A Theory of Credit Scoring and the Competitive Pricing of Default Risk

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Endlessly Preliminary

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- Overall, there is too much unsecured credit (all quantitative work imposes some additional punishments)

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- We measure the additional value of a good reputation.

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- Actions only partially reveal information about type (semi-separating equilibrium).
- Competitive lenders offer loans of different sizes at different prices based on credit scores which account for type dependent

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Equilibrium Models of Bankruptcy

- Full info, Exogenous Punishment: Chatterjee et al. (2007), Livshits et al. (2007), and all the soveriegn default literature
- Asymmetric info, Static Signaling, Exogenous Punishment: Athreya et al. (2009, 2012), Livshits et al. (2015)
- Asymmetric info, Dynamic Signaling, Endogenous Punishment (Reputation): Chatterjee et al. (2008), Mateos-Planas et al. (2017).
- *Important Issue with Asym Info*: Off-Equilibrium-Path Beliefs

Discrete Choice Models

- Estimation of Micro Models *McFadden (1973), Rust (1987).*
- Make sense of behavior in experimental data. QRE. McKelvey and Palfrey (1995,1996)

Properties of Loans and Credit Scores: Han, Keys, and Li (2015), Jagtiani and Li (2015)

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  - Credit scores suffer upon filing for bankruptcy

Source: Han, Keys, and Li (2015), Table 3.

	Having an offer	Credit limit	Spread
VantageScore bins			
550-600	0.100***	-208.749***	1.229***
	(0.009)	(34.383)	(0.181)
600-650	0.155***	-251.378***	2.249***
	(0.010)	(34.182)	(0.186)
650-700	0.223***	-208.157***	1.693***
	(0.011)	(32.610)	(0.203)
700-750	0.252***	-100.237***	0.436**
	(0.011)	(33.416)	(0.188)
750-800	0.285***	102.084***	-0.367*
	(0.011)	(31.987)	(0.191)
800-850	0.292***	326.984***	-0.871***
	(0.012)	(35.524)	(0.190)
850-900	0.264***	577.903***	-0.969***
	(0.012)	(31.360)	(0.185)
900-950	0.254***	714.265***	-0.865***
	(0.011)	(33.499)	(0.192)
> 950	0.267***	809.787***	-0.770***
	(0.010)	(40.664)	(0.178)
Yearly fixed-effects	yes	yes	yes
Monthly fixed-effects	yes	yes	yes
R^2 / pseudo R^2	0.061	0.115	0.341
N	169,692	148,656	217,920

	Having an offer	Credit limit	Spread
Credit hist. attr.			
Bankruptcy filer	-0.068***	-238.100***	0.834***
	(0.008)	(14.106)	(0.111)
Other derog rec.	-0.065***	-130.100***	0.153
	(0.009)	(13.166)	(0.106)
Deep del.	-0.027***	-26.073	0.091
	(0.010)	(19.841)	(0.113)
Recent del.	-0.004***	-31.710***	0.124***
	(0.001)	(2.115)	(0.015)
Num inquiries	0.001	-20.461***	0.011
	(0.001)	(2.610)	(0.011)
Debt-income ratio	-0.001	-9.194***	-0.030***
	(0.001)	(2.715)	(0.007)
Have credit card	0.065***	47.819***	-0.536***
	(0.004)	(12.860)	(0.044)
High util	-0.022***	-13.200	0.454***
	(0.006)	(16.002)	(0.043)
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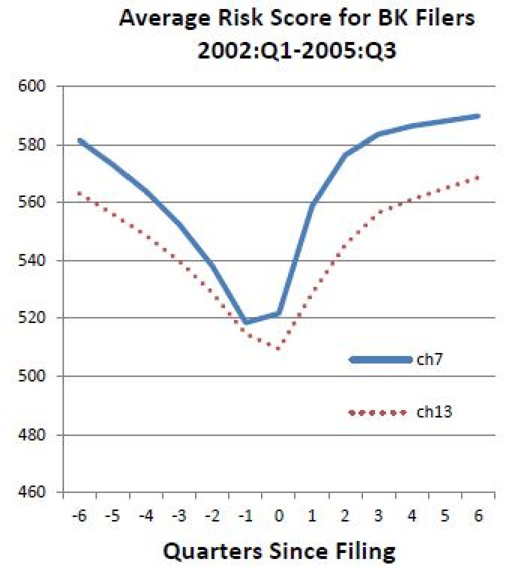
Credit Scores around Default in the Data (Jagtiani and Li (2015))



A Model of Bankruptcy and Reputation

An Aiyagari-Bewley-Huggett-Imrohoroglu type model with preferences

$$\mathrm{E}\left\{\sum_{t=0}^{\infty} \beta_i^t \; u(c_{it})\right\}$$

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- $\psi \in [0, 1]$ assigned (randomly) to nearest two scores in $\mathcal{S}$ via $s' \sim \Gamma^s(s'|\psi)$
- Offer discount loans at prices $q^{(d,a')}(\omega)$

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4. Intermediaries revise type scores from $s \rightarrow \psi^{(d,a')}(\omega)$ via Bayes' rule
5. Next period state β' is drawn from $\Gamma^{\beta}(\beta'|\beta)$, e' drawn from $\Gamma^{e}(e'|e)$, and s' drawn from $\Gamma^{s}(s'|\psi)$

HH Optimization Problem

Taking price and type score functions $f = (q, \psi)$ as given, HH solves

$$V(\epsilon, \beta, \omega | f) = \max_{(d,a') \in \mathcal{F}(\omega | f)} v^{(d,a')}(\beta, \omega | f) + \epsilon^{(d,a')}$$

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Taking price and type score functions $f = (q, \psi)$ as given, HH solves

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subject to (d, a') in the feasible set $\mathcal{F}(\omega | f)$ defined by

$$c^{(d,a')}(\omega | f) = \begin{cases} e + z + a - q^{(0,a')}(\omega) \cdot a' > 0 & \text{for } d = 0, a' \neq 0 \\ e + z - \kappa & \text{for } d = 1, a' = 0 \end{cases}$$

HH Decision Rules

Lemma

Given f, there exists a unique solution $W(\cdot|f)$ to the individual's decision problem in and $W(f)$ is continuous in f.

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- The modal action has highest $v^{(d,a')}(\cdot)$.
- With extreme value distribution, higher α implies lower variance of ϵ, so HH is more likely to take the modal action.

