

Wage Dispersion in the Aiyagari Model

A Progress Report

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March 9, 2018

TODAY'S PLAN

- In the standard [Aiyagari \(1994\)](#) wages or earnings are completely exogenous. How can this be changed?
 - Entrepreneurial (or criminal) activity a la [Quadrini \(2000\)](#)
 - Could add Education, Life-Cycle w/o Learning by Doing
 - Search Frictions and/or Learning by Doing.
- Explore a variety of models á la [Aiyagari \(1994\)](#) where
 - Workers endowment of efficiency units is constant.
 - Getting a job has frictions.
 - Firms create jobs and post wages.
- Wage dispersion and the wealth distribution are endogenous
 - [Aiyagari \(1994\)](#) meets [Burdett and Mortensen \(1998\)](#).
 - Related to [Lise \(2013\)](#), [Hornstein, Krusell, and Violante \(2011\)](#), [Krusell, Mukoyama, and Åđahin \(2010\)](#) [Eeckhout and Sepahsalari \(2015\)](#), [Chaumont and Shi \(2017\)](#),

MODEL 1: PRECAUTIONARY SAVINGS, COMPETITIVE SEARCH

- Jobs are created by firms (plants). A job plus a worker produce one unit of the good.
 - To get a worker firms pay a flow cost \bar{c} to post a vacancy.
 - Jobs are destroyed at rate δ . Workers cannot (won't) quit.
- Households differ in wealth and wages (if any). Households can save. There are no state contingent claims, nor borrowing.
 - If unemployed households produce b and search. If employed they get w and do not search.
 - Matching protocol is competitive search. Workers know what type of wage they are looking for.
- General equilibrium (unimportant): Workers own firms.
- Small equilibrium wage dispersion

MODEL 2. ENDOGENOUS QUILTS: BEAUTY OF EXTREME VALUE SHOCKS

1. Shocks to the utility of working or not working: Some workers quit.
2. Add a (smoothed) quitting motive so that higher wage workers quit less often: Firms may want to pay high wages to retain workers.
3. Conditional on wealth, high wage workers quit less often.
4. But Selection (correlation 1 between wage and wealth when hired) makes wealth trump wages and higher wages imply quit less often: Wage inequality collapses due to firms profit maximization.

MODEL 3. DIFFUSION OF WEALTH & WAGES: MORE EXT. VAL. SHOCKS

1. Reduces the correlation of Wages and Applicants Wages, even if exaggerating wage dispersion
2. Another set of Extreme Value Shocks, this time to the type of market that the unemployed aim to. Difuses the link between wage and wealth.
3. Which reduces/solces the selection problem and justifies paying higher wages for longer tenure.

MODEL 4. ADD ENDOGENOUS PRODUCTIVITY CREATION

1. Firms can spend more to make more productive plants with higher maintenance costs even when idle. Only worth if workers last longer: hence EFFICIENCY WAGES
2. Can be added to a theory of non-linear wages. The work week can

MODEL 5. ADD ON THE JOB SEARCH

1. With extreme value shocks makes a more empirically relevant world than [Burdett and Mortensen \(1998\)](#) or [Chaumont and Shi \(2017\)](#).
There are frictions.
2. Perhaps even Model 6 with Human Capital / Occupation Expertise accumulation.

30 SECONDS OF WHY!!

- We want to build a joint theory of income and wealth inequality suitable for macro purposes:
 - Needs some form of Aiyagari structure
 - We want more than just exogenous income.
 - Exploratory research. We see where we are going.
 - The earlier the input of the audience the more valuable it is.

PRELIMINARY FINDINGS

1. **Model 1** With workers on the job being identical (no endogenous quitting) wage dispersion is about 2-3%. Yet to be disciplined quantitatively. As (in different context) Hornstein, Krusell, and Violante (2011).
2. **Model 2** With endogenous quits (which we wanted it to add dispersion), actual wage dispersion collapses due to selection. Big bad news. Not theorem but quantitative statement. More later.
3. **Model 3** With diffused access of workers to differently waged jobs, wage dispersion returns.
4. Taking Stock: By themselves, wealth differences are not a promising venue for frictional wage dispersion unless perhaps for occupational choice at the beginning of the working life (not today).

ORDER OF EVENTS OF THE MODEL 1

1. Households enter period t with or without a job.
2. **Production & Consumption:** The employed produce z on the job. The unemployed produce b at home. They make consumption-saving decisions.
3. **Job Search:** Potential firms decide whether to enter and if so, the wage w at which to post a vacancy. The unemployed choose wages to apply.
4. **Job Match & Separation:** The employed workers who receive exogenous separation shocks become unemployed. The successfully matched job candidates become employed.
5. Households enter period $t + 1$ with new employment status.

