Wealth Accumulation, On-the-Job Search and Inequality[*]

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Abstract

We study a directed search equilibrium with risk-averse workers who can search on the job and accumulate non-contingent assets under a borrowing limit. On-the-job search generates a wage ladder among homogeneous workers. Search outcomes affect earnings and wealth accumulation. In turn, wealth and earnings affect search decisions by changing the optimal tradeoff between the wage and the matching probability. The calibrated model yields sizable inequality in wages and wealth. Wealth significantly reduces a worker's transition rates from unemployment to employment and from one job to another. The interaction between search and wealth provides important self-insurance as it reduces the pass-through of earnings inequality into consumption by more than 60%. Moreover, we analyze welfare effects of the borrowing limit and the unemployment insurance.

Keywords: Wealth accumulation; On-the-job search; Inequality; Directed search.
JEL classifications: E21; E24; J60.

1. Introduction

For a typical individual, labor income is the chief source of income. Even when the aggregate economy is stable, a worker's labor income fluctuates because of changes in jobs and the labor market status, etc. A worker is unable to perfectly insure against such risks when the market is incomplete for assets contingent on workers' labor market outcomes. However, a worker can accumulate non-contingent assets to partially insure against earnings risks. In addition, a worker can modify job search strategies in order to reduce income risks, e.g., by searching for jobs that are relatively easy to obtain. *In the equilibrium*, how do job search and wealth accumulation interact? What are the implications of these interactions on inequalities in income, wealth and consumption? The objective of this paper is to address such questions with a model where individuals can accumulate assets under a borrowing limit and search in the labor market both off and on the job.

It is well understood that labor income risks affect individuals' consumption and wealth because of incomplete asset markets. Recent literature documents the effect in the reverse direction. Specifically, higher wealth implies a lower job-finding probability for an unemployed worker, a lower transition rate between jobs for an employed worker, and higher future wages. For example, Chetty (2008) reports that individuals who receive larger severance payments stay unemployed for longer periods of time. Herkenhoff et al. (2015) show that increasing individuals' ability to borrow in the U.S. increases unemployment duration significantly. This positive effect of wealth on unemployment duration also exists in France (Algan et al., 2003), Germany (Bloemen and Stancenelli, 2001), and Denmark (Lentz and Tranbaes, 2005). For employed workers, Lise (2013) shows that higher wealth implies a lower transition rate from one job to the next job. Hartung et al. (2018) document that changes in the unemployment insurance affect employed workers' behavior substantially in Germany. Finally, Griffy (2017) finds that relaxing liquidity constraints of unemployed workers results in higher wages in their next job. All this evidence indicates that job search and wealth accumulation interact with each other and this interaction affects an individual's earnings process endogenously.

This interaction should be of first-order importance in macroeconomics, because it naturally connects two major markets that have often been examined separately – the labor market with search frictions and the intertemporal goods market with borrowing constraints. In addition, modeling the interaction between the two markets is useful for understanding economic inequality and, especially, frictional wage inequality among homogeneous workers. Randomness in the outcome of job search induces inequality in earnings and wealth. On the other hand, the ability to accumulate wealth induces individuals to search differently depending on their wealth, as explained later. This interaction between job search and wealth can increase frictional wage inequality and, hence, can help accounting for the large frictional wage inequality in the data that is difficult to be explained by most models (see Hornstein et al., 2011).

An equilibrium model is *necessary* for analyzing the interaction between wealth accumulation and job search. The interaction goes in both directions and affects employed workers as well as unemployed workers. Moreover, if workers' search decisions depend on wealth, firms can respond to such dependence by changing vacancy creation and job offers. This response determines the distribution of wage offers. Because the offer distribution determines the expected return to search, it is a central factor affecting how job search interacts with wealth accumulation. By determining the offer distribution endogenously, an equilibrium generates the mapping from a model's mechanism to the empirical evidence. In contrast, by ignoring firms' responses, a partial-equilibrium analysis is problematic for understanding this mapping. In particular, a typical analysis of a partial equilibrium assumes an offer distribution that is inconsistent with an equilibrium (see the end of section 4.4 for more discussion). Moreover, a partial-equilibrium analysis is problematic for policy analysis because it fails to incorporate how a policy changes the offer distribution.

We integrate an equilibrium model of labor market search into an intertemporal model of consumption and savings. Individuals are risk averse. They can save and borrow with non-contingent assets but face a borrowing limit. Both employed and unemployed workers can search. Search is directed by firms that create vacancies competitively in submarkets that differ in the wage offer and the matching probability. When unemployed, workers have

access to the unemployment insurance that expires with some probability. The possibilities of failure to match and losing the unemployment insurance generate labor income risks. Over time, workers differ in the history of search outcomes. This endogenous heterogeneity induces dispersion in earnings, wealth and consumption. To focus on such heterogeneity induced endogenously by search frictions, we assume that all workers have the same productivity and all jobs are the same.

On the worker side, wealth accumulation and labor market search interact as follows. A worker with high wealth optimally chooses to apply for a high wage and face a low matching probability. If the application fails to yield a match, the worker can still maintain high consumption by decumulating wealth. This stretches the high end of the wage ladder. In contrast, a worker whose wealth is close to the borrowing limit has only a limited scope of using wealth to smooth consumption. To prevent consumption from falling sharply over time, the worker optimally chooses to apply for a low wage in order to obtain the job quickly. This stretches the low end of the wage ladder. Thus, the effect of wealth on search, in turn, increases heterogeneity in the paths of earnings and wealth.

Firms cater to endogenously heterogeneous workers by offering wages conditional on the applicants' wealth. For any given wage offer, firms prefer to hire workers with higher wealth. This preference does not arise from discrimination or any direct contribution of workers' wealth to output. Rather, it arises from firms' rational expectation that an employee with higher wealth will search for another job that has a relatively high wage. Because such a job has a low matching probability for the worker, the employee will separate from the current job with a low probability, which increases the present value of the job to the current employer. Thus, each submarket is indexed by the wage offer and the applicants' wealth. In section 3.2, we will discuss the empirical evidence that firms' hiring can depend on the applicants' wealth. Moreover, we will show that the role of wealth in hiring can be replaced by observables such as the applicants' current wage and labor market status.

The mechanism in this model can qualitatively replicate the aforementioned empirical patterns of the interactions between workers' wealth and labor market dynamics. To

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assess the *quantitative* importance of the mechanism, we calibrate the baseline model and compare the results with two benchmarks in the literature. One benchmark is a "no-search" model similar to Aiyagari (1994), where the labor market is Walrasian but individuals face employment risks and a borrowing limit. The probability of becoming employed and the probability of losing employment are set to be equal to the average counterparts in the baseline model. The other benchmark is a "no-wealth" search model where individuals are hand-to-mouth. This benchmark differs from the canonical search model (e.g., Diamond, 1981, Mortensen, 1982, and Pissarides, 2000) in that search is directed, workers can search on the job and workers are risk averse.

In our baseline model, the computed optimal decisions show that high wealth levels or current earnings significantly reduce the transition rates of workers from unemployment to employment and from one job to another. Alternatively, we use the data generated by the model to regress the transition rates on wealth and current earnings. The regression coefficients are negative and statistically significant. Moreover, the effect of wealth increases frictional wage inequality significantly, as measured by the *mean-min ratio* in equilibrium wages. Hornstein et al. (2011) show that the mean-min ratio in the U.S data is about 2 but is about 1.04 in a variety of search models. In the no-wealth benchmark (but with on-the-job search), the mean-min ratio in wages is 1.413. By introducing wealth, the baseline model increases the mean-min ratio to 1.737. Relative to hand-to-mouth, the ability to accumulate wealth stretches out both ends of the wage ladder, because workers modify their job search strategy according to their wealth as explained earlier.

On-the-job search is important for frictional wage dispersion. Regardless of whether workers can accumulate wealth, on-the-job search induces frictional dispersion by generating a wage ladder among homogeneous workers. Wealth accumulation strengthens this effect. Anticipating the ability to continue to search on the job, low-wealth workers are willing to start employment with low wages. As a worker's wealth increases, the worker will search for higher wages. In Appendix B, we shut down on-the-job search as a counterfactual exercise, which reduces the mean-min wage ratio from 1.737 to 1.172. On the other hand, shutting down wealth accumulation but allowing for on-the-job search, the

mean-min wage ratio falls from 1.737 to 1.413, as reported above.

In the baseline model, the Gini coefficient in wealth is 0.428. This is large given that the model abstracts from all exogenous heterogeneity among workers. Moreover, savings provide significant self insurance against the earnings risk. In the no-search model, the Gini coefficient in consumption is 51.7% of that in earnings. That is, 48.3% of the earnings risk is self-insured by accumulating wealth. Directed search provides additional insurance by reducing the Gini coefficient in consumption to 37.5% of that in earnings.

We analyze the welfare effect of changing the borrowing limit, the replacement rate and the expected duration of the unemployment insurance. In each exercise, the government balances the budget intertemporally by using a wage tax to finance the unemployment insurance. We start the economy in the steady state and treat each policy change as an unanticipated shock. The welfare effect of a policy change is measured by the percentage change in lifetime consumption, taking into account transitional dynamics. We find that moving from an economy with a borrowing limit $\underline{a} = 0$ to one with $\underline{a} = -6.36$ (55% of the average annual income) increases welfare by approximately 0.06%. On the unemployment insurance, the replacement rate that maximizes social welfare is 0.2 if the expected duration of the benefit is kept at 28 weeks. Alternatively, if the replacement rate is kept at 0.5, the optimal expected duration of the unemployment insurance is approximately three months. For each of these two policies, moving from the baseline to the optimal level increases welfare by 0.04% to 0.06%. The relatively small welfare gains reflect the facts that the average unemployment duration is relatively short in the data and that workers can accumulate wealth to insure against a large part of the risk caused by unemployment.

These dynamic welfare results differ significantly from steady-state ones. Opposite to the dynamic effect, the steady-state effect of relaxing the borrowing limit is *negative*, because the relaxed limit leads to lower average wealth and lower average consumption in the steady state. Also, if only the steady state is concerned, the optimal replacement rate is 0. In the steady state, individuals prefer to reduce the wage tax to zero by eliminating the unemployment insurance and use wealth and search to smooth consumption. Moving

from the baseline replacement rate of 0.5 to 0 increases welfare by 1.8% in the steady state. However, when dynamics are taken into account, such a large reduction in the replacement rate is not optimal, because workers need to take low wages and low consumption for a relatively long time in order to accumulate wealth to the level in the new steady state.

Our work builds on several strands of the literature. Models of business cycles have incorporated capital accumulation and undirected search (e.g., Andolfatto, 1996, Merz, 1995). With both directed and undirected search, Shi and Wen (1999) have studied taxes and subsidies. In these models, idiosyncratic risks generated by labor search are completely smoothed either within large households or in a perfect insurance market. Moreover, individuals do not face a borrowing limit. Lentz and Tranaes (2005) and Lentz (2009) analyze the optimal unemployment insurance when wealth affects unemployed workers' search effort. Since these models do not allow for on-the-job search, the equilibrium has a single wage rate. There are also papers that analyze how hidden savings affect unemployment insurance when there is moral hazard, e.g. Werning (2001) and Kocherlakota (2004).

Two papers more closely related to ours are Krusell et al. (2010) and Lise (2013). These papers integrate labor search into a model of precautionary savings in the style of Bewley (1977), Huggett (1993) and Aiyagari (1994). Our paper differs from these papers by incorporating all three features below: on-the-job search, directed search and an equilibrium framework. In contrast, Krusell et al. (2010) and Lise (2013) model search as an undirected process; that is, they assume that a worker is randomly matched with a wage offer instead of choosing the wage to search. In addition, Krusell et al. (2010) do not allow for on-the-job search. Lise (2013) allows for on-the-job search but studies only a partial equilibrium where the offer distribution of wages is exogenous. Our model requires this distribution to be the outcome of firms' optimal response to the market. Earlier in this Introduction, we articulated the importance of an equilibrium model.

Why is it important to incorporate on-the-job search and directed search? On-the-job search is necessary for explaining the large job-to-job transition in the data (e.g., Fallick and Fleischman, 2004). Obtaining the first job is only part of a worker's transition in

the labor market. The ability to search on the job in the future affects a worker's wealth accumulation and current search. Shutting down on-the-job search would eliminate this feedback effect, significantly weaken the interaction between wealth and search (as reported earlier), and fail to explain the empirical evidence that increasing wealth reduces a worker's job-to-job transition rate. Directed search is also intuitive and realistic.[1] In particular, it enables workers to adjust their search target according to their wealth. In addition, directed search simplifies the analysis and the computation by making the equilibrium in the baseline model block recursive (see Shi, 2009). Namely, individuals' decisions and the market tightness do not depend on the large dimensional object – the distribution of workers across individual states. This enables us to compute the dynamic equilibrium exactly, rather than approximate it as in Krusell et al. (2010). Even when block recursivity fails in section 5 on the policy analysis, directed search enables us to tractably compute the dynamic welfare effect of a policy instead of only the steady-state effect.

Our paper is closely related to Herkenhoff (2015) in the spirit but differs in the objectives. He uses a directed search model to study how increasing the access to credit card debt impacts unemployment duration and job recoveries after recessions. The main parts of his analysis assume that only unemployed workers can search, although he discusses in an appendix how to extend the analysis to allow for on-the-job search. Braxton et al. (2018) examine the optimal unemployment insurance when unemployed workers have private information about their own type and have access to credit. Corbae and Glover (2017) analyze the effects of banning employers' use of workers' credit record in the hiring process. In contrast, we emphasize the accumulation of general wealth and on-the-job search.