Type Scoring and Debt Pricing by Intermediaries

It updates the assessment of a HH's type given its actions and observable characteristics using Bayes' rule, ▸ Details

$$\psi^{(d,a')}(\omega) = \Pr(\beta' = \beta_H | d, a', \omega)$$

Perfect competition, deep pockets $\implies$ breakeven pricing:

$$q^{(0,a')}(\omega) = \begin{cases} \frac{p^{(0,a')}(\omega|f)}{1+r+\iota} & \text{if } a' < 0 \\ \frac{1}{1+r} & \text{if } a' \geq 0 \end{cases}$$

where $p(\cdot)$ is the **assessed repayment probability** using both the type score ψ and decision rules σ:

$$p^{(0,a')}(\omega) = \int_{s',e',z'} \Gamma^s(s'|\psi^{(d,a')}(\omega)) \cdot \Gamma^e(e'|e) \cdot H(z') \cdot \left[s'(1 - \sigma^{(1,0)}(\beta_H, \omega')) + (1 - s')(1 - \sigma^{(1,0)}(\beta_L, \omega')) \right]$$

Actual definition of the "data relevant" credit score

- It is the probability that an agent defaults the following period conditional on today's observables

$$\xi_1(\omega) = \sum_{(d,a')\in\mathcal{Y}} \left[p^{(0,a')}(\omega) \cdot \sum_{\beta\in\mathcal{B}} \sigma^{(d,a')}(\beta,\omega) \cdot \frac{\overline{x}(\beta,\omega)}{\sum_{\hat{\beta}\in\mathcal{B}} \overline{x}(\hat{\beta},\omega)} \right]$$

Equilibrium Definition

A **stationary recursive competitive equilibrium** is a vector-valued pricing function q^*, a vector-valued type scoring function ψ^*, a vector-valued quantal response function σ^*, and a steady state distribution $\overline{x}^*$ such that:

- $\sigma^{(d,a')*}(\beta, \omega | f^*)$ satisfies household optimization,
- $q^{(0,a')*}(\omega)$ implies lenders break even with objective likelihood of repayment $p^{(0,a')*}(\omega | f^*)$,
- $\psi_{\beta'}^{(d,a')*}(\omega)$ satisfies Bayes', and
- $\overline{x}^*(\beta, \omega | f^*)$ is stationary.

Theorem

There exists a stationary recursive competitive equilibrium.

Mapping the Model to Data

How to Specify a Particular Economy

In a particularly hard model to solve

- We estimate (pedestrian exactly identified GMM) a four parameter model ($\beta_H, \beta_L, \Gamma_{HH}, \Gamma_{LL}$).

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- Crucially, we estimate the parameters using a model with **observable** patience. So no role for dynamic punishment and the credit score is not a state variable.
- We then move on to solve the model with **dynamic** punishments (the credit score is a state variable) and look to the extent to which those moments are different.

Parameterization and Model Fit

	PARAMETERS	Notation	Value	Value
Selected	CRRA	ν	3	3
	Risk-free rate	r	1.0%	1.0%
	Filing costs to mean income	κ	2.0%	2.0%
	Extreme value scale parameters	α_d, α_a	500,200	500,200
Calibrated	low type discount factor	β_L	0.863	0.863
	high type discount factor	β_H	0.994	0.994
	low β to high β transition probability	$\Gamma^{\beta}(\beta'_H \mid \beta_L)$	0.02	0.02
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			Base	**Dyn**

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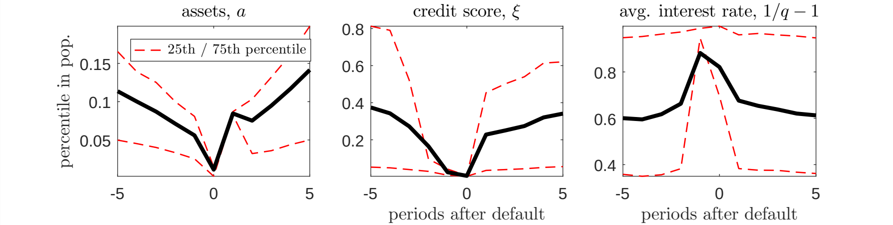
	MOMENTS	**Data**	**Base Model**	**Dyn Model**
Targeted	Total wealth to total income	3.34	3.23	3.27
	Mean wealth to median wealth	2.50	2.63	2.38
	P50 to P30 wealth	5.54	5.67	6.33
	Prob of remaining in P20	0.67	0.69	0.70

- So the targeted moments do not change very much

Dynamics of Debt and Default



- The model replicates the behavior of the bad consequences of bankruptcy filing for credit scores and for credit terms.

- $\uparrow$ HH debt $\implies \downarrow$ credit score $\implies \uparrow$ higher rates
- CS (IR) tanks (spikes) following default ▸ Figure ▸ Panel Construction

Assessing the Role of Dynamic Punishment

1. We compute untargeted static punishment model-implied debt and default statistics and compare with data.
    - Bankruptcy filing rate
    - Average interest rate
    - Median networth to median income
    - Average debt to income
    - Fraction of households in debt
    - Interest rate dispersion
    - Average chargeoff rate

2. We then compute those with dynamic punishment and see whether the amount and type of credit implied is closer to that observed.

Untargeted Moments

MOMENTS	DATA	STATIC Pnshmnt	DYN Pnshmnt
Bankruptcy filing rate	2.10%	3.82%	2.96%
Average interest rate	13.54%	64.48%	32.70%
Average debt to income	2.10%	0.85%	1.12%
Fraction of households in debt	12.88%	13.14%	16.74%
Interest rate dispersion	7.1%	53.57%	42.48%
Average chargeoff rate	3.99%	51.23%	33.29%

- The market for unsecured credit is clearly not good enough.

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- The market for unsecured credit is clearly not good enough.
- Dynamic punishment improves dramatically.
- Still not enough.

How much does Info Asymmetry Matter?

Full information environment:

- ϵ still unobservable and transitory
- β observable $\implies$ **no inference problem**
- obviates type scoring $\implies$ no $\psi(\cdot)$, no *s*

Key insights:

- high (low) β type with full info case face more (less) favorable price schedules than high (low) *s* type in benchmark ▸ Prices
- high (low) β take on more (less) debt to income and default more (less) than in benchmark, important selection effects. ▸ Moments
- on average, HH are slightly better off in full info, but low β types in debt prefer benchmark ▸ Welfare Analysis

How Much is Reputation Worth?

Question: How much must a HH be **compensated** to accept being assigned the lowest possible type score?

