Banking Dynamics and Capital Regulation in General Equilibrium

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A Growth Model around a Banking Industry

- There is a Rep hhold
  - It owns a Mutual Fund that yields dividends
  - It gets utility from deposits
  - It holds bonds (risk free in St St, not necessarily so outside)
  - Some of its members work

- Many Putty Clay firms
  - Start up with bank loans. Become equity firms after Calvo shock.
  - All proceeds go to Mutual Funds

- A Banking Industry.
  - Individual Banks make Loans to firms with maturity $\lambda$
  - Borrow and issue deposits
  - Startup costs paid by Mutual Funds with difficulty (via func $u^b$)

- Mutual Funds
  - Manage Loan firms
  - Own Equity firms
  - Open and own banks with transfer difficulties
1 Steady State
Prices, Aggregate Variables, and Other Objects

- **Prices**
  - Interest rate $q$ for bonds: Safe
  - Interest rate $r^\ell$ for loans: Unsafe
  - Interest rate for deposits $q^D$: Safe because insured by Gov.
  - Wage function $w(k, C)$ (I am using a guess and verify based on logs)

- **Quantities**
  - Employment, and Number of Firms/Plants $N$
  - Capital per Plant $K$
  - Output, Cons, Inv, $C + \delta NK = Y = NAK^\alpha$: Intermediate Inputs
  - Loans $L = (1 - \lambda)NK$ V: (Double check, but similar formula)
  - Deposits $D$
  - Bonds $B$
  - Taxes, Banks Loses $T$

- **Other Elements**
  - A Banking Industry with a measure of banks $x$, new entrants $m^E$, and dividends $C^b$
  - Mutual funds that manage/own all firms
**Bank’s Problem**

\[
V^i(a, \ell) = \max \left\{ 0, W^i(a, \ell) \right\}
\]

\[
W^i(a, \ell) = \max_{\ell_n \geq 0, c \geq 0, b'} \left\{ u^b(c^b) + \beta \sum_{i'} \Gamma_{i,i'} \sum_{\delta'} \pi(\delta') V^{i'}[a'(\delta'), \ell'(\delta')] \right\} \quad \text{s.t.}
\]

\[
(\text{TL}) \quad \ell' = (1 - \lambda) (1 - \delta') \ell + (1 - \delta) \ell_n
\]

\[
(\text{TA}) \quad a' = \left( \lambda + r^\ell \right)(1 - \delta')\ell + r^\ell(1 - \delta)\ell_n - \xi^{i,d} - b'
\]

\[
(\text{BC}) \quad c^b + \ell_n + \xi^{i,n}(\ell_n) + \xi^{i,b}(b') \leq a + q^{i,b}(\ell, \ell_n, b')b' + q^d \xi^{i,d}
\]

\[
(\text{KR}) \quad \frac{\ell_n + \ell - q^d \xi^{i,d} - q^{i,b}(\ell, \ell_n, b')b'}{\omega^r(n + \ell) + \omega^s 1_{b' < 0} b' q^{i,b}(\ell, \ell_n, b')} \geq \theta
\]
Some banks go bankrupt when they cannot roll over debt. Let the default set be $M_i(A, L)$

There is entry of new banks, ($m^E$ is the measure of entrants), occurs as long as the free-entry condition is satisfied:

$$W^E(a^E, \ell^E) = u^b(\kappa^{Eb})$$

- $a^E, \ell^E$ is the prespecified values of new entrants.
- Function $u^b(.)$ translates units of the good into units of the objective function of banks
- $\kappa^{E,b}$ is the opening cost of a new bank.
The definition is exactly like the one in the other paper. But for our purposes we need to link it with the rest of the model.

We proceed by specifying what are inputs to the banks

Given safe interest rate, $1/q$, deposit rate $1/q^d$, loan rate $r^\ell$ and cost of entry $\kappa^Eb$, it yields

- A measure of Banks over their states $x$, including entrants $m^E$, and fraction of loans in hands of failing banks $d^B$.