The paper is organized as follows. In section 2, we describe the model, analyze individuals' optimal decisions and characterize the market tightness. In section 3, we define the equilibrium and explain why it is block recursive. Section 4 calibrates the model to study the quantitative implications. Section 5 studies welfare implications of relaxing the borrowing limit and modifying the unemployment insurance scheme. Section 6 concludes.

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[1] For example, in a survey data in the U.S., Hall and Krueger (2008) document that 84 percent of white, male, non-college workers either "know exactly" or "had a pretty good idea" about how much their current job would pay before the job interviews.

The appendices provide additional materials, including counter-factual exercises.

2. Model of Consumption, Savings and Search

2.1. Environment of the model

Time is discrete and lasts forever. There is a unit measure of risk averse workers whose utility in each period is $u(c)$, where c is consumption. The function u satisfies $u'(c) \in (0, \infty)$ and $u''(c) \in (-\infty, 0)$ $\forall c \in \mathbb{R}_+$, with $u'(0) = \infty$. A worker's discount factor is $\beta \in (0,1)$. A worker can accumulate non-contingent assets denoted as $a \in A = [\underline{a}, \bar{a}]$. The lower bound on asset holdings, $\underline{a}$, can be negative, in which case it is a borrowing limit. The upper bound $\bar{a}$ can be chosen so that it is not binding. The net rate of return on assets is r, which is determined in the world market and taken as given in this model. We assume $\beta(1+r) < 1$ to ensure that individuals' asset levels are bounded in the equilibrium.[2] In this model, ex ante identical workers become endogenously heterogeneous ex post in three aspects: the labor market status, current earnings, and wealth.

A worker's labor market status is $\varepsilon \in \{e, u\}$, where e indicates being employed and u unemployed.[3] Denote a worker's earnings as ω. An employed worker's earnings are the wage $w \in W \equiv [\underline{w}, \bar{w}]$. An unemployed worker's "earnings" are the unemployment insurance b, which can change over the unemployment spell. In the first period after a worker becomes unemployed, the worker receives the unemployment insurance ρw_{-1}, where w_{-1} is the worker's wage prior to unemployment and $\rho \in (0,1)$ is the replacement rate. Thereafter, the insurance benefit expires with probability χ at the end of each period, which we will choose to match the average duration of the unemployment insurance in the data. The two parameters (ρ, χ) approximate the unemployment insurance in reality on which we will conduct policy analysis. The stochastic expiration reduces the state space relative to a deterministic expiration schedule, since the latter will introduce the time till

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[2]As shown by Aiyagari (1994), when individuals face income risks and a borrowing limit, the precautionry saving motive would induce them to accumulate infinite wealth if $\beta(1+r) < 1$ were violated.

[3]We have extended the model to allow a worker to choose whether to participate in the market. Since the extension yields quantitatively similar results to the model presented here, we omit the extension.

expiration as a state variable.[4]

The measure of firms is determined by competitive entry. Firms are separate identities instead of being owned by workers. They are risk neutral but have the same discount factor β as workers. The production technology has constant returns to scale in jobs, and so a firm treats the jobs independently. We refer to a job as a firm, as in most search models of the labor market. All jobs yield a constant stream of output, y, and the production cost is normalized to 0. The vacancy cost per period is $k > 0$. Firms commit to offers, but workers can quit at anytime. In addition, firms cannot respond to the employee's outside offers, which is realistic for most jobs. If an employed worker receives a better offer, the worker quits the current job.[5] In addition to endogenous separation, an existing match can be destroyed by exogenous separation, which occurs with probability $\delta \in (0, 1)$. Exogenous separation is *iid* across matches and time.

In each period, an unemployed worker is able to search with probability one, and an employed worker with probability λ_e.[6] The labor market consists of a continuum of submarkets indexed by the wage offer w and the applicants' wealth a. Each submarket (w, a) has a tightness $\theta(w, a)$, which is the ratio of vacancies to applicants. Taking the tightness function as given, recruiting firms and searching workers choose the submarket to enter. The tightness function is independent of the distribution of workers because the equilibrium is block recursive, as explained in section 3. Moreover, workers who search in submarket (w, a) must have the wealth level a. Conditioning a submarket on applicants' wealth simplifies the analysis of the equilibrium. In section 3.2, we will explore alternative ways to describe the submarkets when wealth is not observable.

Matching is random in each submarket. The matching technology has constant returns to scale in the measure of applicants and vacancies. In any submarket with tightness θ, the matching probability is $p(\theta)$ for an applicant and $q(\theta)$ for a vacancy, where $p(\theta) = \theta q(\theta)$

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[4]To simplify the analysis, we take the unemployment insurance as given in the baseline model. In section 5 on welfare analysis, we will introduce a wage tax to finance the unemployment insurance.

[5]See Postel-Vinay and Robin (2002) for an undirected search model where firms can match an employee's outside offers.

[6]In the baseline exercise, we will set λ_e for the model to match the average rate of job-to-job transition. In a counter-factual experiment, we will set $\lambda_e = 0$ to examine the importance of on-the-job search.

because of constant returns to scale. Also, the matching probabilities satisfy the standard assumptions: $p(\theta), q(\theta) \in [0,1]$, $p'(\theta) \in (0, \infty)$, $q'(\theta) \in (-\infty, 0)$ and $p''(\theta) < 0$ for all θ. Although $p(\theta)$ and $q(\theta)$ are exogenous functions of θ, the tightness function $\theta(w, a)$ is endogenously determined by firms' entry and workers' search decisions. Thus, the matching probabilities are endogenous functions of (w, a).

Each period is divided into five stages: (i) production, (ii) consumption and savings, (iii) search and matching, (iv) exogenous separation, and (v) the expiration of the unemployment insurance. In the production stage, all existing matches produce, and earnings (including the unemployment insurance) are obtained. In the second stage, individuals choose consumption and savings. In the third stage, the opportunity to search is realized according to probability λ_e for each employed worker and to probability 1 for each unemployed worker. Searchers choose the submarket to search, where new matches are formed. In stage (iv), a separation shock destroys a fraction $\delta \in (0, 1)$ of old matches and throws the workers into unemployment. To reduce repetitive accounting, we assume that a newly formed match is not subject to exogenous separation until the next period. Finally, in stage (v), the unemployment insurance expires with probability χ.

We define some objects. A worker's individual state is $s \equiv (\varepsilon, \omega, a)$, which consists of the labor market status ε, current earnings ω, and wealth a. The labor market status lies in the set $E \equiv \{e, u\}$. Earnings lie in the set $W \equiv [\underline{w}, \bar{w}]$ for an employed worker, and in the set $\rho W \cup \{0\}$ for an unemployed worker. Denote $\bar{W} \equiv W \cup \rho W \cup \{0\}$ so that $\bar{W} \ni \omega$. Wealth lies in the set $A \equiv [\underline{a}, \bar{a}]$. Denote the Borel sets as $\mathcal{E}$ for E, $\mathcal{W}$ for $\bar{W}$, and $\mathcal{A}$ for A. The set of individual states is $S \equiv E \times \bar{W} \times A$, whose Borel sets are $\mathcal{S} = \mathcal{E} \times \mathcal{W} \times \mathcal{A}$.

The aggregate state of the economy is $\psi : \mathcal{S} \to [0, 1]$, a distribution of workers over individual states. Let $\Psi(S, \mathcal{S})$ be the space of distribution functions on the measurable space $(S, \mathcal{S})$. Let $\mathcal{T} : \Psi(S, \mathcal{S}) \to \Psi(S, \mathcal{S})$ be the law of motion of the state of the economy. As explained later, the equilibrium in this model is block recursive in the sense that individuals' optimal decisions, value functions and the market tightness are independent of ψ. In particular, the market tightness function is $\theta : W \times A \to \mathbb{R}_+$, as introduced earlier.

2.2. Decisions on consumption and savings

Consider a worker in the state $s = (\varepsilon, \omega, a)$. When choosing consumption and savings in a period, the worker's value function is $V_\varepsilon(\omega, a)$. Let $\hat{a}$ be the choice of savings, i.e., future wealth. The continuation value immediately after the choice is $\beta R_\varepsilon(\omega, \hat{a})$, which is computed later. The optimal choices of consumption and savings solve:

$$V_\varepsilon(\omega, a) = \max_{(c, \hat{a})} \; [u(c) + \beta R_\varepsilon(\omega, \hat{a})] \tag{2.1}$$

$$\text{s.t.} \quad c + \frac{\hat{a}}{1+r} = y_{\min} + \omega + a \quad \text{and} \quad \hat{a} \geq \underline{a}.$$

The first constraint is the budget constraint and the second constraint the borrowing limit. The amount $y_{\min} > 0$ is a small amount of home production that prevents $c \leq 0$ in the case $\omega + a \leq 0$.[7] Denote the policy function as $c_\varepsilon(\omega, a)$ for optimal consumption and $\hat{a}_\varepsilon(\omega, a)$ for optimal savings.

2.3. Unemployed workers' search

Consider an unemployed worker who receives the unemployment insurance b in the period and whose wealth after consumption is $\hat{a}$. The worker chooses a submarket $(\hat{w}, \hat{a})$ to search and receives a match with probability $p(\theta(\hat{w}, \hat{a}))$. If the worker gets a match, the worker will start the next period employed with the value $V_e(\hat{w}, \hat{a})$. If the worker fails to match, the unemployment insurance (if any) will expire with probability χ. Let $R_u(b, \hat{a})$ denote the worker value generated by the optimal choice of search. Then,

$$R_u(b, \hat{a}) \equiv \max_{\hat{w}} \; \{ \, p(\theta(\hat{w}, \hat{a})) V_e(\hat{w}, \hat{a}) \\ + [1 - p(\theta(\hat{w}, \hat{a}))] [\chi V_u(0, \hat{a}) + (1 - \chi) V_u(b, \hat{a})] \, \}. \tag{2.2}$$

Denote the policy function as $\tilde{w}_u(b, \hat{a})$ for the optimal search target. It is useful to rewrite this optimal choice as a function of the worker's wealth at the beginning of the period, a, instead of wealth at the end of the period, $\hat{a}$. Substituting $\hat{a} = \hat{a}_u(b, a)$ from the optimal choice of savings, we have $\hat{w}_u(b, a) \equiv \tilde{w}_u(b, \hat{a}_u(b, a))$. Similarly, we express the tightness of the submarket searched by the unemployed worker as $\hat{\theta}_u(b, a) \equiv \theta(\hat{w}_u(b, a), \hat{a}_u(b, a))$.

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[7]If $\underline{a} > 0$, then $y_{\min} > 0$ is not needed for $c > 0$. If $\underline{a} < 0$, however, an unemployed worker's income can reach 0 with positive probability when the unemployment insurance expires.

The above formulation reveals that search serves as a mechanism of partial self-insurance. A worker adjusts the search target according to the wealth level and earnings to smooth consumption. Consider an unemployed worker with low wealth. The worker expects to find a job in the future with some probability, in which case the income will increase. The risk averse worker would like to smooth consumption by decumulating wealth to transfer some of the higher future income to the present. However, the borrowing limit constrains the worker's ability to do so. When the borrowing constraint is binding, the worker is forced to consume a relatively low amount. If the worker fails to find a job, consumption will likely be even lower in the next period. To reduce the likelihood of such falling consumption, the worker will choose to search for jobs that have relatively high job-finding probabilities. In equilibrium, those jobs are the ones paying relatively low wages. Similarly, for any given wealth, unemployed workers who receive a relatively low unemployment insurance have incentive to apply for lower wages. This income effect on search exists even if the unemployment insurance does not expire, but it is strengthened by the possibility that the unemployment insurance can expire.

2.4. Employed workers' search

The search decision of an employed worker is formulated similarly to an unemployed worker's. Consider a worker employed at wage w whose wealth at the end of the period is $\hat{a}$. If the worker receives an opportunity to search, which occurs with probability λ_e, the worker chooses the submarket $(\hat{w}, \hat{a})$ to search. In this submarket, the worker gets a match with probability $p(\theta(\hat{w}, \hat{a}))$, in which case the continuation value will be $V_e(\hat{w}, \hat{a})$. If the worker fails to find a match, the worker will be hit by exogenous separation with probability δ. In this case, the worker will receive the unemployment insurance ρw, and the continuation value will be $V_u(\rho w, \hat{a})$. If the worker escapes exogenous separation, the continuation value will be $V_e(w, \hat{a})$. Thus, the optimal search decision generates the value $R_e(w, \hat{a})$ to the worker and solves:

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R_e(w, \hat{a}) \equiv \max_{\hat{w}} \quad \{ \lambda_e p(\theta(\hat{w}, \hat{a})) V_e(\hat{w}, \hat{a}) \\
+ [1 - \lambda_e p(\theta(\hat{w}, \hat{a}))] [\delta V_u(\rho w, \hat{a}) + (1 - \delta) V_e(w, \hat{a})] \}
\tag{2.3}
$$

Denote the optimal target wage of search by the policy function $\tilde{w}_e(w, \hat{a})$. Recall that worker's wealth at the beginning of the period is a. We express the optimal search target as $\hat{w}_e(w, a) \equiv \tilde{w}_e(w, \hat{a}_e(w, a))$. The tightness of the submarket searched by the employed worker is $\hat{\theta}_e(w, a) \equiv \theta(\hat{w}_e(w, a), \hat{a}_e(w, a))$.

