Wage Dispersion in the Aiyagari Model

More Inexistent than Preliminary

Per Krusell Jinfeng Luo José-Víctor Ríos-Rull

July 17, 2018

McCalm 2018, Edinburgh

Introduction

- In the standard Aiyagari (1994) wages or earnings are completely exogenous. How can this be changed?
  - Entrepreneurial (or criminal) activity a la Quadrini (2000)
  - Could add Education, Life-Cycle w/o Learning by Doing
  - Search Frictions and/or Learning by Doing.
- We explore a variety of models á la Aiyagari (1994) where
  - Workers endowment of efficiency units is constant.
  - Getting a job has frictions.
  - Firms create jobs and post wages that remain constant for the duration of the job.
- Wage dispersion and the wealth distribution are endogenous
  - Aiyagari (1994) meets Burdett and Mortensen (1998).
  - Related to Lise (2013), Hornstein, Krusell, and Violante (2011), Krusell, Mukoyama, and Åđahin (2010) Eeckhout and Sepahsalari (2015), Chaumont and Shi (2017),

Model 1: Precautionary Savings, Competitive Search

- Jobs are created by firms (plants). A job plus a worker produce one unit of the good.
  - To get a worker firms pay a flow cost $\bar{c}$ to post a vacancy.
  - Jobs are destroyed at rate δ. Workers cannot (won't) quit.
- Households differ in wealth and wages (if working). Households can save. There are no state contingent claims, nor borrowing.
  - If unemployed, households produce b and search. If employed they get w and do not search.
  - Matching protocol is competitive search. Workers know what type of wage they are looking for.
- General equilibrium (unimportant): Workers own firms.
- Small equilibrium wage dispersion

Model 2. Endogenous Quits: Beauty of Extreme Value Shocks

1. Shocks to the utility of working or not working: Some workers quit.

2. Add a (smoothed) quitting motive so that higher wage workers quit less often: Firms may want to pay high wages to retain workers.

3. Conditional on wealth, high wage workers quit less often.

4. But Selection (correlation 1 between wage and wealth when hired) makes wealth trump wages and higher wages imply quit less often: Wage inequality collapses due to firms profit maximization.

Model 3. Diffusion of wealth & wages: More Ext. Val. Shocks

1. Reduces the correlation of Wages and Applicants Wages, even if exaggerating wage dispersion

2. Another set of Extreme Value Shocks, this time to the type of market that the unemployed want to go to (aiming shocks). It difuses the link between wage and wealth.

3. Which reduces/solves the selection problem and justifies paying higher wages for longer tenure.

Model 4. Add Endogenous Productivity creation

1. Firms can spend more to make more productive plants with higher maintenance costs eve when idle. Only worth if workers last longer: hence EFFICIENCY WAGES

2. Can be added to a theory of non-linear wages.

Model 5. Add on the job search

1. With extreme value shocks makes a more empirically relevant world than Burdett and Mortensen (1998) or Chaumont and Shi (2017). There are frictions.

2. Perhaps even Model 6 with Human Capital / Occupation Expertise accumulation.

Aggregate Fluctuations

1. Aggregate shocks can be added with finite cost using advances in Boppart, Krusell, and Mitman (2018)

2. Today we have Models 1-3 and Aggregate Fluctuations in Models 1-2.

Preliminary Findings

1. **Model 1** With workers on the job being identical (no endogenous quitting) (preliminarily) wage dispersion is about 2-3%. As (in different context) Hornstein, Krusell, and Violante (2011).

2. **Model 2** With endogenous quits (which we wanted it to add dispersion), actual wage dispersion collapses due to selection. Big bad news. Not theorem but quantitative statment. More later.

3. **Model 3** With diffused access of workers to differently waged jobs, wage dispersion returns.

4. Taking Stock: By themselves, wealth differences are not a promising venue for frictional wage dispersion unless perhaps for occupational choice at the beginning of the working life (not today).

Preliminary Findings: Aggregate Fluctuations

1. Models 1 and 2 deliver exciting (expected) implications.
    - Large employment variation
    - Smaller wage variation
    - Quiting in Model 2 (early unemployment jump)

2. We are very hopeful about Model 3 as an engine for wage disperion

3. Models 4 and 5 will complete the task

Order of Events of Model 1

1. Households enter period t with or without a job.

2. **Production & Consumption**: The employed produce z on the job. The unemployed produce b at home. They make consumption-saving decisions.

3. **Job Search**: Potential firms decide whether to enter and if so, the wage w at which to post a vacancy. The unemployed choose which wages to apply to.

4. **Job Match & Separation**: The employed workers who receive exogenous separation shocks become unemployed. The successfully matched job candidates become employed. Quitting is (irrelevantly) outlawed.

5. Households enter period $t + 1$ with new employment status.

Household Problem

- An individual is either employed (e) or unemployed (u).
- Individual state: wealth and wage
  - If employed: (a, w)
  - If unemployed: (a)
- Problem of the employed: (Standard)

$$V^e(a, w) = \max_{c, a'} u(c) + \beta \ [(1 - \delta) V^e(a', w) + \delta V^u(a)]$$

$$\text{s.t.} \quad c + a' = a(1 + r) + w, \quad a \geq 0$$

- Problem of the unemployed: Choose which wage to look for

$$V^u(a) = \max_{c, a', w} u(c) + \beta \left\{ \psi^h[\theta(w)] \ V^e(a', w) + [1 - \psi^h(\theta(w))] \ V^u(a') \right\}$$

$$\text{s.t.} \quad c + a' = a(1 + r) + b, \quad a \geq 0$$

Firms Post vacancies at different wages & filling probabilities

- Value of a job with wage w:

$$\Omega(w) = z - w + \frac{1-\delta}{1+r}\Omega(w)$$

  - Affine in w

$$\Omega(w) = (z - w)\frac{1+r}{r+\delta}$$

- Value of posting a vacancy

$$\psi^f[\theta(w)] \;\; \Omega(w)$$

- Free entry condition requires

$$\bar{c} = \psi^f[\theta(w)] \;\; \Omega(w), \qquad \forall w \;\; \text{that are offered}$$

Stationary Equilibrium

- A stationary equilibrium is: $\{V^e, V^u, \Omega, a^e, a^u, w^u, \theta\}$, an interest rate r, and a stationary distribution x over (a, w), s.t.

  1. $\{V^e, V^u, a^e, a^u, w^u\}$ solve households' problems, $\{\Omega\}$ solves the firm's problem.

  2. Zero profit condition holds for active markets

$$\bar{c} = \psi^f[\theta(w)] \quad \Omega(w), \qquad \forall w \text{ that are offered}/$$

  3. An interest rate r clears the asset market

$$\int a \; dx = \int \Omega(w) \; dx.$$

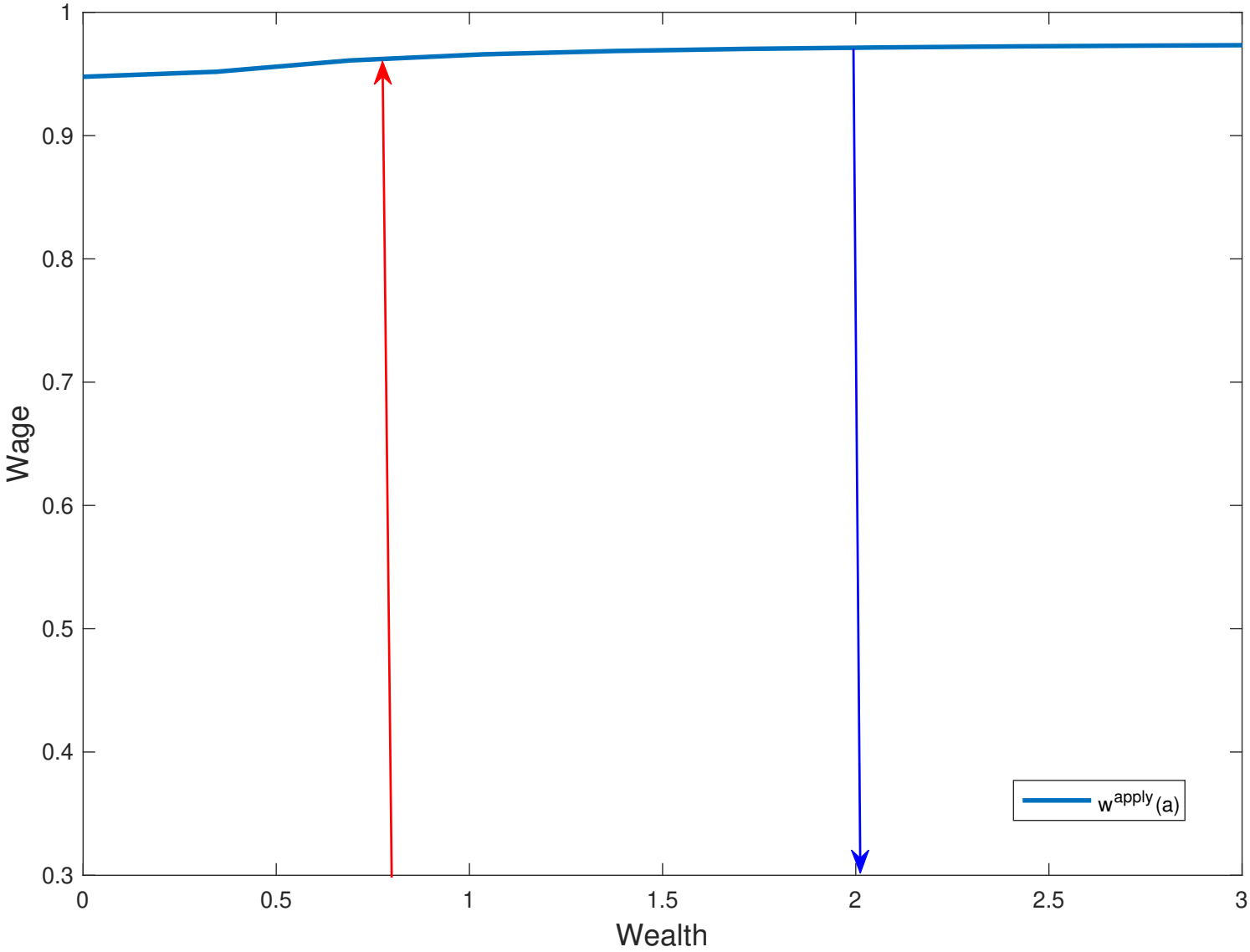
Characterization of a worker's decisions

- The F.O.C for wage applicants

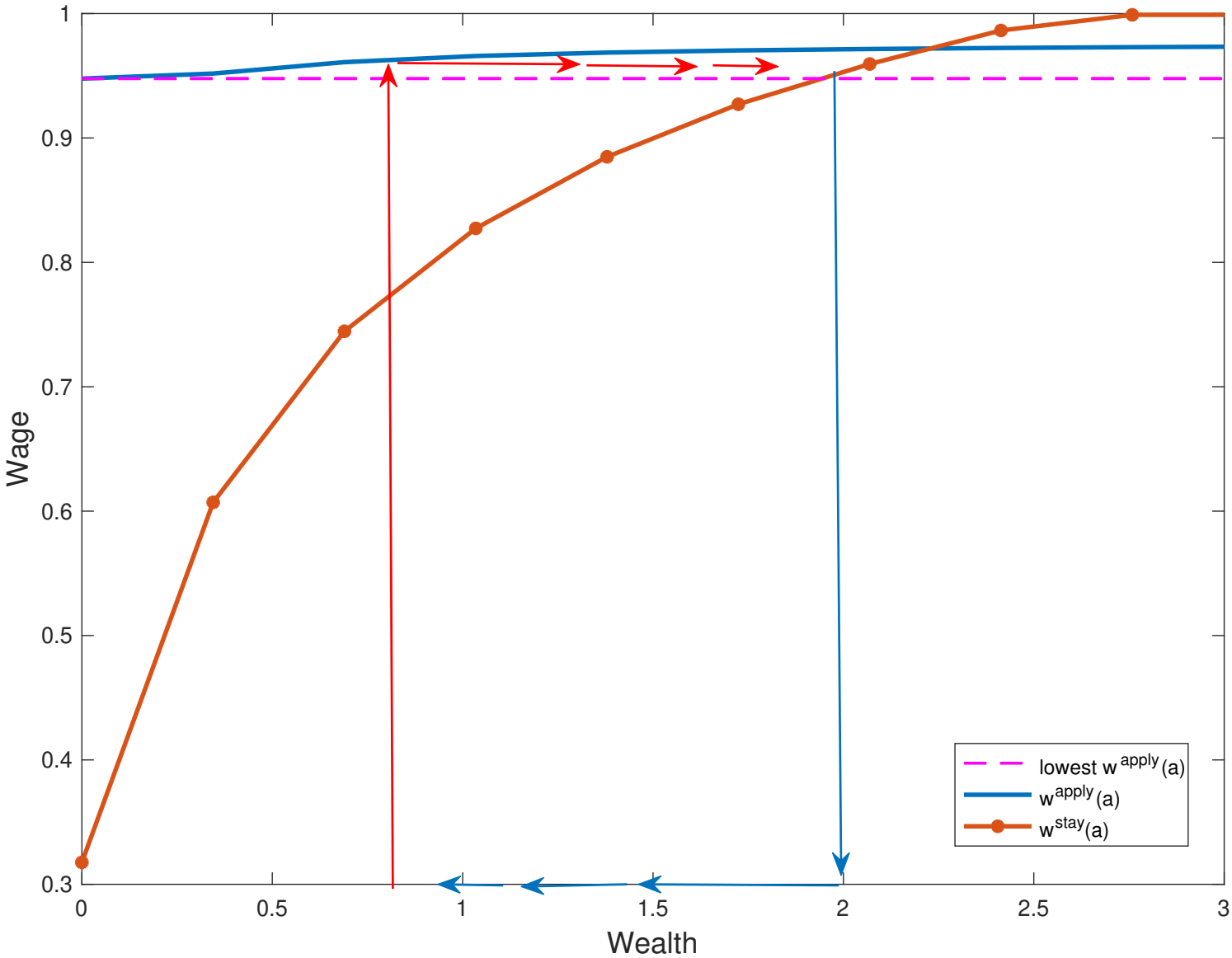
$$\psi^h(w)V_w^e(a',w) = \psi_w^h(w)\left[V^u(a') - V^e(a',w)\right]$$

- Households with more wealth are able to insure better against unemployment risk.
- As a result they apply for higher wage jobs and we have dispersion
- A form of "Precautionary job search".

