

Wages:

$$w = \mu A(k^*)^\alpha + (1-\mu)\overline{b} \tag{6}$$

Banks:

$$\kappa^{E,b} = W^E(a^E, \ell^E) \tag{7}$$

$$d^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell)\in D^i} \ell \; dm^i(a,\ell)}{\sum_{i=1}^{N_\xi} \int \ell \; dm^i(a,\ell)} \tag{8}$$

Market clearing conditions:

$$D = \sum_{i=1}^{N_\xi} \xi^{i,d} \int_{(a,\ell)\notin D_i} dm^i(a,\ell) + \xi^{E,d}\, m^E \tag{9}$$

$$qB = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D^i} q^{i,b}\left(\ell, \ell^{i,n}(a,\ell), b'^i(a,\ell)\right)\, b'^i(a,\ell) dm^i(a,\ell) + q^{E,b} b'^E m^E \tag{10}$$

$$k^* N^n + (1-\gamma)\left\{\lambda(1-d^B) + d^B\right\} K^0 = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D^i} \ell^{i,n}(a,\ell) dm^i(a,\ell) + \ell^{E,n} m^E \tag{11}$$

$$\begin{aligned} Y = C + k^* N^n + \delta_2 kN + \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D^i} \xi^{i,n}\left(\ell^{i,n}(a,\ell)\right) dm^i(a,\ell) + \xi^{E,n}\left(\ell^{E,n}\right) m^E \\ + \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D^i} \xi^{i,b}\left(b'^i(a,\ell)\right) dm^i(a,\ell) + \xi^{E,b}\left(b'^E\right) m^E \\ + \kappa^{E,b} m^E + \kappa^{E,f} N^n + d^B(\zeta^B + \zeta^F) K^0 \end{aligned} \tag{12}$$

Steady state conditions (4)

Laws of motion:

$$Y = \frac{A(k^*)^\alpha N^n}{\delta_1} \tag{13}$$

$$N = \frac{N^n}{\delta_1} \tag{14}$$

$$K^0 = \frac{k^* N^n}{1-(1-\delta_1)\left[1-\gamma\left\{\lambda(1-d^B)+d^B\right\}\right]} \tag{15}$$

Aggregate endogenous variables:

$C, D, B, k^*, K^0, N^n, N, Y, d^B, m(a,\ell), m^E, q, q^d, r^\ell, w$

Parameters:

HHs: $\beta,\ \mu,\ \overline{b},\ \eta^D$

Firms: $\alpha,\ A,\ \kappa^{E,f},\ \zeta^F$

Banks: $u^b(),\ \beta^B,\ \lambda,\ \xi^{i,d},\ \xi^{i,n},\ \xi^{i,b},\ F(\delta'),\ \kappa^{E,b},\ \zeta^B$

- Set Parameters of Banking: $u^b(), \beta^B, \lambda, \xi^{i,d}, \xi^{i,n}, \xi^{i,b}, F(\delta')$ and prices r^ℓ, q^d, q.V: (may come back to this)
- Compute the banking industry equilibrium. Get loans L, deposits D bank dividends C^b, losses T, resources for new entrants $m^E \kappa^{Eb}$.
- Set HH preference parameters $\beta, \overline{b}$, η_D, and the bargaining power μ so that they are consistent with q, the observed consumption-to-deposit ratio and the labor share of 2/3.
- Set Technology A, α as well as δ_2 and ζ^F to solve the firms' problem. Given α and δ_2, adjust A to make sure that all markets clear. V: (I think that λ doesn't matter much so we should set this to get the equity/debt ratio of the nonfinancial sector and a normalize)
- Generate key moments of interest.

Setting the bargaining parameters

- We target labor share and the outside option for workers $\overline{b} = \phi_b w$:

$$LS = \mu + (1-\mu)\frac{\overline{b}N}{Y} = \mu + (1-\mu)\frac{\phi_b w}{A(k^*)^\alpha}$$
$$w = \mu A(k^*)^\alpha + (1-\mu)\overline{b} = \mu A(k^*)^\alpha + (1-\mu)\overline{\phi}_b w$$

- Solving the two conditions simultaneously,

$$\mu = \frac{(1-\phi_b)LS}{1-\phi_b LS}$$
$$w(k^*) = \frac{\mu}{1-(1-\mu)\phi_b} A(k^*)^\alpha$$
$$\overline{b} = \phi_b w(k^*)$$

- $LS = 2/3$ and $\phi_b = 0.9$ imply $\mu = 1/6$.