Similar to an unemployed worker, search serves as a mechanism for an employed worker to partially self-insure, which is reflected by the effects of wealth and earnings on the search decision. An employed worker with relatively low wealth or a relatively low wage has incentive to obtain a wage increase quickly in order to build up wealth for self insurance. To do so, the worker searches for a job that yields a relatively small wage gain but has a higher matching probability for the worker.[8]

2.5. Firms and market tightness

Consider a job that pays wage w and is filled by a worker with wealth a. Current profit of the job is $(y - w)$. The worker will quit for another job with probability $\lambda_e p(\hat{\theta}_e(w, a))$, where $\hat{\theta}_e(w, a)$ is the tightness of the submarket searched by the worker. In addition to the quit for another job, the worker separates exogenously into unemployment with probability δ. When either type of separation occurs, the future value of the job to the firm will be zero. If no separation occurs, the future value of the job to the firm will be $\beta J(w, \hat{a}_e(w, a))$, where $\hat{a}_e(w, a)$ is the worker's future wealth. Thus, the value of the job to the firm satisfies:

$$J(w, a) = y - w + (1 - \delta)[1 - \lambda_e p(\hat{\theta}_e(w, a))]\beta J(w, \hat{a}_e(w, a)). \qquad (2.4)$$

Competitive entry of vacancies determines the tightness in each submarket. Consider submarket $(\hat{w}, \hat{a})$. The matching probability for a vacancy is $q(\theta(\hat{w}, \hat{a}))$. If the firm is matched, production will start in the next period, and the present value of the firm is $\beta J(\hat{w}, \hat{a})$. The flow cost of a vacancy is $k > 0$. For all $(\hat{w}, \hat{a})$, if $\beta J(\hat{w}, \hat{a}) \geq k$, competitive entry of vacancies will equate the expected value of a vacancy to the cost k. If $\beta J(\hat{w}, \hat{a}) < k$,

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[8]Although the effect of the current wage on search exists even when workers are risk neutral, risk aversion strengthens the effect for workers with low wealth because the marginal utility of consumption is higher for such workers.

no vacancy will enter the submarket. Thus, competitive entry of vacancies yields

$$q\left(\theta\left(\hat{w},\hat{a}\right)\right)\beta J\left(\hat{w},\hat{a}\right) \leq k \text{ and } \theta\left(\hat{w},\hat{a}\right) \geq 0, \forall\left(\hat{w},\hat{a}\right),$$

where the two inequalities hold with complementary slackness. We solve:

$$\theta\left(\hat{w},\hat{a}\right) = \begin{cases} q^{-1}\left(\frac{k}{\beta J(\hat{w},\hat{a})}\right) & \text{if } \beta J\left(\hat{w},\hat{a}\right) \geq k \\ 0 & \text{otherwise.} \end{cases} \tag{2.5}$$

Note that $q\left(\theta\right)$ is a decreasing function and $p\left(\theta\right)$ an increasing function. A submarket with relatively high job-finding probability for an applicant must have a relatively large tightness. Because such a submarket has a relatively low matching probability for a vacancy, firms enter such a submarket only if the submarket yields a relatively high firm value J and, hence, has a relatively low wage offer.[9]

3. Equilibrium and Submarkets

3.1. Equilibrium definition and block recursivity

The aggregate state of the economy is a distribution of workers over individual states. Given the distribution of workers at the beginning of a period, ψ, individuals' optimal decisions and matching outcomes induce the distribution of workers at the beginning of the next period, $\hat{\psi}$. We omit the characterization of this transition of the aggregate state because it is cumbersome counting of the flows of workers between states.

Definition 3.1. *An equilibrium consists of the value function of workers, V_ε (for $\varepsilon = e, u$), the firm value function J, policy functions, $(c_\varepsilon, \hat{a}_\varepsilon)$ and $(\hat{w}_u, \hat{w}_e)$, and the transition function of the aggregate state, $\mathcal{T}$, that satisfy the following requirements:*
(i) The value function of workers, $V_\varepsilon : \bar{W} \times A \rightarrow \mathbb{R}$, satisfies (2.1). The corresponding optimal decisions of participation, consumption and savings yield the policy functions $(c_\varepsilon, \hat{a}_\varepsilon)$, and optimal search decisions yield the policy functions $(\hat{w}_u, \hat{w}_e)$.
(ii) The firm's value function, $J : W \times A \rightarrow \mathbb{R}$, satisfies (2.4).

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[9]Features such as an applicant's labor market status, earnings and past employment history can affect the applicants' wealth and the target wage of search. However, given $(\hat{w}, \hat{a})$, these features do not add information to the determination of the market tightness.

(iii) The tightness function θ satisfies (2.5) for all $(\hat{w}, \hat{a}) \in W \times A$.

(iv) The aggregate state transition, $\mathcal{T} : \Psi(S, \mathcal{S}) \to \Psi(S, \mathcal{S})$, is consistent with the policy functions and induces the aggregate state in the next period as $\hat{\psi} = \mathcal{T}(\psi)$.

The equilibrium defined above is a *block recursive equilibrium* (BRE), as defined and analyzed by Shi (2009) and Menzio and Shi (2010, 2011). Namely, value functions, policy functions and the market tightness function in the equilibrium are determined by (i)-(iii) independently of the distribution of workers. We can verify block recursivity as follows. Start with the hypothesis that the value function of a filled job, $J(w, a)$, is independent of ψ. Competitive entry of vacancies into submarkets requires the expected value of a filled job to be equal to the vacancy cost. This requirement determines the market tightness function, $\theta(w, a)$, independently of ψ (see (2.5)). Because matching probabilities in a submarket are only a function of the market tightness, they are also independent of ψ. Given these matching probabilities, individuals can calculate the present value of a job and make their decisions. The resulting value functions and optimal choices are independent of ψ. In particular, the value function of a filled job to a firm, $J(w, a)$, is independent of ψ, which supports the initial hypothesis.

Directed search and competitive entry of vacancies are critical for block recursivity. Because search is directed, a worker chooses to search in the submarket that features the optimal tradeoff between the gain in value and the matching probability. For this decision, the worker does not need to know the distribution of workers, provided that the matching probabilities are independent of the distribution. Because competitive entry of vacancies drives down expected profit of a vacancy to zero in every viable submarket, the tightness and the matching probabilities are indeed independent of the distribution.

Block recursivity reduces the complexity of the equilibrium substantially. If the equilibrium is not block recursive, then the distribution of workers is a state variable relevant for individuals' decisions. This aggregate state has infinite dimension because earnings and wealth lie in intervals. Moreover, the law of motion of this aggregate state is endogenous because it must be consistent with the flows of workers induced by individuals' optimal

decisions. Exactly computing such a non-block recursive equilibrium is not feasible. The approximation in the literature assumes that only a small number of moments of the distribution matter (e.g., Krusell and Smith, 1998, Krusell et al., 2010). In contrast, when the equilibrium is block recursive, individuals' optimal decisions and the market tightness function depend only on the individual state that has a small dimension.

3.2. The role of wealth in the description of submarkets

In the modeling above, submarkets are described by the wage offer and applicants' wealth. Why does a firm care about an applicant's wealth in addition to the offer? An employee's wealth affects the survival probability of a match, because the employee's probability of quitting for another job is $\lambda_e p(\hat{\theta}_e(w, a))$. For any given current wage w, a worker with high wealth has incentive to search for a high wage. Since a high wage offer comes with a relatively low matching probability for the worker, the worker will succeed in getting the outside offer with a low probability. Thus, the employee's wealth increases the survival probability of the match and, hence, increases the expected value of the match to the firm. For any wage offer, a recruiting firm prefers to hire a worker with a high level of wealth.

If wealth is not observable, how can the equilibrium be implemented? One way is to assume that firms can elicit information about applicants' wealth. An applicant is not able to show more wealth than what he has. If an applicant chooses to hide some wealth, the worker does not gain once the worker gets the applied job, but the worker increases the risk of getting the job if other applicants to the job report wealth truthfully. Thus, it is incentive compatible for the applicant to show wealth truthfully. In the majority of states in the U.S., employers are allowed to check job applicants' credit records before hiring, and they do. In a survey, the Society for Human Resource Management (2012) found that 60 percent of the managers in 2010 checked job applicants' credit records before hiring.[10] Although a credit record may not precisely identify a worker's wealth, it can be highly informative of a worker's wealth.

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[10]The question asked in the survey is: "Does your organization, or an agency hired by your organization, conduct credit background checks for any job candidates by reviewing the candidates' consumer reports?"

Another way to implement the equilibrium without assuming observable wealth is to include an applicant's *current job*, defined as $\eta \equiv (\varepsilon, \omega)$, in the description of a submarket. The additional information may be available from an applicant's resume and can be used as a qualification for a job. The following lemma provides the conditions under which the offer wage and the applicants' current job together are sufficient for describing submarkets (see Appendix A for a proof):

Lemma 3.2. *Suppose that a worker's search choice is not observable to the worker's current employer unless search results in a match and that a worker's wealth is not observable. However, the information about a worker's current job η is publicly observable and contractible. Assume that (i) for any given (ε, ω), the policy function $\tilde{w}_\varepsilon(\omega, \hat{a})$ is one-to-one of $\hat{a}$, and (ii) the policy functions $\tilde{w}_\varepsilon(\omega, \hat{a})$ and $\hat{a}_\varepsilon(\omega, a)$ are single-valued. Then, the equilibrium where submarkets are described by $(\hat{w}, \eta)$ is the same as the equilibrium where wealth $\hat{a}$ is observable and submarkets are described by $(\hat{w}, \hat{a})$.*

Under the assumption that $\tilde{w}_\varepsilon(\omega, \hat{a})$ is one-to-one of $\hat{a}$ for any given (ε, ω), a wage $\hat{w}$ is searched only by applicants with a particular wealth level $\hat{a}$ for any given current job $\eta = (\varepsilon, \omega)$. Such self-selection in the search process implies that recruiting firms are able to infer the applicant's wealth from the wage offer and an applicant's current job. Given this inference, a firm that succeeds in recruiting a worker can forecast perfectly the worker's future search choices and, hence, the separation rate in the future. Thus, a recruiting firm's expected value in a submarket is a function of only the wage offer $\hat{w}$ and the applicants' current job η. Then, competitive entry of vacancies determines the tightness in each submarket as a function of only $(\hat{w}, \eta)$. Once an applicant's wealth is inferred from $(\hat{w}, \eta)$, the applicant's past jobs and the duration at the current job do not add any relevant information to the description of the submarket, since they affect only the path by which the worker has reached the particular wealth level.

The above exposition explains why the two elements $(\hat{w}, \eta)$ are sufficient for describing a submarket. When wealth is not observable, the two elements are also necessary for separating the applicants into different submarkets. It is possible that two employed workers

can differ in both wealth and current earnings (or the labor market status) in a particular way that induces them to search for the same wage. If both succeed in matching, they will have the same wage. But because they differ in wealth, they will search for different wages in the future. This implies that the two firms matched with the two workers will face different separation rates in the future and, hence, different firm values. If submarkets were described only by the offer wage, the two workers could not be separated.

Despite Lemma 3.2, we will continue to describe submarkets by the offer wage and applicants' wealth. It is difficult to analytically verify the assumptions (i) and (ii) in the lemma, although they hold in the quantitative analysis later. Also, the dimension of submarkets $(\hat{w}, \hat{a})$ is lower than $(\hat{w}, \eta)$ by one.

4. Quantitative Analysis

4.1. Calibration

The utility function and the matching probabilities have the following forms:

$$u(c,l) = \frac{c^{1-\sigma}}{1-\sigma}, \quad p(\theta) = [1+\theta^{-\gamma}]^{-\frac{1}{\gamma}}, \quad q(\theta) = \frac{p(\theta)}{\theta}.$$

The matching probability function, $p(\theta)$, is a Dagum (1975) function, with $\gamma > 0$. If a submarket has a measure N of applicants and tightness θ, the function $p(\theta)$ implies that the measure of matches is:

$$\mathcal{M}(N, \theta N) = p(\theta) N = \frac{N \times (\theta N)}{[(N)^{\gamma} + (\theta N)^{\gamma}]^{\frac{1}{\gamma}}}.$$

The model is calibrated to the monthly frequency. We normalize $y = 1$. Other parameters and calibration targets are listed in Table 1. The interest rate is $r = 0.327\%$ per month or 4% annually. Since the distance between the interest rate r and the discount rate $\frac{1}{\beta} - 1$ is an important determinant of wealth distribution, we set β for the model to match the ratio of total income to total wealth of 15% in Survey of Consumer Finance (SCF) of 2007. With the borrowing limit $\underline{a} = -6.36$, the model generates a fraction of the population with negative wealth of 7.8%, as in the SCF of 2007 (see Kennickell, 2012). The curvature of the utility function is set at $\sigma = 2$ as in most macro calibration.

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On the labor market, the exogenous separation rate $\delta = 0.026$ matches the average transition rate from employment to unemployment in the Current Population Survey (CPS). For an employed worker, the probability of having the search opportunity in a period, λ_e, is chosen to yield a monthly job-to-job transition rate 0.029, which lies in the range $[0.022, 0.032]$ in recent empirical evidence (e.g., Fallick and Fleischman, 2004, Hornstein et al., 2011). We adjust γ to target the average elasticity of the job-finding probability of unemployed workers with respect to average market tightness for unemployed workers. This elasticity is 0.32 in the calibration, which lies in the range $[0.27, 0.5]$ in the literature (see Shi, 2018). Home production in the absence of the unemployed benefit is set at $y_{\min} = 0.05$ to prevent individuals from reaching $c = 0$. The replacement ratio of the unemployment benefit is set to a realistic value $\rho = 0.5$. In our model, the expected duration of the unemployment benefit is $1 + \frac{1}{\chi}$, since χ is the expiration probability of the benefit in each period after receiving the benefit in the first period of unemployment. Calibrating this expected duration to a common value in reality, 6.5 (i.e., 28 weeks), yields $\chi = 0.182$. The vacancy cost is set at $k = 0.288$ to match an unemployment rate of 6.5%.