How does the Model Work



How does the Model Work



Look at a Standard Economy:

- CRRA Utility Function

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \qquad \sigma = 2$$

- Period is a quarter $\beta = .99$
- Average job duration: 5 years ($\delta = 0.05$)
- Home production: 30% of market production ($b = 0.3z$) (low end)
- Vacancy Posting Cost: 50% of period job output (large) ($\bar{c} = 0.5z$). Firms are valuable
- Cobb-Douglas Matching Function

$$M(u, v) = \chi u^{\eta} v^{1-\eta}$$

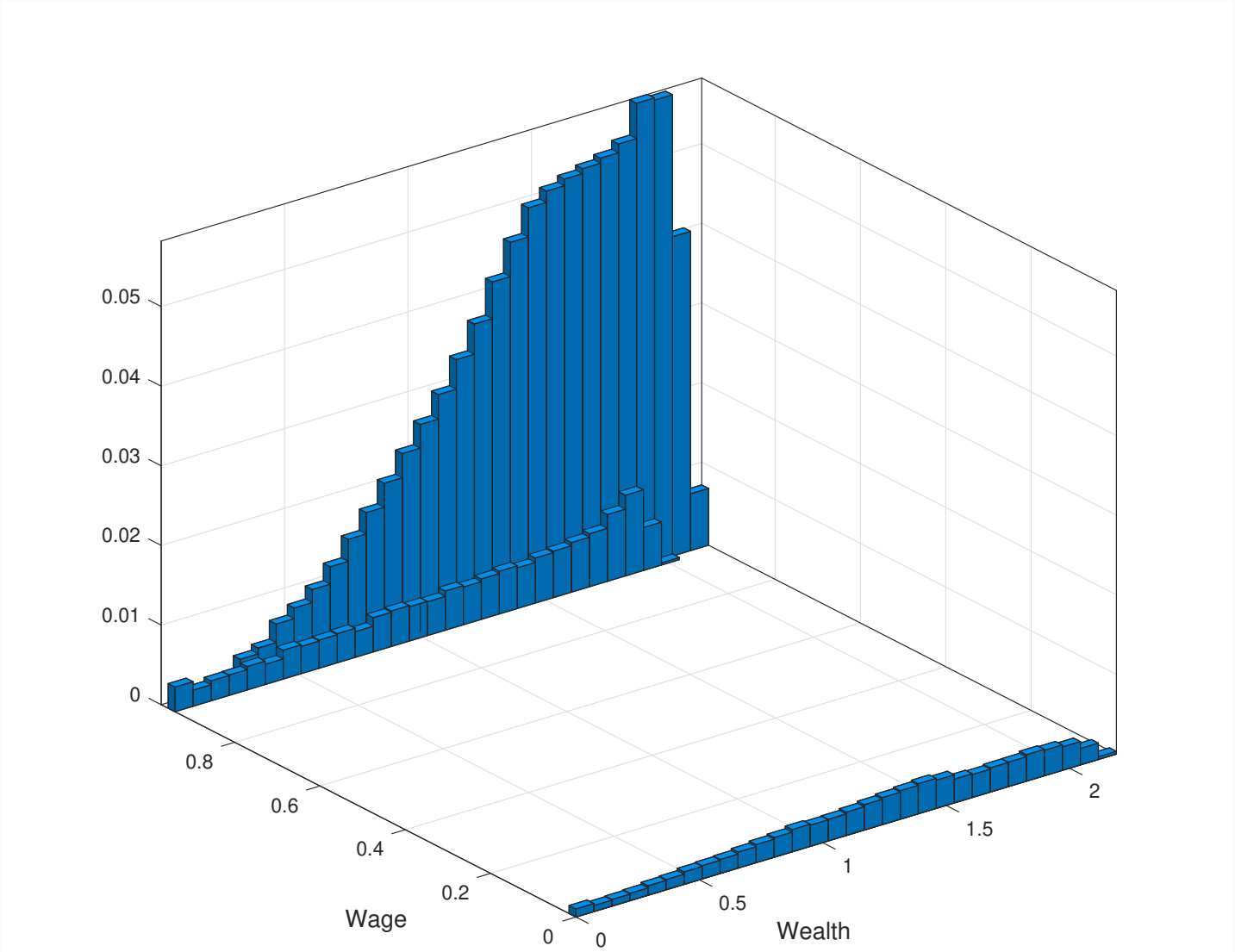
	σ	β	χ	η	δ	z	b	$\bar{c}$
Value	2	0.99	0.675	0.72	0.05	1	0.3	0.5

Key Model Statistics: Benchmark

	Notation	Benchmark
Interest Rate	r	0.24%
Unemployment Rate	u	7.18%
Unemployment Duration	$\chi\theta^{\eta-1}$	1.54
Employment Duration	$\frac{1}{\delta}$	20
Wage Mean-min Ratio	$\frac{\bar{w}}{w^{min}}$	1.0165
Wage Max-min Ratio	$\frac{w^{max}}{w^{min}}$	1.0255

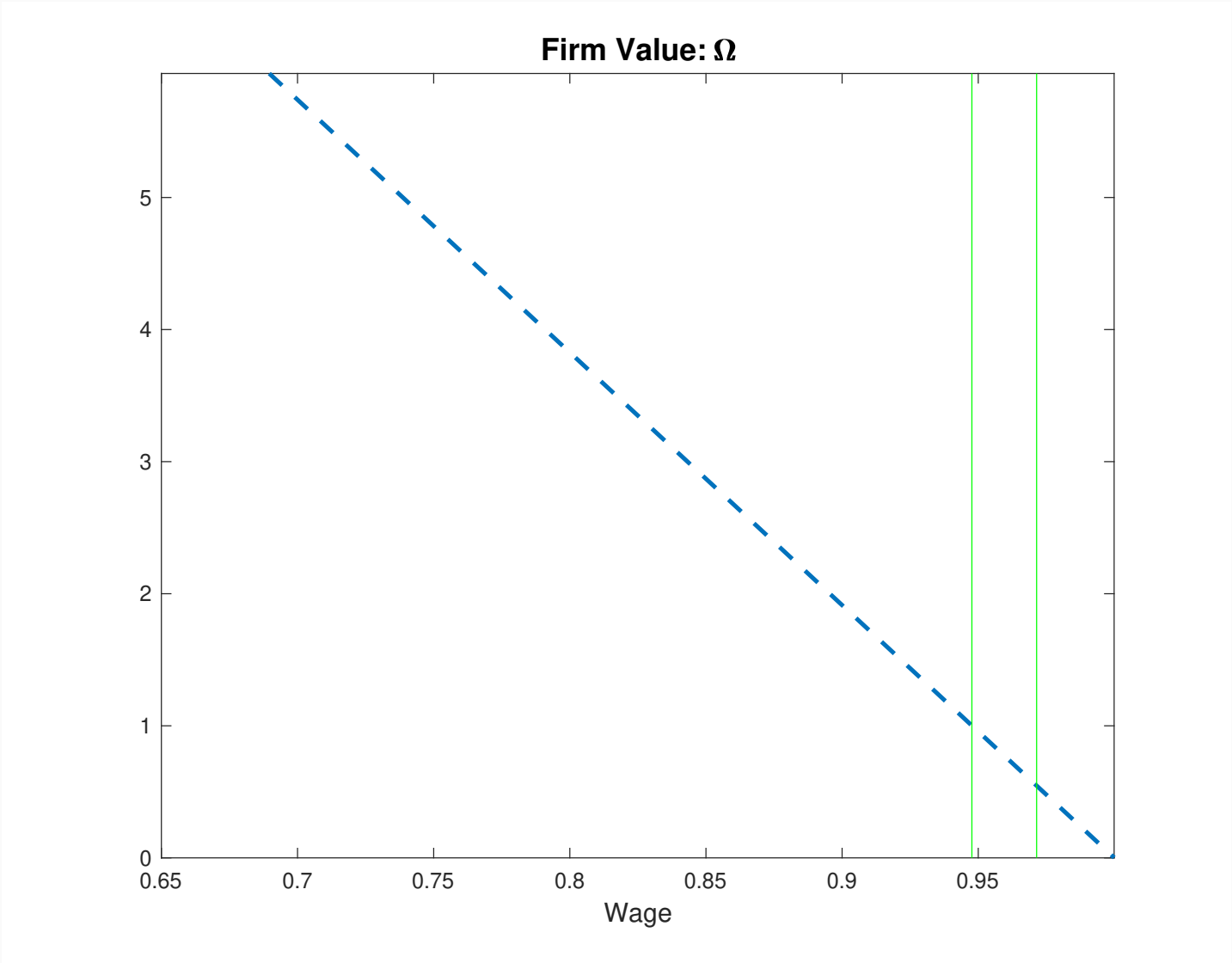
The Distribution of Wealth and Wages

- Small total wealth
- Very small wealth dispersion (honest hard work only)



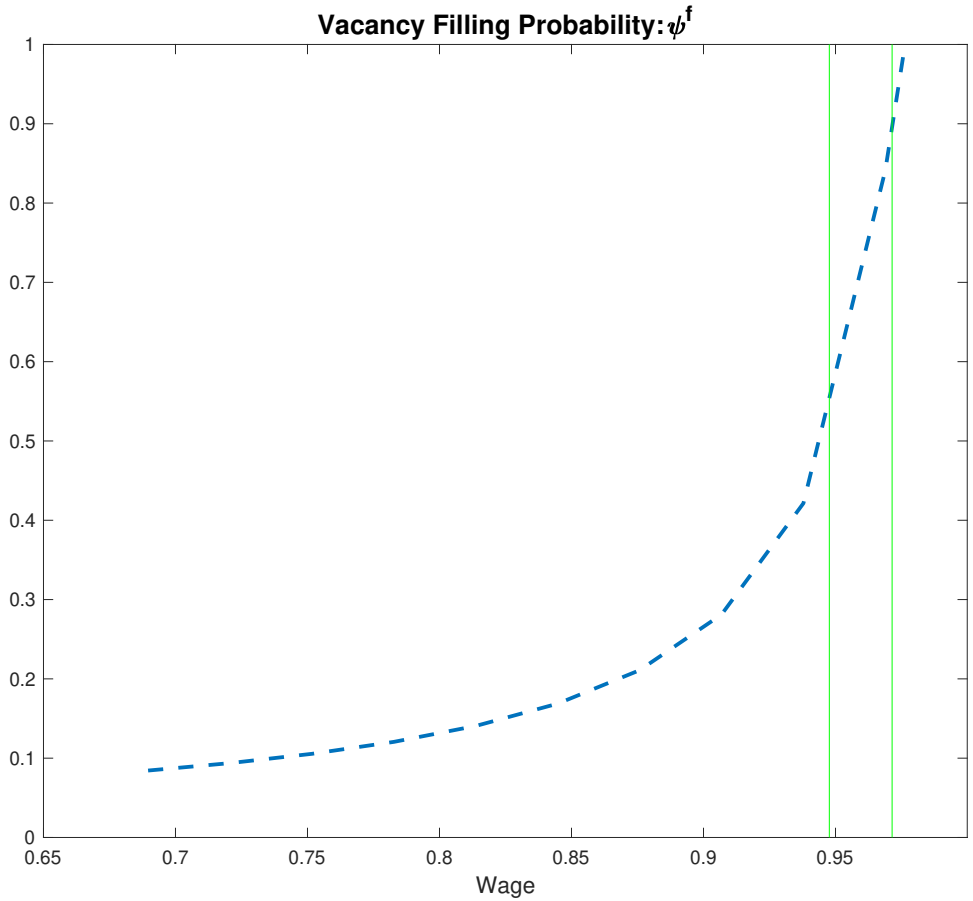
Firms Value Function

- Firm value: $\Omega(w) = (z - w) \times$ discounted duration



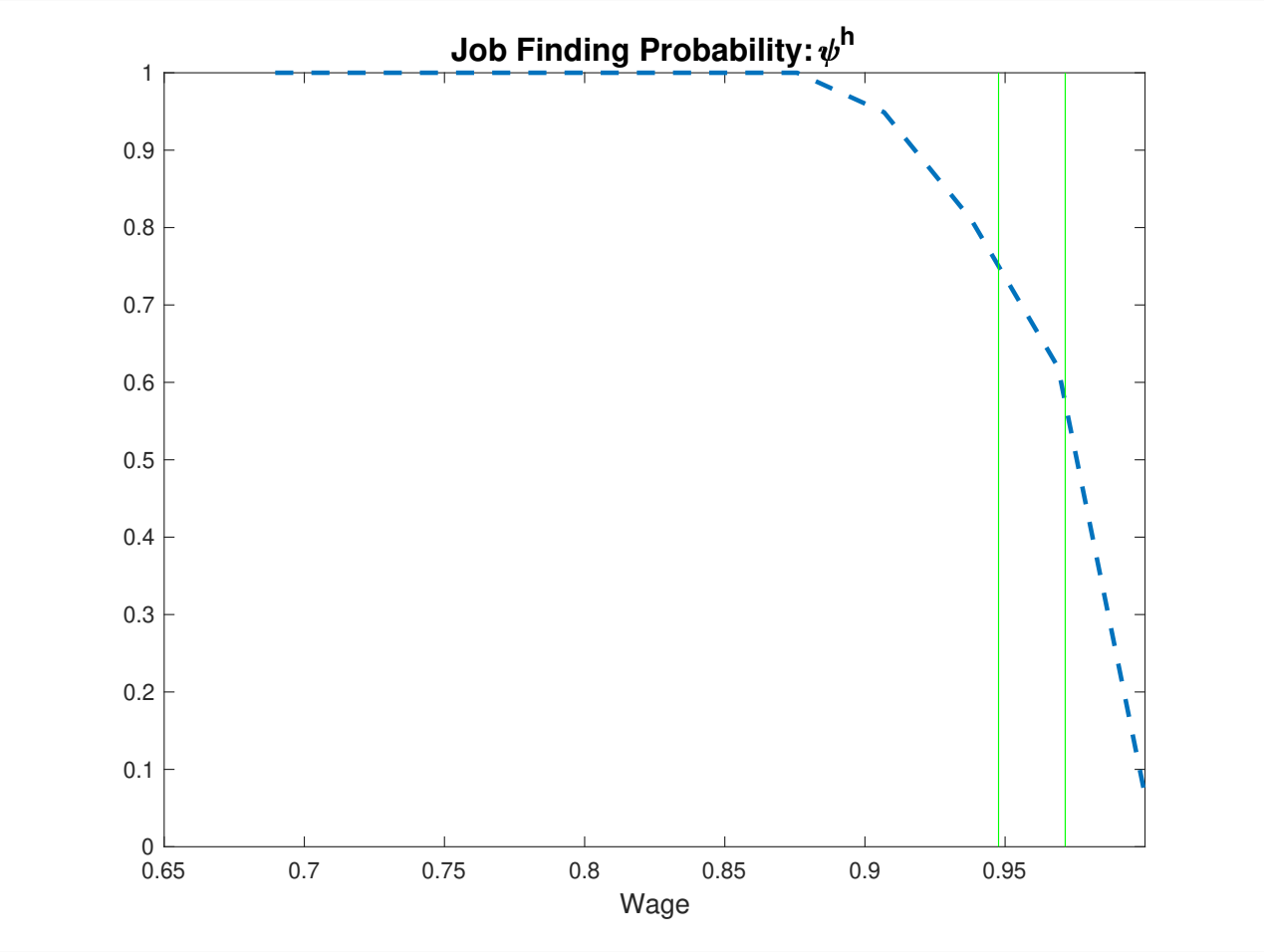
Job filling Probabilities

- A large equilibrium probability variation (0.6–0.9) for a narrow range of wages 2.5%
- Differences in job finding rate are an insufficient rationale for wage dispersion.