- Total Quantity of Bonds $B$
- Total Quantity of Deposits $D$
- Total Dividends $C^b$
- Total Loses $T$ to be covered by government
- Total resources needed by new entrants $m^E\kappa^Eb$
**Investment and firms: Putty-Clay**

- Under Free Entry, One-Worker Putty-Clay Plants arise: \( y = A k^\alpha \).
- Firms get destroyed with probability \( \delta \). From the point of view of banks \( \delta \sim \gamma_\delta \), with mean \( \delta_1 \).
- Financed with Bank loans of stochastic maturity \( \lambda \). Upon arrival of Maturity, becomes Equity firm. Mutual Fund pays loan.
- All cash flows of firms end up in Mutual Funds.
- Extensive margin: There are \( N^n \) new firms each period.
- Intensive margin: Each period firms invest \( k \) units.
- Total amount of new loans is \( L^n = k N^n \).
- Employment or the number of plants is
  \[
  N' = (1 - \delta_1)N + N^n.
  \]
- Output is
  \[
  Y' = (1 - \delta_1)Y + N^n A k^\alpha.
  \]
Investment and firms: Financing

- Firms must borrow 100% of their investment $k$ from a bank.

- If the Bank does not fail (prob $1 - d^B$), then with probability $1 - \lambda$, the firm continues to be debt-financed and pays interest $kr^\ell$; with probability $\lambda$, a loan terminates. With probability $\gamma$, the firm chooses refinancing by banks. Otherwise, the mutual fund pays $(1 + r^\ell)k$ at the beginning of next period, and the firm becomes an Equity firm.

- If the bank fails (prob $d^B$), we assume that the loan also terminates with prob $\gamma$ and the Mutual pays the government $k(1 + r^\ell + \zeta^F)$. \textbf{V:} What happens with prob $(1 - \lambda)$?

- $d^B$ is the endogenous fraction of loans held by defaulting banks:

$$d^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell) \in D_i} \ell \ dm_i(a,\ell)}{\sum_{i=1}^{N_\xi} \int \ell \ dm_i(a,\ell)}$$
Value of firms: There are measures \( m^0(k, r^\ell) \) and \( m^1(k) \) of them

- Given capital \( k \), the maintenance cost \( \delta_2 \), interest rate \( r^\ell \), wage \( w(k) \), and the repayment cost \( \zeta^F \) when banks default, the value of a loan firm is

\[
\Pi^0(k, r^\ell) = A k^\alpha - w(k) - (r^\ell + \delta_2)k + (1 - d^B)(1 - \lambda)q(1 - \delta_1)\Pi^0(k, r^\ell) \\
+ q(1 - \delta_1) \left\{ \lambda(1 - d^B) + d^B \right\} (1 - \gamma)\Pi^0(k) \\
+ q(1 - \delta_1) \left[ \lambda(1 - d^B) + d^B \right] \gamma [-k + \Pi^1(k)] - q(1 - \delta_1)d^B\gamma \zeta^F k
\]

- The value of an equity firm is

\[
\Pi^1(k) = A k^\alpha - w(k) - \delta_2 k + q(1 - \delta_1)\Pi^1(k)
\]

- Letting \( R(k) = A k^\alpha - w(k) \), \( \Pi^0 < \Pi^1 \) due to loan repayment costs:

\[
\Pi^1(k) = \frac{R(k) - \delta_2 k}{1 - q(1 - \delta_1)}
\]
\[
\Pi^0(k, r^\ell) = \frac{R(k) - \delta_2 k}{1 - q(1 - \delta_1)} - \frac{r^\ell + q(1 - \delta_1)\gamma \left[ \lambda(1 - d^B) + d^B + d^B \zeta^f \right]}{1 - q(1 - \delta_1) \left[ 1 - \gamma \left\{ \lambda(1 - d^B) + d^B \right\} \right]} k
\]
• Given the expected value, a firm chooses the size of capital:

\[ k^* = \arg \max_k \{ q \Pi^0(k, r^\ell) - \kappa^{Ef} \} \]

• With FOC

\[ k^* = \left\{ \frac{(1 - \mu)\alpha A}{r^\ell + q(1-\delta_1)\gamma[\lambda(1-d^B)+d^B+d^B\zeta^f][1-q(1-\delta_1)] + \delta_2} \right\}^{\frac{1}{1-\alpha}} \]

• Firms enter until profits are zero:

\[ \kappa^{E,f} = q\Pi^0(k^*; r^\ell) \]
**Outcome of Investment Decisions**

- Given $r^\ell, q, d^B, L^n, \delta_1$ and wage function $w(k)$

- Pose parameters of firm problem: $\delta_2, A, \alpha, \mu, \bar{b}$

- Yields $k, w, N$, new firms $\delta_1 N$, that satisfy

  1. Wage equation
  2. FOC of firms
  3. Zero Profit Condition
  4. Feasibility: $Y = A N k^\alpha = C + I +$ costs of starting firms and operating banks
  5. $I = (\delta_1 + \delta_2)kN$
Mutual Funds