HOUSEHOLD PROBLEM

- An individual is either employed (e) or unemployed (u).
- Individual state: wealth and wage
 - If employed: (a, w)
 - If unemployed: (a)
- Problem of the employed: (Standard)

$$V^e(a, w) = \max_{c, a'} u(c) + \beta [(1 - \delta)V^e(a', w) + \delta V^u(a)]$$

$$\text{s.t. } c + a' = a(1 + r) + w, \quad a \geq 0$$

- Problem of the unemployed: Choose which wage to look for

$$V^u(a) = \max_{c, a', w} u(c) + \beta [\psi^h(\theta(w))V^e(a', w) + (1 - \psi^h(\theta(w)))V^u(a')]$$

$$\text{s.t. } c + a' = a(1 + r) + b, \quad a \geq 0$$

FIRMS POST VACANCIES AT DIFFERENT WAGES AND FILLING PROBABILITIES

- Value of a job with wage w :

$$\Omega(w) = z - w + \frac{1 - \delta}{1 + r} \Omega(w)$$

- Affine in w

$$\Omega(w) = (z - w) \frac{1 + r}{r + \delta}$$

- Value of posting a vacancy

$$\psi^f[\theta(w)] \Omega(w)$$

- Free entry condition requires

$$\bar{c} = \psi^f[\theta(w)] \Omega(w), \quad \forall w \text{ that are offered}$$

EQUILIBRIUM

- A stationary equilibrium is: $\{V^e, V^u, \Omega, a^e, a^u, w^u, \theta\}$, an interest rate r , and a stationary distribution x over (a, w) , s.t.
 1. $\{V^e, V^u, a^e, a^u, w^u\}$ solve households' problems, $\{\Omega\}$ solves the firm's problem.
 2. Zero profit condition holds for active markets

$$\bar{c} = \psi^f[\theta(w)] \quad \Omega(w), \quad \forall w \text{ that are offered/}$$

3. An interest rate r clears the asset market

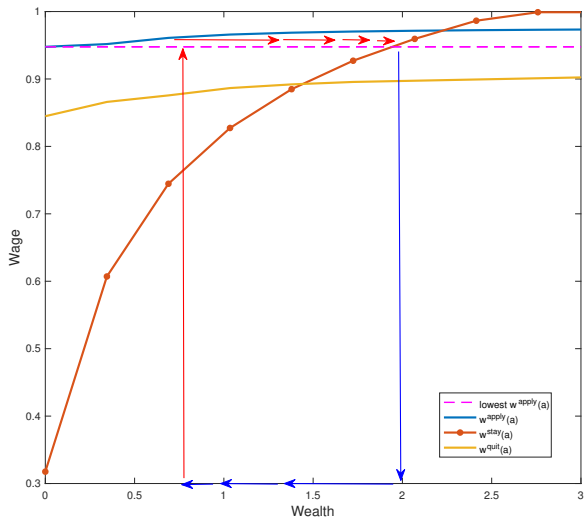
$$\int a \, dx = \int \Omega(w) \, dx.$$

- The F.O.C for wage applicants

$$\psi^h(w) V_w^e(a', w) = \psi_w^h(w) [V^u(a') - V^e(a', w)]$$

- Households with more wealth are able to insure better against unemployment risk.
- As a result they apply for higher wage jobs and we have dispersion
- A form of “Precautionary job search”.

HOW DOES THE MODEL WORK



LOOK AT A STANDARD ECONOMY:

- CRRA Utility Function

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad \sigma = 2$$

- Period is a quarter $\beta = .99$
- Average job duration: 5 years ($\delta = 0.05$)
- Home production: 30% of market production ($b = 0.3z$) (low end)
- Vacancy Posting Cost: 50% of period job output (large) ($\bar{c} = 0.5z$).
Firms are valuable
- Cobb-Douglas Matching Function

$$M(u, v) = \chi u^\eta v^{1-\eta}$$

	σ	β	χ	η	δ	z	b	\bar{c}
Value	2	0.99	0.675	0.72	0.05	1	0.3	0.5

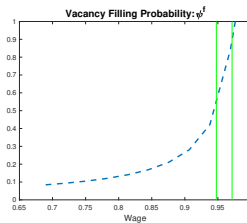
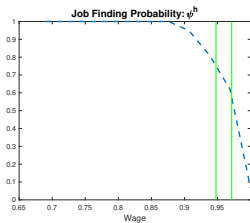
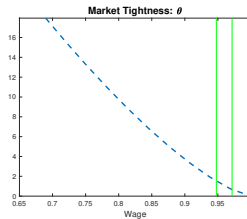
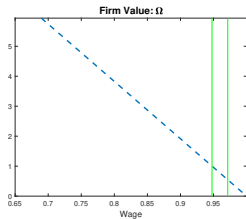
KEY MODEL STATISTICS: BENCHMARK

	Notation	Benchmark
Interest Rate	r	0.24%
Unemployment Rate	u	7.18%
Unemployment Duration	$\chi\theta^{\eta-1}$	1.54
Employment Duration	$\frac{1}{\delta}$	20
Wage Mean-min Ratio	$\frac{\bar{w}}{w^{min}}$	1.0165
Wage Max-min Ratio	$\frac{w^{max}}{w^{min}}$	1.0255