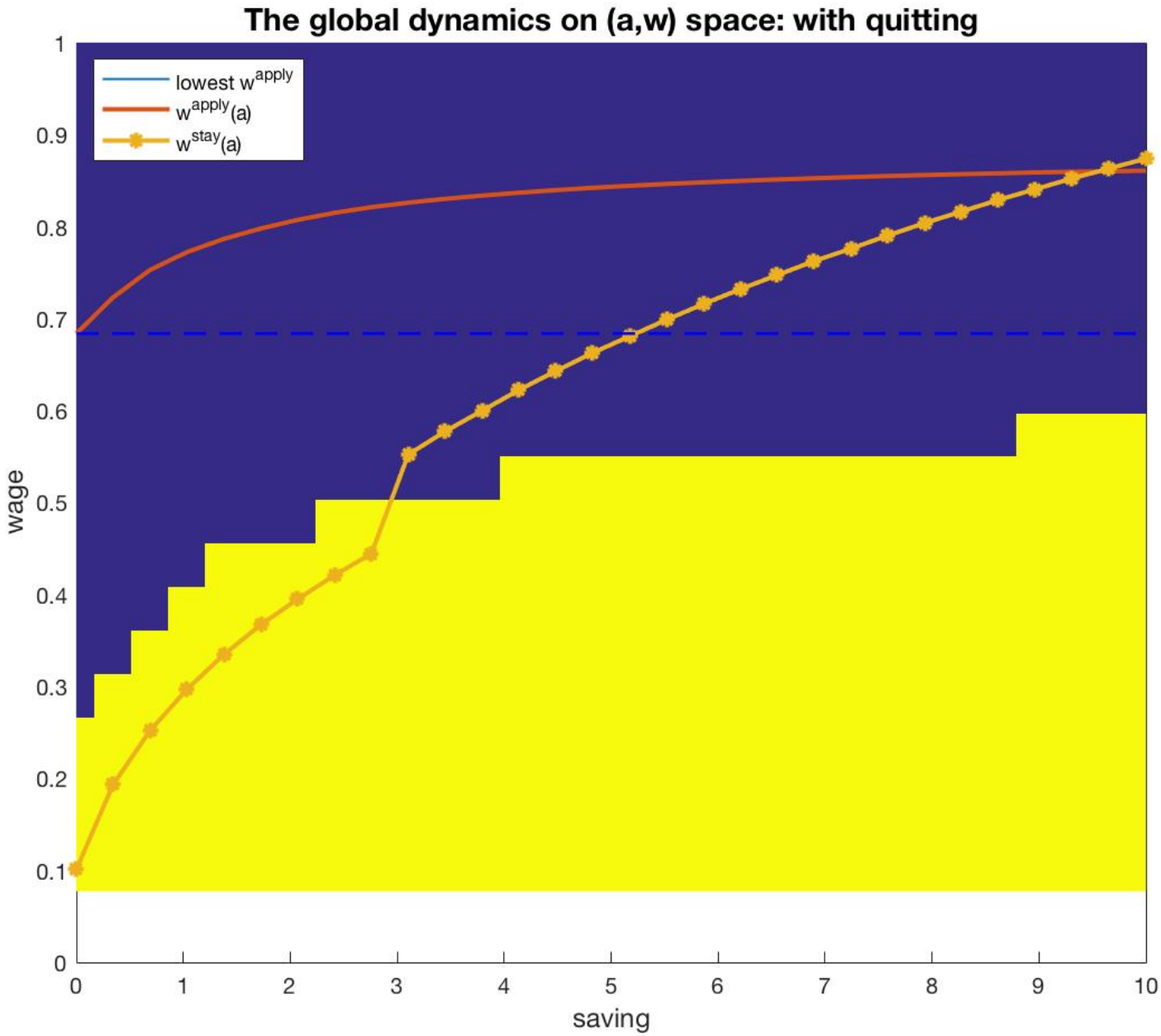


Job Finding Probabilities

- Job finding rate implied by vacancy filling rate
- Differences in job filling rate is an insufficient rationale for large wage dispersion.



Right to quit does not change anything



Summary of Model 1

- We have a standard Aiyagari model plus a competitive labor search.
- Precautionary job search motive causes richer people to apply for higher wage jobs.
- Quantitatively wage dispersion due to this is small.
- Firms do not have enough rewards to pay different wages. Model 2 attempts to fix this.
- Workers also need to be able to coexist with higher wage dispersion. Model 3 works towards this.

Model 2: Add Incentive to Quit to get a flatter $\Omega(w)$

- Suppose at the beginning of the period employed workers receive a pair of i.i.d shocks $\{\epsilon^e, \epsilon^u\}$ depending on quitting decisions.
- Value of the employed right before receiving the shocks:

$$\widehat{V}^e(a, w) = \int \max\{V^e(a, w) + \epsilon^e, V^u(a) + \epsilon^u\} dF^\epsilon$$

  V^e and V^u are values after quitting decision as described before.

- If shocks are Type-I Extreme Value dbtn (Gumbel), then $\widehat{V}$ has a closed form and the ex-ante quitting probability $q(a, w)$ is

$$q(a, w) = \frac{1}{1 + e^{\alpha[V^e(a,w) - V^u(a)]}}$$

- Hence higher wages imply longer job durations

Model 2: Value of the firm

- Probability of retaining a worker with tenure j at wage w is $\ell^j(w)$.
  (One to one mapping between wealth and tenure)

- The firm's value

$$\Omega^j(w) = \ell^j(w)\left[z - w + \frac{1-\delta}{1+r}\Omega^{j+1}(w)\right]$$

- Solving forward

$$\Omega^0(w) = (z-w)\sum_{\tau=0}^{\infty}\left[\left(\frac{1-\delta}{1+r}\right)^{\tau}\prod_{i=0}^{\tau}\ell^i(w)\right] = (z-w)\ Q(w)$$

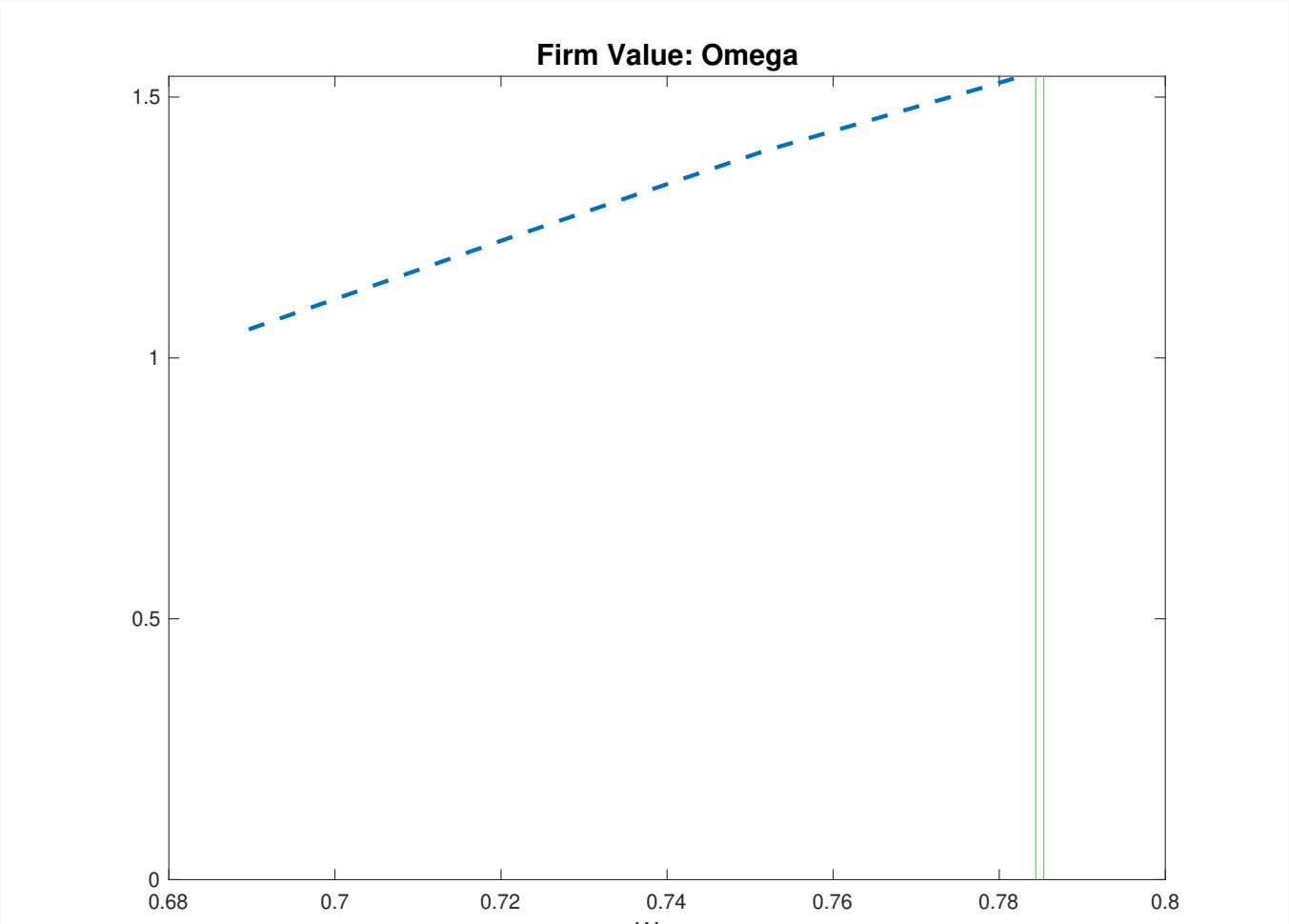
- Only equilibrium object relevant for the firm is $Q(w)$. Rest is unchanged.

Model 2: Time-line

- Household enters period t with or without a job: {E, U}.
- **Job Posting**: Potential firms decide whether to enter and if so, the market (w) at which to post a vacancy.
- **Quitting**: E draw shocks $\{\epsilon^e, \epsilon^u\}$ and make quitting decision.
- **Production and Consumption**: E quitters and U produce b at home, and choose $\{a', w\}$; E non-quitters produce y on the market, and choose $\{a'\}$.
- **Job Search**: E quitters and U who successfully find jobs become E, otherwise becomes U.
- **Separation**: E non-quitters who receive δ shock become U, otherwise stay as E.

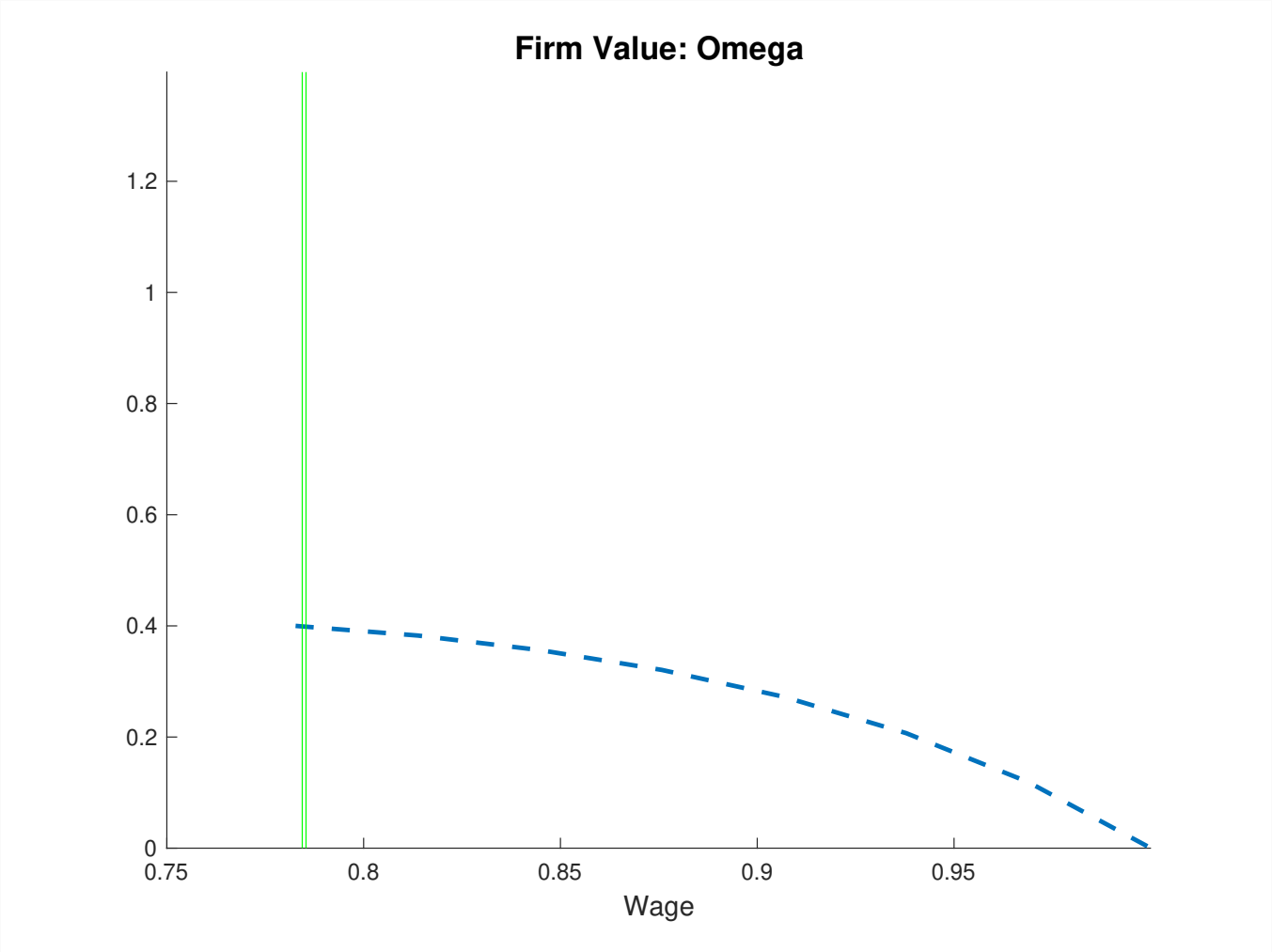
Value of the firm: same worker but different wage: Poor

- For very poor people, employment duration increases fast when wage goes up.
- Despite wage is increasing while output is fixed, firm value is increasing!



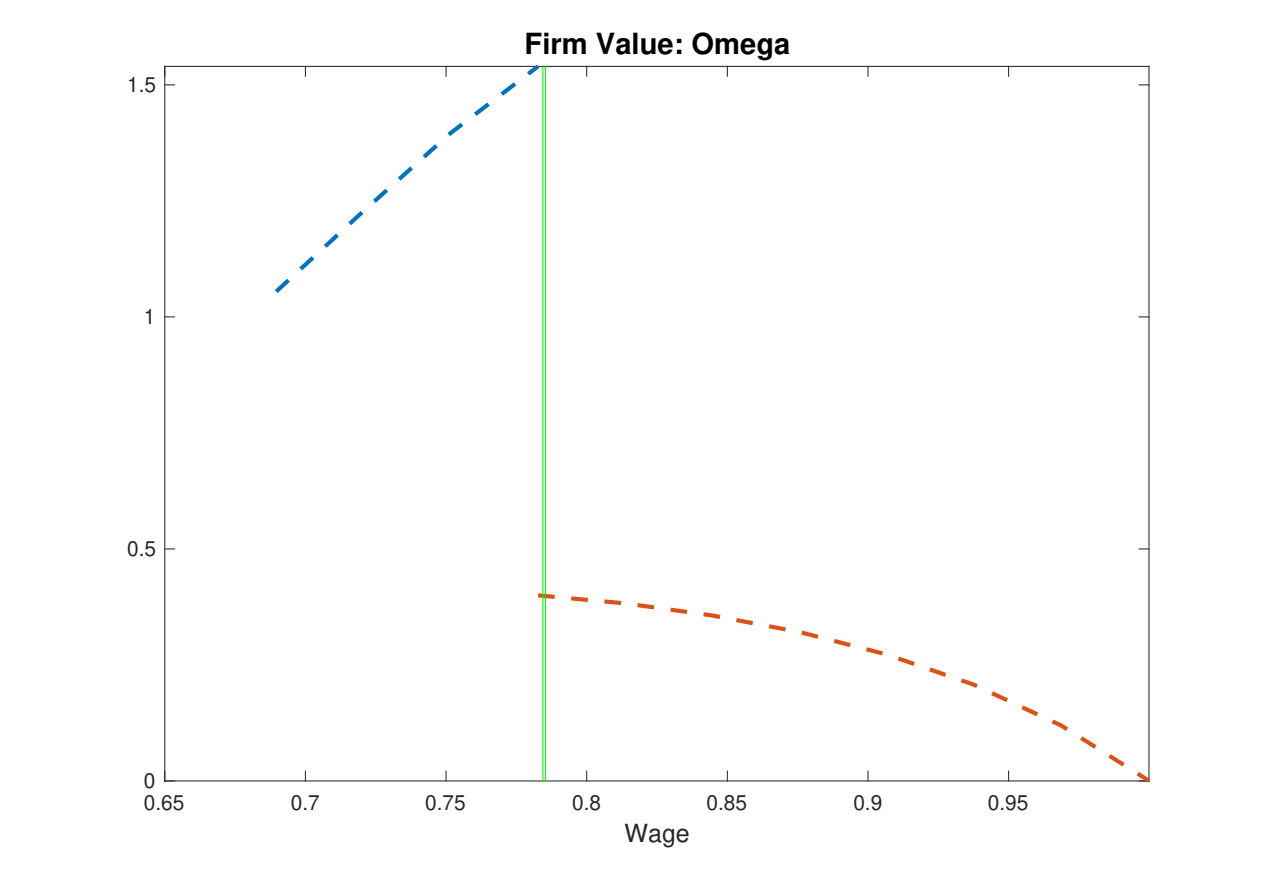
Value of the firm: same worker but different wage: Rich

- For very rich people, employment duration increases not so fast.
- Firm value is decreasing in wages.