- Households own Mutual Funds which in turn own firms and banks, but do not trade its shares, just passively receive its dividends.
- Mutual Funds create banks and receive its dividends. Even though, banks assess the dividends according to function $u^b()$. Its cash flow is

$$
\pi^b = \sum_{i=1}^{N_z} \int_{(a, \ell) \notin D_i} c^{i,b}(a, \ell) \, dm^i(a, \ell) + (c^{E,b} - \kappa^{E,b}) m^E
$$

- Mutual Funds manage Loan-firms and own Equity Firms:

$$
\pi^f = Y - \mu Y - (1 - \mu) \bar{b} N - r^e K^0 - (1 - d^B) \lambda K^0 - d^B (1 + \zeta^F) K^0 - \kappa^{E,f} N^n
$$

$$
= \int_{k, r^e} \left[ R^0(k, r^e) - kr^e - (1 - d^B) \lambda k - d^B (1 + \zeta^F) k \right] \, dm^0(k, r^e) + \int_k R^1(k) \, dm^1(k) - \kappa^{E,f} N^n
$$
Outcome of Mutual Funds

• By Aggregation we get Profits to be Distributed to Households. It needs
  1. New Banks Creation
  2. Profits and loses from Banks $C^b$
  3. Cash Flow net of Interest from Loan firms (not zero because of fixed costs)
  4. Loan Repayment
  5. Profits from Equity Firms
Wage Determination

- A bargaining process between the firm and the worker. \( V: \) (We may change this to get more wage rigidity and avoid the Shymer puzzle)

- The bargaining process is repeated every period and if unsuccessful neither firm nor worker can partner with anybody else within a period. We assume that the financial obligations to the bank by the firm do not disappear. Let \( \mu \) be the bargaining weight of the worker and \( \bar{b} \) is workers’ outside option. Then, we have

\[
\begin{align*}
\tilde{w}^0(k) &= \tilde{w}^1(k) = \mu A k^{\alpha} + (1 - \mu)\bar{b}
\end{align*}
\]

- Total (per capita) Labor Income paid in the Economy are

\[
\begin{align*}
W \ N &= N \int \left[ \mu A k^{\alpha} + (1 - \mu)\bar{b} \right] \ di = \mu Y + (1 - \mu)\bar{b}N
\end{align*}
\]
Household

\[ v(a) = \max_{c, b', d'} u(c, d') + \beta v(a') \quad \text{s.t.} \]

\[ c + q^d d' + qb' = a + WN + (1 - N)b + \pi^f + \pi^B - T \]

\[ a' = d' + b' \]

where \( T \) is the taxes needed to pay for bank losses. FOCs:

\[ u_c = \frac{\beta u_c'}{q} \]

\[ u_d = q^d u_c - \beta u_c' \]
The cost of deposit insurance is the amount of deposits that defaulting banks owe minus liquidated capital.

\[ T = N_\xi \sum_{i=1}^{N_\xi} \int_{(a, \ell) \in D} dm^i(a, \ell) - K_0 d^B (1 - \zeta^B) \]

where \( \zeta^B \) is the fraction that the government is unable to recover during the liquidation process.
Output of Household Problem

- Given safe interest rate, $1/q$, deposit rate $1/q^d$, Taxes $T$, wages $W$, Profits $\Pi$, and Bonds $B$, Employment $N$ we obtain

1. Consumption $C$

2. Deposits $D$
Deposits

\[ D' = \sum_{i=1}^{N_\xi} \xi^d_i \int_{(a, \ell) \notin D_i} dm^bi(a, \ell) + \xi^{dE} m^E \]

Bonds

\[ qB' = \sum_{i=1}^{N_\xi} \int_{(a, \ell) \notin D_i} q^{ib} \left( \ell, \ell^{in}(a, \ell), b''(a, \ell) \right) b''(a, \ell) dm^i(a, \ell) + q^{Eb} b'^E m^E \]
Market clearing (continued): V: How does NIPA treat $F$? Intermediate goods?