Table 1: Baseline calibration parameters

Parameter	Baseline	Target
β	0.996617	total income as % of total wealth = 15%.
r	0.327%	annual interest rate = 4%
$\underline{a}$	-6.36	fraction of population with $a < 0$ = 7.8%
δ	2.6%	separation rate into unemployment in CPS
σ	2.0	standard in macro calibration
γ	0.65	elasticity of p_u to θ_u lies in $[0.27, 0.50]$
λ_e	0.23	EE transition rate lies in $[2.2, 3.2]\,\%$
k	0.288	unemployment rate = 0.065
ρ	0.5	replacement rate = 0.5
χ	0.182	expected UI duration = 28 weeks

With the identified parameters, we solve the equilibrium and simulate the model (for the procedure, see Appendix C in the supplementary appendix). We will analyze the computed policy functions in the next subsection and the transition rates in subsection 4.3. Subsection 4.4 will analyze the distribution and inequality.

4.2. Policy functions, value functions and market tightness

A critical feature of the equilibrium is that an individual's decision on consumption and savings interacts with search decision in the labor market. We analyze this interaction using the computed policy functions.

Figure 1 depicts workers' optimal search decision (the top panel), the difference between next period's wealth and current wealth (the middle panel), and consumption (the bottom panel), all as functions of current wealth. In each panel, the green dashed line is for an unemployed worker. The other three lines are for employed workers at three levels of current earnings: low $w \in (0, \hat{w}_u(0, a))$ (the black solid line), medium $w \in (\hat{w}_u(0, a), \bar{w})$ (the red dashed line), and high $w = \bar{w}$ (the blue line).[11] For earnings at the maximum, the target wage for search stays constant at this maximum over all wealth levels, as depicted by the blue line in the top panel. For all earnings lower than the maximum, the target wage for search is increasing in wealth if wealth is moderate or low. This policy function for the search target becomes flat when wealth is high. Thus, the effect of wealth on job search is strong at low wealth levels and dissipates as wealth increases.

These features of the optimal search target are intuitive. For a worker with low wealth, the borrowing limit is either binding or will be binding if the wage does not increase soon. In both cases, future consumption is expected to fall. To partially insure against this outcome, the worker tries to obtain a wage increase quickly by search. The optimal target wage for search is low because only low-wage jobs have high job-finding probabilities. An increase in wealth reduces the likelihood that the borrowing limit will be binding soon. This enables the worker to tolerate a lower job-finding probability and, hence, to search for higher wages. That is, the target wage as a function of wealth is positively sloped at low wealth levels. As wealth keeps increasing, the effect diminishes, and so the target wage policy function becomes less steep. When wealth is sufficiently high, the worker is perfectly self-insured against income risks, in which case further increases in wealth do not affect

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[11]The low wage used in Figure 1 is lower than $\hat{w}_u(0, a)$ and, hence, is out of the (steady-state) equilibrium. Nevertheless, it is useful to include this low wage in Figure 1 since it may occur during the transition to the steady state equilibrium.
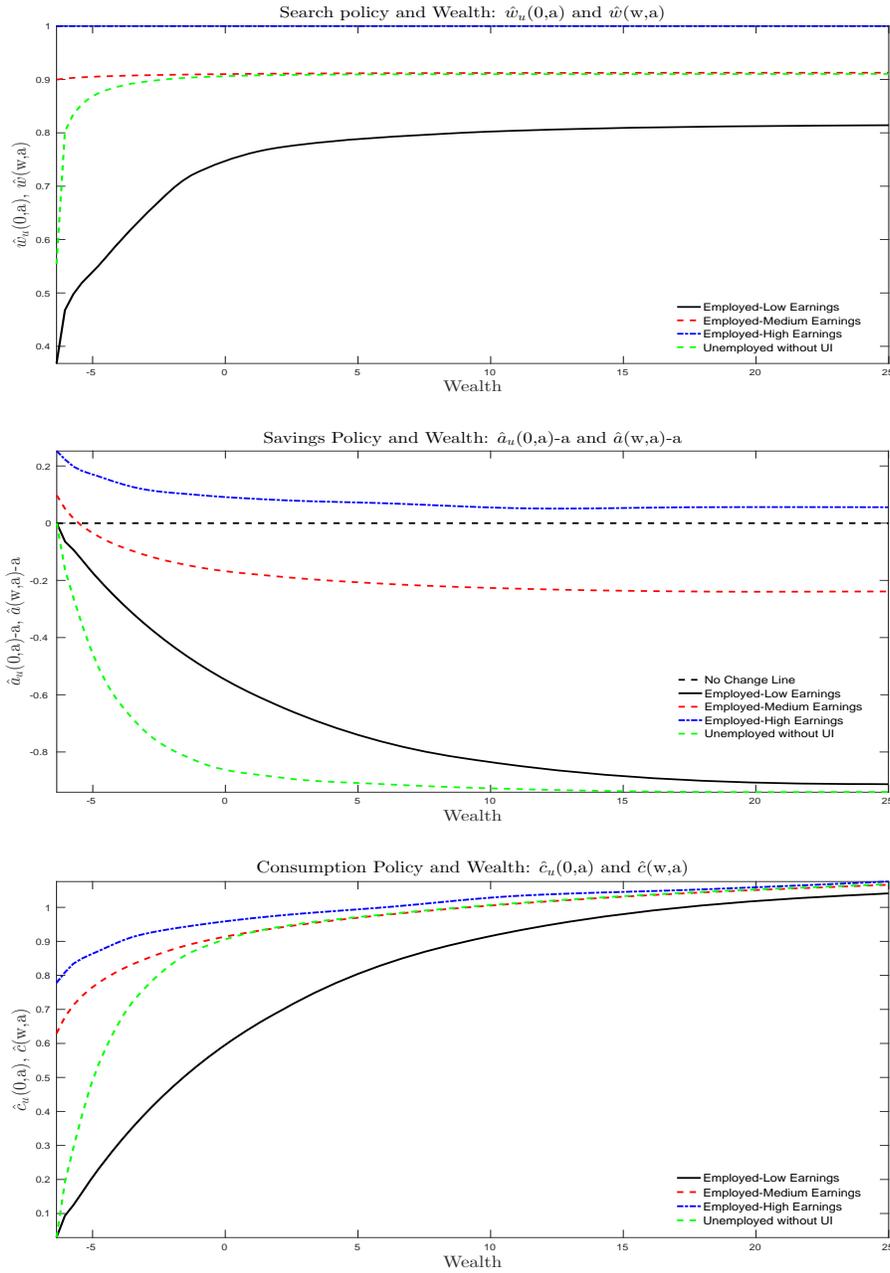
the optimal target wage for search.

Figure 1. Optimal search target, future wealth and current consumption
as functions of wealth for earnings fixed at low $w \in (b, \hat{w}_u(a))$,
medium $w \in (\hat{w}_u(a), \bar{w})$, and high $w = \bar{w}$.

The middle panel in Figure 1 shows that, as wealth increases, the difference between
future and current wealth decreases in current wealth. The decreasing difference implies
different dynamics of wealth for workers with different earnings, because the sign of the

difference depends on earnings. For a worker with high earnings, the difference between future and current wealth is above zero. The worker is accumulating wealth as a precaution for exogenous job separation into unemployment. As wealth increases toward the level of perfect self-insurance, the difference between future and current wealth decreases toward zero. In contrast, for a worker with low earnings, the difference between future and current wealth is negative. The worker is decumulating wealth in order to maintain smooth consumption. As wealth decreases, the difference between future and current wealth increases but remains negative. The absolute value of this difference declines. For a worker with medium earnings, the dynamics of wealth depend on current wealth. If current wealth is lower than a threshold, the worker accumulates wealth over time; if current wealth is higher than the threshold, the worker decumulates wealth over time.

The bottom panel in Figure 1 shows consumption as an increasing function of wealth. The slope of the consumption function represents the marginal propensity to consume out of wealth, which decreases as wealth increases to improve the ability to self-insure. When wealth approaches the level of perfect self-insurance, consumption becomes constant, and so the marginal propensity to consume approaches zero. Moreover, for any given wealth, workers with low earnings have lower consumption and, to smooth consumption, these workers consume more out of their wealth proportionally. Reflecting this effect of current earnings, the consumption function is lower but steeper for workers with low earnings than for workers with high earnings.

Figure 2 depicts employed workers' target wage for search (the top panel), the difference between future and current wealth (the middle panel), and consumption (the bottom panel), all as functions of current earnings. In each panel, the three lines correspond to three wealth levels: low (the black solid line), medium (the red dashed line) and high (the blue dot-dashed line). These panels confirm the above analysis. Relative to workers with high earnings, workers with low earnings search for lower wages, decumulate wealth less to smooth consumption, and have higher propensities to consume out of wealth. These differences narrow as wealth increases. When wealth is high, consumption change little with earnings and the difference between future and current wealth becomes positive for

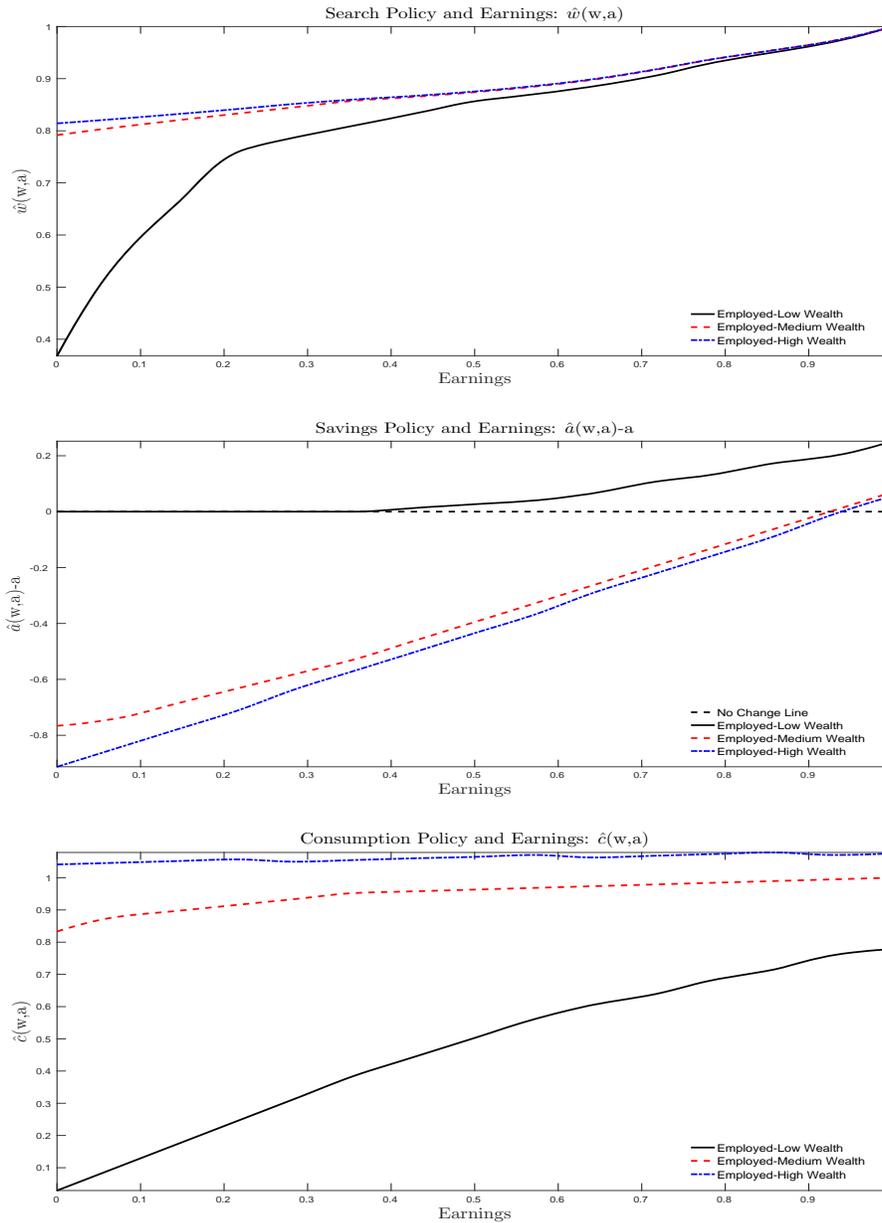all wealth levels because precautionary savings motives are stronger.

Figure 2. Optimal search target, future wealth and current consumption
as functions of earnings for wealth fixed at low, medium and high levels.