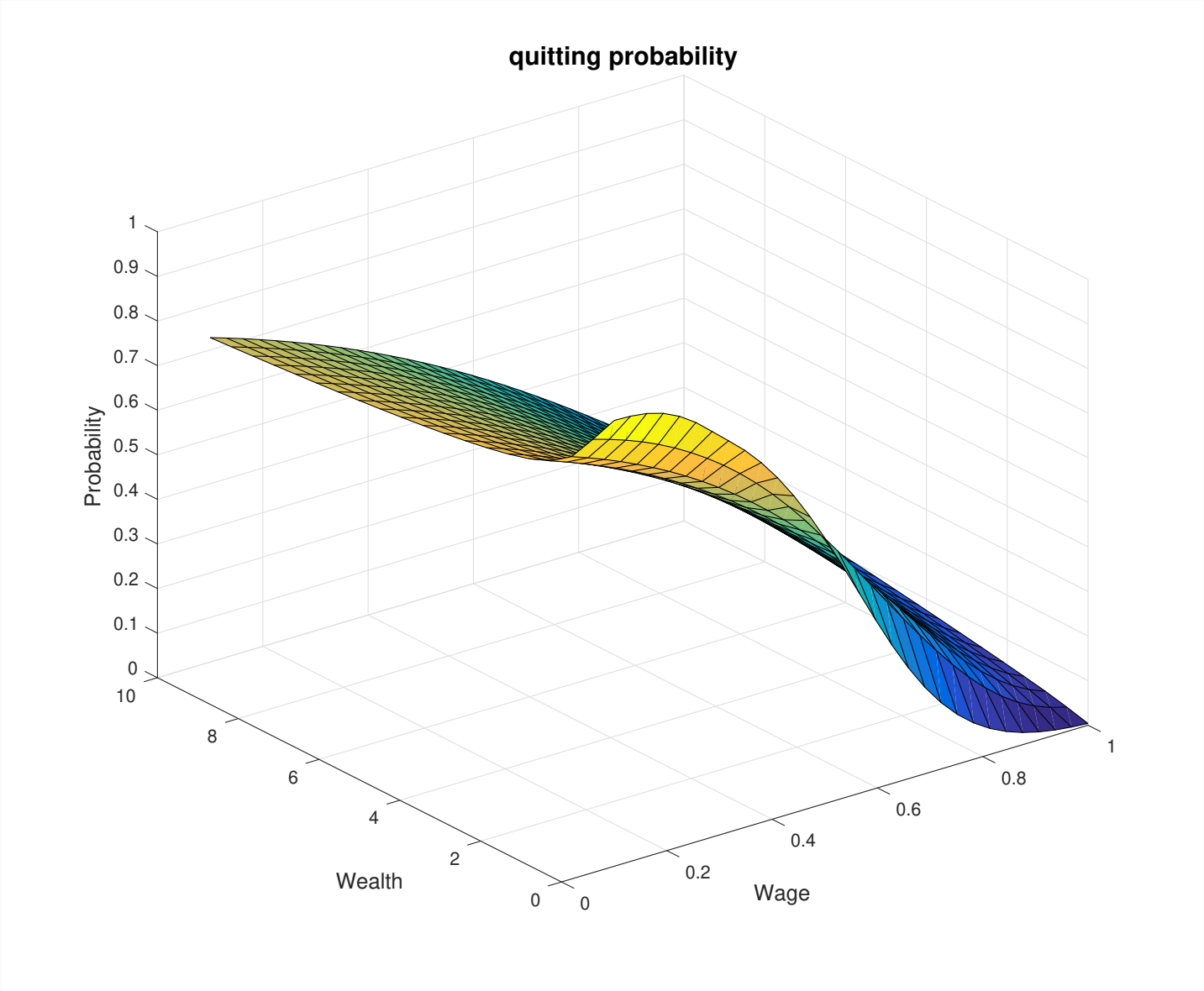


Value of the firm: Accounting for Worker Selection

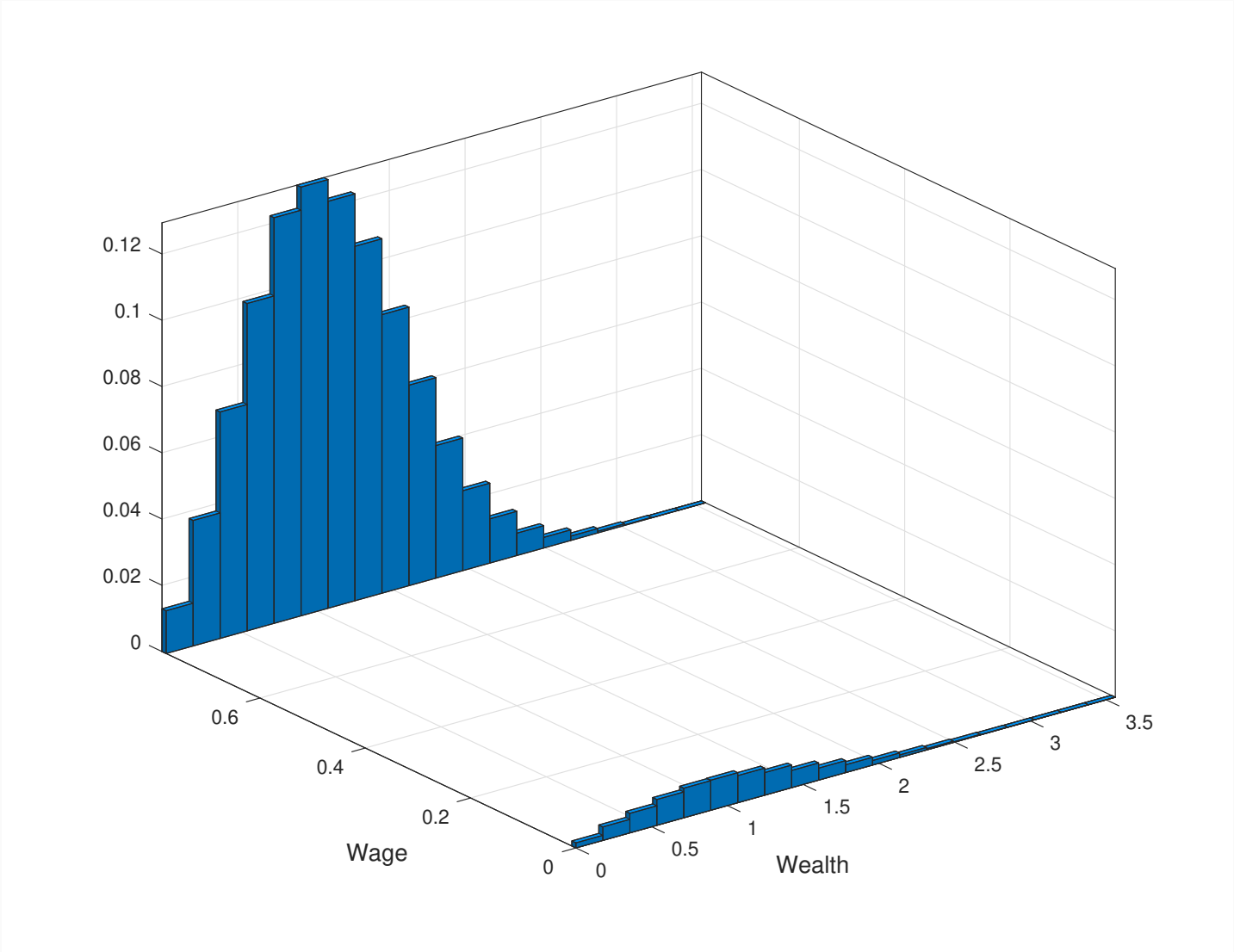
- Large drop from below to above equilibrium wages.
- In Equilibrium wage dispersion COLLAPSES due to selection.



Quitting Probability



Collapsed Wage Dispersion



Effect of Quitting: The Mechanism

- Two forces shape the wage dispersion
  - People quit less at higher paid jobs, which enlarge the spectrum of wages that firms are willing to pay (for a given range of vacancy filling probability).
  - However, by paying higher wages, firms attract workers with more wealth.
  - Wealthy people quit more often, shrink the employment duration.
  - In the equilibrium, the wage gaps is narrow and the effect of wealth dominates.

Model 3: Diffuse Wealth on Jobs

- Now try to diffuse wealth for each wage level.
- We introduce another dose of extreme value shocks to different job matches.
- At the beginning of the period, the unemployed look for jobs subject to shocks to potential matches

$$V^u(a) = \int \max_w \left[\psi^h(w) \widehat{V}^e(a, w) + (1 - \psi^h(w)) \widehat{V}^u(a) + \epsilon^w \right] dF^\epsilon$$

- The employed choose whether to quit as before

$$V^e(a, w) = \int \max\{ \widehat{V}^e(a, w) + \epsilon^e, \widehat{V}^u(a) + \epsilon^u \} dF^\epsilon$$

- $\widehat{V}^e(a, w)$ and $\widehat{V}^u(a)$ are after-job-market values.

Model 3: Diffuse Wealth on Jobs

- After the job market, the employed face the problem

$$\widehat{V}^e(a, w) = \max_{c, a' \geq 0} u(c) + \beta \left[(1 - \delta) V^e(a', w) + \delta V^u(a') \right]$$
$$\text{s.t.} \quad c + a' = a(1 + r) + w$$

- The unemployed face the problem

$$\widehat{V}^u(a) = \max_{c, a' \geq 0} u(c) + \beta V^u(a')$$
$$\text{s.t.} \quad c + a' = a(1 + r) + b$$

Model 3: Value of the Firm

- The value of a new firm with wage w and tenure j is again

$$\Omega^0(w) = (z - w) \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{\tau} \prod_{i=0}^{\tau} \ell^i(w) \right] = (z - w)\ Q(w)$$

- where $\ell^j(w)$, the probability of retaining a worker with tenure j at wage w, is now

$$\ell^j(w) = \int \left[1 - q^e(g^{e,j}(a, w), w) \right] \pi(w; a) \mathrm{d}x^u(a)$$

- $\pi(w; a)$ is the logit choice density of wage for some given wealth level a

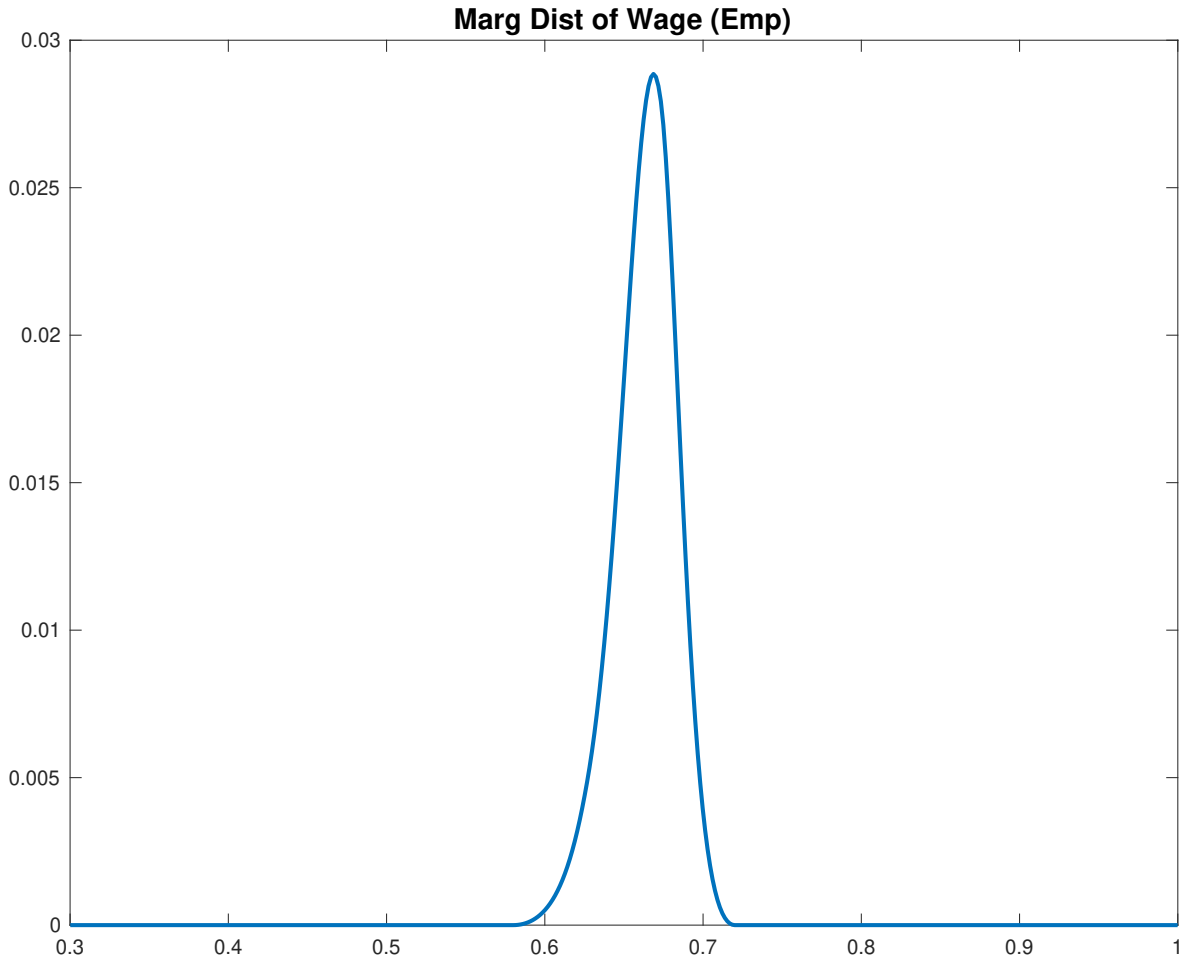
$$\pi(w; a) = \frac{\exp\left\{ \alpha^w \left[\psi^h(w) \widehat{V}^e(a, w) + (1 - \psi^h(w)) \widehat{V}^u(a) \right] \right\}}{\int \exp\left\{ \alpha^w \left[\psi^h(\tilde{w}) \widehat{V}^e(a, \tilde{w}) + (1 - \psi^h(\tilde{w})) \widehat{V}^u(a) \right] \right\} \mathrm{d}\tilde{w}}$$

Model 3: Time-line

- Household enters period t with or without a job: {E, U}.
- V^e, V^u defined here.
- **Quitting**: E draw shocks (ϵ^e, ϵ^u) and make quitting decisions. Those who quit become U' and those who stay become E'.
- **Job Search & Match**: Potential firms decide whether to enter and if so, the market (w) at which to post a vacancy; U receive match specific shocks $\{\epsilon^w\}$ and choose the wage level w to apply. Those who successfully find jobs become E', otherwise becomes U'.
- $\widehat{V}^e, \widehat{V}^u, \{\Omega^j\}$ defined here.
- **Production & Consumption**: U' produce b at home, E' produce y on the market; they then choose consumption today and wealth level tomorrow $\{c, a'\}$.
- **Separation**: E non-quitters who receive δ shock become U', otherwise stay as E'.

