Default and Aggregate Fluctuations in Growth Economies

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Very Preliminary

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Introduction

- **We explore the role of consumer credit in shaping the properties of business cycles.**

- **In our environment consumers can and do file for consumer bankruptcy as they do in the U.S. (Chatterjee, Corbae, Nakajima, and Ríos-Rull (2001)). In recessions credit availability interacts with and difficults economic activity.**

- **We want to know whether by explicit exploring this channel we get different answers about business cycles than with standard models.**

- **We want to know what features of business cycles interact the most with credit frictions.**

The How

- **We build a heterogeneous agents model with**

  - **U.S. bankruptcy regulations.**

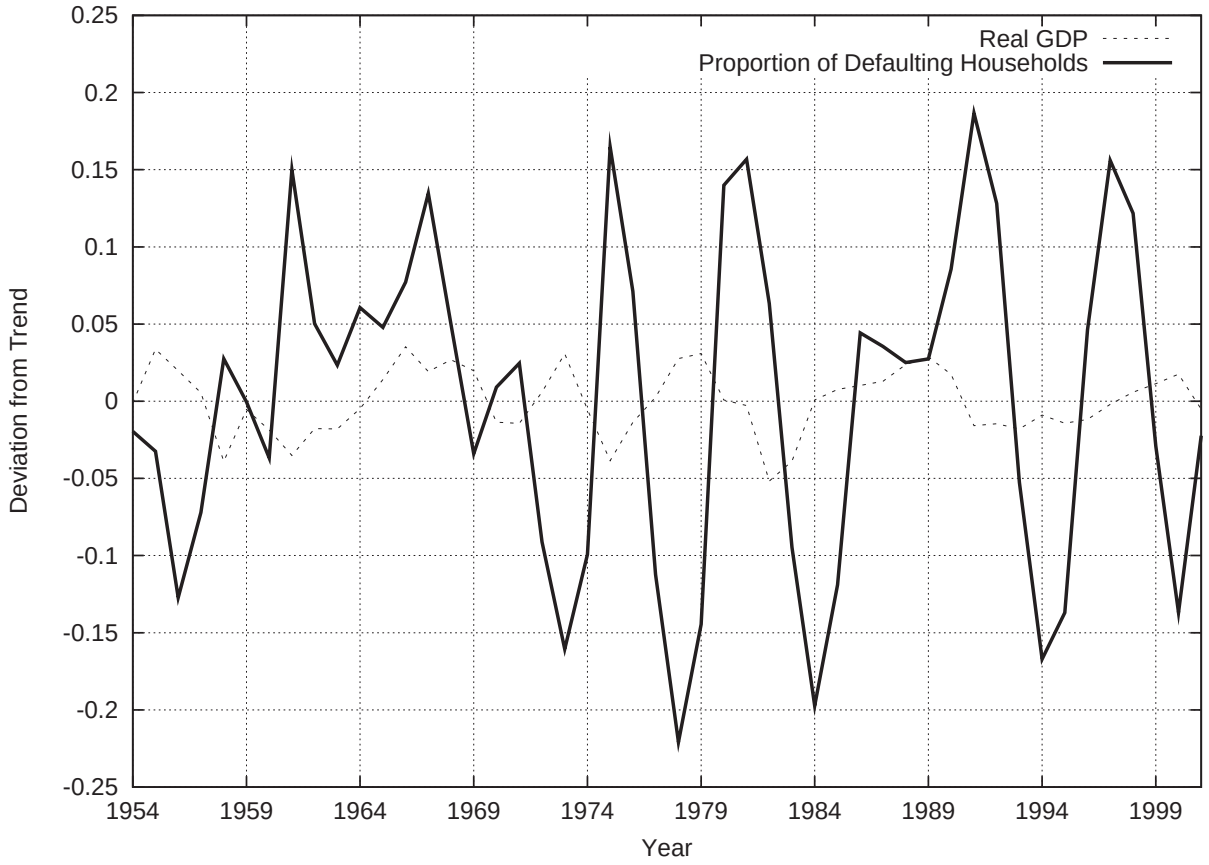
  - **A competitive loan industry with free entry (where lenders can offer any menu of loan sizes and borrowing rates, and expected profit of any lenders is zero in equilibrium).**

  - **The production structure of the growth model mapped into a modern economy.**

  - **A variety of aggregate and idiosyncratic shocks that trigger fluctuations.**

Bankruptcies in U.S.