New loans

$$k^* N^n + (1 - \gamma) \left\{ \lambda (1 - d^B) + d^B \right\} K^0 = \sum_{i=1}^{N\xi} \int_{(a,\ell) \notin D_i} \ell_i^n (a, \ell) dm_i (a, \ell) + \ell_E^n m_E$$

Goods

$$Y = C + kN^n + \delta_2 kN +$$

$$+ \sum_{i=1}^{N\xi} \int_{(a,\ell) \notin D_i} \xi^n_i \left( \ell_i^n (a, \ell) \right) dm_i (a, \ell) + \xi^n_E \left( \ell_E^n \right) \quad \text{(Bank loan issuance costs)}$$

$$+ \sum_{i=1}^{N\xi} \int_{(a,\ell) \notin D_i} \xi_b \left( b_i'(a, \ell) \right) dm_i (a, \ell) + \xi^b_E \left( b_E' \right) \quad \text{(Bank bond issuance costs)}$$

$$+ \kappa^b_E m_E + \kappa^f_E N^n \quad \text{(Entry costs)}$$

$$+ d^B (\zeta^B + \zeta^F) K^0 \quad \text{(Bank default costs)}$$
Households: $u(C, D, N) = \log(C) + \eta^D \log(D)$,

$$q = \beta \quad (1)$$

$$\frac{\eta^D C}{D} = q^d - \beta \quad (2)$$

Firms:

$$k^* = \left\{ \frac{(1 - \mu)A}{\left[ r^\ell + q(1 - \delta_1) \gamma \{ \lambda(1 - d^B) + d^B + d^B \zeta^f \} \right] [1 - q(1 - \delta_1)] + \delta_2} \right\}^{\frac{1}{1 - \alpha}} \quad (3)$$

$$\kappa^{E_F} = q \, \Pi^0 \quad (4)$$

$$\Pi^0 = \frac{(1 - \mu) \left( A(k^*)^\alpha - \bar{b} \right) - \delta_2 k^*}{1 - q(1 - \delta_1)}$$

$$- \frac{r^\ell + q(1 - \delta_1) \gamma \left[ \lambda(1 - d^B) + d^B + d^B \zeta^f \right]}{1 - q(1 - \delta_1) [1 - \gamma \{ \lambda(1 - d^B) + d^B \}]} k^* \quad (5)$$
**Steady state conditions (2)**

**Wages:**

\[ w = \mu A(k^*)^\alpha + (1 - \mu)b \quad (6) \]

**Banks:**

\[ \kappa^{E,b} = W^E(a^E, \ell^E) \quad (7) \]

\[ dB = \frac{\sum_{i=1}^{N} \int_{(a,\ell)\in D} \ell \ dm^i(a,\ell)}{\sum_{i=1}^{N} \int \ell \ dm^i(a,\ell)} \quad (8) \]
Market clearing conditions:

\[ D = \sum_{i=1}^{N_\xi} \xi_{i,d} \int_{(a,\ell) \notin D_i} dm^i(a, \ell) + \xi_{E,d} m^E \]  \hfill (9)

\[ qB = \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin D_i} q^{i,b}(\ell, \ell^{i,n}(a, \ell), b'^{i}(a, \ell)) b'^{i}(a, \ell) dm^i(a, \ell) + q^{E,b} b'^E m^E \]  \hfill (10)

\[ k^* N^n + (1 - \gamma) \{ \lambda (1 - d^B) + d^B \} K^0 = \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin D_i} \ell^{i,n}(a, \ell) dm^i(a, \ell) + \ell^{E,n} m^E \]  \hfill (11)

\[ Y = C + k^* N^n + \delta_2 kN + \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin D_i} \xi_{i,n} \left( \ell^{i,n}(a, \ell) \right) dm^i(a, \ell) + \xi_{E,n} \left( \ell^{E,n} \right) m^E \]

\[ + \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin D_i} \xi_{i,b} \left( b'^{i}(a, \ell) \right) dm^i(a, \ell) + \xi_{E,b} \left( b'^E \right) m^E \]

\[ + \kappa_{E,b} m^E + \kappa_{E,f} N^n + d^B (\zeta^B + \zeta^F) K^0 \]  \hfill (12)
**Steady state conditions (4)**

Laws of motion:

\[ Y = \frac{A(k^*)^\alpha N^\delta}{\delta_1} \quad (13) \]

\[ N = \frac{N^n}{\delta_1} \quad (14) \]

\[ K^0 = \frac{k^* N^n}{1 - (1 - \delta_1) [1 - \gamma \{\lambda (1 - d^B) + d^B\}]} \quad (15) \]

Aggregate endogenous variables:

\[ C, D, B, k^*, K^0, N^n, N, Y, d^B, m(a, \ell), m^E, q, q^d, r^\ell, w \]