Answer: Define for each state (β, ω) a number τ such that

$$W(\beta, e, z, a, s) = W(\beta, e, z, a + \tau(\beta, e, z, a, s), s_{\min})$$

Aggregating, we find:

$\overline{\tau}$ (%)	agg.	$a < 0$	$s = s_{\max},\ a < 0$	$s = s_{\max},\ a = a_{\min}$
agg.	0.015	0.139	0.613	0
β_H	0.020	0.216	0.586	3.5
β_L	0.012	0.088	0.847	2.0

- small numbers in aggregate reflect small fraction in debt

How much is still missing?

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- Earnings are now affected by your reputation in amount λ

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$$y = z + e + s\lambda - (1 - s)\lambda = z + e + 2s\lambda - \lambda$$

Model Fit with additional value to a good reputation

	PARAMETERS	Notation	Static Model	Dyn Model	$\lambda = 2\%$ Model
Selected	CRRA	ν	3	3	3
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Implied fraction of H types 50%

Untargeted Moments

MOMENTS	DATA	STATIC Model	DYN Model	$\lambda = 2\%$ Model
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Average interest rate	13.54%	64.48%	33.25%	27.34%
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Fraction of households in debt	12.88%	13.14%	16.74%	15.40%
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- Depending on the target statistic that we use for identification the value of reputation (by linear extrapolation) the additional value of a good reputation is in the range of $2\lambda = [5\% - 20\%]$
- Similar logic gives a value of dynamic punishment for credit purposes only of somewhere around 20% of earnings.

Conclusion

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Developed model of unsecured consumer credit in which

- agents have option to **default**, and do so in equilibrium
- unobservable preference shocks impose an **inference problem** on intermediaries who price debt
- **credit scoring** helps solve this problem

Calibrated the model to key credit market moments to show

- default behavior by credit score closely matches data
- **asymmetric info** expands the fraction of economy in debt (selection effects matter), but reduces welfare relative to **full info**.
- **reputation** matters in that many borrowers would require significant compensation to be labeled as “bad"

Appendix

Budget Feasibility and Actions

Set of all **possible** default and asset choices:

$$\mathcal{Y} = \left\{(d, a') : (d, a') \in \{0\} \times \mathcal{A} \text{ or } (d, a') = (1, 0)\right\}$$

Given observable state ω and a set of equilibrium functions f the set of **feasible** actions is

$$\mathcal{F}(\omega | f) \subseteq \mathcal{Y}$$

that contains all actions $(d, a') \in \mathcal{Y}$ such that $c^{(d,a')} > 0$
Consumption is pinned down by the budget constraint:

$$c^{(d,a')} = \begin{cases} e + z + a - q^{(0,a')}(\omega) \cdot a' & \text{for } d = 0, a' < 0 \\ e + z + a - a'/(1 + r) & \text{for } d = 0, a' \geq 0 \\ e + z - \kappa & \text{for } d = 1, a' = 0 \end{cases}$$

Existence of a Solution to HH Problem

Theorem

Given f, there exists a unique solution $W(f)$ to the individual's decision problem and $W(f)$ is continuous in f.

Sketch of proof:

- Apply Contraction Mapping Theorem defining the operator $(T_f)(W) : \mathbb{R}^{B+|\Omega|} \to \mathbb{R}^{B+|\Omega|}$.
- To prove continuity of $W(f)$, show that the operator T_f is continuous in f.

  Follows given continuity of

  - u with respect to c,
  - $c^{(d,a')}$ with respect to q for $(d, a') \in \mathcal{F}(\omega|f)$ and
  - Γ^s with respect to ψ.
- Since $\mathbb{R}^{M+K}$ is a Banach space, then apply Theorem 4.3.6 in Hutson and Pym (1980).

▸ Back to HH policies ▸ Back to theorems

Extreme Value Shocks 101

Extreme Value Distribution with location parameter $= 0$ and scale parameter $\frac{1}{\alpha}$ where higher α implies lower variance.

- Can show that $\frac{\partial \sigma^{(d,a')}(\beta,\omega)}{\partial \alpha}$ takes the sign of

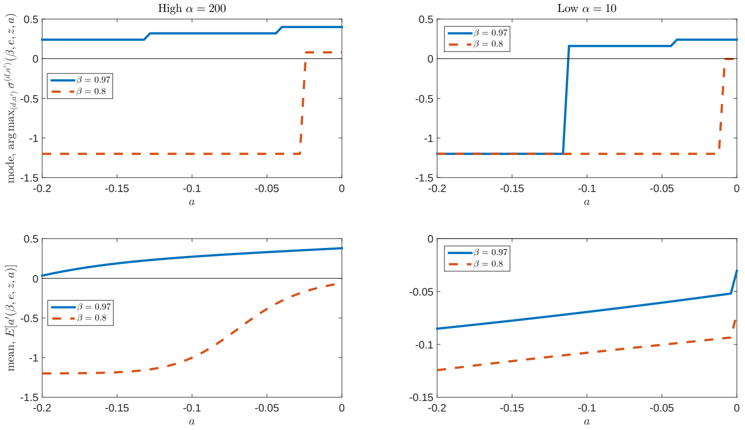
$$\sum_{(\tilde{d},\tilde{a}')\in\mathcal{F}(\omega)} \left[v^{(d,a')}(\beta,\omega) - v^{(\tilde{d},\tilde{a}')}(\beta,\omega)\right] \cdot \exp\left\{\alpha \cdot \left(v^{(d,a')}(\beta,\omega) + v^{(\hat{d},\hat{a}')}(\beta,\omega)\right)\right\}$$

  so more likely to take optimal action the lower is variance.
- From the formula for $\sigma(\cdot)$ we have

$$\arg\max_{(d,a')\in\mathcal{F}(\omega)} \sigma^{(d,a')}(\beta,\omega) = \arg\max_{(d,a')\in\mathcal{F}(\omega)} v^{(d,a')}(\beta,\omega),$$

  so the optimal action **without extreme value shocks** is the **modal action** in our paper.