- **While there has been a drastic increase in the # consumer bankruptcies, the cyclical properties look stable over the whole sample and they are that much more volatile than output, and slightly countercyclical or zero.**



U.S. Economy: Annual Cyclical Statistics (1954-2001)

Variable		SD%/	Cross-Correlation of Y with				
	SD%	SD%Y	X(t-2)	X(t-1)	X(t)	X(t+1)	X(t+2)
Output	**2.13**	1.00	0.02	0.52	**1.00**	0.52	0.02
Consumption	**1.24**	0.59	-0.07	0.46	**0.88**	0.63	0.24
Investment	**7.05**	3.32	0.11	0.51	**0.89**	0.23	-0.32
Aggregate Hours	**2.26**	1.11	-0.25	0.28	**0.91**	0.57	-0.11
Filing HHs	**10.61**	4.99	0.05	-0.11	**-0.26**	0.06	0.47

Bankruptcy is... from (Chatterjee et al. 2001)

- **We look at Chapter 7 bankruptcies (the most popular by far, a little over a million each year). An indebted person files for bankruptcy, and upon successful completion of the process (a very easy thing that lasts three or four months):**
  - **the person's assets above a certain level (varies by state) are liquidated,**
  - **the person's debts disappear, and creditors lose any rights to recover the debts by future income,**
  - **the person gets to keep its future income, and**
  - **the person cannot file again for seven years,**
  - **after ten years, the bad credit history disappears.**

We Interpret Bankruptcy as...

- **With a good credit history, an agent can borrow and file for bankruptcy.**
- **Upon bankruptcy:**
  - **Its debts disappear; its creditors lose any future claims to debts.**
  - **In the filing period, the agent cannot save and must consume all of its current earnings.**
  - **Its credit history turns bad.**
- **With a bad credit history:**
  - **The agent cannot borrow but can save.**
  - **It suffers some inconveniences (bonded credit cards) that we model as a proportional γ loss of earnings.**
  - **Upon termination of the punishment period (10 years), the agent's credit history turns good.**

The Model

- There is a continuum of households that are subject to persistent, aggregate shocks z, as well as to uninsured, persistent, idiosyncratic shocks (the actual model also has demographics).

- There are idiosyncratic shocks to preferences θ, to efficiency units of labor e, to the parameters that govern future distributions of efficiency units of labor ϵ, and to asset destruction λ.

- A household decides:
(i) how much to work, save and consume, and
(ii) (if it is an option) whether to default or not.

- Free entry in the credit market. Firms in the credit industry operate at zero costs. All loans are one–period loans.

- The bankruptcy scheme is that of the U.S.

Household Problem [1]

- **Households are infinitely-lived and maximize expected discounted sum of period utilities with idiosyncratic multiplicative shocks θ.**

- **Aggregate states are:**
(i) z: aggregate shock, which follows a Markov process, and
(ii) x: distribution of households over assets and shocks.

- **Individual states are:**
(i) ϵ: shock that determines the c.d.f of eff units of labor.
(ii) $e \in E = [\underline{e}, \bar{e}]$: eff units of labor which are drawn from the distribution that depends on ϵ. The cdf is then $F(e|\epsilon)$.
(iii) θ: Shock to marginal utility: $\theta\, u(c, h)$.
(iv) λ: Asset destruction shock.
(v) b: credit history, either GOOD (0) or BAD (1)
(vi) $a \in L = \{a_{\min}, \cdots, 0, \cdots, a_{\max}\}$: Asset

- **$s = (\epsilon, \theta, \lambda)$ follows a Markov process. The process can depend on aggregate shocks.**

Case 1: Non-Delinquent and Non-Defaulting

- **Conditional on NOT DEFAULTING, and on V being concave in a, households solve the following concave problem:**

$$\xi_n(z,x,s,e,0,a) = \max_{c,h,a'} \left\{ \theta\, u(c,h) + \beta \sum_{z',s'} \Gamma_{z's'|zs}\; V(z',x',s',0,a') \right\}$$

$$\begin{aligned} c + a'Q &\leq a\,R + h\,e\,w(z,x) \\ x' &= \varphi(z,x) \end{aligned}$$

Notice that for convenience of notation $R = (1 + r(z,x) - \delta)\; a \geq 0$, while $R = 1$ when $a < 0$ (equity).
Also, $Q = 1$ when $a' \geq 0$, and $Q = q(z,x,s,a')$ if $a' < 0$ (uncontingent debt).