Now we examine how the firm value of a filled job and the market tightness depend on
the wage and the applicant's wealth. In Figure 3, the left panels depict the dependence on
an applicant's wealth, where the applicant's current earnings are set to three levels with
the same legends and colors as in Figure 1. The right panels depict the dependence on the

wage offer, where the applicant's wealth is set to be three levels with the same legends and colors as in Figure 2. Not surprisingly, the firm value of a filled job is a decreasing function of the wage offer for any given wealth level of the applicant. Anticipating the low value of a filled job, not many firms enter the submarket to offer the high wage. Thus, the market tightness is also a decreasing function of the wage offer.
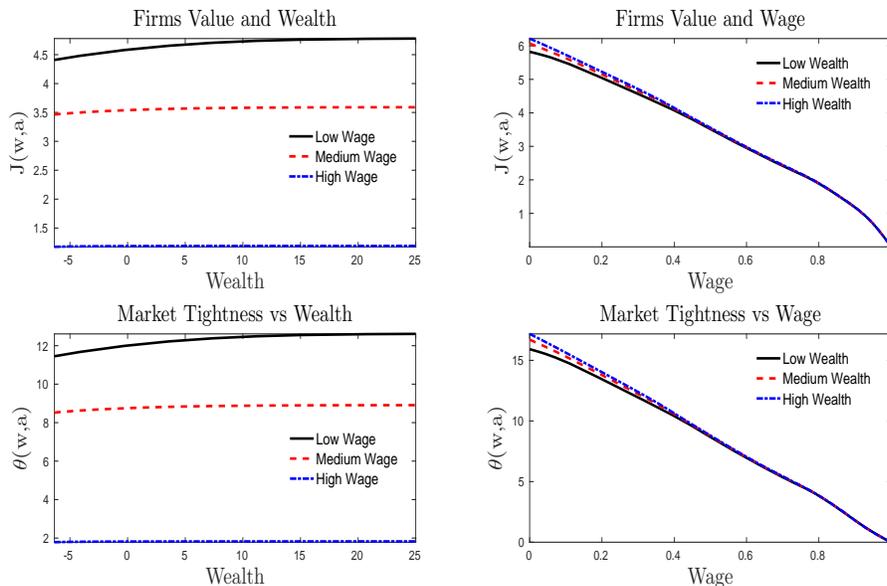

Figure 3. Firm value and market tightness as functions
of worker's wealth and the wage offer

For any given wage offer, the firm value of a filled job and the tightness of the submarket offering the wage are increasing functions of an applicant's wealth, provided that the applicant's current wage is lower than the maximum. That is, for any given wage offer, a recruiting firm prefers an applicant with high wealth to an applicant with low wealth. This preference arises from the effect of a worker's wealth on search decisions. When a worker's wealth is high, it is not urgent for the worker to obtain a wage increase for self insurance. As a result, the worker will search for higher wages that are less likely to be obtained. This reduces the endogenous separability probability of the worker from the current job and, hence, increases the firm value of the job filled by the worker. Anticipating this higher value of a job filled by a wealthier worker, more vacancies enter the submarket to attract such workers. The tightness increases in this submarket. These effects of a worker's wealth

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weaken as the wage offer increases. When the wage offer is at the maximum, a worker employed at such a wage is not expected to move to another job. In this limit, the firm value of a filled job and the tightness of the submarket offering the maximum wage are independent of the worker's wealth.[12]

4.3. Worker transition

By affecting workers' search decisions, wealth affects the transition rates of workers in the labor market. To gauge the significance of this effect, we use the model simulated data to run the following cross-sectional regressions:

$$UE_i = \beta_0^{ue} + \beta_a^{ue} a_i + \varepsilon_1,$$
$$EE_i = \beta_0^{ee} + \beta_a^{ee} a_i + \beta_w^{ee} w_i + \varepsilon_2.$$

UE is the monthly transition rate from unemployment to employment, EE is the transition rate directly from one job to another job, and the subscript i indicates individuals. To check for robustness, we replace the regressors with log regressors in the above regressions, i.e., a_i with $\ln a_i$ and w_i with $\ln w_i$. Table 2 lists the coefficients of the two types of regressions.

Table 2. Effects of an applicant's wealth and
wage on job transition rates

Regressors\Coefficients	β_a^{ue}	β_a^{ee}	β_w^{ee}
Level regressors	-0.0017	-0.00005	-0.5496
(a_i, w_i)	[10.09]	[2.85]	[169.82]
Log regressors	-0.0039	-0.0002	-0.5424
$(\ln a_i, \ln w_i)$	[4.15]	[2.49]	[164.66]

Note: The numbers in [.] are Newey-West adjusted t statistics.

The two regressions yield similar results. All regression coefficients are statistically significant. Wealth affects both the UE and EE rates negatively. This finding is consistent with the analysis in section 4.2. Namely, wealthier workers apply for better paid jobs and face a lower job-finding probability. Similarly, a worker's current wage negatively affects the transition rate to another job. For any given wealth, a worker's current wage increases the target wage for search and reduces the matching probability.

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[12]At very low wage and wealth levels, the benefit to a firm of increasing the wage to retain a worker may be even higher than the direct cost of the wage increase. In this case, an increase in the wage offer improves the payoff to both a recruiting firm and an applicant. Thus, submarkets with such low wages are not active in the equilibrium.

4.4. Distribution of workers and inequality

Table 3 reports measures of inequality in earnings, income, wealth and consumption. Income is equal to earnings plus interest income on assets. The results in the baseline model, reported in the first column, are compared with those in two benchmarks:

(i) The "no-search" model (the second column in Table 4): This is similar to Aiyagari (1994), where the labor market is frictionless but individuals face employment risks and a borrowing limit.[13] In each period, a job is destroyed exogenously with probability δ and the worker is unemployed. An unemployed worker becomes employed with probability $p_u \in (0, 1)$. The probability p_u is set to be equal to the average job-finding probability of an unemployed worker in the baseline model. The wage is the competitive level that sets the profit of a vacancy to zero.

(ii) The "no-wealth" (the third column in Table 4): This is a model where individuals are hand-to-mouth. As in the baseline model, employed and unemployed workers can search, search is directed, and workers are risk averse.

Appendix D in the supplementary appendix describes these two benchmarks in more detail. Note that in the baseline and the no-wealth model, all wage inequality is frictional in the sense that it is caused by search frictions. Since all workers have the same ability and preferences, and all jobs produce the same amount of output, all workers would have the same wage if search frictions were absent in the labor market as in the no-search model.

The mean-min wage ratio, proposed by Hornstein et al. (2011) as a measure of wage inequality, is the ratio of the average wage earned by an employed worker to the lowest wage in the equilibrium. Our model generates a mean-min wage ratio of 1.737. To put this number in perspective, it is useful to know that the mean-min wage ratio is no more than 1.04 in a variety of search models as demonstrated by Hornstein et al. (2011).[14]

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[13]This benchmark differs from the model in Aiyagari (1994) primarily in that the interest rate is exogenous rather than endogenous.

[14]Shi (2018) constructs a directed model of on-the-job search that can generate a mean-min ratio of 2.6. In that model, the cost of posting a vacancy is increasing and convex in the capital stock of the vacancy. In the equilibrium, firms create jobs with a low capital stock for unemployed workers and invest to increase the capital stock later.

Wage inequality in our model is higher for two reasons: (*i*) unemployed workers without unemployment insurance are willing to take low wages; and (*ii*) the search decision interacts with wealth. Ignoring wealth accumulation, as in the no-wealth model, results in lower frictional wage dispersion. As explained in section 4.2, on the one hand, the desire for self insurance motivates unemployed workers with low wealth to take jobs that pay low wages. This expands the left tail of the equilibrium wage distribution. On the other hand, as wealth increases, a worker becomes better insured and can take the chance of searching for higher wages. This expands the right tail of the wage distribution. It is worth noting that Krusell et al. (2010) also analyze search with wealth accumulation but they do not allow for on-the-job search. Their model generates a mean-min ratio of 1.02 when it is calibrated as in Shimer (2005).[15] The presence of on-the-job search in our model is important for the higher dispersion, as examined further in Appendix B. Not allowing for on-the-job search reduces the mean-min ratio from 1.737 to 1.172.

Table 3. Inequality measures (not targeted)

	Baseline	No-search	No-wealth	Wage-Tax
Mean-min wage ratio	1.737	1.000	1.413	1.690
Gini: wealth	0.428	0.333	——	0.418
Gini: earnings	0.050	0.037	0.113	0.050
Gini: income	0.054	0.046	0.113	0.053
Gini: consumption	0.019	0.019	0.113	0.019
$\frac{\text{Gini: consumption}}{\text{Gini: earnings}}$	0.375	0.517	1.000	0.373

Table 3 also reports Gini coefficients in wealth, earnings, income, and consumption.[16] The baseline model shows more earnings and income inequality than the no-search model. With no search, there is no dispersion in wages, and so earnings inequality is only driven by the uncertainty in whether workers are employed. Despite having higher inequality in earnings and income, the baseline model generates a lower ratio of the Gini in consumption to the Gini in earnings than the no-search model. This ratio can be interpreted as the proportion of earnings risks that are passed through to consumption. The pass-through

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[15]They get a mean-min ratio of 1.0002 if home production is calibrated as in Hagedorn and Manovskii (2008).

[16]Since some households hold negative wealth we adjust the Gini coefficient in wealth following Chen, Tsaur and Rhai (1982).

rate is 37.5% higher in the no-search model. Thus, individuals in the baseline model are able to shield consumption better from fluctuations in earnings than in the no-search model. This better insurance comes from individuals' ability to choose which jobs to apply for as well as the amount of savings. Moreover, if individuals are hand-to-mouth, as in the no-wealth model, then all earnings risks are passed through to consumption. Comparing this ratio in the no-search and the no-wealth models, we see that savings provide significant self insurance against the earnings risk. In the no-search model, the Gini coefficient in consumption is 51.7% of that in earnings. In other words, 48.3% of the earnings risk is self insured by accumulating wealth.
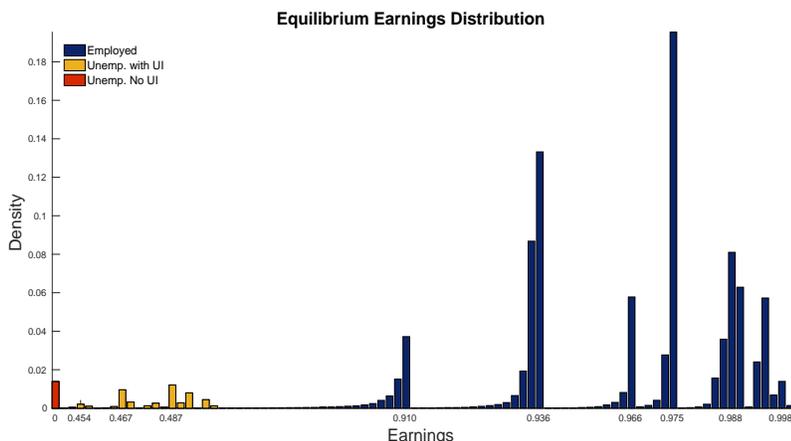

Figure 4. Equilibrium earnings distribution

It is informative to go beyond the statistics of inequality to examine the entire distributions of wages and wealth. Figure 4 shows the equilibrium density of wages. The support of the distribution is endogenous and discrete. This support excludes the income $y_{\min}$ and the interest income on assets. The red bar at zero earnings shows the mass of workers who are unemployed without unemployment insurance. The orange bars show the density of unemployed workers receiving the unemployment insurance which is dispersed because it is proportional to the wage prior to unemployment. Each blue bar shows the mass of workers at an equilibrium wage, which ranges from approximately 0.6 to almost 1. Most employed workers are employed at a few values of wages. The shorter bars to the left of the tall bars are the densities of workers who have low wealth and apply for relatively low

wages in order to have higher matching probabilities. Those workers are the ones for whom the wealth effect on job search decision is strong.

Table 4. Fraction of workers accumulating
and decumulating wealth

current wealth \ future wealth	$\hat{a} \leq a$	$\hat{a} > a$
$a > \bar{a}$	11.4%	36.6%
$a \leq \bar{a}$	12.4%	39.6%

Table 4 shows the fraction of individuals that are accumulating and decumulating wealth according to their position with respect to the average asset holdings in equilibrium ($\bar{a} = 6.37$). The elements on the main diagonal of the table show the fraction of the population that push the wealth distribution towards the mean. The two elements off the diagonal show the fraction of the individuals that are spreading the wealth distribution away from the mean. In total, 49% of the population is spreading wealth away from the mean, with about $\frac{3}{4}$ of them pushing the right tail of the distribution. In addition, 51% of the population is contracting the distribution towards the mean.
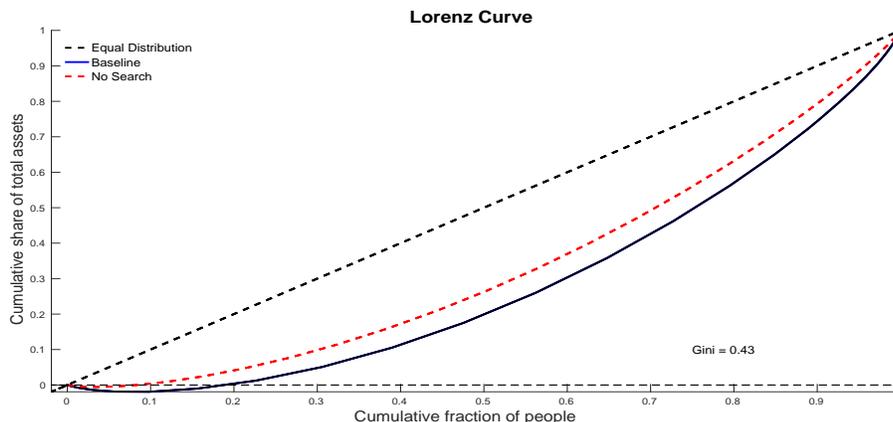

Figure 5. Lorenz curve of wealth

The above behavior of savings affects the wealth distribution in equilibrium. Figure 5 plots the Lorenz curve and Figure 6 the density function of wealth, in the baseline model and in the model with no search. Relative to the baseline, the no-search model generates higher average wealth and a wealth distribution less skewed to the left than the baseline. In particular, the fraction of people with negative wealth is smaller in the baseline model.