Parameters:

HHs: \( \beta, \mu, \overline{b}, \eta^D \)

Firms: \( \alpha, A, \kappa^{E,f}, \zeta^F \)

Banks: \( u^b(), \beta^B, \lambda, \xi^{i,d}, \xi^{i,n}, \xi^{i,b}, F(\delta'), \kappa^{E,b}, \zeta^B \)
Algorithm to get Steady State

• Set Parameters of Banking: $u^b()$, $\beta^B$, $\lambda$, $\xi^i_d$, $\xi^i_n$, $\xi^i_b$, $F(\delta')$ and prices $r^\ell$, $q^d$, $q$.
V: (may come back to this)

• Compute the banking industry equilibrium. Get loans $L$, deposits $D$ bank dividends $C^b$, losses $T$, resources for new entrants $m^E \kappa^Eb$.

• Set HH preference parameters $\beta$, $\overline{b}$, $\eta_D$, and the bargaining power $\mu$ so that they are consistent with $q$, the observed consumption-to-deposit ratio and the labor share of $2/3$.

• Set Technology $A$, $\alpha$ as well as $\delta_2$ and $\zeta^F$ to solve the firms’ problem. Given $\alpha$ and $\delta_2$, adjust $A$ to make sure that all markets clear.
V: (I think that $\lambda$ doesn’t matter much so we should set this to get the equity/debt ratio of the nonfinancial sector and a normalize)

• Generate key moments of interest.
• We target labor share and the outside option for workers $\bar{b} = \phi_b w$:

$$LS = \mu + (1 - \mu) \frac{bN}{Y} = \mu + (1 - \mu) \frac{\phi_b w}{A(k^*)^\alpha}$$

$$w = \mu A(k^*)^\alpha + (1 - \mu) \bar{b} = \mu A(k^*)^\alpha + (1 - \mu) \phi_b w$$

• Solving the two conditions simultaneously,

$$\mu = \frac{(1 - \phi_b)LS}{1 - \phi_b LS}$$

$$w(k^*) = \frac{\mu}{1 - (1 - \mu)\phi_b} A(k^*)^\alpha$$

$$\bar{b} = \phi_b w(k^*)$$

• $LS = 2/3$ and $\phi_b = 0.9$ imply $\mu = 1/6$. 
**Endogenous Variables**

- $\beta = q$ by (1)
- $N^n = \delta_1 \overline{N}$ by (14), where $\overline{N} = 0.9$.
- The banking industry equilibrium gives $L^n$: back out $k^*$ from (11).
- Set $A$ so that the loan demand (3) is equal to the loan supply.
- $Y = Ak^*\overline{N}$ and $I = (\delta_1 + \delta_2)k^*N$.
- Compute $K^0$ from (15)
- $C$ is determined as a residual in (12)
• For simplicity, we ignore various intermediate costs for now

• Consumption-deposit ratio:

\[
\frac{C}{D} = \frac{C \cdot L^n}{L^n \cdot D} = k^* \delta_1 \bar{N} \left[ 1 + \frac{(1 - \gamma) \left\{ \lambda(1 - d^B) + d^B \right\}}{1 - (1 - \delta_1)(1 - \gamma \left\{ \lambda(1 - d^B) + d^B \right\})} \right] \frac{L^n}{D}
\]

\[
= \frac{A(k^*)^{\alpha - 1} - \delta_1 - \delta_2}{\delta_1 \left[ 1 + \frac{(1 - \gamma) \left\{ \lambda(1 - d^B) + d^B \right\}}{1 - (1 - \delta_1)(1 - \gamma \left\{ \lambda(1 - d^B) + d^B \right\})} \right]} \frac{L^n}{D}
\]

\[
= \left[ \frac{1}{k^* \delta_1 \left[ 1 + \frac{(1 - \gamma) \left\{ \lambda(1 - d^B) + d^B \right\}}{1 - (1 - \delta_1)(1 - \gamma \left\{ \lambda(1 - d^B) + d^B \right\})} \right]} - \frac{\delta_1 + \delta_2}{\delta_1 \left[ 1 + \frac{(1 - \gamma) \left\{ \lambda(1 - d^B) + d^B \right\}}{1 - (1 - \delta_1)(1 - \gamma \left\{ \lambda(1 - d^B) + d^B \right\})} \right]} \right] \frac{L^n}{D}
\]