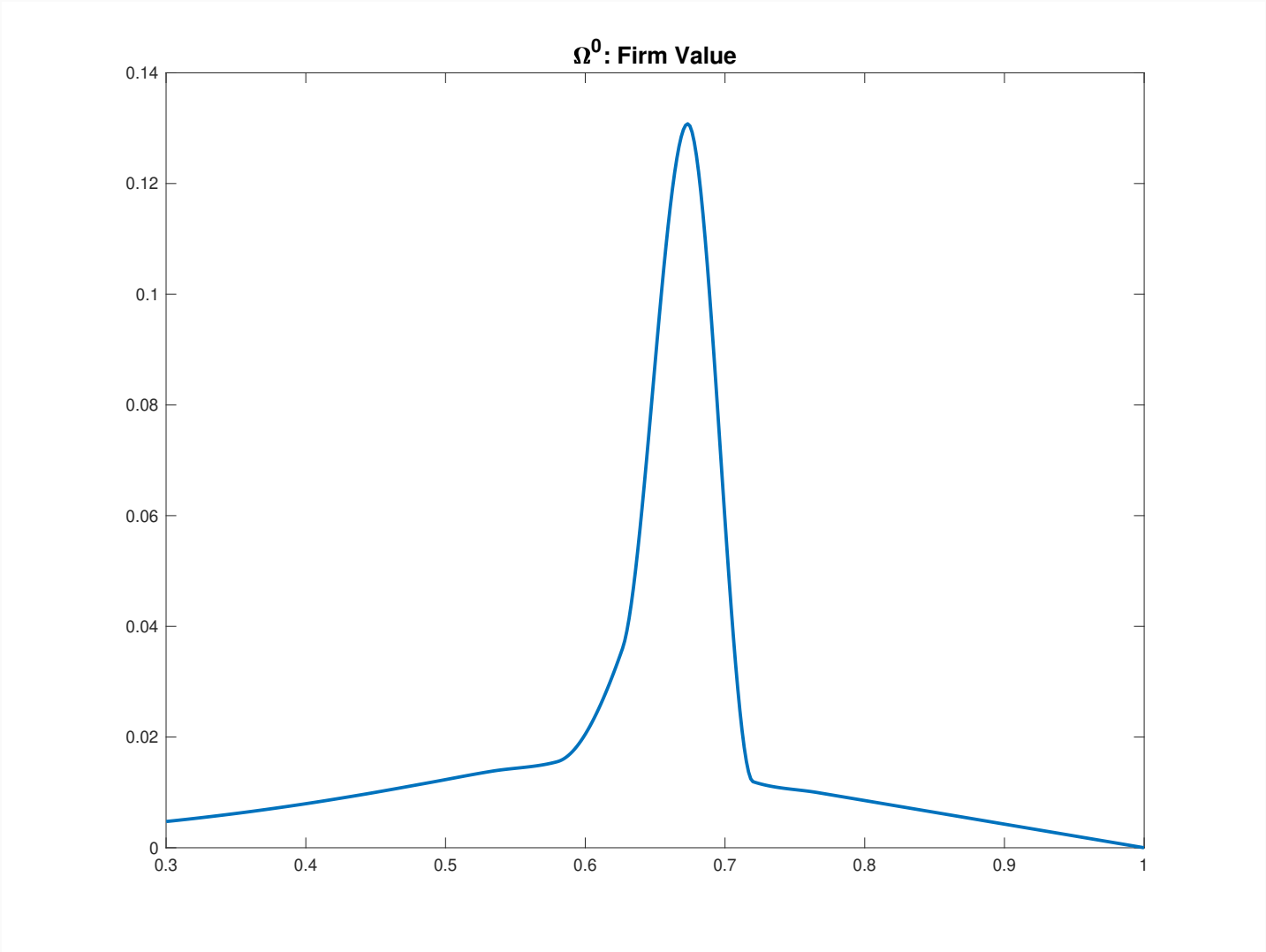
Results: Restored Wage Dispersion

- Wage dispersion is restored due to wage applying shocks.
- Scale parameter of the wage applying shocks: $\alpha^w = 0.5$



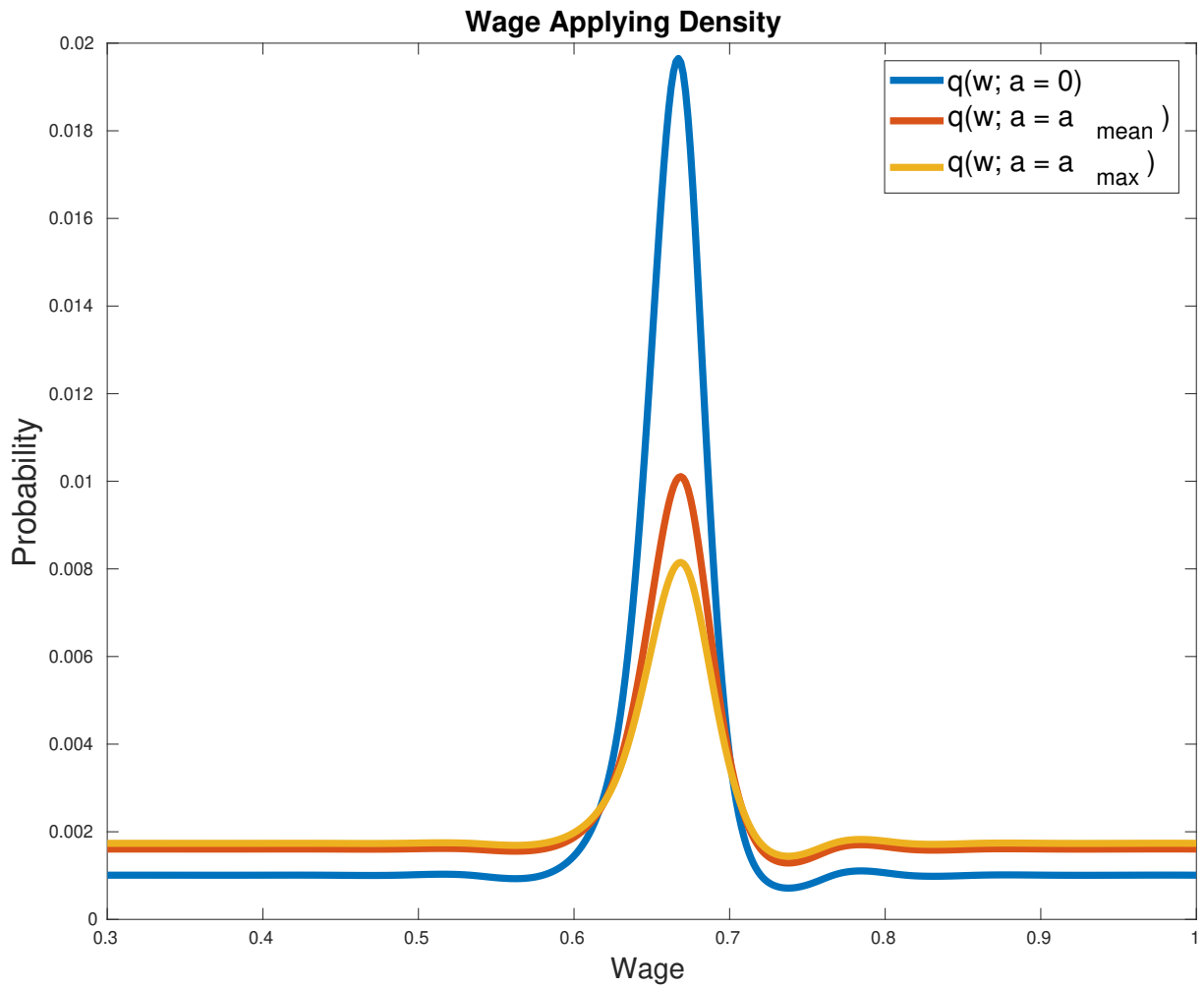
Results: Smooth Firm Value

- Firm value $\Omega^0(w)$ has no sharp drop.
- Selection effect is smoothed.



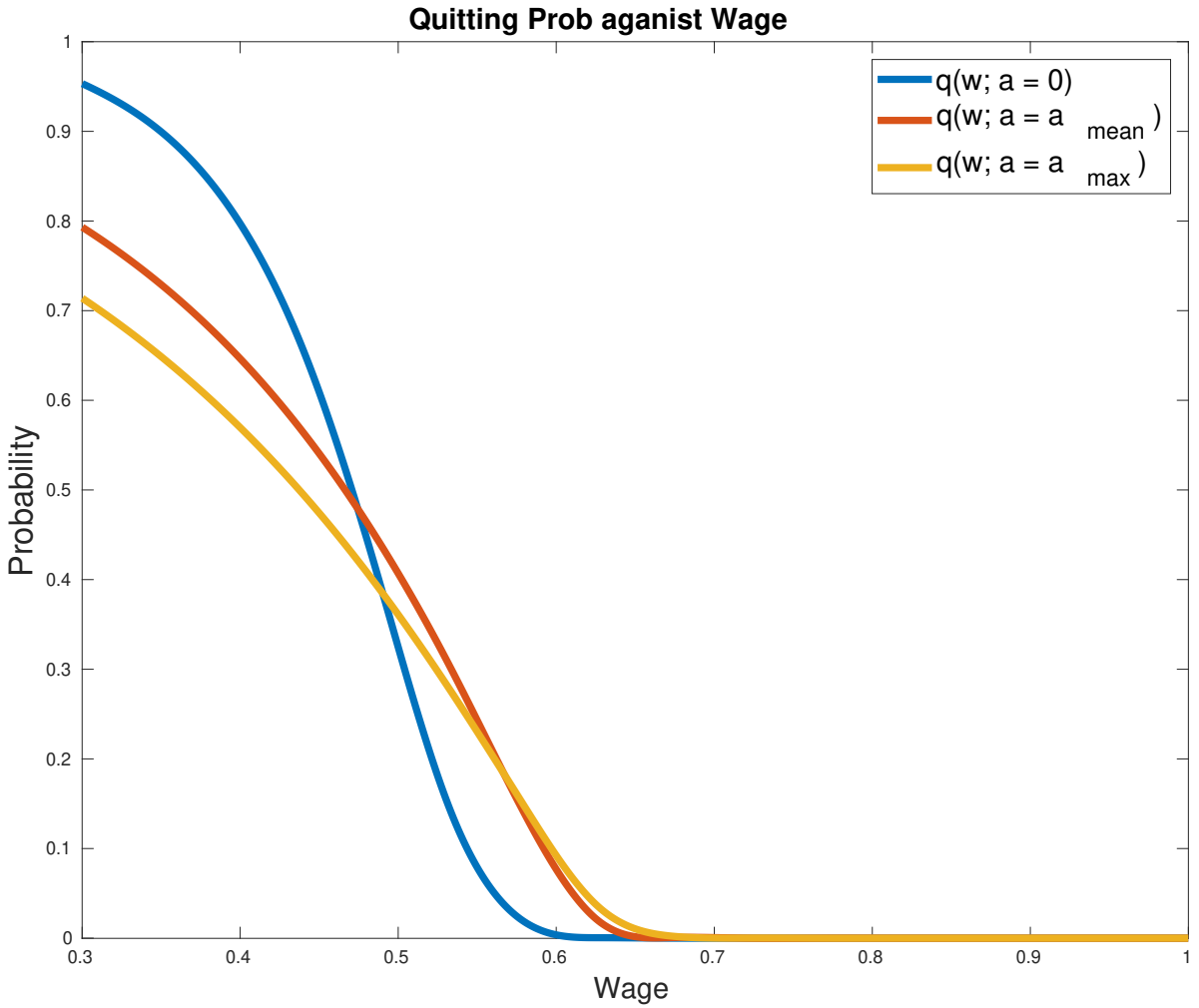
Results: Wage Applying Density

- Wage applying density for agents with different level of wealth a.
- Wage dispersion is almost due to the shock ϵ^w.
- Wage applying is more dispersed for the rich.



Results: Quitting Probability

- Quitting probability for agents with different level of wealth a.
- Quitting happens when poor agents in low-paid jobs get rich.



Results: Key Model Statistics

	Notation	Benchmark
Interest Rate (fixed)	r	0.24%
Unemployment Rate	u	13.87%
Quitting Rate	$\bar{q}$	0.28%
Mean Wage	$\bar{w}$	0.66
Wage Mean-min Ratio	$\frac{\bar{w}}{w^{min}}$	1.12
Wage Max-min Ratio	$\frac{w^{max}}{w^{min}}$	1.21

Aggregate Fluctuations

What is needed?

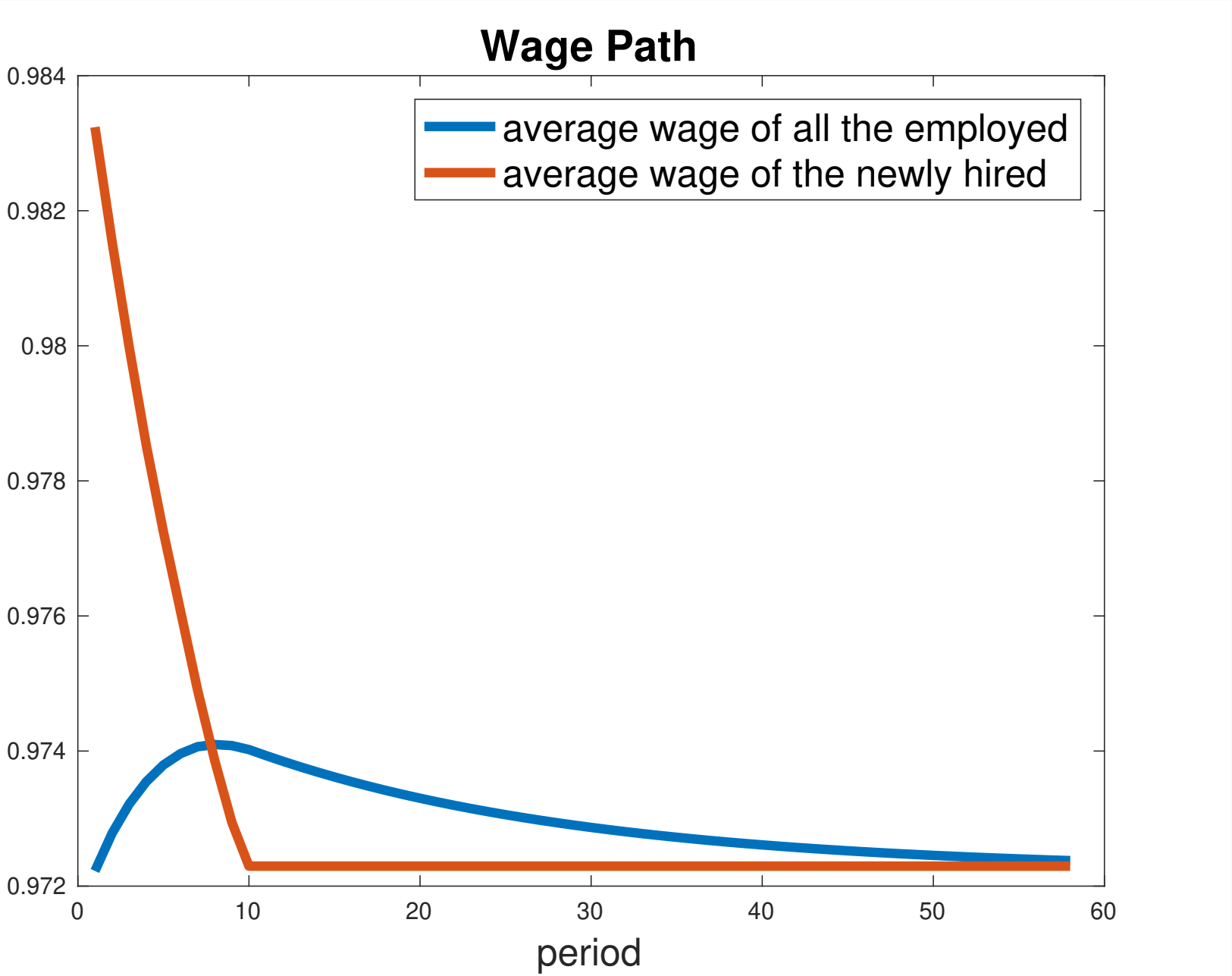
- Two steps
  1. Compute the TRUE impulse response to an MIT Shock
  2. Use this path as a dynamic linear approximation to generate fluctuations (Boppart, Krusell, and Mitman (2018))

- The transition is a large but doable problem:
  - Firms need to know functions $Q_t(w)$ at each stage (no block recursivity)
  - Households need to know $\phi_t^h(w)$ job finding probabilities every period.
  - Also need to know sequence of interest rates
- So it is a second order difference functional equation.
- This is why we are still having trouble but we will finish it.