Case 2: Non-Delinquent and Defaulting

• **Conditional on DEFAULTING, and on V being concave in a, households solve the following concave problem:**

$$\xi_d(z,x,s,e,0,a) = \max_{c,h} \left\{ \theta\, u(c,h) + \beta \sum_{z',s'} \Gamma_{z's'|sz}\, V(z',x',s',1,a') \right\}$$

$$\begin{aligned} c &\leq h\, e\, w(z,x) \\ x' &= \varphi(z,x) \end{aligned}$$

Case 3: Delinquent

• **Delinquent households solve the following concave (as long as V is concave) problem**

$$\xi(z,x,s,e,1,a) = \max_{c,h,a'} \left\{ \theta\, u(c,h) + \beta \sum_{z's'b'} \Gamma_{z's'|sz}\, \pi_{bb'} V(z',x',s',b',a') \right\}$$

$$\begin{aligned} c + a' &\leq a\,(1 + r(z,x) - \delta) + h\, e\, w(z,x) \\ a' &\geq 0 \\ x' &= \varphi(z,x) \end{aligned}$$

Solving the Value Function

- **Fortunately integrating ξ preserves concavity.**

$$V(z,x,s,b,a) = \int_E \max_{0,1}\{\xi_d(z,x,s,e,0,a), \xi_n(z,x,s,e,1,a)\}\ dF(e|s)$$

- **The solution is (typically, but not always) to default only below certain threshold of earnings that depends on all other variables. Conditional on the default decision, the decision rules are monotonic.**

- **At this stage, we also obtain the probability of default**

$$p(z,x,s,a) = \int_E \operatorname*{argmax}_{0,1}\{\xi_d(z,x,s,e,0,a), \xi_n(z,x,s,e,1,a)\}\ dF(e|s)$$

Unsecured Credit Industry

- **The lending firms are competitive, have zero costs and free entry. Their problem is static.**

- **Firms do offer different prices for each type and each debt level so their expected profits are zero for each loan type.**

- **More specifically, the prices of bonds satisfy:**

$$q(z,x,s,a') = \sum_{z's'} \Gamma_{s'z'|sz} \; r(z',\varphi(z,x)) \; [1-p(z',\varphi(z,x),s',a')]$$

- **Note that actual profits may be positive or negative depending on tomorrow's aggregate state. Recessions may lower relevant rates of return but may increase the likelihood of default.**

Equilibrium

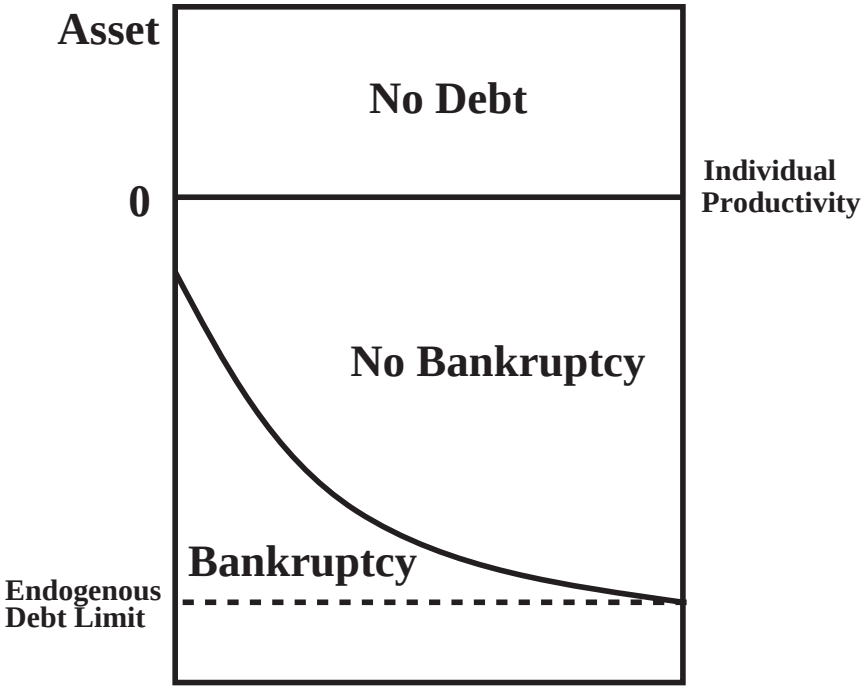
- **Given forecasting function $\varphi(z,x)$ for the distribution of agents, and pricing functions $r(z,x), w(z,x), q(z,x,s,a')$, the value function $V(z,x,s,b,a)$ solves agents' problems.**