• With \( K/Y = 3, \ L^n/D = 0.9, \ \delta_1 = 0.02, \ \delta_2 = 0.08, \ \gamma = \lambda = 0.5, \ d^B = 0, \) consumption-deposit ratio is about 5.4.
2 Equilibrium in Terms of Sequences
Existing bank's problem given $V_{t+1}, r_t^\ell, q_t^d, \theta_t$

$$V_t^i(a, \ell) = \max \{0, W_t^i(a, \ell)\}$$

$$W_t^i(a, \ell) = \max_{\ell^n \geq 0, c \geq 0, b'} u^b(c^b) + \beta^b \sum_i \Gamma_i \sum_{\delta'} \left\{ \pi_t(\delta') V_{t+1}^i[a'(\delta'), \ell'(\delta')] \right\} \ s.t.$$

\begin{align*}
(TL) & \quad \ell' = (1 - \lambda) (1 - \delta') \ell + (1 - \bar{\delta}) \ell^n \\
(TA) & \quad a' = (\lambda + r_t^\ell)(1 - \delta') \ell + \lambda(1 - \bar{\delta}) \ell^n - \xi_i^i, d - b' \\
(BC) & \quad c^b + \ell^n + \xi_i^i, n(\ell^n) + \xi_i^i, b'(b') \leq a + q_t^{i, b}(\ell, \ell^n, b')b' + q_t^d \xi_i^i, d \\
(KR) & \quad \frac{\ell^n + \ell - q_t^d \xi_i^i, d - q_t^{i, b}(\ell, \ell^n, b')b'}{\omega_t^t(n + \ell) + \omega_t^s 1_{b' < 0} b' q_t^{i, b}(\ell, \ell^n, b')} \geq \theta_t
\end{align*}

$\pi_t$ is an exogenous aggregate shock.

$\theta_t$ is exogenous. A feedback rule to be considered in the next step.
ENTRY AND EXIT OF BANKS

- Entry condition:
  \[ W_t^E (a^E, \ell^E) = u^b (\kappa^{E,b}) \]  
  (16)

- A fraction of loans destroyed by bank default:
  \[
  d^B_{t-1} = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell) \in M_t} \ell \, dm^i_{t-1}(a, \ell)}{\sum_{i=1}^{N_\xi} \int \ell \, dm^i_{t-1}(a, \ell)}
  \]  
  (17)
EQUITY-FINANCED FIRM’S VALUE GIVEN $\Pi_{t+1}^1, q_t$

- The value is

$$\Pi_t^1(k) = A_t k^\alpha - w_t(k) - \delta_2 k + q_t(1 - \delta)\Pi_{t+1}^1(k)$$  \hspace{1cm} (18)

- The wage is given by

$$w_t(k) = \mu A_t k^\alpha + (1 - \mu)\bar{b}$$  \hspace{1cm} (19)
The value of bank-financed firm is

$$
\Pi_t^0(k) = Ak^\alpha - w(k) - (r_t^\ell + \delta_2)k + (1 - d_t^B)(1 - \lambda)q_t(1 - \delta_1)\Pi_{t+1}^0(k) \\
+ q_t(1 - \delta_1) \left\{ \lambda(1 - d_t^B) + d_t^B \right\} (1 - \gamma)\Pi_{t+1}^0(k) \\
+ q_t(1 - \delta_1) \left[ \lambda(1 - d_t^B) + d_t^B \right] \gamma \left[ -k + \Pi_{t+1}^1(k) \right] \\
- q_t(1 - \delta_1)d_t^B \gamma \zeta F_{k^*} \tag{20}
$$

Given $q_t$ and $\Pi_{t+1}^0$, entrants choose $k_t^*$:

$$
k_t^* = \arg \max_k \left\{ q_t\Pi_{t+1}^0(k) - \kappa^{E,f} \right\} \tag{21}
$$

Entry occurs until firms break even ex-ante

$$
q_t\Pi_{t+1}^0(k_t^*) = \kappa^{E,f} \tag{22}
$$
**Aggregate output, investment and capital stock**

- Aggregate output:
  
  \[ Y_t = A_t (k_t^*)^\alpha N_t^n + (1 - \delta_1) Y_{t-1} \]  
  \[ (23) \]

- Aggregate investment:
  
  \[ I_t = k_t^* N_t^n + \delta_2 K_{t-1} \]  
  \[ (24) \]

- Aggregate capital:
  
  \[ K_t = k_t^* N_t^n + (1 - \delta_1) K_{t-1} \]  
  \[ (25) \]