Endogenous variables

- $\beta = q$ by (1)
- $N^n = \delta_1 \overline{N}$ by (14), where $\overline{N} = 0.9$.
- The banking industry equilibrium gives L^n: back out k^* from (11).
- Set A so that the loan demand (3) is equal to the loan supply.
- $Y = Ak^* \overline{N}$ and $I = (\delta_1 + \delta_2) k^* N$.
- Compute K^0 from (15)
- C is determined as a residual in (12)

Calibration

- For simplicity, we ignore various intermediate costs for now
- Consumption-deposit ratio:

$$
\begin{aligned}
\frac{C}{D} = \frac{C}{L^n}\frac{L^n}{D} &= \frac{Y - I}{k^* \delta_1 \overline{N}\left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]}\right]}\frac{L^n}{D} \\
&= \frac{A(k^*)^{\alpha-1} - \delta_1 - \delta_2}{\delta_1\left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]}\right]}\frac{L^n}{D} \\
&= \left[\frac{1}{\frac{K}{Y}\delta_1\left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]}\right]} - \frac{\delta_1 + \delta_2}{\delta_1\left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]}\right]}\right]\frac{L^n}{D}
\end{aligned}
$$

- With $K/Y = 3$, $L^n/D = 0.9$, $\delta_1 = 0.02$, $\delta_2 = 0.08$, $\gamma = \lambda = 0.5$, $d^B = 0$, consumption-deposit ratio is about 5.4.

2 Equilibrium in Terms of Sequences

Existing bank's problem given $V_{t+1}, r_t^\ell, q_t^d, \theta_t$

$$V_t^i(a,\ell) = \max\left\{0, W_t^i(a,\ell)\right\}$$

$$W_t^i(a,\ell) = \max_{\ell^n \geq 0, c \geq 0, b',} u^b(c^b) + \beta^b \sum_i \Gamma_{i,i'} \sum_{\delta'} \left\{\pi_t(\delta') V_{t+1}^{i'}[a'(\delta'), \ell'(\delta')]\right\} \text{ s.t.}$$

$$(TL) \qquad \ell' = (1-\lambda)\,(1-\delta')\,\ell + (1-\bar{\delta})\ell^n$$

$$(TA) \qquad a' = (\lambda + r_t^\ell)(1-\delta')\ell + \lambda(1-\bar{\delta})\ell^n - \xi^{i,d} - b'$$

$$(BC) \qquad c^b + \ell^n + \xi^{i,n}(\ell^n) + \xi^{i,b}(b') \leq a + q_t^{i,b}(\ell, \ell^n, b')b' + q_t^d \xi^{i,d}$$

$$(KR) \qquad \frac{\ell^n + \ell - q_t^d \xi^{i,d} - q_t^{i,b}(\ell,\ell^n,b')b'}{\omega_t^r(n+\ell) + \omega_t^s\, 1_{b'<0} b' q_t^{i,b}(\ell,\ell^n,b')} \geq \theta_t$$

π_t is an exogenous aggregate shock.

θ_t is exogenous. A feedback rule to be considered in the next step.

- Entry condition:

$$W_t^E(a^E, \ell^E) = u^b(\kappa^{E,b}) \tag{16}$$

- A fraction of loans destroyed by bank default:

$$d_{t-1}^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell)\in M_t^i} \ell \; dm_{t-1}^i(a,\ell)}{\sum_{i=1}^{N_\xi} \int \ell \; dm_{t-1}^i(a,\ell)} \tag{17}$$

Equity-finaced firm's value given Π^1_{t+1}, q_t

- The value is

$$\Pi^1_t(k) = A_t k^\alpha - w_t(k) - \delta_2 k + q_t(1-\delta)\Pi^1_{t+1}(k) \tag{18}$$

- The wage is given by

$$w_t(k) = \mu A_t k^\alpha + (1-\mu)\overline{b} \tag{19}$$

- The value of bank-financed firm is

$$\begin{aligned}\Pi^0_t(k) = Ak^\alpha - w(k) - (r^\ell_t + \delta_2)k + (1-d^B_t)(1-\lambda)q_t(1-\delta_1)\Pi^0_{t+1}(k) \\ + q_t\,(1-\delta_1)\left\{\lambda(1-d^B_t) + d^B_t\right\}(1-\gamma)\Pi^0_{t+1}(k) \\ + q_t(1-\delta_1)\left[\lambda(1-d^B_t) + d^B_t\right]\gamma\left[-k + \Pi^1_{t+1}(k)\right] \\ - q_t(1-\delta_1)d^B_t\gamma\zeta^F k \quad (20)\end{aligned}$$