- **Given forecasting function $\varphi(z,x)$, the bond price function $q(z,x,s,a')$ satisfies the expected zero profit condition of lending firms.**

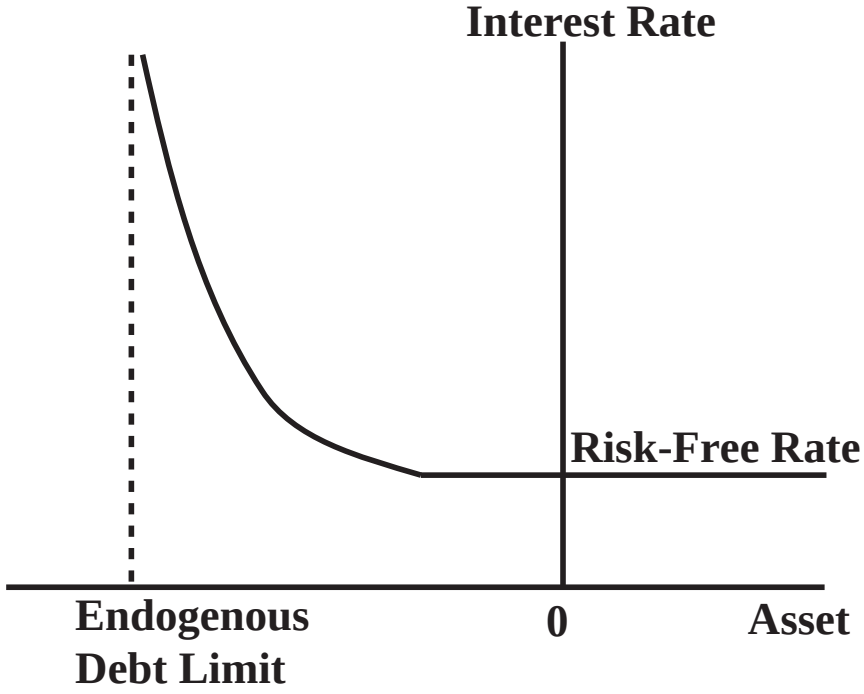
- **Given forecasting function $\varphi(z,x)$, pricing functions $r(z,x), w(z,x)$ are generated by marginal productivities of factors of production which as in growth models come from CRS technology.**

- **Forecasting function $\varphi(z,x)$ is generated by the optimal choices of households.**

Typical Bankruptcy Set and Interest Rate



(a) **Bankruptcy Set**



(b) **Interest Rate**

Approximation Method and Computation Method

- **We follow the insight of Krusell and Smith (1998) and especially Krusell and Smith (1997) to approximate forecasting functions.**

- **Specifically:**

  - **We pick a set of statistics $S = \{K, B^-, \mu^-\}$ that forecast prices and future aggregate states accurately enough.**

  - **We substitute $x' = \varphi(z, x)$ by $S' = \tilde{\varphi}(z, S)$.**

  - **We set an initial guess for $\tilde{\varphi}(z, S)$, solve the optimal decisions of households and firms, run a simulation and update the guess with a new regression.**

  - **We continue this procedure until we find a fixed point of the forecasting functions.**

Putting the Model to Work

- **First, we calibrate the deterministic version of the model to U.S. non cyclical data:**
  - **Average Macroeconomic Statistics**
  - **Distributional Statistics**
  - **Recent Bankruptcy Facts**
- **Then we specify the aggregate shocks and calibrate the parameters associated with aggregate shock to match U.S. business cycle statistics (output volatility and the fact that recessions are shorter).**
- **As our baseline model, we use only shocks to TFP, which means that there are no distributional effect of aggregate shocks.**

Mapping the Model to Data

Statistic	Target	Model
Basic Aggregate Targets		
Wealth to Output Ratio	3.32	3.32
Labor Share	0.64	0.64
Prop of Hours Spent on Working	0.31	0.32
Cross-Sect (St Dev Log Cons / St Dev Log Hours)	5.00	5.23
Distribution Related Targets		
Population Turnover Rate	2.5%	2.5%
Earnings Gini	0.61	0.62
Wealth Gini	0.80	0.71
Default Related Targets		
Households filing Bankruptcy	0.54%	0.46%
Average Length of Punishment	7 years	7 years
Households with Zero or Negative Assets	9.9%	11.8%
Debt to Output Ratio	1.2	0.8

Distributional Statistics

Statistic	U.S. Economy	Model Economy
Earnings Gini	0.61	0.62
Earnings Held by 1st Quintiles	-0.002	0.02
Earnings Held by 2nd Quintiles	0.04	0.04
Earnings Held by 3rd Quintiles	0.13	0.08
Earnings Held by 4th Quintiles	0.23	0.20
Earnings Held by 5th Quintiles	0.60	0.65
Wealth Gini	0.80	0.71
Wealth Held by 1st Quintiles	-0.003	0.003
Wealth Held by 2nd Quintiles	0.01	0.05
Wealth Held by 3rd Quintiles	0.05	0.09
Wealth Held by 4th Quintiles	0.12	0.14
Wealth Held by 5th Quintiles	0.82	0.72

U.S. and Model Economy: Cyclical Statistics

Variable	SD%/ SD%Y	Cross-Correlation of Y with X(t-2)	X(t-1)	X(t)	X(t+1)	X(t+2)
U.S. Economy (48 Periods: 1954-2001)						
Output	**1.00**	0.02	0.52	**1.00**	0.52	0.02
Consumption	**0.59**	-0.07	0.46	**0.88**	0.63	0.24
Investment	**3.32**	0.11	0.51	**0.89**	0.23	-0.32
Earnings	**1.05**	-0.16	0.39	**0.91**	0.71	0.23
Aggregate Hours	**1.11**	-0.25	0.28	**0.91**	0.57	-0.11
Filing HHs	**4.99**	0.05	-0.11	**-0.26**	0.06	0.47
Hours per Worker	**0.20**	0.08	0.37	**0.58**	-0.25	-0.68
Baseline Model Economy (48 Periods)						
Output	**1.00**	-0.16	0.15	**1.00**	0.15	-0.16
Consumption	**0.25**	-0.37	-0.12	**0.77**	0.53	0.25
Investment	**2.71**	-0.11	0.20	**0.99**	0.07	-0.23
Earnings	**1.00**	-0.16	0.15	**1.00**	0.15	-0.16
Hours	**0.15**	0.08	0.33	**0.83**	-0.25	-0.47
Filing HHs	**0.75**	0.25	-0.01	**-0.61**	-0.18	-0.41
Labor Input	**0.07**	0.11	0.34	**0.79**	-0.29	-0.49

• Standard business cycles features

– **Consumption fluctuates less than output and is procyclical.**

– **Investment is much more volatile than output and highly procyclical.**

– **Hours fluctuate much less than output and is procyclical, perhaps even more than data (Kydland and Prescott, Hansen). Productivity is more procyclical than the measured Solow residual.**

• **Business cycle properties of the number of bankruptcies:**

– **Number of bankruptcies fluctuates much more than output. The volatility in the model is similar to that in the data.**

– **Number of bankruptcies is countercyclical as in data.**

What Affects Cyclical Properties of Bankruptcies?

- **Agents receive higher labor income in expansions (uniformly in our baseline model).**

- **But agents look forward. So it is better to be delinquent in expansions than in recessions.**

Does the Existence of Loans matter for Business Cycles?

A Straight Comparison to a No Loans Economy

Economy With Loans	St Dev Relt Y SD%Y	Correlation with Y
Output	1.00	1.00
Consumption	0.25	0.77
Investment	2.71	0.99
Earnings	1.00	1.00
Hours	0.15	0.83
Labor Input	0.07	0.79
Filing HHs	0.75	-0.61
Economy Without Loans		
Output	1.00	1.00
Consumption	0.26	0.79
Investment	2.64	0.99
Earnings	1.00	1.00
Hours	0.14	0.83
Labor Input	0.07	0.77

- It matters Very little

Different Types of Business Cycles

• **Non-uniform aggregate shocks: Recessions hit particularly hard on some. Three ways to implement this idea.**

1. **Countercyclical Earnings Variance as reported by Storesletten, Telmer and Yaron (2000)**

2. **Larger elasticity of hours worked (Cross-Sect (St Dev Log Cons / St Dev Log Hours)=2.5)**

3. **Recessions are periods of asset destruction (small business failures and other).**

Cyclical Properties of an Economy with Countercyclical Earnings Variance as Storesletten, Telmer and Yaron

Variable	St Dev Relt Y	Correlation with Y
Output	1.00	1.00
Consumption	0.30	0.88
Investment	2.57	0.99
Earnings	1.00	1.00
Hours	0.32	0.96
Labor Input	0.04	-0.27
Filing HHs	11.89	-0.28

- Hours are too volatile and Labor Input Contracylical !!!

Economies with Countercyclical Earnings Variance

Economy with Loans	St Dev Relt Y	Correlation with Y
Output	1.00	1.00
Consumption	0.30	0.88
Investment	2.57	0.99
Earnings	1.00	1.00
Hours	0.32	0.96
Labor Input	0.04	-0.27
Filing HHs	11.89	-0.28
Economy Without Loans		
Output	1.00	1.00
Consumption	0.30	0.87
Investment	2.53	0.99
Earnings	1.00	1.00
Hours	0.32	0.96
Labor Input	0.04	-0.43

- Still Loans Matter Very little

Cyclical Properties of the Economy with Smaller Countercyclical Earnings Variance (1/6 th of previous)

	with 1/6 of STY		with 1/2 of STY	
	St Dev Y	Cor Y	St Dev Y	Cor Y
Output	1.00	1.00	1.00	1.00
Consumption	0.26	0.79	0.28	0.80
Investment	2.69	0.99	2.64	0.99
Earnings	1.00	1.00	1.00	1.00
Hours	0.16	0.85	0.23	0.91
Labor Input	0.06	0.69	0.05	-0.28
Filing HHs	1.01	-0.76	8.99	-0.40
HHs in Debt	0.30	0.26	0.58	-0.51

- Hours vary more than in baseline.
- More sensible behavior of the Labor Input.
- And Still no action from loans

Higher Elasticity of Hours Relt C-S St Dev of 2.4 & STY

With Loans	St Dev Relt Y	Correlation with Y
Output	1.00	1.00
Consumption	0.21	0.61
Investment	2.82	0.99
Earnings	1.00	1.00
Hours	0.72	0.92
Labor Input	0.12	-0.04
Filing HHs	31.16	-0.18
HHs in Debt	6.58	-0.24
Without Loans		
Output	1.00	1.00
Consumption	0.33	0.93
Investment	2.40	0.99
Earnings	1.00	1.00
Hours	0.67	0.96
Labor Input	0.09	-0.15

- Now Loans matter but the volatility of filings is still too high and the labor input too negatively correlated

Economies with Asset Destruction Shocks

With Loans	St Dev Relt Y	Correlation with Y
Output	1.00	1.00
Consumption	0.29	0.80
Investment	2.69	0.99
Earnings	1.00	1.00
Hours	0.14	0.73
Labor Input	0.07	0.71
Filing HHs	3.65	-0.97
HHs in Debt	0.94	-0.91
Without Loans		
Output	1.00	1.00
Consumption	0.28	0.80
Investment	2.67	0.99
Earnings	1.00	1.00
Hours	0.14	0.75
Labor Input	0.07	0.72
Filing HHs	37.65	-1.00
HHs in Debt	37.32	-1.00

- Not much either.

Conclusions

- **Whether we model economies with endogenous lending and bankruptcy levels does not seem to change a lot the aggregate business cycles behavior of the economy, except if**

1. **The elasticity of substitution is quite high and**

2. **The cross-sectional variance of earnings is quite countercyclical.**

Conclusions

- **Whether we model economies with endogenous lending and bankruptcy levels does not seem to change a lot the aggregate business cycles behavior of the economy, except if**

1. **The elasticity of substitution is quite high and**

2. **The cross-sectional variance of earnings is quite countercyclical.**

- **The details of modeling how recessions affect different households have different implications for aggregate business cycle statistics. We still need to learn a lot about this.**

Filers

Computation [1]

- **Prices (w, r, and q for each type of households and level of debt) are no longer independent of x, so households do need to use the information to forecast prices, a much harder problem.**

- **We follow the insight of (Krusell and Smith 1998) and especially (Krusell and Smith 1997) and we approximate forecasting functions for:**
  - **Capital stock in the next period,**
  - **Debt stock in the next period,**
  - **Average discount price of debt in the next period,**
  - **Amount of defaulted debt**
  - **Prices of bonds for each type.**

- **These are sufficient information to forecast prices. We iterate on these functions.**

Computation [2]

- **Iterating on these things involves among other things solve for market clearing of many commodities each period a very long problem.**

- **We use piecewise linear and/or splines to interpolate and integrate value functions. Interpolation is very useful.**

- **It turns out that simulating large samples of agents is not too good because of sampling error, so we approximate densities.**

- **We use f90 and a 9 node Athlon 1.4GHz Beowulf cluster.**

Default Options

- **Household credit history, $b \in \{0, 1\}$.**
- **Default decision, $d \in \{0, 1\}$.**
- **If $h = 0$ (good credit history), choosing $d = 0$, implies a standard problem.**
- **If $b = 0$ (good credit history), choosing $d = 1$, implies**
  - **$a = 0$ (debt is wiped clean)**
  - **$a' = 0$ (cannot save in same period you default).**
- **If $b = 1$, (the household has a bad credit history).**
  - **$a' \geq 0$ (cannot borrow).**
  - **$b' = 0$ with probability $1 - \eta$.**
  - **$b' = 1$ with probability η.**

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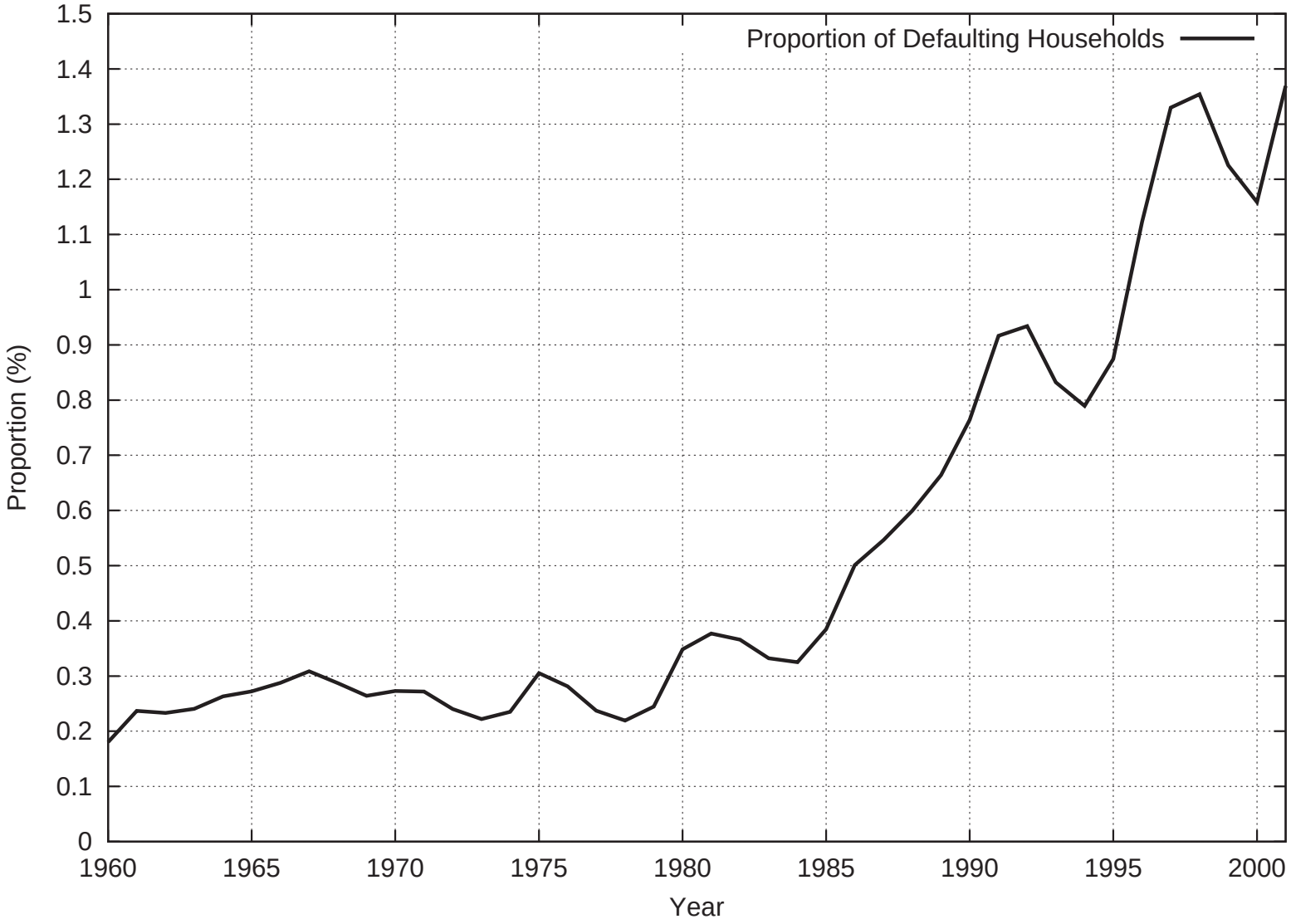
Cyclical Properties of the Economy with Smaller Countercyclical Earnings Variance

Variable		SD%/	Cross-Correlation of Y with				
	SD%	SD%Y	X(t-2)	X(t-1)	X(t)	X(t+1)	X(t+2)
Output	2.04	1.00	-0.17	0.33	1.00	0.33	-0.17
Consumption	0.57	0.28	-0.49	-0.04	0.80	0.67	0.31
Investment	5.40	2.64	-0.10	0.39	0.99	0.23	-0.27
Earnings	2.04	1.00	-0.17	0.33	1.00	0.33	-0.17
Total Asset	0.69	0.34	-0.55	-0.54	-0.16	0.64	0.77
Labor Share	0.00	0.00	-0.07	0.05	-0.23	0.20	-0.07
Net Capital Return	0.24	0.12	0.01	0.46	0.95	0.11	-0.39
Hours	0.48	0.23	0.08	0.50	0.91	0.02	-0.47
Labor Input	0.09	0.05	0.60	0.40	-0.28	-0.72	-0.69
Filing HHs	18.36	8.99	0.64	0.08	-0.40	-1.01	-0.22
HHs in Debt	1.19	0.58	0.12	-0.51	-0.51	-0.36	-0.73

Cyclical Properties of the Economy with Smaller Countercyclical Earnings Variance and without Loan/Default

Variable		SD%/	Cross-Correlation of Y with				
	SD%	SD%Y	X(t-2)	X(t-1)	X(t)	X(t+1)	X(t+2)
Output	2.21	1.00	-0.31	-0.02	1.00	-0.02	-0.31
Consumption	0.55	0.25	-0.37	-0.24	0.89	0.30	-0.10
Investment	5.72	2.59	-0.29	0.02	1.00	-0.08	-0.34
Earnings	2.21	1.00	-0.31	-0.02	1.00	-0.02	-0.31
Total Asset	0.56	0.25	-0.15	-0.41	-0.34	0.68	0.51
Labor Share	0.00	0.00	-0.15	-0.11	-0.00	-0.42	-0.02
Net Capital Return	0.26	0.12	-0.24	0.07	0.98	-0.18	-0.39
Hours	0.48	0.22	-0.22	0.12	0.95	-0.24	-0.42
Labor Input	0.08	0.03	0.37	0.35	-0.60	-0.58	-0.14

Surge in Bankruptcies



U.S. Economy: Annual Cyclical Statistics 1979-2001

Variable		SD%/	Cross-Correlation of Y with				
	SD%	SD%Y	X(t-2)	X(t-1)	X(t)	X(t+1)	X(t+2)
Output	2.02	1.00	0.06	0.52	1.00	0.52	0.06
Consumption	1.39	0.69	-0.17	0.41	0.87	0.70	0.35
Investment	6.93	3.42	0.36	0.59	0.86	0.15	-0.35
Earnings	2.16	1.07	-0.06	0.39	0.91	0.72	0.30
Labor Share	0.90	0.44	-0.28	-0.24	-0.07	0.56	0.59
Aggregate Hours	2.19	1.08	0.03	0.46	0.94	0.52	0.00
Hours per Worker	0.40	0.20	0.27	0.38	0.50	-0.36	-0.60
Filing HHs	11.89	5.87	-0.31	-0.18	-0.05	0.46	0.65

The Technical Issue of Dealing with Aggregate Shocks

- **The distribution of agents over assets and shocks ia a state variable, which is a huge dimensional object.**

- **Two ways have been used to deal with this problem.**

  - **Specify the model so that prices are independent of the type distribution (*i.e.* essentially exogenous) as Diaz-Gimenez, Prescott, Fitzgerald, and Alvarez (1992) did. We followed this approach (Nakajima and Ríos-Rull (2002))line and discovered that it has bad properties because it requires recessions to be perfectly forecastable.**

  - **Summarize the type distribution by some statistics, as in Krusell and Smith (1998). This is what we do. In our case we keep track of more moments of the distribution: total assets, number of borrowers, and the amount borrowed.**