- Aggregate capital held by bank-financed firms:
  
  \[ K_0^t = k_t^* N_t^n \]

  \[ + \left[ (1 - d_{t-1}^B)(1 - \lambda) + (1 - \gamma) \left\{ \lambda(1 - d_{t-1}^B) + d_{t-1}^B \right\} \right] (1 - \delta_1) K_{t-1}^0 \]  
  \[ (26) \]
• Consumption Euler equation:

\[ u_{c,t} = \beta \frac{u_{c,t+1}}{q_t} \]  \hspace{1cm} (27)

• Consumption-deposit marginal condition:

\[ u_{d,t} = q_t^d u_{c,t} - \beta u_{c,t+1} \]  \hspace{1cm} (28)
Market clearing conditions

\[ D_t = \sum_{i=1}^{N} \xi^{i,d} \int_{(a,\ell) \notin M_{t-1}^{i}} dm_{t-1}^{i}(a, \ell) + \xi^{E,d} m_{t}^{E} \]  \hspace{1cm} (29)

\[ k_{t}^{*} N_{t}^{n} + (1 - \gamma) \left\{ \lambda (1 - d_{t-1}^{B}) + d_{t-1}^{B} \right\} K_{t-1}^{0} = \sum_{i=1}^{N} \int_{(a,\ell) \notin M_{t}^{i}} \ell_{i}^{n}(a, \ell) dm_{t-1}^{i}(a, \ell) + \ell_{t}^{E,n} m_{t}^{E} \]  \hspace{1cm} (30)

\[ Y_{t} = C_{t} + k_{t}^{*} N_{t}^{n} + \delta_{2} K_{t-1} \]

\[ + \sum_{i=1}^{N} \int_{(a,\ell) \notin M_{t}^{i}} \xi^{i,n} \left( \ell_{i}^{n}(a, \ell) \right) dm_{t-1}^{i}(a, \ell) + \xi^{E,n} \left( \ell_{t}^{E,n} \right) m_{t}^{E} \]

\[ + \sum_{i=1}^{N} \int_{(a,\ell) \notin M_{t}^{i}} \xi^{i,b} \left( b_{i}^{t}(a, \ell) \right) dm_{t-1}^{i}(a, \ell) + \xi^{E,b} \left( b_{t}^{E} \right) m_{t}^{E} \]  \hspace{1cm} (31)
Equilibrium objects to be computed

- Aggregate prices: $r_t^e, q_t, q_t^d$

- Endogenous aggregate states: $Y_{t-1}, K_{t-1}, K_{t-1}^0, m_{t-1}^i(a, \ell), d_{t-1}^B$

- Other endogenous aggregate variables: $I_t, C_t, D_t, N_t^n, k_t^*, m_t^E$

- Banking industry decisions:
  $\{c_t^{i,b}(a, \ell), \ell_t^{i,n}(a, \ell), b_t'^i(a, \ell), M_t^i, c_t^E, \ell_t^{E,n}, b_t'^E, q_t^{i,b}(\ell, \ell^n, b')\}$

- Exogenous aggregate variables: $\theta_t, A_t, \pi_t$

- $B_t$ can be computed once we know the equilibrium path.
The economy is in steady state in $t = 1$ and $t \geq T$

Banks’ problem do not depend on endogenous aggregate quantities, but firms’ problem depend on $d^B_t$. [This isn’t the case if a policy rule reacts to, say, aggregate output. But, we can still use what we do here to generate an initial guess.]

Firm-entry conditions determine $r^\ell_t$, given $q_t$: This process is inexpensive, as opposed to finding $q^d_t$ given $q_t$ and $r^\ell_t$ from the bank-entry condition

Thus, our approach is to guess $\{q_t\}_{t=1}^T$, $\{q^d_t\}_{t=1}^T$, $\{d^B_t\}_{t=1}^T$, $\{m^E_t\}_{t=1}^T$ and $\{N^n_t\}_{t=1}^T$, and gradually adjust these objects to meet market-clearing conditions
**Solution Algorithm: Solving Backwards**

Guess \( \{q_t, q_t^d, d_t^B\}_{t=1}^{T-1} \) and start with \( V_T, \Pi_T^0 \) and \( \Pi_T^1 \). For \( t = T - 1, \ldots, 2 \),