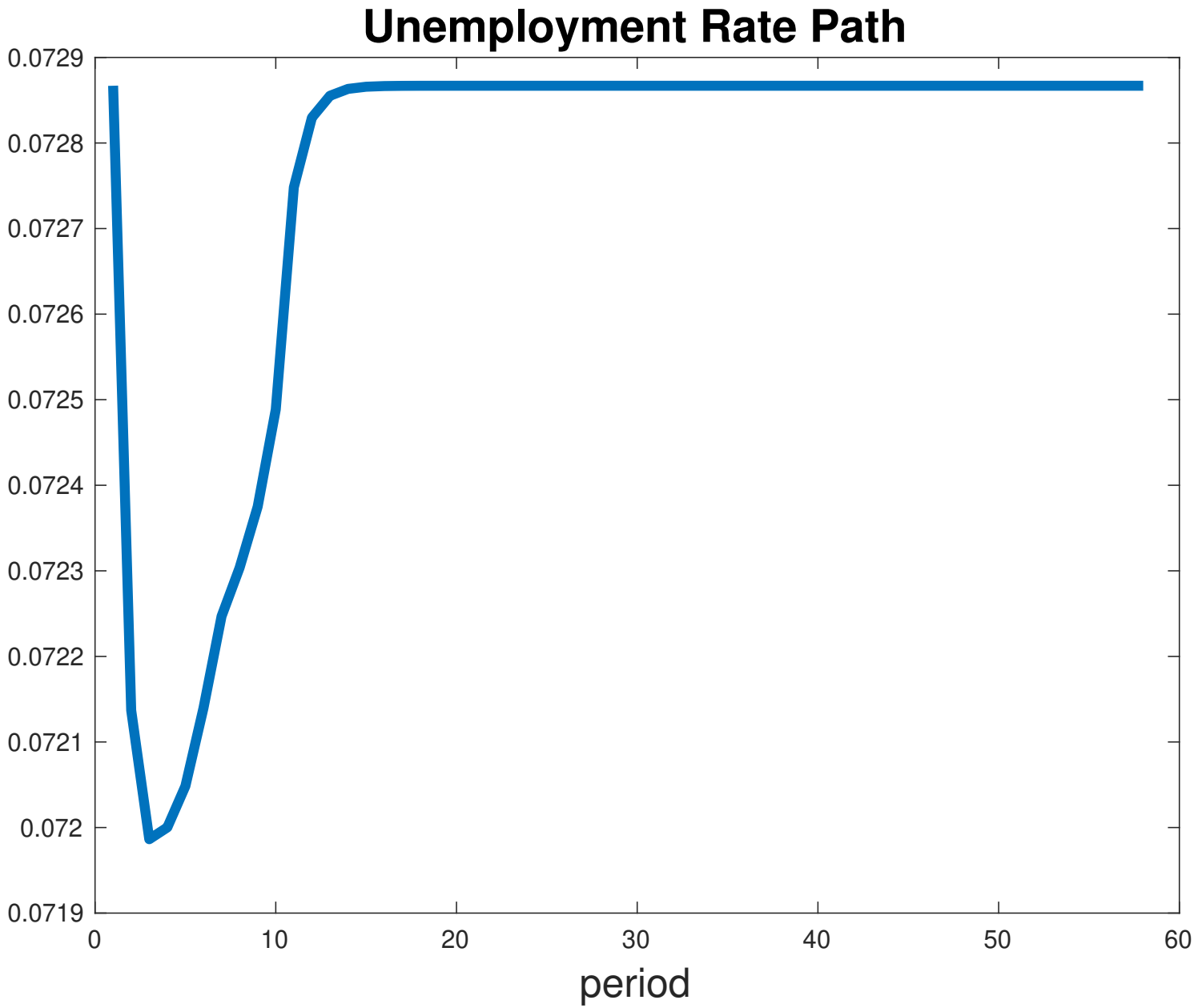
Model 1. 5% Productivity Shock ($\rho = .9$)

- Recall that there is no quitting.
- Wages of existing workers cannot adjust
- We compute the TRUE impulse response to an MIT Shock (This is the object that can be used to generate fluctuations via Boppart, Krusell, and Mitman (2018)
- The outcome is
  - Average wages don't move much
  - Employment moves more (not so much of Shimer puzzle)
  - Newly hired Wage Distribution Shifts upward
  - No quits

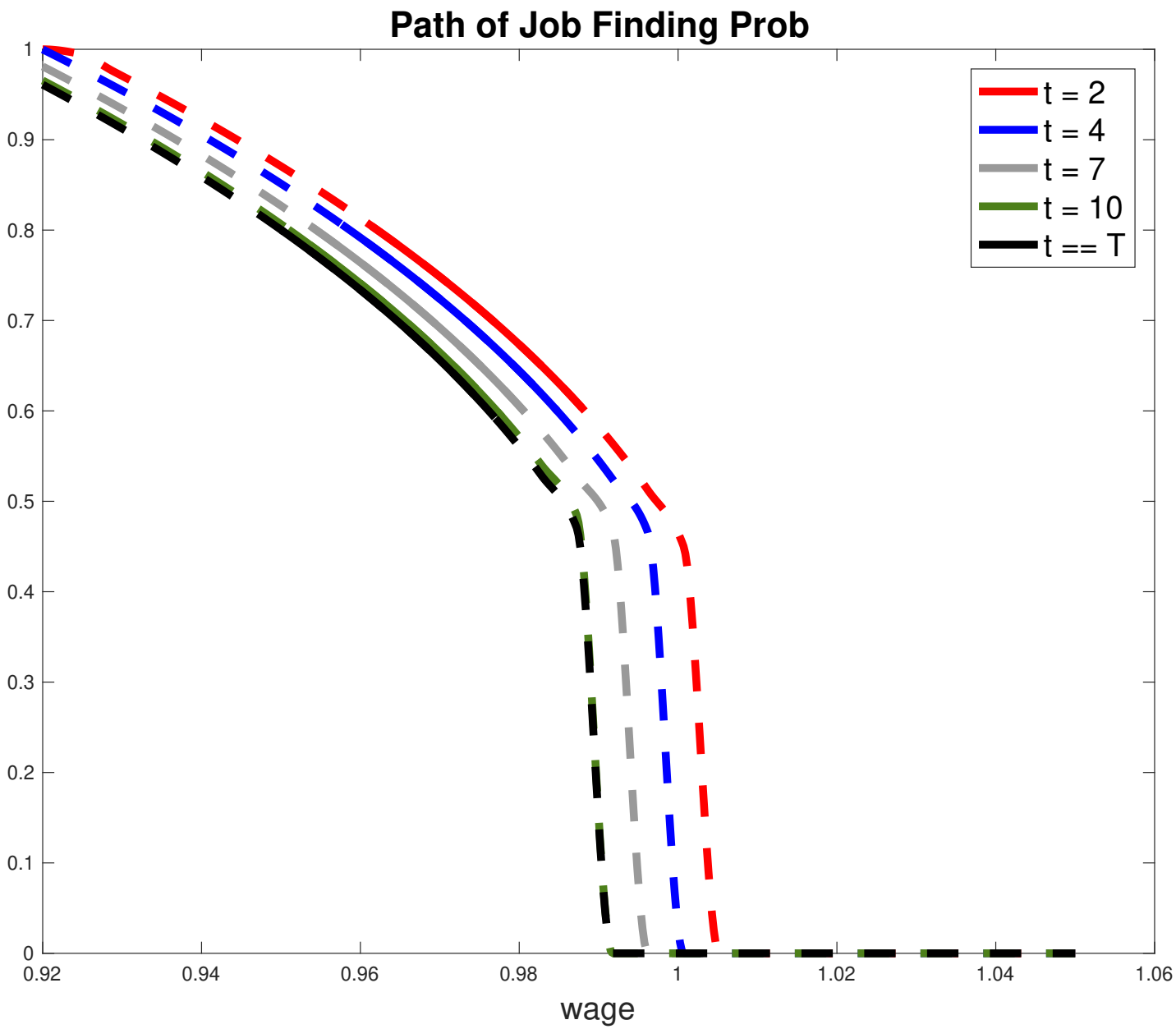
Model 1. 5% Productivity Shock ($\rho = .9$)



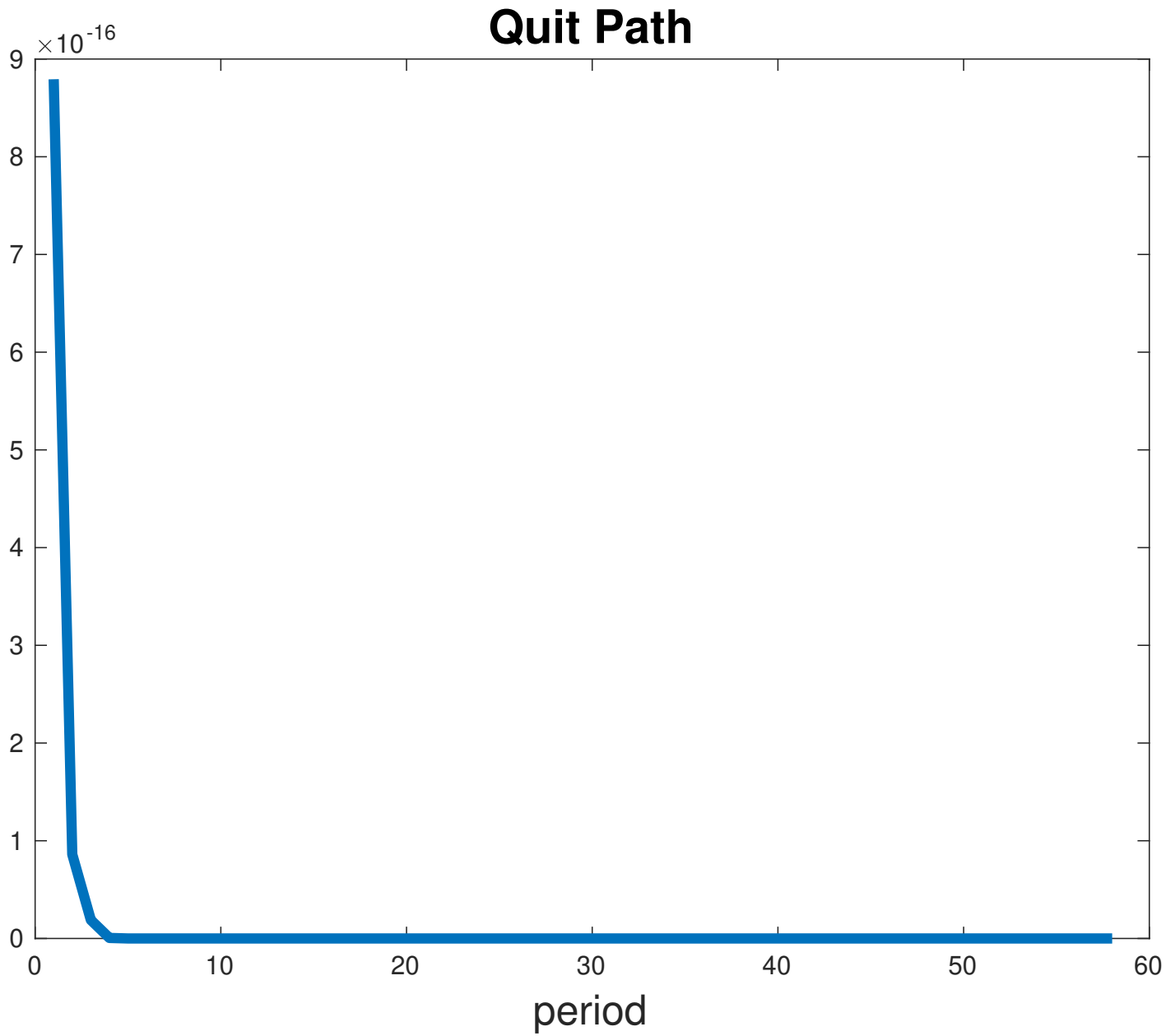
Model 1. 5% Productivity Shock ($\rho = .9$)



Model 1. 5% Productivity Shock ($\rho = .9$)



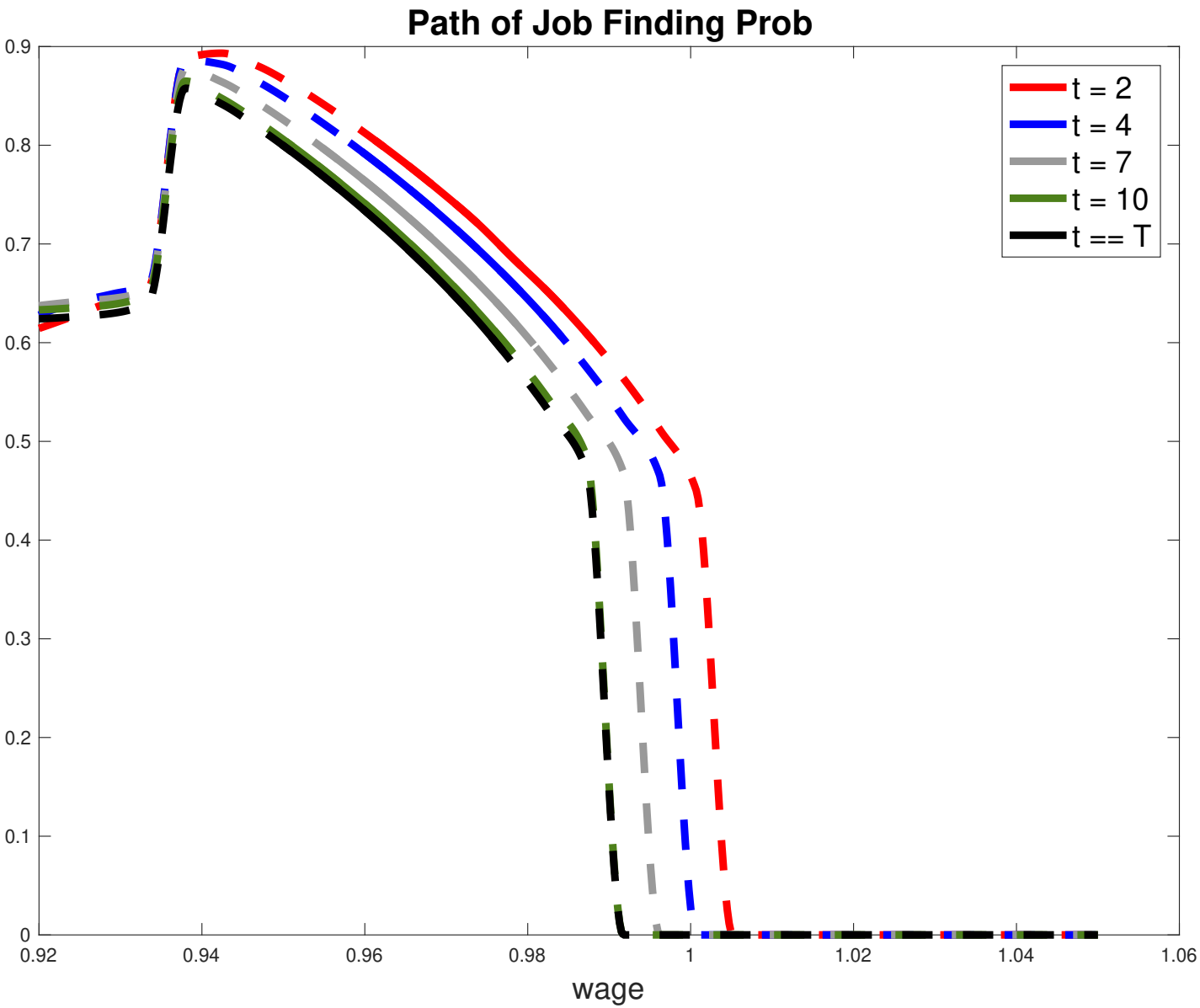
Model 1. 5% Productivity Shock ($\rho = .9$)