Figure: Impact of Extreme Value Shocks



▸ Back

Bayesian Type Assessment Updating and Pricing Details

Probability that an agent will be of type β' tomorrow given by Bayes rule:

$$\psi_{\beta'}^{(d,a')}(\omega) = \sum_{\beta} \Gamma^{\beta}(\beta'|\beta) \cdot \frac{\sigma^{(d,a')}(\beta,\omega) \cdot s(\beta)}{\sum_{\hat{\beta}} \left[\sigma^{(d,a')}(\hat{\beta},\omega) \cdot s(\hat{\beta})\right]}$$

for each type β. Since ψ may not lie on the grid $\mathcal{S}$, for the two nearest grid points $s'_i \leq \psi^{(d,a')}(\omega) \leq s'_j$ compute

$$\chi(\psi) = \frac{s'_j - \psi}{s'_j - s'_i} \implies \Gamma^s(s'|\psi) = \begin{cases} \chi(\psi) & \text{if } s' = s'_i \\ 1 - \chi(\psi) & \text{if } s' = s'_j \\ 0 & \text{otherwise} \end{cases}$$

Then repayment probabilities given by:

$$\begin{aligned} p^{(0,a')}(\omega) \quad &= \quad \sum_{s',e',z'} \Gamma^s(s'|\psi^{(d,a')}(\omega)) \cdot \Gamma^e(e'|e) \cdot H(z') \\ &\quad \cdot \left[s'(1-\sigma^{(1,0)}(\beta_H,\omega')) + (1-s')(1-\sigma^{(1,0)}(\beta_L,\omega'))\right] \end{aligned}$$

Back

Cross-sectional Distribution

Let $x(\beta,\omega|f)$ be the measure of individuals in state (β,ω) today for a given set of equilibrium functions f.

The distribution evolves according to

$$x'(\beta',\omega'|f) = \sum_{(\beta,\omega)\in\mathcal{B}\times\Omega} T^*(\beta',\omega'|\beta,\omega;f)\cdot x(\beta,\omega|f). \tag{1}$$

where

$$\begin{aligned} T^*(\beta',\omega'|\beta,\omega;f) &= \sigma^{(d,a')}(\beta,\omega|f)\cdot\Gamma^s(s'|\psi^{(d,a')}(\omega)) \\ &\quad \cdot\Gamma^\beta(\beta'|\beta)\cdot\Gamma^e(e'|e)\cdot H(z') \end{aligned} \tag{2}$$

An invariant distribution is a fixed point $\overline{x}(\cdot)$ of (1).

▸ Existence ▸ Back to equilibrium definition

Existence of an Invariant Distribution

Lemma

There exists a unique invariant distribution $\overline{x}$.

Sketch of proof: Use Theorem 11.2 in Stokey and Lucas (1989) to establish this result.

$\overline{x}$ is critical for computing cross-sectional moments

- map model to data

No other equilibrium objects – functions f, the value function $V(\cdot)$ or the decision rule $\sigma^{(\cdot)}(\cdot)$ – take $x(\cdot)$ as argument

- simplifies computation

▸ Back to distribution ▸ Back to theorems

Existence of Equilibrium – pt. 1

Theorem

There exists a stationary recursive competitive equilibrium.

Sketch of proof:

- Let f be the vector composed by stacking $q \in [0,1]^K$ and $\psi \in [0,1]^M$ so $f \in [0,1]^{K+M}$ and let

$$W = W(f) : [0,1]^{K+M} \to \mathbb{R}^{B+|\Omega|}$$

  be the solution established in Theorem 2.

- Given W, use (**??**) to construct the vector-valued function

$$v = J_1(W) : \mathbb{R}^{B+|\Omega|} \to \mathbb{R}^M$$

- Given v, use (15) to construct the vector-valued function

$$\sigma = J_2(v) : \mathbb{R}^M \to (0,1)^M$$

Existence of Equilibrium - pt. 2

- Given σ and ψ, use the mapping in (**??**) to construct the vector-valued function

$$p = J_3(\sigma, \psi) : (0,\ 1)^M \times [0, 1]^M \rightarrow [0, 1]^{|\mathcal{A}_{--} \times \Omega|}.$$

- Given p and σ, use the mapping in (16) and (16) to construct the $K + M$ vector

$$f_{\text{new}} = (q_{\text{new}}, \psi_{\text{new}}) = J_4(p, \sigma) : [0, 1]^{|\mathcal{A}_{--} \times \Omega|} \times [0, 1]^M \rightarrow [0, 1]^{K+M}.$$

- Let $J(f) : [0,\ 1]^{K+M} \rightarrow [0,\ 1]^{K+M}$ be the composite mapping $J_4 \circ J_3 \circ J_2 \circ J_1 \circ W$. By Theorem 2 $W(f)$ is continuous and the functions J_i, $i \in \{1, 2, 3, 4\}$ are also continuous. Hence J is a continuous self-map.
- Since $[0,\ 1]^K$ is a compact and convex subset of $\mathbb{R}^K$, the existence of $f^* = J(f^*)$ is guaranteed by Brouwer's FPT.

▸ Pt. 1 ▸ Back to equilibrium definition

Theorems

1. **HH solution:** Given f, there exists a unique solution $W(f)$ to the individual's decision problem in (14) to (**??**) and $W(f)$ is continuous in f. ▸ Existence of HH solution

2. **Stationary distribution:** There exists a unique invariant distribution $\overline{x}$. ▸ Existence of stationary distribution

3. **Equilibrium existence:** There exists a stationary recursive competitive equilibrium. ▸ Existence of equilibrium

Computational Algorithm and Estimation

Algorithm: tiered-loop grid search

1. create grids for β, e, z, a, s (earnings calibrated outside model)
2. start with initial guesses of $f = f_i$
3. compute feasible set $\mathcal{F}(e, z, a, s | f_i)$
4. value function iteration $\implies \sigma(\beta, e, z, a, s | f_i)$
5. $\sigma \implies f_{i+1}$
6. if $\max\{|f_{i+1} - f_i\} < tol$, continue; else, go back to 2
7. compute x, moments

Estimation: 2-stage SMM

1. set $W_0 = I_5$, embed above algorithm in DFBOLS optimization procedure of Zhang et al. (2010) to get parameter estimates $\hat{\theta}_0$
2. simulate $N \times T$ panel from the model under $\hat{\theta}_0$ to compute efficient weighting matrix W^*, repeat stage 1 procedure to get final estimates $\hat{\theta}^*$ and standard errors from W^*

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Parameterization Details

Grid	Size	Range	Details
β	2	$\{0.89, 0.97\}$	bivariate type $\implies$ scalar ψ, Γ
e	3	[0.58,1.74]	Floden and Lindé (2009)
z	3	{-0.182,0,0.182}	$z = +/- \sqrt{3/2 \times 0.0421}$
a	151	[-0.25,7.0]	50 neg + 100 pos
s	50	$[0.04, 0.90]$	$[\Gamma^{\beta}(\beta'_L\|\beta_H), 1 - \Gamma^{\beta}(\beta'_L\|\beta_H)]$