- Given q_t and Π^0_{t+1}, entrants choose k^*_t:

$$k^*_t = \arg\max_k \left\{q_t\Pi^0_{t+1}(k) - \kappa^{E,f}\right\} \tag{21}$$

- Entry occurs until firms break even *ex-ante*

$$q_t\Pi^0_{t+1}(k^*_t) = \kappa^{E,f} \tag{22}$$

- Aggregate output:

$$Y_t = A_t(k_t^*)^\alpha N_t^n + (1-\delta_1)Y_{t-1} \tag{23}$$

- Aggregate investment:

$$I_t = k_t^* N_t^n + \delta_2 K_{t-1} \tag{24}$$

- Aggregate capital:

$$K_t = k_t^* N_t^n + (1-\delta_1)K_{t-1} \tag{25}$$

- Aggregate capital held by bank-financed firms:

$$\begin{aligned} K_t^0 &= k_t^* N_t^n \\ &+ \left[(1-d_{t-1}^B)(1-\lambda) + (1-\gamma)\left\{\lambda(1-d_{t-1}^B) + d_{t-1}^B\right\}\right](1-\delta_1)K_{t-1}^0 \end{aligned} \tag{26}$$

- Consumption Euler equation:

$$u_{c,t} = \beta \frac{u_{c,t+1}}{q_t} \tag{27}$$

- Consumption-deposit marginal condition:

$$u_{d,t} = q_t^d u_{c,t} - \beta u_{c,t+1} \tag{28}$$

Market clearing conditions

$$D_t = \sum_{i=1}^{N_\xi} \xi^{i,d} \int_{(a,\ell)\notin M^i_{t-1}} dm^i_{t-1}(a,\ell) + \xi^{E,d}\, m^E_t \tag{29}$$

$$\begin{aligned} k^*_t N^n_t + (1-\gamma)\left\{\lambda(1-d^B_{t-1}) + d^B_{t-1}\right\} K^0_{t-1} \\ = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin M^i_t} \ell^{i,n}_t(a,\ell) dm^i_{t-1}(a,\ell) + \ell^{E,n}_t m^E_t \end{aligned} \tag{30}$$

$$\begin{aligned} Y_t = C_t + k^*_t N^n_t + \delta_2 K_{t-1} \\ + \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin M^i_t} \xi^{i,n}\left(\ell^{i,n}_t(a,\ell)\right) dm^i_{t-1}(a,\ell) + \xi^{E,n}\left(\ell^{E,n}_t\right) m^E_t \\ + \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin M^i_t} \xi^{i,b}\left(b'^{i}_t(a,\ell)\right) dm^i_{t-1}(a,\ell) + \xi^{E,b}\left(b'^{E}_t\right) m^E_t \end{aligned} \tag{31}$$

Equilibrium objects to be computed

- Aggregate prices: r_t^ℓ, q_t, q_t^d
- Endogenous aggregate states: Y_{t-1}, K_{t-1}, K_{t-1}^0, $m_{t-1}^i(a,\ell)$, d_{t-1}^B
- Other endogenous aggregate variables: I_t, C_t, D_t, N_t^n, k_t^*, m_t^E
- Banking industry decisions: $\{c_t^{i,b}(a,\ell), \ell_t^{i,n}(a,\ell), b_t'^i(a,\ell), M_t^i, c_t^E, \ell_t^{E,n}, b_t'^E, q_t^{i,b}(\ell,\ell^n,b')\}$
- Exogenous aggregate variables: θ_t, A_t, π_t
- B_t can be computed once we know the equilibrium path.

Solution algorithm: outline

- The economy is in steady state in $t = 1$ and $t \geq T$
- Banks' problem do not depend on endogenous aggregate quantities, but firms' problem depend on d_t^B. [This isn't the case if a policy rule reacts to, say, aggregate output. But, we can still use what we do here to generate an initial guess.]
- Firm-entry conditions determine r_t^ℓ, given q_t: This process is inexpensive, as opposed to finding q_t^d given q_t and r_t^ℓ from the bank-entry condition
- Thus, our approach is to guess $\{q_t\}_{t=1}^T$, $\{q_t^d\}_{t=1}^T$, $\{d_t^B\}_{t=1}^T$, $\{m_t^E\}_{t=1}^T$ and $\{N_t^n\}_{t=1}^T$, and gradually adjust these objects to meet market-clearing conditions