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These contrasts indicate that the option to modify job search is quantitatively important for wealth accumulation. Because workers do not have this option in the no-search model, they need to rely more on wealth as self-insurance. This increases the average wealth level and reduces the mass of workers whose wealth is close to the borrowing limit.
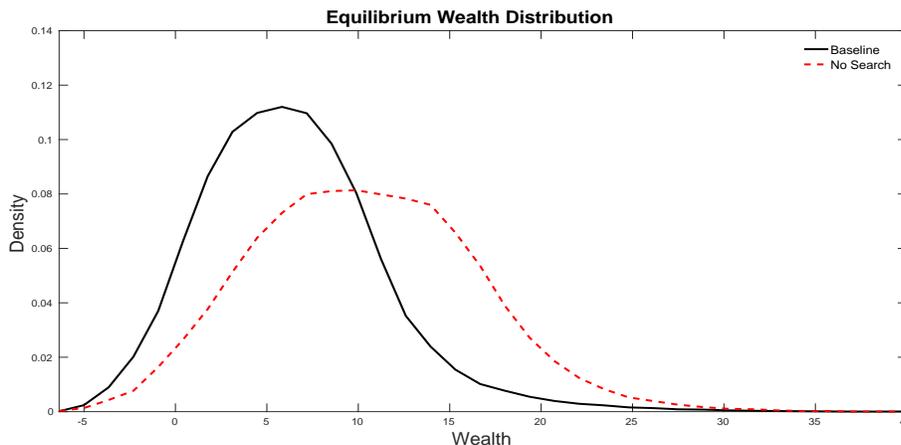

Figure 6. Wealth distribution

In Appendix B we conduct counter-factual exercises. The results can be summarized as follows. First, shutting down on-the-job search (OJS) reduces the mean-min wage ratio from 1.737 to 1.172. OJS is important for wage dispersion by reducing the minimum wage in the equilibrium.[17] Second, without OJS, average wealth is higher is higher than in the baseline model. As job search decisions are less useful for consumption smoothing they need to stock more wealth for insurance. However, average wealth without OJS (but with unemployed search) is still lower and the wealth distribution is more skewed to the left than in the no-search benchmark (see Figure 9 in Appendix B).

To conclude this section, we echo the discussion in the Introduction on the importance of using an equilibrium model for the analysis, which determines the offer distribution of wages endogenously. To be specific, we discuss the paper by Lise (2013) who incorporates OJS to examine the effect of wealth on job search under an exogenously given offer distribution of wages. He estimates the offer distribution and other parameters by matching the

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[17]Allowing for wealth accumulation but not for on-the-job search, Krusell et al. (2010) find that the mean-min wage ratio is only 1.023, which is even smaller than 1.04 that Hornstein et al. (2011) find in models without wealth.

model and the data on the observables such as the distribution of employed workers over wages and workers' transition rates. This estimated distribution of wage offers is generically inconsistent with firms' optimal decisions on vacancy creation and wage offers. If Lise's (2013) model were extended to endogenize firms' decisions, the resulting offer distribution of wages would have much narrower dispersion than he estimated with the partial-equilibrium model. To see this, note that Lise (2013) leaves the value of an unemployed worker's home production, b, to be estimated rather than calibrated. The estimated value is $b = 0$, in contrast to $b = 0.5w_{-1}$ in our calibration. Thus, as other partial-equilibrium models, Lise's (2013) model generates large dispersion in employed wages by forcing the value of home production to be unrealistically low.[18] Moreover, since search is undirected in Lise (2013), the value $b = 0$ implies that unemployed workers who differ in wealth have the same transition rate into employment and accept the same distribution of wages. These implications are clearly inconsistent with the empirical evidence discussed in the Introduction on how wealth affects unemployment duration. Furthermore, a partial-equilibrium model is not suitable for policy analysis, to which we turn in the next section.

5. Welfare Analysis

In this section, we analyze welfare effects of changing the borrowing limit, the replacement rate and the expected duration of the unemployment benefit. For each policy, we compute the dynamic effect, in addition to the steady-state effect, and study the optimal policy.

5.1. Welfare measure and the computational method

We measure the welfare effect in terms of the percentage change in consumption in the lifetime, as in Lucas (1987). In all experiments, case 0 refers to the benchmark and case 1 to the case after the change. Let $\left[c_{i,t}^{j}\right]_{t=0}^{\infty}$ be the consumption path of individual i in case $j \in \{0, 1\}$. When the economy moves from case 0 to case 1, the welfare effect on individual

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[18]Hornstein et al. (2011) make the same observation.

i, denoted μ_i, is given by

$$\sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}^0 \left(1 + \mu_i\right)\right) = \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}^1\right). \tag{5.1}$$

The welfare measure μ_i is the percentage increase in lifetime consumption that individual i obtains when the economy moves from case 0 to case 1. Similarly, the aggregate welfare effect, denoted as $\bar{\mu}$, is defined as follows:

$$\int_0^1 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}^0 \left(1 + \bar{\mu}\right)\right) \, \mathrm{d}i = \int_0^1 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}^1\right) \, \mathrm{d}i. \tag{5.2}$$

A policy can change the sum of the unemployment insurance. To compare welfare in a meaningful way, it is necessary to require the unemployment insurance to be financed. We assume that the government uses a proportional wage income tax rate, τ_w, to finance the unemployment benefit in the present value. Given the tax rate, the tax revenue needed to finance the unemployment benefit depends on the distribution of workers over wages and the unemployment rate. This dependence makes the equilibrium fail to be block recursive, because the tax rate affects households' decisions on consumption, savings and search. Despite this failure, we compute equilibrium dynamics exactly rather than using the typical method of approximation in Krusell and Smith (1998). It is important to compute equilibrium dynamics instead of only the steady state. As shown later, the welfare effect of a policy along the transitional path differs significantly in the magnitude, and sometime in the sign, from the steady-state effect.

Given this importance of the dynamic equilibrium, it is helpful to explain how we compute such an equilibrium without encountering the typical problem of dimensionality of the state space. The tractability comes from the property that the equilibrium is block recursive for any *given* tax rate. Exploring this property, we iterate on the tax rate. Starting with an arbitrarily given tax rate, we compute the dynamic equilibrium without imposing the requirement of a balanced budget for the government. Since the tax rate is treated as a parameter in this step, the equilibrium is block recursive as defined in section 3.1 and can be computed relatively quickly. Then, we calculate the present value of the

government surplus. If this surplus is not zero, we adjust the tax rate and recompute the equilibrium until the government's intertemporal surplus becomes zero.

5.2. Welfare effect of changing the borrowing limit

The presence of the borrowing limit reduces the extent of consumption smoothing. How much better off would workers be if their borrowing limit were relaxed? To answer this question we take the steady state in the baseline model with $\underline{a} = 0$ as case 0. Then, we relax the borrowing limit to a new level $\underline{a} < 0$, which is case 1.[19] This change is treated as a permanent shock. We compute the transitional dynamics and assess the welfare effect. The tax rate τ_w is adjusted to balance the government budget intertemporally.
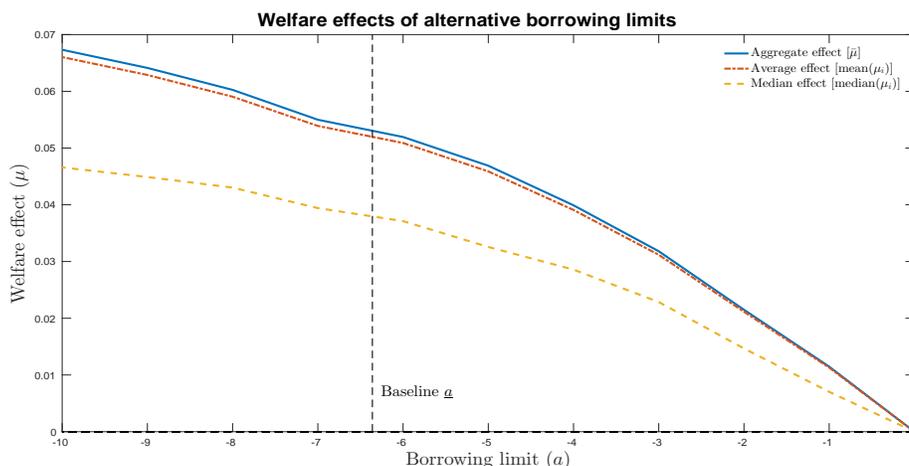

Figure 7: Welfare consequences of changes in the borrowing limit.

Figure 7 shows the welfare effects measured by the aggregate welfare change, $\bar{\mu}$ (the blue solid line), and also the mean and median of individual welfare changes, μ_i (the red and orange dashed lines, respectively). The horizontal axis shows the relaxed borrowing limit. The vertical axis shows welfare changes in percentage points. The vertical dashed line signals the borrowing limit in our baseline calibration of the model. Relaxing the borrowing limit increases welfare monotonically. In terms of magnitudes, moving from an

[19]It is also meaningful to conduct the experiment in the opposite direction by treating the steady state of an economy with $\underline{a} < 0$ as case 0 and tightening the borrowing limit. However, such tightening of the limit imposes negative consumption on individuals who are constrained by the borrowing limit in the initial steady state.

economy to with $\underline{a} = 0$ to one with $\underline{a} = -6.36$ (55% of the average annual income), as in the baseline calibration, increases welfare by approximately 0.06%. A likely reason for the welfare gain to be small is that workers are able to increase wealth above the limit relatively quickly by modifying job search.

All welfare gains from relaxing the borrowing limit occur during the transition as the extra borrowing capacity enables workers to maintain higher consumption in bad times. Once the economy reaches the new steady state, workers have reduced their average asset holdings and exhausted the benefits of moving to the economy with a larger borrowing limit. In fact, the steady-state welfare effect of relaxing the borrowing limit is negative. Thus, correctly assessing welfare gains of relaxing borrowing constraints requires precise computation of the transition to the new steady state, as we do here.

5.3. Alternative schedules of the unemployment insurance

We examine the welfare effects of changing the generosity and the expected length of the unemployment insurance. The generosity of the benefit is parameterized by the replacement rate ρ and the expected length of the benefit by $1/\chi$, where χ is the probability that the benefit expires. As in the previous exercise, the tax rate τ_w adjusts to balance the government budget intertemporally.

Figure 8 shows the effects of changing the replacement rate ρ from the baseline value, 0.5. The upper panel depicts the welfare effect in the steady state, and the lower panel depicts the welfare effect when transitional dynamics are taken into account. In both panels the horizontal axis is the value of ρ to which the policy changes from 0.5, while the vertical axis is the welfare change in percentage points. The figures show the aggregate welfare effect (blue solid line), the average welfare effect (red dashed line) and the median welfare effect (orange dashed line).

Steady state comparisons (the upper panel) show that the optimal replacement rate is $\rho = 0$, with $\tau_w = 0$. Moving from $\rho = 0.5$ to $\rho = 0$, aggregate steady-state welfare increases by approximately 1.8%. In the steady state, workers prefer to receive no unemployment

insurance in order to pay no tax on wages. Instead of the unemployment insurance, they prefer to rely on wealth accumulation and job search decisions as self-insurance mechanisms. Relative to the baseline with $\rho = 0.5$, the new steady state with $\rho = 0$ has more job creation since firms can pay lower gross wages and yet deliver higher after-tax wages.

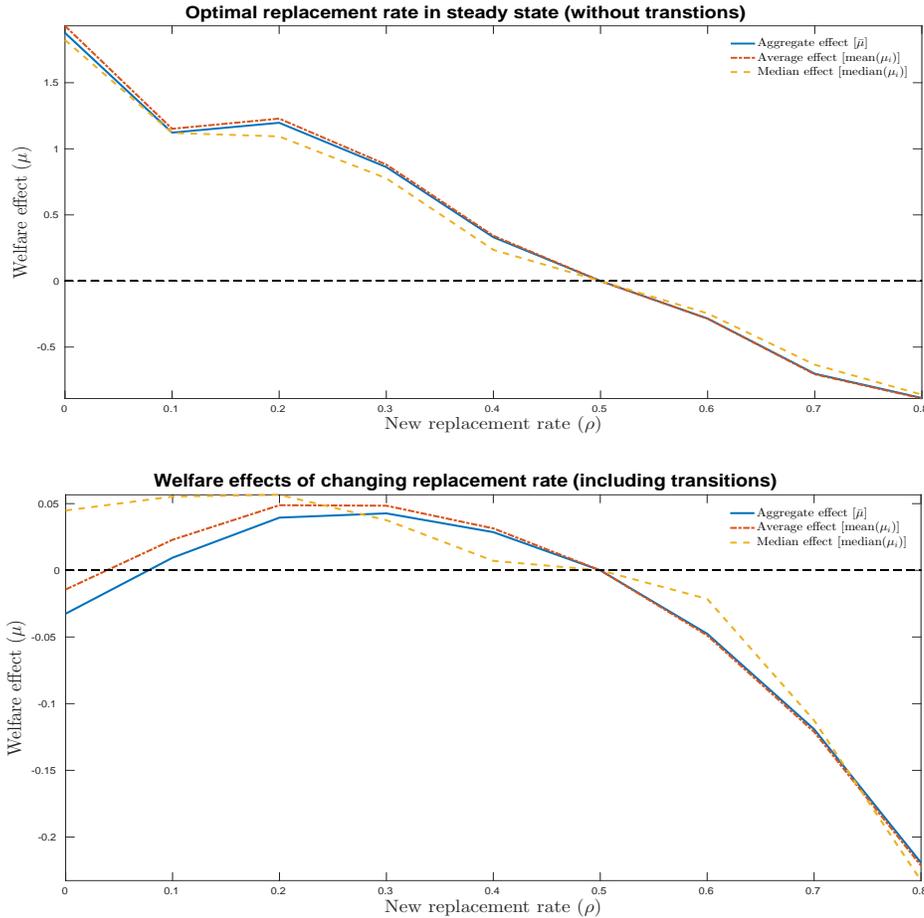**Optimal replacement rate in steady state (without transtions)**

Figure 8: Welfare effects of changes in the replacement rate.