Earnings details

	e	$\Gamma^e(e'\|e)$		
		e'_1	e'_2	e'_3
e_1	0.575	0.818	0.174	0.004
e_2	1.000	0.174	0.643	0.174
e_3	1.739	0.004	0.174	0.818

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Definitions of Key Model Moments

Default rate $= \sum_{\beta,\omega} \sigma^{(1,0)}(\beta,\omega) \cdot x(\beta,\omega)$

Median net worth to median income - straightforward

Fraction of HH in debt $= \sum_{\beta,e,z,s} \sum_{a<0} x(\beta,e,z,a,s)$

Average debt to income ratio
$= \sum_{\beta,e,z,a<0,s} \frac{a}{e+z+(1/q(\cdot)-1)\cdot a} \cdot \frac{x(\beta,e,z,a,s)}{\sum_{\hat{\beta},\hat{e},\hat{z},\hat{a}<0,\hat{s}} x(\hat{\beta},\hat{e},\hat{z},\hat{a},\hat{s})}$

Average chargeoff rate $\frac{\text{total debt}}{\text{total debt defaulted}}$, where

$$\begin{aligned} \text{total debt} &= \sum_{a<0} a \cdot \left(\sum_{\beta,e,z,s} x(\beta,e,z,a,s) \right) \\ \text{total debt defaulted} &= \sum_{a<0} a \cdot \left(\sum_{\beta,e,z,s} \sigma^{(1,0)}(\beta,e,z,a,s) \cdot \frac{x(\beta,e,z,a,s)}{\sum_{\hat{\beta},\hat{e},\hat{z},\hat{s}} x(\hat{\beta},\hat{e},\hat{z},a,\hat{s})} \right). \end{aligned}$$

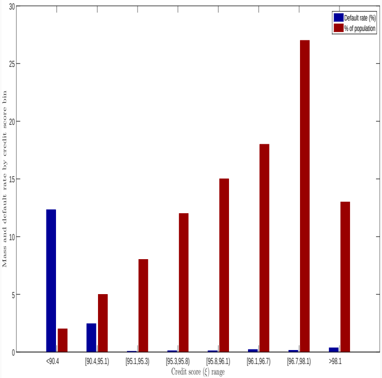
▸ Back

Targeted Moments

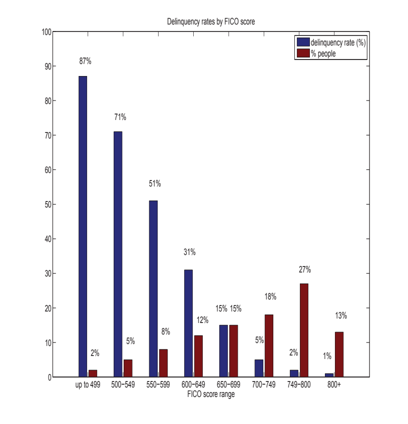
Moment [Source]	**Data**	**Model**
Default rate (%) - Chatterjee et al. (2007)	0.54	0.53
Average interest rate (%) - Chatterjee and Eyigungor (2009)	11.35	9.98
Median net worth / median income - Chatterjee and Eyigungor (2009)	1.28	2.13
Fraction of households in debt (%) - Chatterjee et al. (2007)	6.73	8.24
Average debt-to-income ratio (%)	0.67	0.64

Credit Score Distribution

Model



Data (TransUnion)



- compute "credit scores" within the model by integrating choice-specific default probabilities over choice probabilities

▸ Model credit score details ▸ Back

Credit Scores

Mapping from model "credit scorecard" to analog of real world "credit score" is not trivial

- can't just use type score: β_L types have higher propensity to default $\implies$ priced out $\implies$ default less...so who's the "bad type"?
- can't just use the repayment probability: p is action-specific, not like FICO or anything...

Proper procedure: **integrate** over actions, conditional on going into debt

- define **credit score function** $\xi(\cdot)$ as

$$\xi(\omega) = \sum_{a'<0} \frac{p^{(0,a')} \cdot \sum_{\beta} \left(\sigma^{(0,a')}(\beta, \omega) \cdot x(\beta, \omega) \right)}{\sum_{\hat{\beta}, \hat{a}' < 0} \left(\sigma^{(0,\hat{a}')}(\hat{\beta}, \omega) \cdot x(\hat{\beta}, \omega) \right)}$$

Construction of Panel

In order to construct the figures that map out the prices and states before and after default, we follow the procedure:

1. draw a set of $N = 5000$ initial conditionals for (β, ω) from the stationary distribution $\overline{x}(\cdot)$
2. for $T = 100$ periods, use the decision rule $\sigma(\cdot)$ and the exogenous transitions to map HH's flows through states
3. isolate all the default events, and the HH's state in $t_{-} = 5$ periods before and t_{+} periods after
4. average over all relevant variables and compute desired confidence intervals

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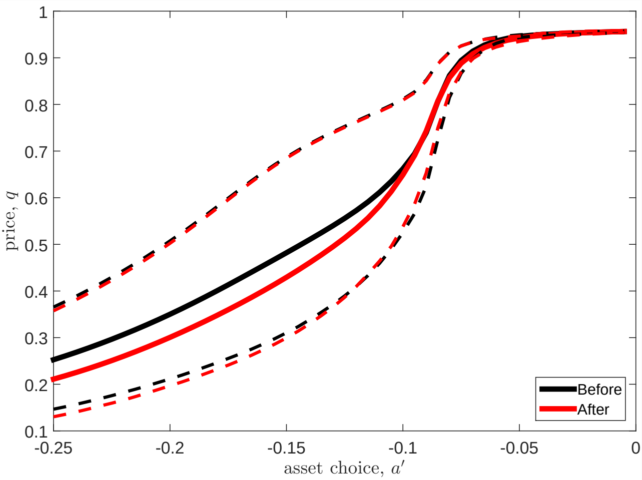
Impact of Default on Price Menu



- default raises entire menu of interest rates. ▸ Construction ▸ Back

Asymmetric vs. Full Information: Selection Effects

Why do interest rates rise under full information?

- While High type continue to default less than Low type, High type default relatively more under full info while Low type default relatively less as High types try to maintain their reputation with asymmetric info.

- While the pricing menus reflect lower default probabilities for High types, High types "select" relatively more debt resulting in higher relative default rates and interest rates.

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Welfare analysis

Q:: How much more **consumption per period** must an agent receive in the asymmetric info economy to be indifferent with the full info economy?