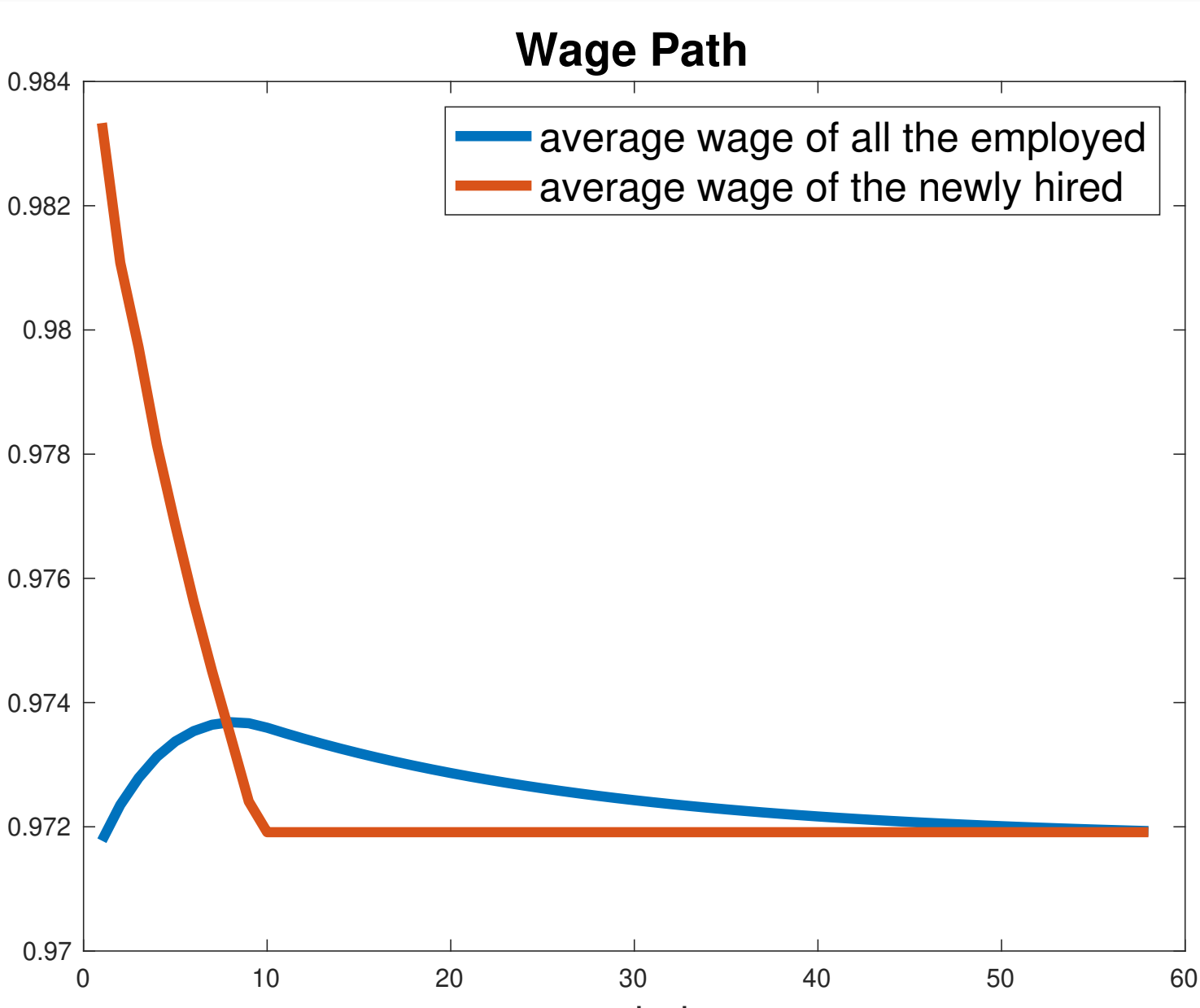
Model 1.5 5% Productivity Shock (HIGHER $\rho = .99$)

- Now there is quitting but not a lot.
- Wage Dispersion Shrinks shrinks a tiny bit
- Quitting becomes noticeable, but barely

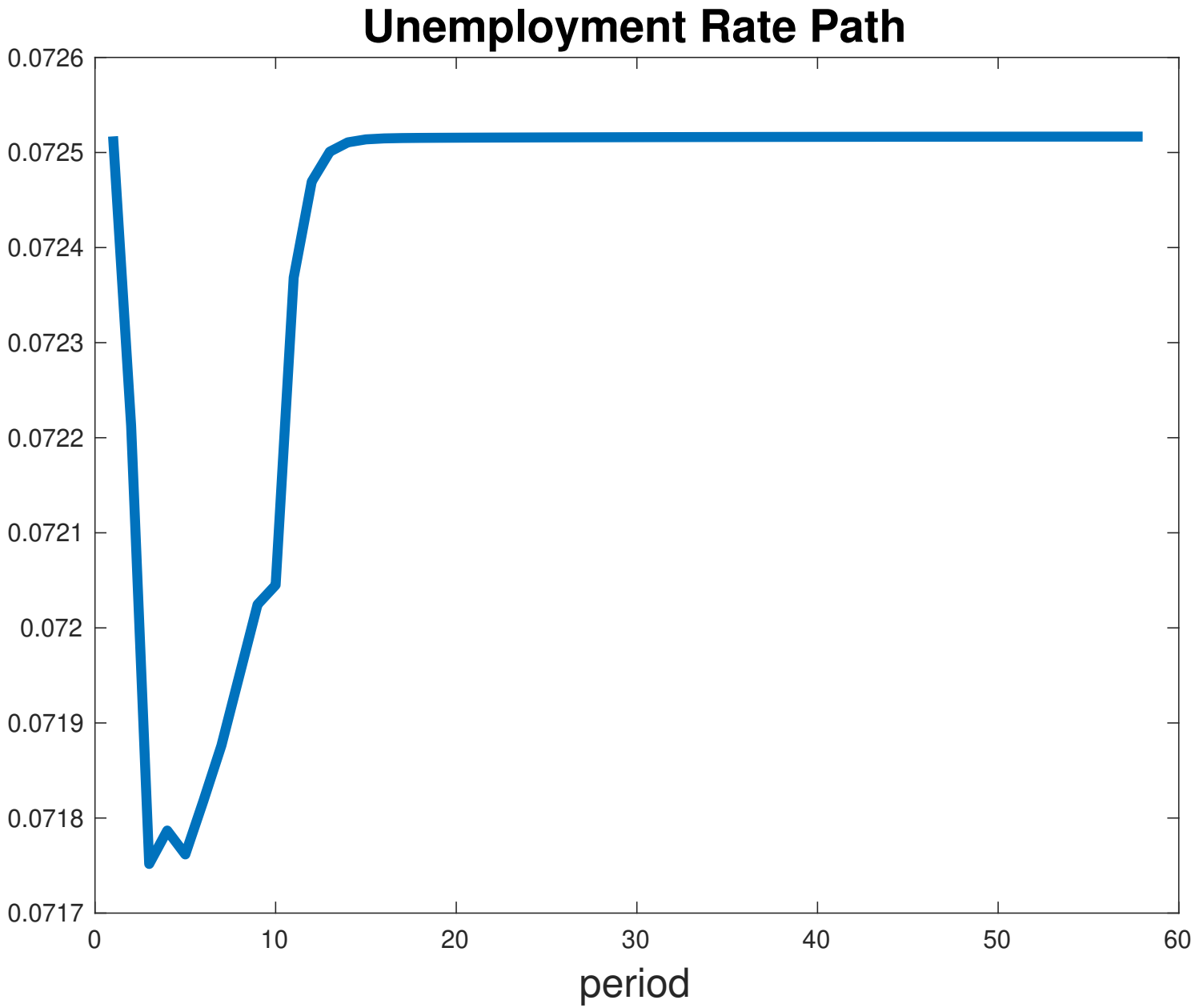
Model 1.5 5% Productivity Shock ($\rho = .99$)



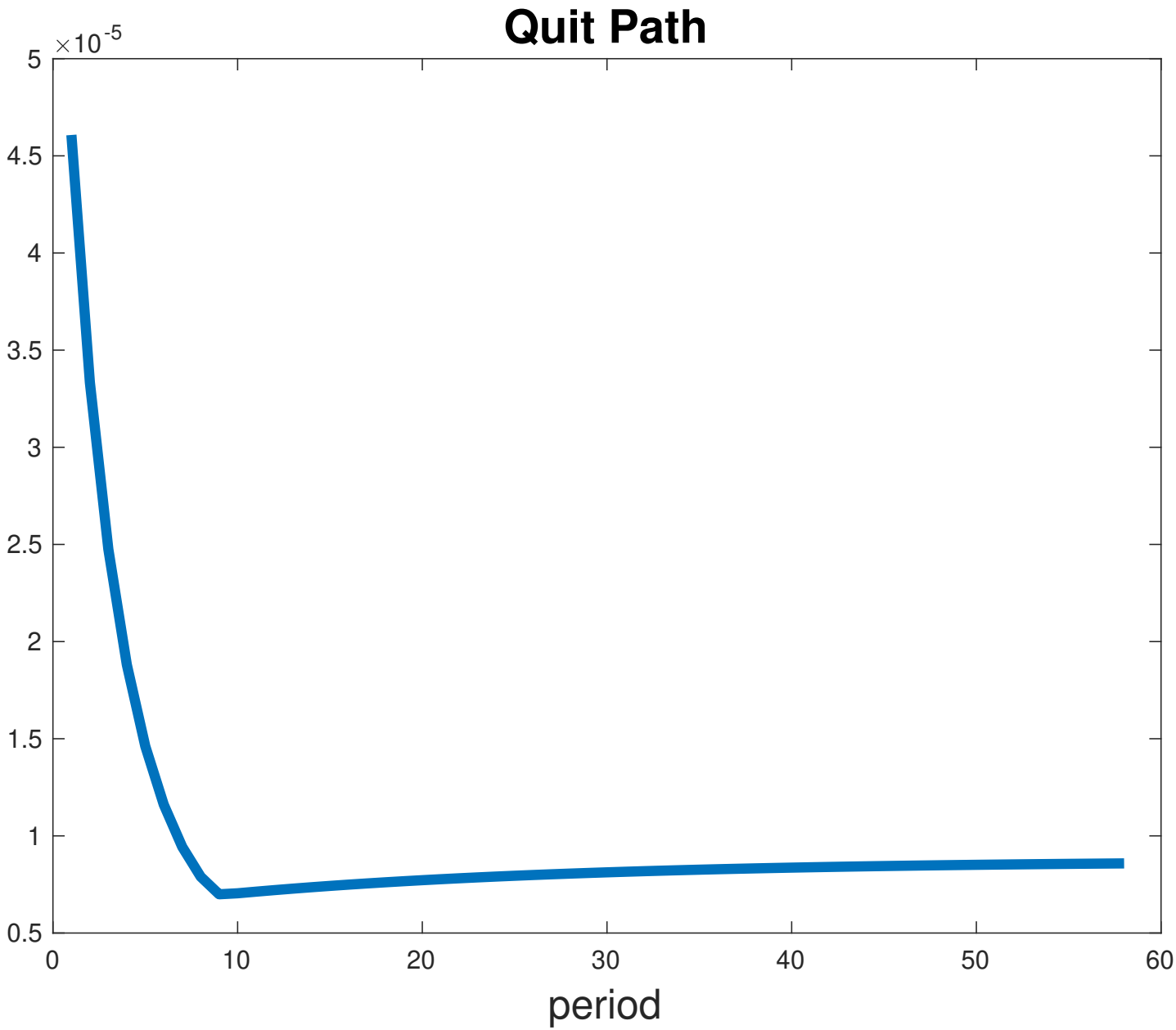
Model 1.5 5% Productivity Shock ($\rho = .9$) Average Wages: .1% Rltv Change



Model 1.5 5% Productivity Shock (ρ.99)



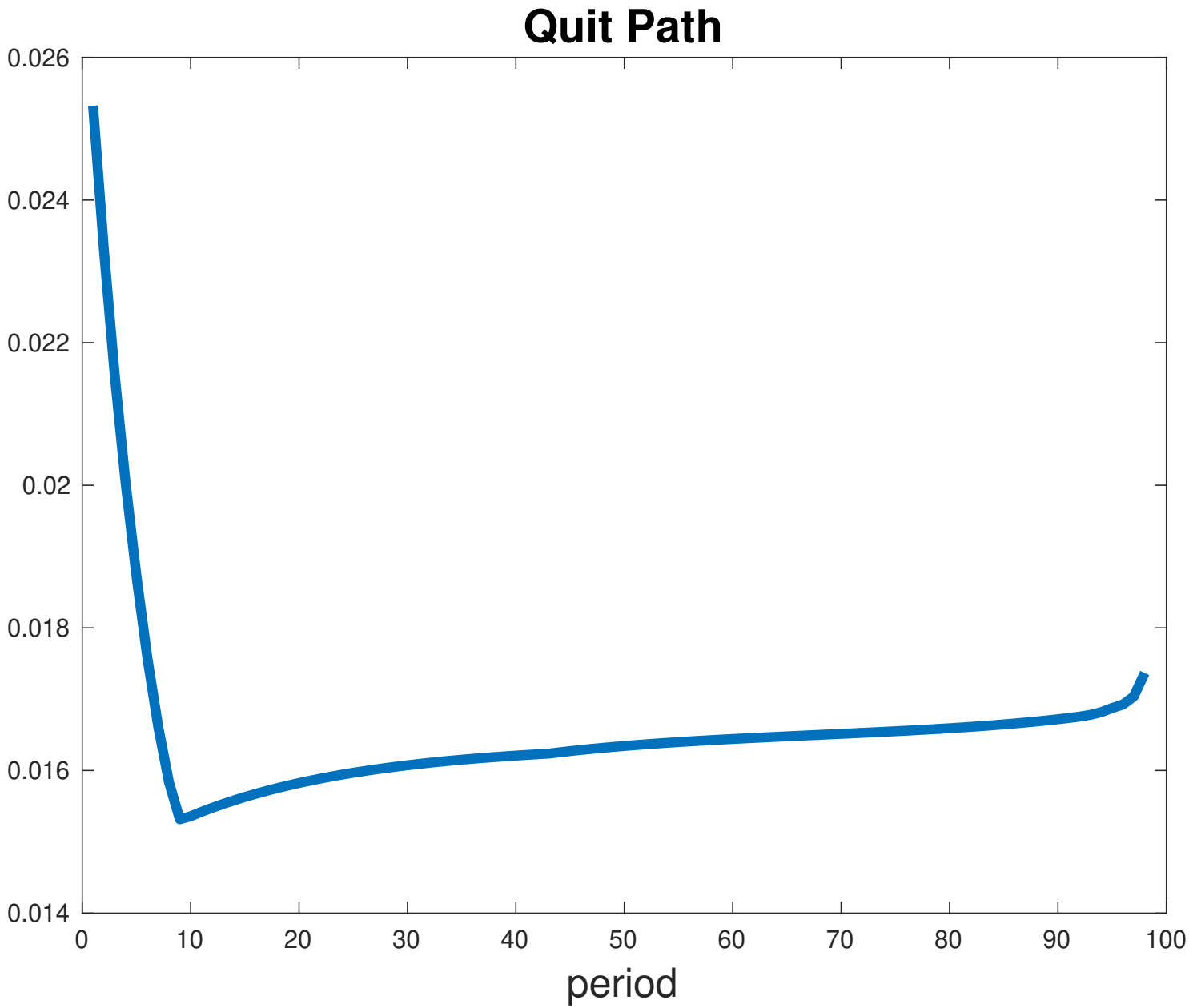
Model 1.5 5% Productivity Shock (ρ.99)