The dynamic effect of ρ differs significantly from the steady-state effect. Taking transitional dynamics into account, the lower panel in Figure 8 shows that aggregate welfare is maximized at $\rho \approx 0.2$ with an associated wage tax of $\tau_w = 1.01\%$. Moving from the baseline calibration to this optimal replacement rate increases aggregate welfare by approximately 0.04%. Although this optimal replacement rate is lower than in the baseline calibration, it is significantly positive. Why do not workers want to move all the way to an economy with $\rho = 0$ which is optimal if only the steady state is concerned? The reason

is that to build enough insurance to compensate for the zero replacement rate, workers have to accumulate wealth and apply to lower wages over the transition path. Although employed workers pay lower wage taxes if $\rho = 0$, consumption is lower for a sufficiently long time while workers accumulate enough wealth to self-insure against fluctuations in labor earnings. In the steady state with $\rho = 0.2$, the aggregate level of wealth is approximately 60% larger than in the steady state under the baseline calibration. In contrast, in steady state with $\rho = 0$, aggregate wealth is approximately two times as much as in the steady state in the baseline calibration. Taking into account the transition cost of accumulating assets significantly reduces the welfare gain from the zero replacement rate.

In addition, we consider changes in the expected length of the unemployment insurance by changing the expiration probability χ and adjusting the wage tax rate τ_w to keep the government budget balanced intertemporally. The main message of this exercise is similar to the previous one. Steady-state comparisons show that the optimal probability of losing the unemployment insurance in each period is $\chi = 1$. In the steady state, workers prefer to pay as low taxes as possible and accumulate wealth to self-insure against unemployment risks. In contrast, taking into account transitional dynamics, the optimal probability of losing the unemployment insurance in each period is $\chi \approx 0.5$, which is higher than in the baseline calibration ($\chi = 0.182$) but less than one. The value $\chi = 0.5$ implies that the optimal expected duration of the unemployment benefit, $1 + \frac{1}{\chi}$, is 3 months instead of 28 weeks as in the baseline calibration. Moving from $\chi = 0.182$ to $\chi = 0.5$, aggregate welfare increases by approximately 0.06%.

6. Conclusion

We have studied a search equilibrium with risk-averse workers who can accumulate non-contingent assets under a borrowing limit. Workers can search both on and off the job, and search is directed. Workers with high wealth optimally choose to search for high wages and, in the case of failing to match, they decumulate wealth to smooth consumption. Workers with low wealth optimally choose to search for low wages in an attempt to get a job

quickly to build up wealth. Firms create vacancies and offer different wages to cater to these differential needs of workers. In the other direction, search frictions affect wealth accumulation by generating heterogeneity in search outcomes and earnings. After calibrating the model, we have found that wealth significantly reduces worker transitions in the labor market. The baseline equilibrium generates significantly higher inequalities in earnings and wealth than both the model without search and the model without wealth. However, the interaction between wealth and search provides important self-insurance against earnings risk as it reduces the pass-through of earnings inequality into consumption by more than 60%. Moreover, we have analyzed welfare effects of the borrowing limit and the unemployment insurance. Taking transitional dynamics into account, these effects are substantially different from comparing steady states.

At least two extensions of this model are worth pursuing. First, firms can post dynamic contracts instead of a fixed wage. With dynamic contracts, firms have incentive to backload wages to increase retention, as analyzed by Burdett and Coles (2003) and Shi (2009). This force can stretch the upper tail of the wage distribution. Also, an unemployed with low wealth may be willing to accept even lower wages than in the baseline model in the expectation of wage increases in a contract. This force can stretch the lower tail of the wage distribution. Although both forces can widen frictional wage dispersion, their quantitative importance is yet to be determined. Moreover, shocks to match-specific productivity and/or work effort can be introduced as in Tsuyuhara (2016) and Lamadon (2016). Second, extending the model to study the business cycle seems a natural exercise. When aggregate shocks are present, individuals may have further incentives to accumulate assets to smooth consumption. The computational advantage provided by block recursivity of the baseline equilibrium makes it tractable to study business cycles.

Appendix

A. Proof of Lemma 3.2

Maintain assumptions (i) and (ii) in the lemma. We first prove that whenever a worker succeeds in a match, the worker's wealth at the time of the match can be inferred from the offer wage $\hat{w}$ and the worker's current job $\eta = (\varepsilon, \omega)$. Then, we prove that after being matched with a worker, a firm can perfectly compute the worker's future wealth and search targets in the entire duration of the worker's employment with the firm. This implies that the firm value of a job filled at wage $\hat{w}$ with a worker η is a function of $(\hat{w}, \eta)$ only. With this result, competitive entry of vacancies into submarkets determines the tightness in each submarket as a function of only $(\hat{w}, \eta)$, thus closing the loop of the argument.

Consider a worker with the current job $\eta = (\varepsilon, \omega)$ who chooses to search for wage $\hat{w}$. For this choice to be the optimal, it must be given by the policy function of optimal search, i.e., $\hat{w} = \tilde{w}_\varepsilon(\omega, \hat{a})$, where $\hat{a}$ is the wealth level at the time of search. Because the policy function $\tilde{w}_\varepsilon(\omega, \hat{a})$ is assumed to be one-to-one for any given ω, its inverse for wealth exists for any η. Write this inverse as $\hat{a} = h(\hat{w}, \eta)$ for some function h.

Now consider a match between a firm offering $\hat{w}$ and a worker from a job $\eta = (\varepsilon, \omega)$. In period τ on the new job, let a_τ be the worker's wealth at the beginning of the period and w_τ the wage searched by the worker. The firm does not directly observe w_τ, unless the worker succeeds in the new search. The firm does not directly observe a_τ either. However, the firm can infer (a_τ, w_τ) perfectly as follows. Start at $\tau = 1$. The worker's wealth at the beginning of the period is the same as the wealth level at the time of forming the new match in the previous period. By the calculation above, this is equal to $a_1 = h(\hat{w}, \eta)$. After consumption in period 1 on the new job, the worker's wealth will be $\hat{a}_1 = \hat{a}_e(\hat{w}, a_1)$. Since this is also the wealth that the worker will bring into the next period if the worker stays at the job, then $a_2 = \hat{a}_e(\hat{w}, a_1)$. In period 1, the worker optimally searches for wage $w_1 = \tilde{w}_e(\hat{w}, a_2)$. Substituting a_1, we have $a_2 = h_2(\hat{w}, \eta) \equiv \hat{a}_e(\hat{w}, h(\hat{w}, \eta))$ and $w_1 = W_1(\hat{w}, \eta) \equiv \tilde{w}_e(\hat{w}, h_2(\hat{w}, \eta))$. Moving to $\tau = 2$, the worker's wealth after consumption

will be $a_3 = \hat{a}_2 = \hat{a}_e(\hat{w}, a_2)$ and the optimal search target will be $w_2 = \tilde{w}_e(\hat{w}, a_3)$. These can be rewritten as $a_3 = h_3(\hat{w}, \eta) \equiv \hat{a}_e(\hat{w}, h_2(\hat{w}, \eta))$ and $w_2 = W_2(\tilde{w}, \eta) \equiv \tilde{w}_e(\hat{w}, h_3(\hat{w}, \eta))$. By induction, in any arbitrary period τ on the job, the worker's wealth after production and consumption will be $a_{\tau+1} = h_{\tau+1}(\hat{w}, \eta) \equiv \hat{a}_e(\hat{w}, h_\tau(\hat{w}, \eta))$ and the optimal search target will be $w_\tau = W_\tau(\hat{w}, \eta) \equiv \tilde{w}_e(\hat{w}, h_{\tau+1}(\hat{w}, \eta))$. Because the functions $\hat{a}_e(\hat{w}, a)$ and $\tilde{w}_e(\hat{w}, a)$ are assumed to be single-valued, the sequence $\{(a_\tau, w_\tau)\}_{\tau \geq 1}$ is uniquely calculated from the worker's current wage $\hat{w}$ and the previous job η.

With the above inference on the worker's future wealth and search targets, the firm can calculate the value of a job at wage $\hat{w}$ filled by an applicant whose current job is $\eta = (\varepsilon, \omega)$. At the beginning of any arbitrary duration τ in the new match, the firm value is

$$J_{alt}(\hat{w}, \eta, \tau) \equiv J(\hat{w}, h_\tau(\hat{w}, \eta)),$$

where the subscript alt indicates an alternative expression for J. The dependence of J_{alt} on the worker's previous job η and the duration τ on the current job does not mean that the submarket searched by the firm's employee in period τ is indexed by (η, τ). Rather, this submarket is indexed by only the offer wage w_τ and the worker's current job $\hat{\eta} \equiv (e, \hat{w})$. Firms recruiting in submarket $(w_\tau, \hat{\eta})$ know that an applicant's wealth is $a_{\tau+1} = h(w_\tau, \hat{\eta})$, where $\hat{\eta} = (e, \hat{w})$. The expression J_{alt} is only for the current employer's internal calculation of the value of a filled job. To compute J_{alt}, note that submarket $(w_\tau, \hat{\eta})$ has tightness $\theta_{alt}(\hat{w}, \eta, \tau) \equiv \theta(w_\tau, a_{\tau+1})$, where $w_\tau = W_\tau(\hat{w}, \eta)$ and $a_{\tau+1} = h_{\tau+1}(\hat{w}, \eta)$. The function J_{alt} obeys the following Bellman equation:

$$
\begin{aligned}
& J_{alt}(\hat{w}, \eta, \tau) \\
= \; & y - \hat{w} + (1 - \delta)\left[1 - \lambda_e p(\theta_{alt}(\hat{w}, \eta, \tau))\right]\beta J_{alt}(\hat{w}, \eta, \tau + 1).
\end{aligned}
$$

In particular, the firm value immediately after filling the job is $J_{alt}(\hat{w}, \eta, 1)$, which is a function of only $(\hat{w}, \eta)$.

Finally, in submarket $(\hat{w}, \eta)$, competitive entry of vacancies implies:

$$q(\theta) J_{alt}(\hat{w}, \eta, 1) \leq k \text{ and } \theta \geq 0,$$

where the two inequalities hold with complementary slackness. This condition determines

the tightness θ as a function of only the offer wage $\hat{w}$ and the applicants' current job $\eta = (\varepsilon, \omega)$. **QED**

B. Counter-factual Exercises

We conduct two counter-factual exercises in turn. First, we shut down on-the-job search (OJS) by setting $\lambda_e = 0$ to show that OJS is important for frictional wage dispersion. Second, we study welfare consequences of changing the replacement rate in the no-wealth model. In each exercise, we recalibrate the model to match the targets in Table 1, provided that they remain valid, except for two parameters. We keep the values of borrowing limit, $\underline{a}$, and the discount factor, β, as in the baseline calibration. These two parameters are key determinants of the wealth distribution as the distance of the interest rate and discount rate determines incentives to save, and the borrowing limit determines how much credit workers can access. The main change is in the vacancy cost k. The recalibrated value of k and other results of the counter-factual exercise are reported in Table 5.

Table 5. Results of counter-factual exercises

	Baseline	No OJS ($\lambda_e = 0$)	Data
Vacancy cost k	0.29	0.44	——
Unemployment rate	6.50%	6.50%	6.5%
UE transition rate	2.6%	2.6%	2.6%
EE transition rate	2.89%	0%	2.2-3.2%
Elasticity of p_u to θ_u	0.32	0.33	0.27-0.50
Mean-min wage ratio	1.737	1.172	1.7-2.0
Gini: wealth	0.43	0.38	0.80
Gini: earnings	0.05	0.04	0.61
Gini: income	0.05	0.04	0.55
Gini: consumption	0.02	0.02	0.25
Corr(earnings income)	0.99	0.99	0.72
Corr(earnings, wealth)	0.09	0.07	0.47
Corr(income, wealth)	0.19	0.15	0.60

When on-the-job search (OJS) is shut down, the mean-min wage ratio falls significantly from 1.737 to 1.172. This result shows that most of the frictional wage dispersion in the baseline model comes from on-the-job search. On-the-job search widens wage dispersion

mainly by reducing wages that unemployed workers choose to search for. Unemployed workers are willing to lower their search target in the expectation that they can search for higher wages after being employed.