Answer: Construct consumption equivalents:

$$\lambda(\beta,\omega) = \left[\frac{W^{FI}(\beta,\omega^{FI})}{W(\beta,\omega)}\right]^{\frac{1}{1-\gamma}} - 1$$

Aggregating, we find:

$\overline{\lambda}$ (%)	agg.	$a<0$	$s=\underline{s}$	$s=\overline{s}$	$s=\underline{s},\ a<0$	$s=\underline{s},\ a<0$
agg.	0.038	0.020	0.020	0.047	0.016	0.048
β_H	0.063	0.076	0.040	0.050	0.052	0.114
β_L	0.021	-0.003	0.019	0.023	0.014	0.004

Note that Low type in debt actually benefit from asymmetric info.

Consumption Equivalent Derivation

For each (β, ω), define fraction $\lambda(\beta, \omega)$ by which consumption will have to be increased each period to be indifferent between the benchmark and full information economies

Given benchmark value (up to shocks)

$$W(\beta, \omega) = \int V(\epsilon, \beta, \omega) dG(\epsilon)$$

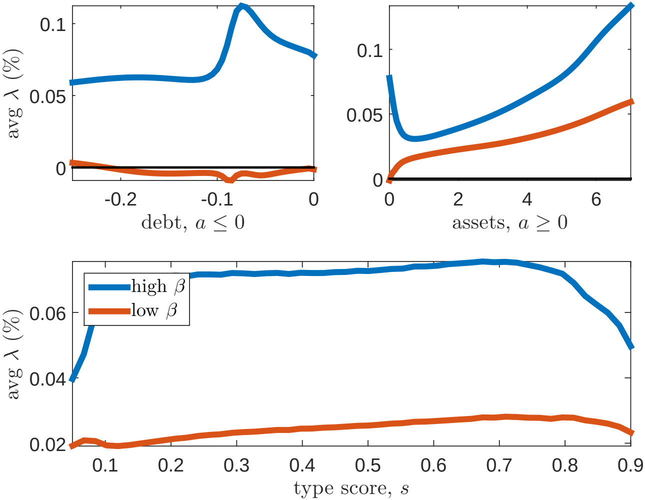
and an analogous value $W^{FI}(\beta, \omega^{FI})$, we can write

$$W^{FI}(\beta, \omega^{FI}) = E_{\beta,\omega}\left[\sum_{t=0}^{\infty} \beta_t^t u\left(c_t^*(1 + \lambda(\beta, \omega))\right)\right],$$

where c_t^* is optimal consumption in the benchmark. Solving for $\lambda(\cdot)$ yields the expression in the main text

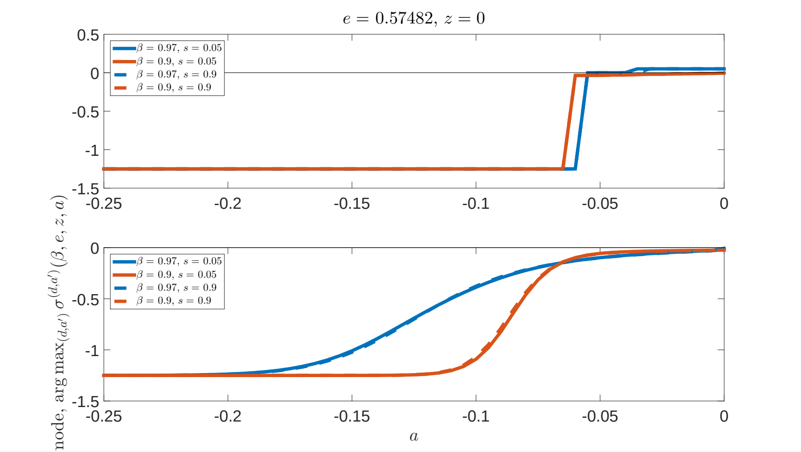
▸ Back ▸ Across the state space

Welfare Across the State Space



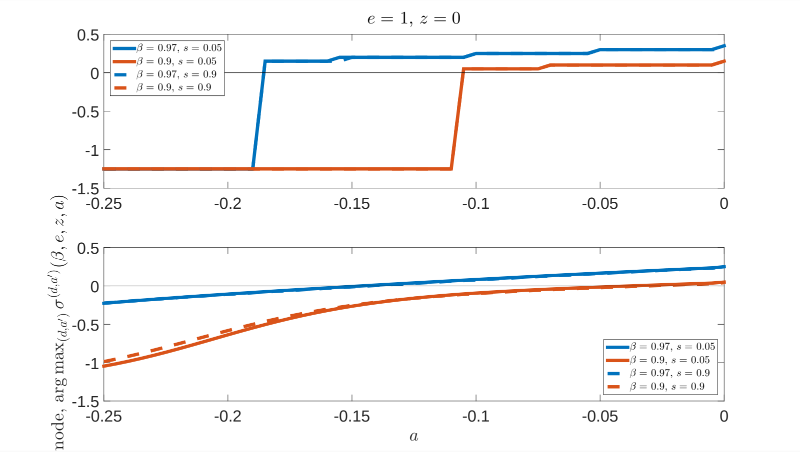
Define $\overline{\lambda}(\beta, a)$ ($\overline{\lambda}(\beta, s)$) to be the average λ for agents with β, a (s) Back

Benchmark Model: HH Policies



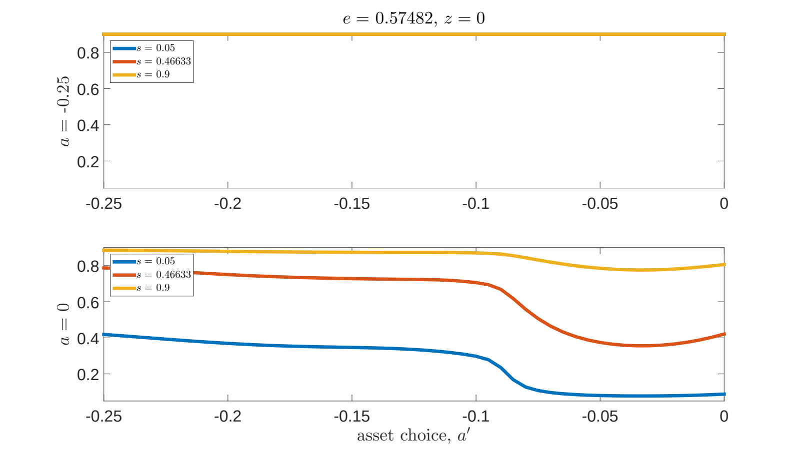
- almost complete separation on β
- minimal differences across s for fixed β