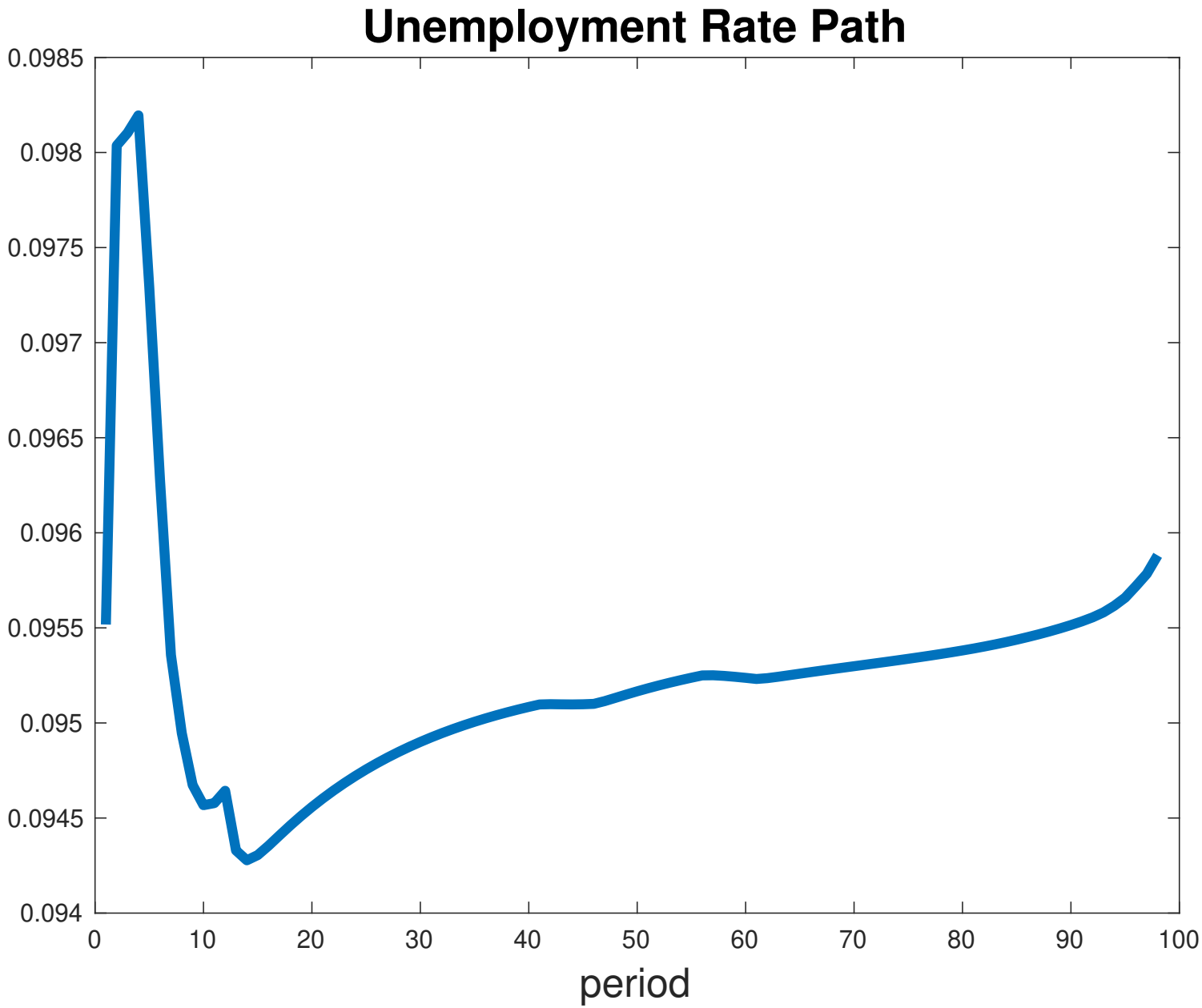
Model 2. 5% Productivity Shock (persistence $\rho = .9$) $\alpha = 3$

- High variance of Extreme Value Shocks and Persistence of Productivity
- Now There is serious quitting.
- Unemployment jumps up before falling (an artifact of no job to job transitions)
- But we are still having convergence trouble
- Within hiring period wage dispersion shrinks but large wage dispersion across workers hired at different times

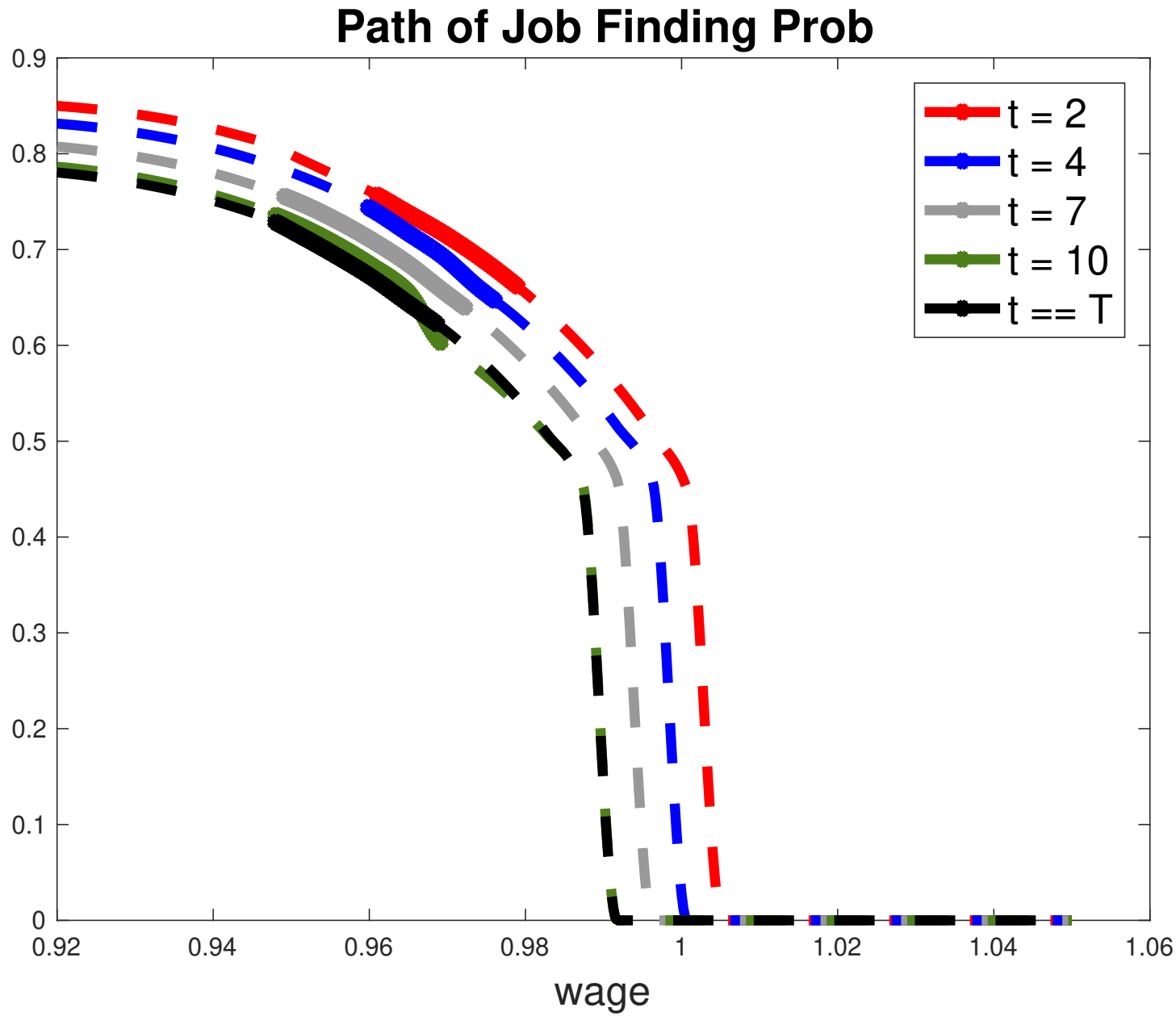
Model 2. 5% Productivity Shock (ρ.9) $\alpha = 3$



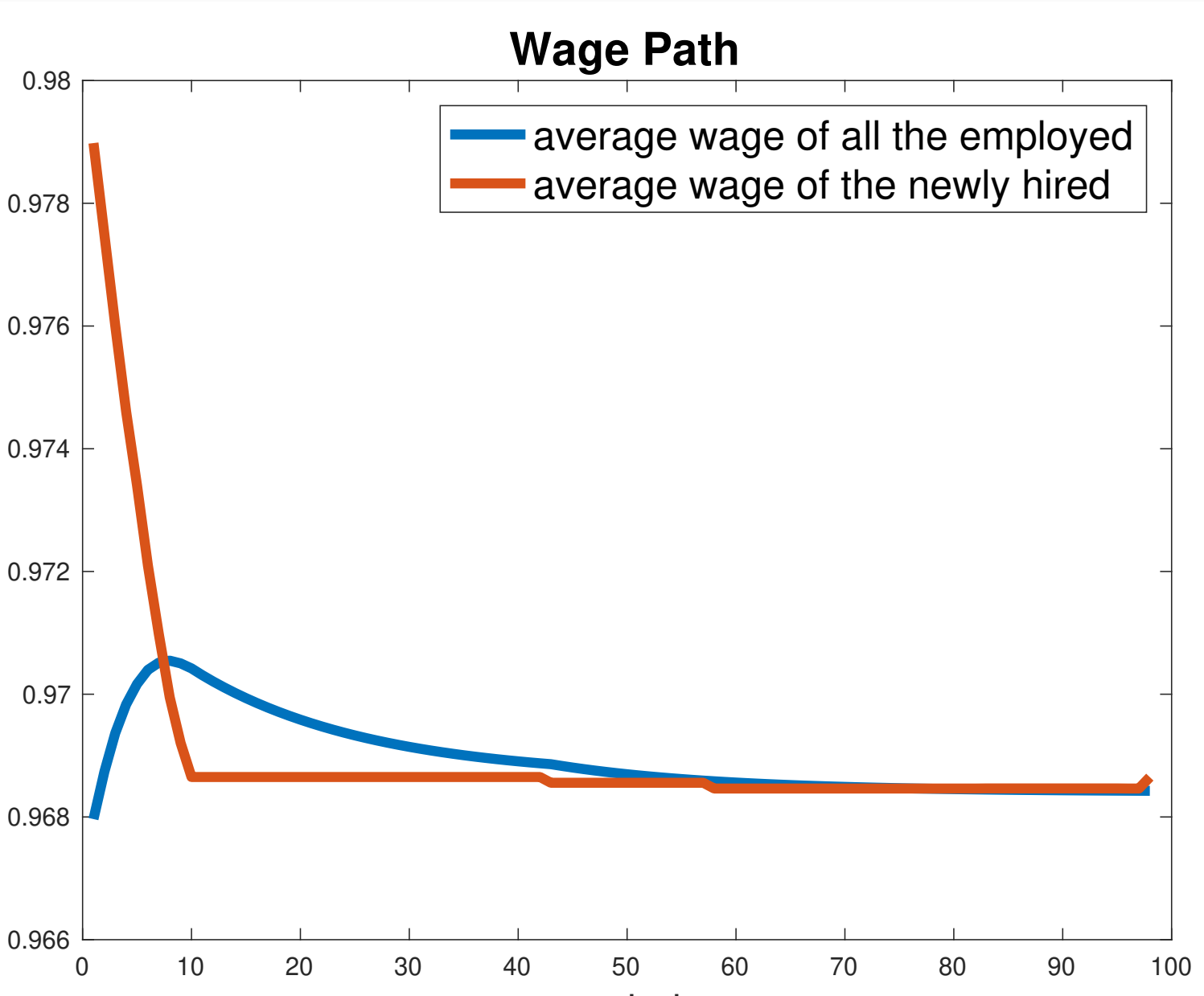
Model 2. 5% Productivity Shock (ρ.9) $\alpha = 3$



Model 2. 5% Productivity Shock (ρ.9) $\alpha = 3$



Model 2. 5% Productivity Shock (ρ.9) Average Wages: .1% Rltv Change



Conclusions

- We are developing the tools and exploring models that marry the two main branches of modern macro:
  1. Aiyagari based models with movements in Consumption and investment and interest rates
  2. Labor search Models that worry about job creation, turnover and wage determination
  3. Needs to use the tools of Empirical Micro to soften the correlations between wages and wealth.
- It can be done
- We are getting procyclical
  - Quits (Employment after a lag)
  - Investment (in this version only in the form of vacancy postings)
  - Consumption
- Long ways to go (exciting set of continuation projects) (efficiency wages, on the job search)

References

Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *Quarterly Journal of Economics* 109 (3):659–684.

Boppart, Timo, Per Krusell, and Kurt Mitman. 2018. "Exploiting MIT shocks in heterogeneous-agent economies: the impulse response as a numerical derivative." *Journal of Economic Dynamics and Control* 89 (C):68–92. URL https://ideas.repec.org/a/eee/dyncon/v89y2018icp68-92.html.

Burdett, Kenneth and Dale Mortensen. 1998. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review* 39:257–273.

Chaumont, Gaston and Shouyong Shi. 2017. "Wealth Accumulation, On the Job Search and Inequality." Https://ideas.repec.org/p/red/sed017/128.html.

Eeckhout, Jan and Alireza Sepahsalari. 2015. "Unemployment Risk and the Distribution of Assets." Unpublished Manuscript, UCL.

Hornstein, Andreas, Per Krusell, and Gianluca Violante. 2011. "Frictional Wage Dispersion in Search Models: A Quantitative Assessment." *American Economic Review* 101 (7):2873–2898.

Krusell, Per, Toshihiko Mukoyama, and AyÅ§egÃijl Åđahin. 2010. "Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations." *Review of Economic Studies* 77 (4):1477–1507. URL https://ideas.repec.org/a/oup/restud/v77y2010i4p1477-1507.html.

Lise, Jeremy. 2013. "On-the-Job Search and Precautionary Savings." *The Review of Economic Studies* 80 (3):1086–1113. URL +http://dx.doi.org/10.1093/restud/rds042.

Quadrini, Vincenzo. 2000. "Entrepreneurship, Saving, and Social Mobility." *Review of Economic Dynamics* 3 (1):1–40.