Solution algorithm: solving backwards

Guess $\{q_t, q_t^d, d_t^B\}_{t=1}^{T-1}$ and start with V_T, Π_T^0 and Π_T^1. For $t = T-1, \dots, 2$,

1. Given r_t^ℓ, q_t, d_t^B, Π_{t+1}^0 and Π_{t+1}^1, compute firms' value functions, (18) and (20), where r_t^ℓ is pinned down by the entry condition (22) given q_{t-1}:

$$
\begin{aligned}
q_{t-1}\Pi_t^0(k_{t-1}^*; r_t^\ell) &= \kappa^{E,f} \\
k_{t-1}^* &= \arg\max_k \Pi_t^0(k; r_t^\ell)
\end{aligned}
$$

2. Solve the bank's problem given q_t^d, r_t^ℓ, q_t and V_{t+1}
3. Using (21), compute k_t^* given q_t and Π_{t+1}^0
4. Using (27) and (28), compute C_t and D_t given q_t and q_t^d

Solution algorithm: integrating forward

In each h-th iteration, do the following for $t = 2, \dots, T-1$, given Y_1, K_1, K_1^0, the decision rules of HHs, banks and firms, and $\{m_t^{E,(h)}, N_t^{n,(h)}\}_{t=1}^{T}$:

1. Aggregate banks' decisions using $m_{t-1}^i(a, \ell)$
2. Aggregate output: $Y_t = A_t (k_t^*)^\alpha N_t^{n,(h)} + (1-\delta_1) Y_{t-1}$
3. Using the goods MCC (31), compute $N_t^{n,*}$:

$$\begin{aligned} Y_t =& C_t + k_t^* N_t^{n,*} + \delta_2 K_{t-1} + \text{loan issuance costs given } m_t^{E,(i)} \\ &+ \text{WSF issuance costs given } m_t^{E,(i)} \end{aligned}$$

4. Given $N_t^{n,*}$, compute $m_t^{E,*}$ using the loan MCC (30):

$$\begin{aligned} & k_t^* N_t^{n,*} + (1-\gamma)\left\{\lambda(1-d_{t-1}^B) + d_{t-1}^B\right\} K_{t-1}^0 \\ &= \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin M_t^i} \ell_t^{i,n}(a,\ell) dm_{t-1}^i(a,\ell) + \ell_t^{E,n} m_t^{E,*} \end{aligned}$$

5. Update the distribution of banks $m_t^i(a, \ell)$, based on banks' decisions and $m_{t-1}^i(a, \ell)$: we can get $d_t^{B,*}$ in this process

Solution algorithm: updating the discount price of deposit

- The deposit MCC (29) implies excess demand for deposits:

$$X_t^d = \sum_i \xi^{i,d} \int_{(a,\ell)\notin M_t^i} dm_{t-1}^i(a,\ell) + \xi^{E,d} m_t^{E,(h)} - D_t \tag{32}$$

- For $\lambda^d < 0$, the updating algorithm for q_t^d is:

$$q_t^{d,(h+1)} = (1 + \lambda^d X_t^d) q_t^{d,(h)} \tag{33}$$

- An intuition here is to make deposit more expensive when its demand exceeds supply

Solution algorithm: updating the discount price of risk-free assets

- From (16), the excess bank-entry condition is:

$$X_t^v = W_t^E(a^E, \ell^E) - \kappa^{E,b} \tag{34}$$

- For $\lambda^v < 0$, the updating algorithm for q_t is:

$$q_t^{(h+1)} = (1 + \lambda^v X_t^d) q_t^{(h)} \tag{35}$$

- An intuition here is to make an entry more costly when the net value of entry is positive

Solution algorithm: updating other guesses

- Updating of d_t^B:

$$d_t^{B,(h+1)} = \gamma^q d_t^{B,*} + (1-\gamma^q) d_t^{B,(h)}$$

- Updating of the measure of bank and firm entry :

$$m_t^{E,(h+1)} = \gamma^m m_t^{E,*} + (1-\gamma^m) m_t^{E,(h)}$$

$$k_t^* N_t^{n,(h+1)} + (1-\gamma)\left\{\lambda(1-d_{t-1}^{B,(h+1)}) + d_{t-1}^{B,(h+1)}\right\} K_{t-1}^{0,(h+1)} = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin M_t^i} \ell_t^{i,n}(a,\ell) dm_{t-1}^i(a,\ell) + \ell_t^{E,n} m_t^{E,(h+1)}$$