GRAPH

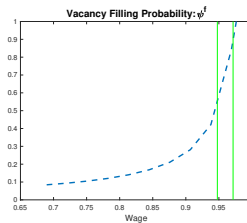
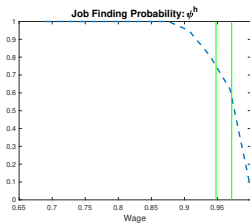
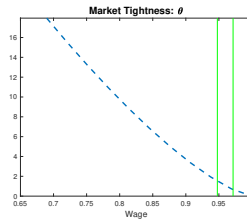
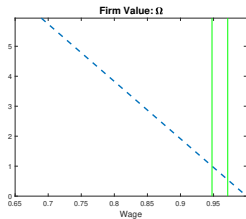
- Small total wealth (no surprise)
- Very small wealth dispersion (only job losing)

FIRMS VALUE FUNCTION



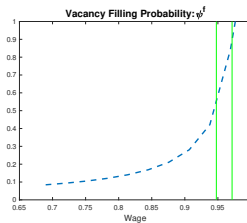
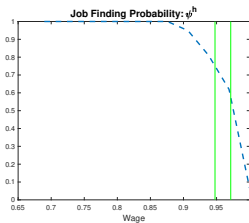
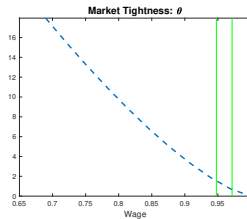
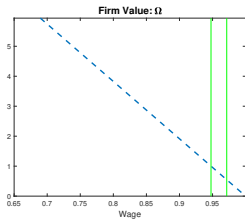
- Affine in wages

JOB FILLING PROBABILITIES



- A large equilibrium probability variation (0.6–0.9) for a narrow range

JOB FINDING PROBABILITIES



- With small differences in wealth

- We have a standard Aiyagari model plus a competitive labor search.
- Precautionary job search motive causes richer people to apply for higher wage jobs.
- Quantitatively wage dispersion due to this is small.
- Firms do not have enough rewards to pay different wages. Model 2 attempts to fix this.
- Workers also need to be able to coexist with higher wage dispersion. Model 3 works towards this.

MODEL 2: ADD INCENTIVE TO QUIT TO GET A FLATTER $\Omega(w)$

- Suppose at the beginning of the period employed workers receive a pair of i.i.d shocks $\{\epsilon^e, \epsilon^u\}$ depending on quitting decisions.
- Value of the employed right before receiving the shocks:

$$\hat{V}^e(a, w) = \int \max\{V^e(a, w) + \epsilon^e, V^u(a) + \epsilon^u\} dF^\epsilon$$

V^e and V^u are values after quitting decision as described before.

- If the shocks follow Type-I Extreme Value distribution, then the ex-ante quitting probability $q(a, w)$ is

$$q(a, w) = \frac{1}{1 + e^{\alpha[V^e(a, w) - V^u(a)]}}$$

- Hence higher wages imply longer job durations

MODEL 2: VALUE OF THE FIRM

- Probability of retaining a worker with tenure j at wage w is $\ell^j(w)$.
(One to one mapping between wealth and tenure)
- The firm's value

$$\Omega^j(w) = \ell^j(w) \left[z - w + \frac{1 - \delta}{1 + r} \Omega^{j+1}(w) \right]$$

- Solving forward

$$\Omega^0(w) = (z - w) \sum_{\tau=0}^{\infty} \left[\left(\frac{1 - \delta}{1 + r} \right)^{\tau} \prod_{i=0}^{\tau} \ell^i(w) \right] = (z - w) Q(w)$$

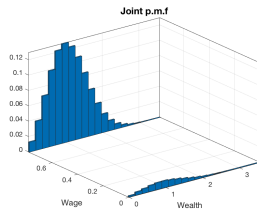
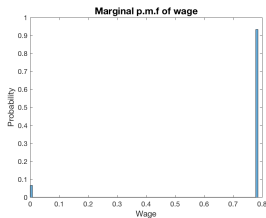
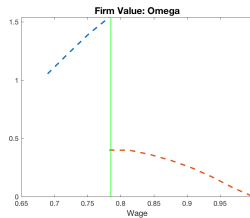
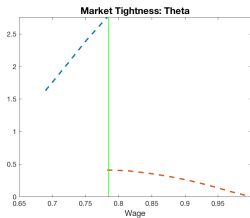
- Only equilibrium object relevant for the firm is $Q(w)$. Rest is unchanged.

- Value of firm increasing in wage!!!

- Value of firm increasing in wage!!!

- In Equilibrium wage dispersion COLLAPSES due to selection

TIGHTNESS, FIRM VALUE AND DISTRIBUTIONS



- Two forces shape the wage dispersion
 - People quit less at higher paid jobs, which enlarge the spectrum of wages that firms are willing to pay.
 - However, by paying higher wages, firms attract workers with more wealth.
 - Wealthy people quit more often, shrink the employment duration.
 - In the equilibrium, the wage gaps is narrow and the effect of wealth dominates.

MODEL 3: KILL THE TIGHT RELATION BETWEEN WAGES AND WEALTH

- We introduce another dose of extreme value shocks to different job matches.
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