1. Given \( r_t^L, q_t, d_t^B, \Pi_{t+1}^0 \) and \( \Pi_{t+1}^1 \), compute firms’ value functions, (18) and (20), where \( r_t^L \) is pinned down by the entry condition (22) given \( q_{t-1} \):

   \[
   q_{t-1} \Pi_t^0(k_{t-1}^*, r_t^L) = \kappa_{E,f}
   \]

   \[
   k_{t-1}^* = \arg \max_k \Pi_t^0(k; r_t^L)
   \]

2. Solve the bank’s problem given \( q_t^d, r_t^L, q_t \) and \( V_{t+1} \)

3. Using (21), compute \( k_t^* \) given \( q_t \) and \( \Pi_{t+1}^0 \)

4. Using (27) and (28), compute \( C_t \) and \( D_t \) given \( q_t \) and \( q_t^d \)
In each $h$-th iteration, do the following for $t = 2, \ldots, T - 1$, given $Y_1$, $K_1$, $K_1^0$, the decision rules of HHs, banks and firms, and $\{m_{t}^{E,(h)}, N_{t}^{n,(h)}\}_{t=1}^{T}$:

1. Aggregate banks’ decisions using $m_{t-1}^{i}(a, \ell)$

2. Aggregate output: $Y_t = A_t(k_t^*)^{\alpha} N_{t}^{n,(h)} + (1 - \delta_1) Y_{t-1}$

3. Using the goods MCC (31), compute $N_{t}^{n,*}$:

   \[ Y_t = C_t + k_t^* N_{t}^{n,*} + \delta_2 K_{t-1} + \text{loan issuance costs given } m_{t}^{E,(i)} + \text{WSF issuance costs given } m_{t}^{E,(i)} \]

4. Given $N_{t}^{n,*}$, compute $m_{t}^{E,*}$ using the loan MCC (30):

   \[ k_t^* N_{t}^{n,*} + (1 - \gamma) \left\{ \lambda(1 - d_{t-1}^B) + d_{t-1}^B \right\} K_{t-1}^0 = \sum_{i=1}^{N_{\xi}} \int M_{t} \ell_{t-1}^{i,n}(a, \ell) dm_{t-1}^{i}(a, \ell) + \ell_{t}^{E,n} m_{t}^{E,*} \]

5. Update the distribution of banks $m_{t}^{i}(a, \ell)$, based on banks’ decisions and $m_{t-1}^{i}(a, \ell)$: we can get $d_{t}^{B,*}$ in this process
Solution algorithm: updating the discount price of deposit

- The deposit MCC (29) implies excess demand for deposits:

\[ X_t^d = \sum_i \xi^{i,d} \int_{(a,\ell) \notin M_t} dm_{i-1}^i(a, \ell) + \xi^{E,d} m^E_t \cdot (h) - D_t \]  

(32)

- For \( \lambda^d < 0 \), the updating algorithm for \( q_t^d \) is:

\[ q_{t, (h+1)}^d = (1 + \lambda^d X_t^d) q_{t}^d \]

(33)

- An intuition here is to make deposit more expensive when its demand exceeds supply
Solution Algorithm: Updating the Discount Price of Risk-Free Assets

- From (16), the excess bank-entry condition is:

\[ X_t^v = W_t^E(a^E, \ell^E) - \kappa^{E,b} \]  \hspace{1cm} (34)

- For \( \lambda^v < 0 \), the updating algorithm for \( q_t \) is:

\[ q_t^{(h+1)} = (1 + \lambda^v X_t^d) q_t^{(h)} \]  \hspace{1cm} (35)

- An intuition here is to make an entry more costly when the net value of entry is positive
Solution algorithm: updating other guesses

- Updating of $d_t^B$:

$$d_t^{B,(h+1)} = \gamma^q d_t^{B,*} + (1 - \gamma^q) d_t^{B,(h)}$$

- Updating of the measure of bank and firm entry:

$$m_t^{E,(h+1)} = \gamma^m m_t^{E,*} + (1 - \gamma^m) m_t^{E,(h)}$$

$$k_t^* N_t^{n,(h+1)} + (1 - \gamma) \left\{ \lambda (1 - d_{t-1}^{B,(h+1)}) + d_{t-1}^{B,(h+1)} \right\} K_{t-1}^{0,(h+1)}$$

$$= \sum_{i=1}^{N_\xi} \int_{(a,\ell) \in M_t^i} \ell_t^{i,n} (a, \ell) dm_{t-1}^i (a, \ell) + \ell_t^{E,n} m_t^{E,(h+1)}$$