Benchmark Model: HH Policies



- almost complete separation on β
- minimal differences across s for fixed β

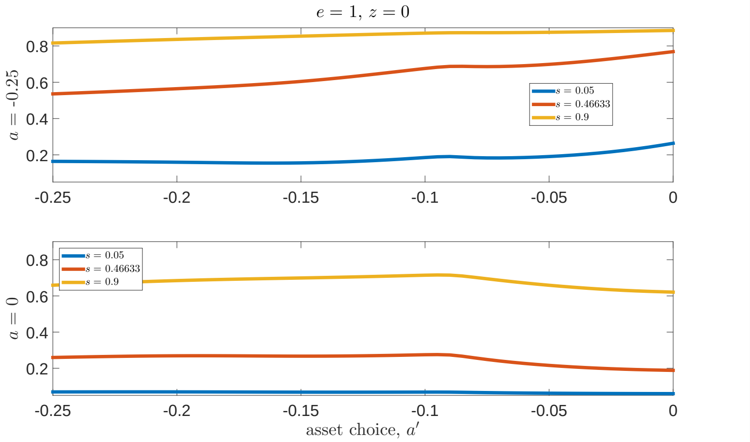
Benchmark Model: Type Scores



- **low earnings**: choice matters for reputation with low wealth

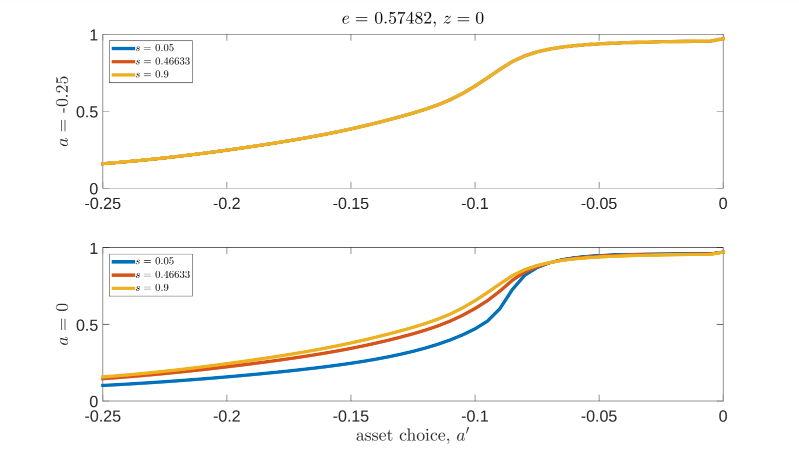
▸ Back ▸ Policies ▸ Prices

Benchmark Model: Type Scores



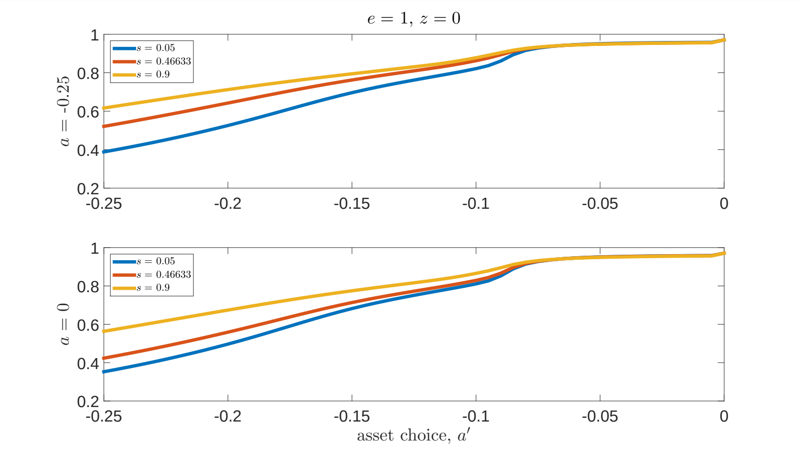
- **high earnings**: deeper debt lowers reputation, more if already low
- deeper debts affect score more adversely

Benchmark Model: Prices



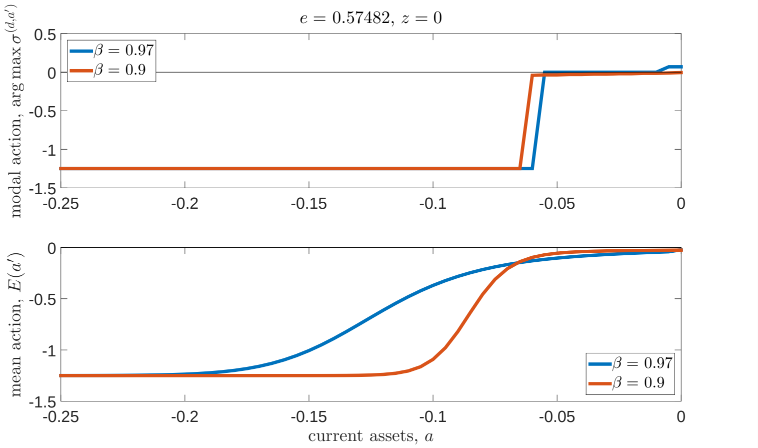
- type score s seems only to matter for low a agents
- effect greater for agents whose (e, z) doesn't compensate

Benchmark Model: Prices



- type score s seems only to matter for low a agents
- effect greater for agents whose (e, z) doesn't compensate

Full Information: HH Policies



- fairly strong **separation**
- modal (mean) action for β_H weakly (strictly) above β_L

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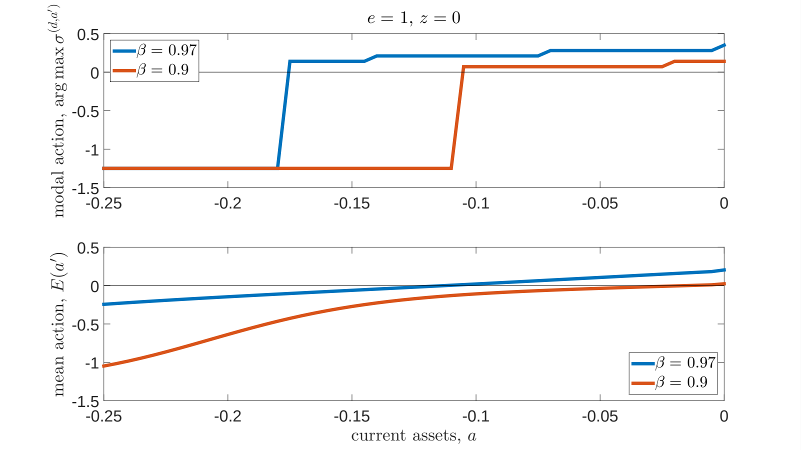
Prices