The wealth distribution also changes. Figure 9 shows the density functions of wealth in the baseline model, the model with no OJS and the model with no search. Relative to the baseline, the wealth distribution with no OJS shifts to the right. As job search decisions are less useful as an insurance mechanism, workers need to accumulate larger amounts of wealth to insure against unemployment risk.
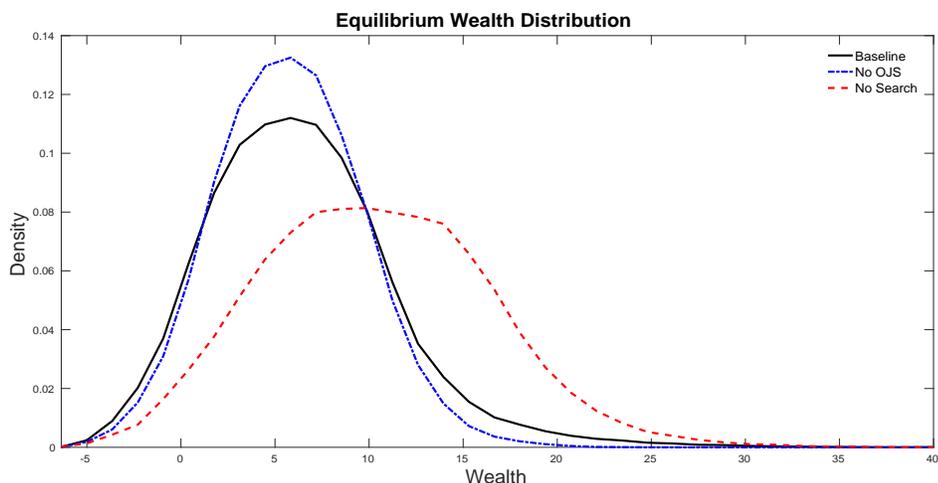

Figure 9. Wealth distribution: baseline vs. two benchmarks

Although workers can use job search decisions as an insurance mechanism, as discussed in section 4.4, the more important insurance mechanism is wealth accumulation. In addition, as shown in section 5.3, if workers could choose they would like to reduce the size of the unemployment insurance and self-insure by accumulating wealth, instead. Figure 10 shows welfare effects of changes in the replacement rate and wage taxes that keep the government budget balanced in the present value. As in section 5.3, we report the aggregate, the average, and the median welfare change.

The top panel shows steady-state comparisons while the bottom panel shows the effects taking into account transitional dynamics. In both cases, welfare is monotonically increasing in the replacement rate. In other words, absent wealth as an insurance mechanism, the

welfare effect of the unemployment insurance program is opposite to the one in the baseline model. Moreover, welfare losses are large (35%) if we eliminate the unemployment insurance as workers do not have any instrument to increase consumption in the current period when they are unemployed. In this case, taking into account the transitions does not make as a big difference as in the baseline model. This is because the change in the replacement rate takes place from one period to the next one and, in contrast to the baseline, there is no asset that can be used to adjust toward the new steady state smoothly.
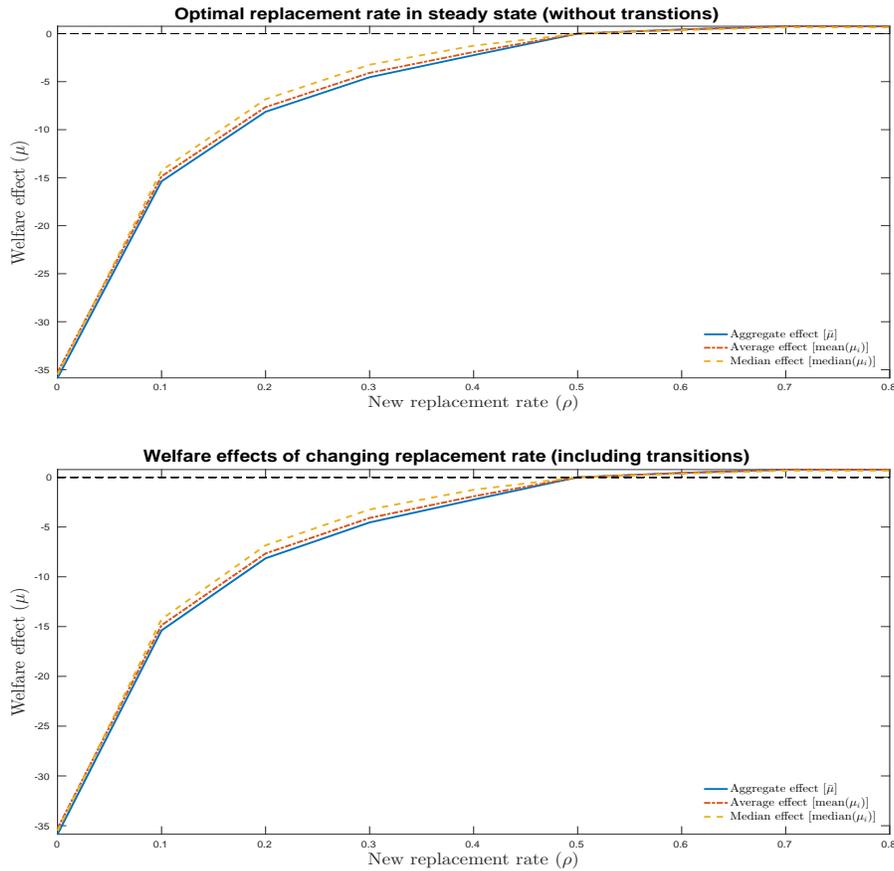

Figure 10.   Welfare effects of changes in the replacement rate in the no-wealth model.

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Supplementary Appendix for
"Wealth Accumulation, On-the-Job Search and Inequality"
Gaston R. Chaumont and Shouyong Shi

C. Solution Algorithm and Simulation

We solve the model using a nested fixed-point algorithm. First, we define the grids for the spaces of wealth and wages. Wealth is set to be $[\underline{a}, \bar{a}] = [0, 75]$, which is larger than the equilibrium support of the wealth distribution. Wages are bounded between the home production level b and the total flow of productions per period y. In equilibrium, there will be no wages outside those bounds. The steps of the algorithm are as follows:

(1) Guess initial value functions of workers and firms.

(2) Given the value function of firms, solve for the market tightness that is consistent with competitive entry of firms.

(3) Given the tightness function, solve a worker's optimization problem and compute optimal savings, consumption, and job search decisions. This step is done by iterating on the worker's value function until convergence, using the shape-preserving Cubic Hermite interpolation to calculate the policy functions.

(4) Given workers' policy functions computed in (3), calculate separation rates of employed workers and update the value function of the firm in each submarket.

(5) Iterate on the value function of the firm until convergence.

Once the value and policy functions are solved, we simulate the model to obtain the distribution of individuals, using $N = 100,000$ individuals and $T = 1,100$ time periods (months). We discard the first 100 periods to avoid dependence on initial conditions. The average of the last 100 periods of the simulations is used to calculate the stationary distribution.[20] To simulate the evolution of individuals' states, we take random draws for separation shocks, the search opportunity (consistent with λ_e) and matching shocks that determine which individuals in each submarket are matched. Also, we start the economy by assigning a random state to each of the N workers. Then, we use the equilibrium

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[20]The distribution appears stationary at most after period 400 in all of the exercises that we have conducted. So, choosing 100 periods is not relevant for the results presented in this paper. We could use only the distribution in the last period and get about the same numbers. However, using more periods allows us to reduce errors.

optimal policy functions and these random shocks to compute the endogenous evolution of the state of each individual.

D. Benchmark Models

In this appendix we present two benchmark models to compare with the baseline model in section 2. The first benchmark is a no-search model where search frictions do not exist in the labor market but individuals are still exposed to employment uncertainty and a borrowing limit. This model is similar to the model in Aiyagari (1994). The second benchmark is a no-wealth model where individuals are hand-to-mouth and search frictions exist in the labor market. This model is the standard model of directed search augmented with on-the-job search and risk aversion.

D.1. No-search model

In this benchmark model, the search stage in each period is replaced by an *iid* employment shock that exogenously determines the employment status of a worker. The shock makes an unemployed worker employed with probability $p_u \in (0, 1)$ and keeps the worker unemployed with probability $1 - p_u$. The probability p_u is set to be equal to the average job-finding probability of an unemployed worker in the baseline model. As in the baseline model, a job is hit by an *iid* separation shock with probability $\delta \in (0, 1)$, in which case the worker becomes unemployed. All employed workers earn the wage $\bar{w}$ that sets the profit of a vacancy to zero, and they do not change jobs. There is competitive entry of vacancies, and the vacancy cost is k per period. Individuals face a borrowing limit.

For a worker with the labor market status $\varepsilon \in \{e, u\}$, the optimal decisions on consumption and savings still solve (2.1) that induces the value function $V_\varepsilon(\omega, a)$. For an unemployed worker with $(b, \hat{a})$ immediately before the employment shock, the value $R_u(b, \hat{a})$ now is

$$R_u(b, \hat{a}) \equiv p_u V_e(\bar{w}, \hat{a}) + (1 - p_u)[\chi V_u(0, \hat{a}) + (1 - \chi) V_u(b, \hat{a})]$$

Since an employed worker does not search on the job, the worker value after consumption and savings is given as $R_e(\bar{w}, \hat{a})$.

Because employed workers do not search on the job, the probability of a match being destroyed is independent of the employee's wealth. This implies that the firm value of a filled job is independent of the employee's wealth and obeys:

$$J(\bar{w}) = y - \bar{w} + (1 - \delta)\beta J(\bar{w}).$$

Competitive entry of vacancies requires $\beta J(\bar{w}) = k$. Solving $J(\bar{w})$ from the above equation, we can determine the competitive wage by

$$\frac{y - \bar{w}}{1 - (1 - \delta)\beta} = \frac{k}{\beta}.$$

An equilibrium can be defined by adapting the definition in section 3.1. In particular, the market tightness function is irrelevant and the wage rate is given above.

D.2. No-wealth model

In this benchmark, workers cannot accumulate wealth, and so consumption is equal to earnings. As in the baseline model, there are search frictions, workers can search off and on the job, and search is directed. There is competitive entry of vacancies into submarkets. The timing of events in a period is the same as in the baseline model. Since $a = \hat{a} = 0$, we eliminate $(a, \hat{a})$ from all value functions, policy functions, and the market tightness function. With these changes, the optimal choices of consumption and savings solve (2.1) and induce the value function $V_\varepsilon(\omega)$. For an unemployed worker, optimal choices of search and participation solve (2.2) that yields the value $R_u(b)$. For an employed worker, the optimal choice of search solves (2.3) that yields the value $R_e(w)$. The firm value of a filled job is given by (2.4) and the tightness of each submarket by (2.5), with the modified notation. An equilibrium can be defined by adapting the definition in section 3.1.

E. Savings Policy Function for All (ω, a)

For the baseline model, we depict the savings policy function for all levels of equilibrium earnings and wealth. In Figure 11, the green dashed line is the savings policy of unemployed workers without unemployment insurance, expressed as $\hat{a}_u(0, a) - a$. The red shaded area is savings policies of unemployed workers receiving unemployment insurance, b, at all

equilibrium values of b, expressed as $\hat{a}_u(b,a) - a$. The blue shaded area is savings policies of employed workers at all equilibrium wages, expressed as $\hat{a}(w,a) - a$.
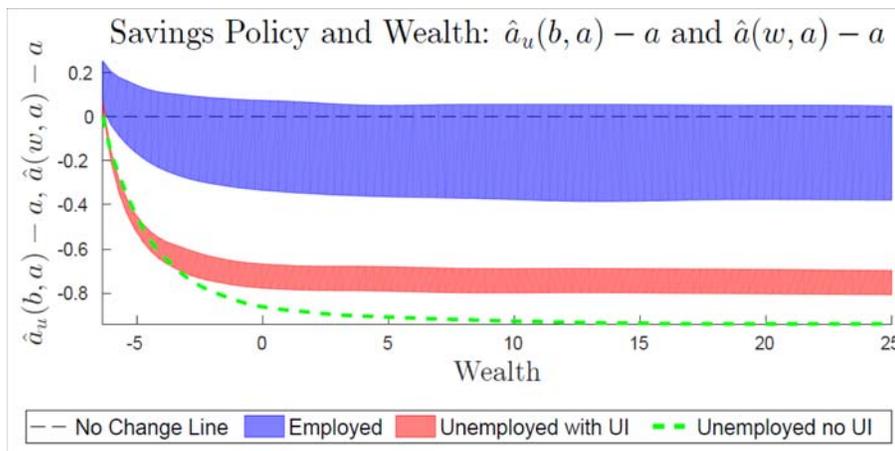

Figure 11. Equilibrium saving policy

Unemployed workers decumulate wealth at almost all wealth levels, since $\hat{a}_u(b,a) - a < 0$ for all (b,a), except for a small fraction of unemployed workers with unemployment insurance that are very close to the borrowing limit. Those few unemployed workers accumulate some wealth in case they lose their unemployment insurance before they find a new job. Most unemployed workers reduce their asset positions to smooth consumption under the expectation of finding a job and getting an increase in income in the future. In contrast, a large part of the blue shaded area in Figure 11 lies above zero, which means that many employed workers accumulate wealth. The savings motive comes from the precaution for exogenous separation into unemployment. In the equilibrium, even the lowest wage of an employed worker is much higher than home production in unemployment. Losing such high earnings represents a large risk which an employed worker wants to insure against by savings. This motive of precautionary savings is particularly strong when an employed worker has low wealth. Even when wealth is high, an employed worker still accumulates wealth if earnings are high. An employed worker decumulates wealth when wealth is high and earnings are low. In this case, the motive of reducing wealth to smooth consumption dominates the motive of precautionary savings. This is because for those workers it is more likely to find a better job in the future than to be separated into unemployment.

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