Full Information: HH Policies



- fairly strong **separation**
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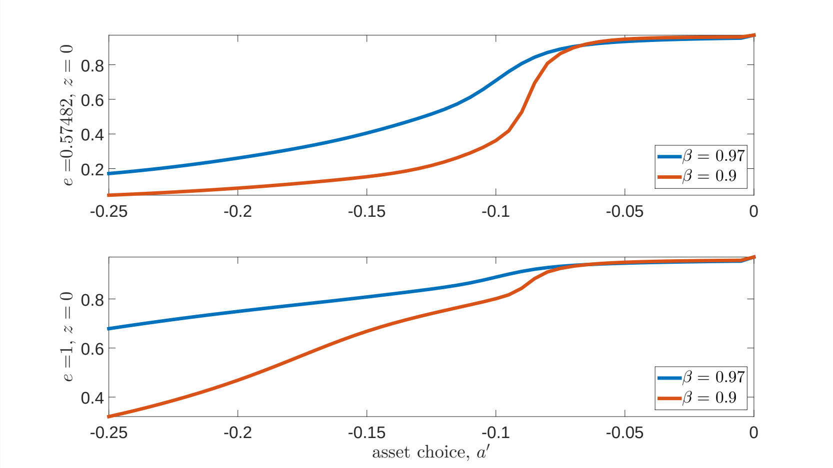
Prices

Full Information: Prices



- $q(a', \beta_H)$ uniformly above $q(a', \beta_L)$, more so far in debt

Construction of Full Info vs. Benchmark Price Schedules

- in the benchmark, prices given by $q^{(0,a')}(\omega)$
- under full information, prices given by $q^{FI\,(0,a')}(\omega^{FI})$

Want to compare the "average" price schedule faced by each β type across all s in benchmark to "average" price schedule faced by each β type in full info case $\rightarrow$ **how to do this?**

- fix the distribution x^* from the benchmark model and compute average prices for each action according to

$$\overline{q}^{(0,a')}(\beta, s) = \sum_{e,z,a} q^{(0,a')}(e,z,a,s) \cdot \frac{x^*(\beta,e,z,a,s)}{\sum_{\hat{e},\hat{z},\hat{a}} x^*(\beta,\hat{e},\hat{z},\hat{a},s)}$$

$$\overline{q}^{FI\,(0,a')}(\beta) = \sum_{\omega} q^{FI\,(0,a')}(\omega) \cdot \frac{x^*(\beta,\omega)}{\sum_{\hat{\omega}} x^*(\beta,\hat{\omega})}$$

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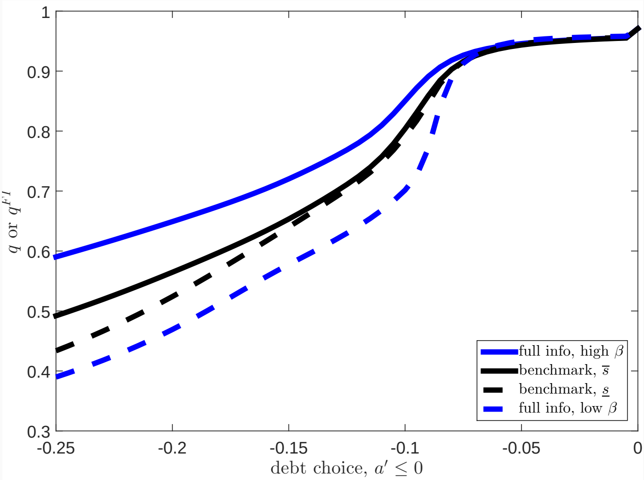
Asymmetric vs. Full Information: Moments

	def. rate	int. rate	$\frac{\text{med net worth}}{\text{med income}}$	frac. in debt	$\frac{\text{debt}}{\text{income}}$
Data	0.54%	11.35%	1.28	6.73%	0.67%
Bench					
agg.	0.53	9.98	2.13	8.24	0.64
β_H	0.39	10.06	2.80	5.24	0.44
β_L	0.61	9.92	1.76	10.22	0.77
Full info					
agg.	0.45	11.61	2.20	7.98	0.61
β_H	0.42	12.94	2.92	5.02	0.45
β_L	0.50	10.77	1.83	9.86	0.72

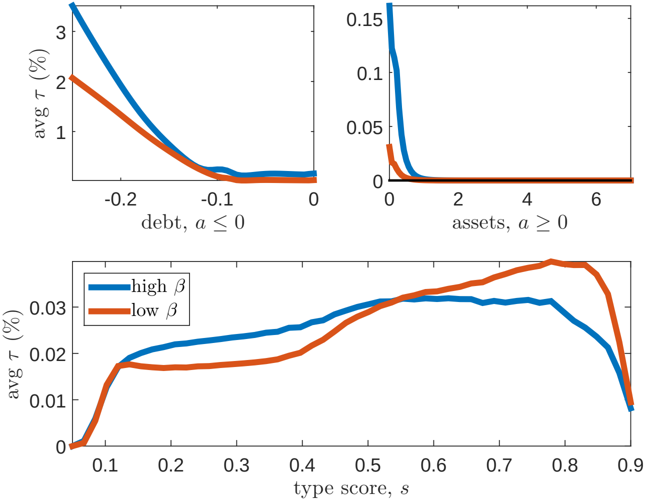
- under full info, β_L types who drive default rate get less debt
- selection affects important for interest rates (high types choose

Asymmetric vs. Full Information: Pricing Schedules



- more dispersion in price schedules with full info ▸ Construction
- high type (i.e. high score) still face lower interest rates ▸ Back

Value of Reputation across the State Space



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Static vs. Dynamic Costs of Default

Question: What happens if there are **no static costs of default**?

Answer: Set $\eta = 0$, re-solve model:

Moment	Data	$\eta = 9.8\%$	$\eta = 0$
Default rate (%)	0.54	0.53	2.63
Average interest rate (%)	11.35	9.98	57.73
Median net worth / median income	1.28	2.13	2.20
Fraction of households in debt (%)	6.73	8.24	6.69
Average debt-to-income ratio (%)	0.67	0.64	0.82
$\overline{\tau}$ (%)	–	0.015	0.212

- $\downarrow \eta \rightarrow \uparrow$ default $\rightarrow \uparrow$ interest rates $\rightarrow$ high willingness to pay for a high type score.

Some Related Literature

Quantitative Models of Bankruptcy: