

Course in Heterogeneity: Econ 081

III: OLG Models with an Application to Inter-generational Redistribution in the Great Recession

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Based on joint work with Andrew Glover, Jonathan Heathcote and Dirk Krueger

1 Introduction



- OLG means there is always entering new agents.
- Which requires for good record keeping that some exit: death.
- Two ways of doing it
 - Every period agents get one period older: $i = i + 1$. Standard.
 - Sometimes convenience dictates that agents age stochastically or just die (Yaari (1965), Blanchard (1985)):

$$i' = \begin{cases} i + 1 & \text{with probability } \lambda \\ i & \text{with probability } (1 - \lambda) \end{cases}$$

- We will look at the first. The second is like the BIHA economies.
- We will look at the first. The second is like the BIHA economies.



- Agents live for I periods and then they die
- Care about $\sum_{i=1}^I \beta_i u(c_i, \ell_i)$
- They have a life cycle endowment of efficiency units of labor. $\{\epsilon_i\}_{i=1}^I$ and one unit of time per period.
- Technology

$$Y = z K^\theta L^{1-\theta}$$

- Aggregate Labor: $L = \sum_i \epsilon (1 - \ell_i)$
- Aggregate Capital: $K = \sum_i a_i$



- State space (i, a, A) , where a is individual wealth held by the household.

$$\begin{aligned}v_i(a, A) &= \max_{c \geq 0, y, \lambda, a'} \{u(c) + \beta_{i+1} v_{i+1}(a', A')\} \\c + a' &= (1 - \ell_i) \varepsilon_i(A) + a R(A) \\A' &= G(A) \\a_1 &= 0.\end{aligned}$$

- Policy functions $c_i(a, A), a'_i(a, A), \ell_i(a, A)$



- Factor markets

$$\begin{aligned}w(A) &= (1 - \theta)z K(A)^\theta L(A)^{-\theta} \\ R(A) &= \theta z K(A)^{\theta-1} L(A)^{1-\theta}\end{aligned}$$

- Aggregation

$$\begin{aligned}K(A) &= \sum_{i=1}^I A_i, \\ L(A) &= \sum_{i=1}^I \epsilon_i [1 - \ell_i(A_i, A)].\end{aligned}$$

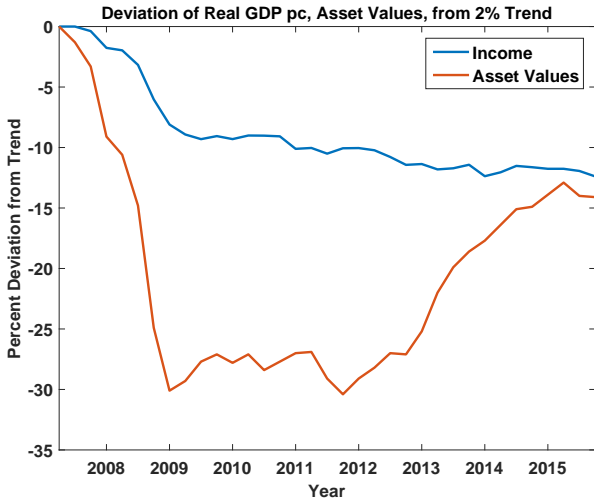
- Law of Motion: $A'_1 = 0$ and

$$A_{i+1}' = G_{i+1}(A,) = a'_i(A_i, A).$$

2 Great Recession and Assets Redistribution



- Salient features of the great recession:
 - Large fall in output and labor incomes.
 - Larger fall in asset prices (stocks, houses).
- Research Question: What are the distributional consequences for households at different stages of the life cycle?

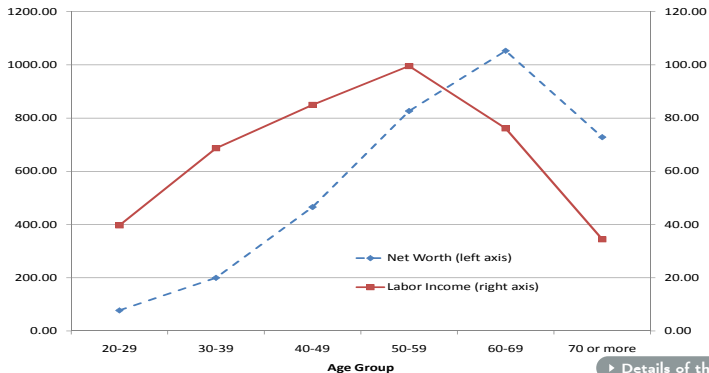




- Why focus on age dimension?
 - Labor income and wealth vary substantially by age.
 - Portfolio composition (risky versus riskless assets) varies substantially by age.
 - Labor income losses in great recession vary substantially by age.
- (1) - (3) \implies Wealth and welfare losses vary substantially by age.



Figure: Labor Income and Net Worth by Age, SCF 2007 (\$1,000)



► Details of the Data

MOTIVATING FACTS: PORTFOLIO SHARES BY AGE FROM 2007 SC



Age Head	(1) Stk	(2) Res. RE	(3) Non bus.	(4) Non RE	(5) Risky NW	(6) Bond +CD	(7) Car	(8) Oth.	(9) Debt	(10) Safe NW
All	30.3	47.0	12.9	3.8	94.0	17.0	3.5	4.2	-18.6	6.0
20-29	13.2	77.7	43.3	1.3	135.5	13.7	15.3	4.5	-68.9	-35.5
30-39	26.3	96.5	12.7	5.0	140.4	13.8	9.7	4.2	-68.2	-40.4
40-49	30.4	57.6	12.6	3.8	104.4	15.2	4.4	4.5	-28.5	-4.4
50-59	32.7	42.4	13.5	3.7	92.4	17.0	2.8	4.0	-16.1	7.7
60-69	32.2	35.6	13.4	4.1	85.3	17.5	2.4	4.7	-9.9	14.7
70+	27.1	39.8	9.0	3.3	79.2	19.3	1.8	3.7	-3.9	20.8

Risky Net Worth (5) is equal to sum of columns (1)+(2)+(3)+(4). Safe Net Worth (10) is sum of columns (6)+(7)+(8)+(9). Total Net Worth is sum of (5)+(10)

MOTIVATING FACTS: CAPITAL LOSSES BY AGE GROUP



Infl. adj. capital losses from 2007:2 to 2009:1-2013:4 (\$1,000, 2007)								
Age of Head	Stocks	Res. RA	Nonc. bus.	Nonres. prop.	Total	(%)net worth	(%) inc.	Total/2009Q1
All	30.6	64.4	15.1	6.5	116.5	21.0	139.6	154.5
20-29	1.9	14.8	7.1	0.3	24.0	31.1	61.9	24.5
30-39	9.5	47.5	5.4	3.0	65.4	32.8	93.7	73.0
40-49	25.7	66.1	12.3	5.4	109.6	23.5	117.3	139.8
50-59	49.1	86.4	23.6	9.4	168.5	20.4	142.8	232.3
60-69	61.5	92.4	29.8	13.3	197.0	18.7	180.6	278.9
70+	35.9	71.4	13.8	7.4	128.5	17.6	223.2	173.9

- Capital losses concentrated among older households



	(%)
pc earnings	-9.8
20-29	-14.3
30-39	-12.6
40-49	-10.3
50-59	-11.1
60-69	-6.0
70+	-1.4

- Current earnings losses concentrated among younger households



- Why focus on age dimension?
 - Labor income and wealth vary substantially by age.
 - Portfolio composition (risky versus riskless assets) varies substantially by age.
 - Labor income losses in great recession vary substantially by age.
- (1) - (3) \implies Wealth and welfare losses vary substantially by age.



- The Approach
 - Construct and compute a quantitative OLG model with aggregate risk.
 - Calibrate it to life cycle facts from 2007 SCF.
 - Engineer a great recession.
- Questions:
 - Can model generate magnitude of asset price declines as observed in the data?
 - Can the model generate realistic age profile of asset portfolios?
 - How are wealth and welfare losses from great recession distributed across different age cohorts?

3 Model



- Labor income and asset prices driven by **aggregate shock** $z \in Z = \{z_n, z_r, z_d\}$.
- z follows Markov process with transition matrix $\Gamma_{z,z'}$.
- **Technology**

$$Y(z) = zK^\theta L^{1-\theta} = z$$

- Supply of fixed factor (land, capital) normalized to $K = 1$. Labor income (wages) equals $w(z) = (1 - \theta)z$. Capital income equals θz .
- **Households** live for I periods. Supply one unit of time, relative labor efficiency (income) $\{\varepsilon_i(z)\}_{i=1}^I$. Normalize $\sum_i \varepsilon_i(z) = L = 1$.
- Time discount factors $\{\beta_i\}_{i=1}^I$ vary with age. Utility function $u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$. Wealth distribution $A = \{A_i\}_{i=1}^I$. No bequests.
- **Market Structure**: Ownership shares of K traded at price $p(z, A)$. Exogenous net supply B of corporate bonds, price $q(z, A)$.



- Can re-interpret the model as explicit model of housing. Assume:
 - Fixed supply 1 of perfectly divisible houses. Competitive rental markets.
 - Cobb Douglas utility over non-durables, housing services $\frac{(c^\nu s^{1-\nu})^{1-\sigma}}{1-\sigma}$
 - Households can freely invest in three assets: bonds, stocks, houses.
- Results: rents are proportional to dividends, housing prices proportional to stock prices.
- Thus model with housing has exactly the same asset pricing and welfare implications as our model without explicit housing.



- In recession labor incomes fall because real wages $w(z) = (1 - \theta)z$ fall, whereas hours worked $L = 1$ remain constant.
- Could equivalently assume that labor income in recession falls due to reduction in hours worked $L(z)$:

$$Y(z) = L(z)^{1-\theta}$$

- As long as $L(z_r)/L(z_n) = (z_r/z_n)^{\frac{1}{1-\theta}}$ model with TFP shocks z and model with aggregate shocks to hours worked $L(z)$ (or aggregate shocks to unemployment) are isomorphic.



- Exogenous net supply B of corporate bonds. Unit supply of shares.
- Aggregate state of the economy (z, A) , where $A = (A_1, \dots, A_I)$ denotes the beginning of period wealth distribution across age cohorts.
- Stock price $p(z, A)$, bond price $q(z, A)$.
- Stocks pay dividends $d(z, A) = \theta z - [1 - q(z, A)] B$
- Aggregate (start of period) wealth: $W(z, A) = p(z, A) + d(z, A) + B$



- State space (i, a, z, A) , where a is the individual share of total wealth $W(z, A)$ held by the household.

$$\begin{aligned}
 v_i(a, z, A) &= \max_{c \geq 0, y, \lambda, a'} \left\{ u(c) + \beta_{i+1} \sum_{z' \in Z} \Gamma_{z, z'} v_{i+1}(a', z', A') \right\} \\
 c + y &= \varepsilon_i(z)w(z) + W(z, A)a \\
 a'W(z', A') &= \left(\lambda \frac{p(z', A') + d(z', A')}{p(z, A)} + (1 - \lambda) \frac{1}{q(z, A)} \right) y \\
 A' &= G(z, A, z')
 \end{aligned}$$

- Policy functions $c_i(a, z, A)$, $y_i(a, z, A)$, $\lambda_i(a, z, A)$ and $a'_i(a, z, A, z')$.



- Labor market: wages $w(z) = (1 - \theta)z$ and $\sum_{i=1}^I \varepsilon_i(z) = L = 1$.
- Financial Markets: Share prices $p(z, S)$ and bond prices $q(z, A)$

$$\sum_{i=1}^I y_i(A_i, z, A) \lambda_i(A_i, z, A) = p(z, A)$$

$$\sum_{i=1}^I y_i(A_i, z, A) [1 - \lambda_i(A_i, z, A)] = q(z, A)B$$

- Law of Motion: $A'_1 = 0$ and $A'_{i+1} = G_{i+1}(z, A, z') = a'_i(A_i, z, A, z')$.

▶ Back to Model



- States $z \in Z = \{z_n, z_r, z_d\}$. Normal times $z_n = 1$, Great Recession $z_r < 1$, Great Depression $z_d < z_r$.
 - Set z_r s.t. transition from z_n to z_r involves output decline of **9.84%** (average 2009-2013 deviation from 2% growth trend).
 - Set z_d s. t. output in z_d is **28.9%** below z_n , (average 1932-1936 deviation from trend).
- Transition matrix Γ
 - Impose (perhaps arbitrary) restrictions $\Gamma_{n,d} = \Gamma_{r,r} = \Gamma_{d,r} = 0$. Note: makes markets sequentially complete with two assets.
 - Choose $\Gamma_{n,r}, \Gamma_{r,d}$ such that unconditional probability of Great Recession is **13.7%** and Great Depression is **2.84%** (as estimated from Maddison data, 1800-2010.)

$$z = \begin{pmatrix} 1.0000 \\ 0.9016 \\ 0.7109 \end{pmatrix}, \Gamma_{z,z'} = \begin{pmatrix} 0.835 & 0.165 & 0.000 \\ z & 0.793 & 0.000 & 0.207 \\ 1.000 & 0.000 & 0.000 \end{pmatrix}$$

z'

4 Example



- Key assumptions:
 - Households only productive when young: $\varepsilon_1 = 1, \varepsilon_2 = \varepsilon_3 = 0$.
 - Households derive no utility from consumption when young. By construction young save everything.
 - Only stocks are traded: $B = 0$.
 - Aggregate shock can only take two values: $Z = \{z_r, z_n\}$.
- State (z, A) where $A = A_3$ is share of assets held by old. Share of wealth held by middle-aged is $1 - A$.
- Only middle-aged make meaningful decision: how many of their shares to sell.
- Note: wealth distribution irrelevant in Rep. Agent model or 2 period OLG model.



- Measure of asset price collapse:

$$\xi(A) = \frac{\log(p(z_r, A)/p(z_n, A))}{\log(z_r/z_n)}$$

Note: in RA economy with $CRRA = \sigma$, iid z shocks: $\xi^{RA} = \sigma$.

- Choice of middle-aged: purchase shares $A' = G(z, A)$, at $p(z, A)$
- Consumption when middle aged and old:

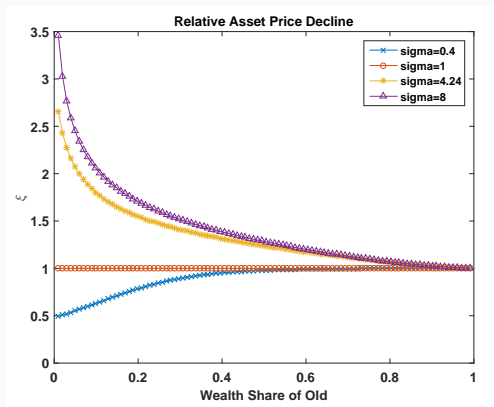
$$\begin{aligned} c_m(z, A) &= (1 - A)(p(z, A) + \theta z) - G(z, A)p(z, A) \\ c_o(z, A; z', A') &= G(z, A)p(z', A') \end{aligned}$$

- Euler equation

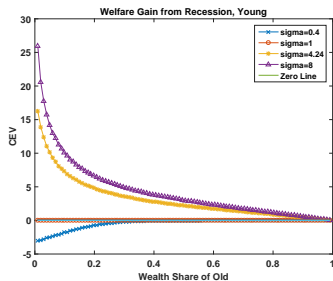
$$\begin{aligned} & u' [(1 - A)(p(z, A) + \theta z) - G(z, A)p(z, A)] \\ &= \beta \sum_{z'} \Gamma_{z, z'} \frac{[p(z', A') + \theta z']}{p(z, A)} u' [G(z, A)p(z', A')] \end{aligned}$$



- Solution is pair of functional equations in the unknown functions $p(z, A)$, $G(z, A)$.
 - Consumption, welfare can be calculated from $p(z, A)$, $G(z, A)$.
 - Note: for log-utility complete analytical characterization of RCE:
 - Asset prices are proportional to output z , that is $\xi = 1$.
 - Wealth distribution $(1 - A, A)$ does not respond to shock z .
 - Consumption of all generations move one for one with z .
 - If z is iid, then young are exactly indifferent between being born into a Great Recession and being born into normal times.
- ▶ More on the Log-Case
- Now: display (numerical) solution for $\sigma \neq 1$. Other parameters consistent with calibration of full model (e.g. income falls 9.84%)



- The more households dislike consumption fluctuations (the higher σ) the larger is the fall in p relative to z in the recession.
- When $IES = 1/\sigma < 1$ a larger wealth share of the middle-aged (smaller A) translates into greater asset price collapse $\xi(A)$.



- Welfare measured as % consumption equivalent variation (positive numbers indicate welfare gains from recession).
- Welfare consequences mirror the elasticity of asset prices to output. Young can easily win from Great Recession. But in the simple model:
 - Young do not value consumption in Great Recession.
 - Young not disproportionately affected by labor income declines.
 - Middle-aged (and old) only have access to risky assets.



Let $\sigma = 1$ and $\varepsilon_i(z) = \varepsilon_i \forall z$. Then there exists a recursive competitive equilibrium such that

- The distribution of wealth $A = \bar{A} = (\bar{A}_1, \dots, \bar{A}_l)$ is constant over time: $\forall z, z', i = 1, \dots, l - 1$

$$\begin{aligned}G_{i+1}(z, \bar{A}, z') &= a'_i(z, \bar{A}, z', \bar{A}_i) = \bar{A}_{i+1} \\G_1(z, \bar{A}, z') &= 0 \quad \forall z, z'\end{aligned}$$

- Aggregate wealth is proportional to the aggregate shock: $\forall z$

$$\rho(z, \bar{A}) + q(z, \bar{A})B = z\Psi$$

- Asset Portfolios are identical across age groups:

$$\lambda_i(z, \bar{A}, \bar{A}_i) = \lambda(z) = \frac{\rho(z)}{z\Psi} \quad \forall z, \forall i = 1, \dots, l - 1.$$

- Consumption and savings at each age are given by:

$$\begin{aligned}c_i(z, \bar{A}, \bar{A}_i) &= z \left[(1 - \theta)\varepsilon_i + \theta\bar{A}_i + (\bar{A}_i - \bar{A}_{i+1}) \Psi \right], \\y_i(z, \bar{A}, \bar{A}_i) &= z\bar{A}_{i+1}\Psi \quad \forall z, \forall i = 1, \dots, l - 1.\end{aligned}$$



Let $\sigma = 1$ and $\varepsilon_j(z) = \varepsilon_j \forall z$. Then there exists a recursive competitive equilibrium with the following properties:

- Stock and bond prices are given by

$$\begin{aligned} p(z, \bar{A}) &= p(z) = z^\Psi - B \frac{z}{R} \sum_{z' \in Z} \Gamma_{z, z'} \frac{1}{z'} \\ q(z, \bar{A}) &= q(z) = \frac{z}{R} \sum_{z' \in Z} \Gamma_{z, z'} \frac{1}{z'} \quad \forall z. \end{aligned}$$

where $R = (\Psi + \theta)/\Psi$.

- The equity premium is given by

$$R \sum_z \frac{\Pi_z}{z} \left\{ \frac{\left(\sum_{z' \in Z} \Gamma_{z, z'} z' - \left(\sum_{z' \in Z} \Gamma_{z, z'} \frac{1}{z'} \right)^{-1} \right)}{1 - \frac{B}{R\Psi} \sum_{z' \in Z} \Gamma_{z, z'} \frac{1}{z'}} \right\}$$

▶ back



If z is iid then for all $z \in Z$

$$\begin{aligned} p(z) &= z \left(\Psi - \frac{B}{R} \sum_{z' \in Z} \Pi_{z'} \frac{\mathbf{1}}{z'} \right) \\ q(z) &= z \left(\frac{\mathbf{1}}{R} \sum_{z' \in Z} \Pi_{z'} \frac{\mathbf{1}}{z'} \right) \end{aligned}$$

and the average equity premium is given by

$$R \frac{\left(\sum_z \frac{\Pi_z}{z} \sum_z \Pi_z z - \mathbf{1} \right)}{\left(\mathbf{1} - \frac{B}{R\Psi} \sum_z \frac{\Pi_z}{z} \right)}$$

In the limit as $\Gamma_{z,z} \rightarrow \mathbf{1} \forall z$ (perfectly persistent shocks), $q(z) \rightarrow R^{-1}$ and $p(z) \rightarrow z\Psi - BR^{-1}$.

▶ back

5 Calibration



- Model period 10 years. Agents enter at age 20, live for 6 periods.
- Aggregate endowment process $z \in Z = \{z_n, z_r, z_d\}$, $\Gamma_{z,z'}$ derived directly from aggregate time series data. In Great Recession (z_r) output falls 9.84%.
- Life cycle profiles $\{\beta_i, \varepsilon_i(z)\}$ chosen so that model with $z = z_n$ matches life cycle earnings and net worth profiles from 2007 SCF.
- Choose $(\theta = 30\%, B = 0.07)$ s.t. model matches 2007 SCF aggregate wealth to earnings ratio (7.88), share of risky assets (91.8%).
- Choose $\sigma = 4.24$ s.t. model ξ lines up with Great Recession $\xi = \Delta W / \Delta z = 26.8\% / 9.84\% = 2.7$. Why need low IES $1/\sigma$?



- States $z \in Z = \{z_n, z_r, z_d\}$. Normal times $z_n = 1$, Great Recession $z_r < 1$, Great Depression $z_d < z_r$.
 - Set z_r s.t. transition from z_n to z_r involves output decline of **9.84%** (average 2009-2013 deviation from 2% growth trend).
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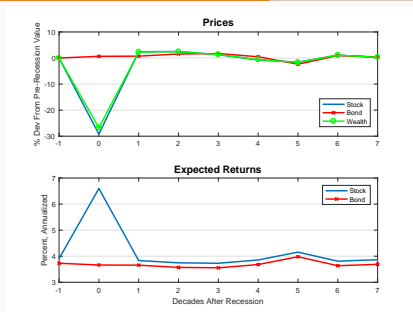
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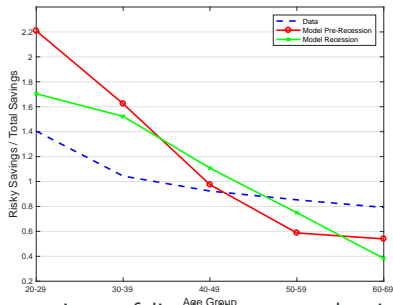


- Estimate age-specific earnings declines (relative to aggregate trend) from 2007 to 2010 using CPS data to obtain $\{\varepsilon_i(z_r)\}_{i=1}^I$.

	(%)
20-29	-14.3
30-39	-12.6
40-49	-10.3
50-59	-11.1
60-69	-6.0
70+	-1.4
Average	-9.8



- Thought experiment: Following long period of normal times, Great Recession for 10 years with $\Delta z = 9.8\%$, then recovery.
- p falls by 29.2% ($\sigma > 1$ is key), price of bonds q barely moves.
 - Positive expected consumption growth (q should fall)
 - But: Increase in income risk \implies precautionary savings up. Keeps q from falling, risk free rate from rising (as in actual Great Recession).



- Share of risky assets in portfolio declines strongly with age. Why?
 - Markets sequentially complete \implies All households *born prior to recession* share recession consumption risk perfectly.
 - For same risk exposure, young require more leveraged portfolios.
- Portfolio age profile flattens in model Great Recession: Fear of Great Depression curbs appetite of young for risky assets in Great Recession.
- Endogenous portfolio shares depend too strongly on age. Will consider model with exogenous (factual) portfolios.



- Welfare measured as percentage change in consumption (in all future dates, states) under no-recession scenario needed to make households indifferent between current state being z_n and z_r .

Age	Δ Welf.
20-29	-1.07%
30-39	-4.78%
40-49	-5.69%
50-59	-7.48%
60-69	-9.61%
70+	-10.00%

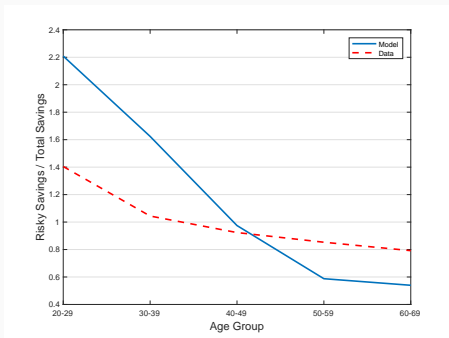


- Immediate age-specific consumption response to recession symmetric (-10%) across generations alive prior to recession.
- Newborns see smaller consumption drop (relative to no recession (-7.0%) percent. Permanent consumption advantage in future.



- Three welfare impacts from Great Recession in baseline model:
 1. Reduced PDV of future labor earnings
 2. Reduced value of asset portfolio on impact
 3. Gains from future asset price recovery
- Now: Partial equilibrium with *constant* q 's. Goal: isolate effect 3.
 - Counterfactual A: Hold wealth distribution constant at onset of recession. Only effect 1.
 - Counterfactual B: Reduction in age-specific wealth implied by asset price fall. Effects 1 and 2.

Age	Benchmark	A (Eff 1.)	B (Eff. 1. & 2.)
20-29	-1.07	-6.53	-6.53
30-39	-4.78	-7.19	-14.03
40-49	-5.69	-6.90	-17.40
50-59	-7.48	-6.55	-16.33
60-69	-9.61	-3.38	-11.27
70+	-10.00	-1.88	-10.00



- Now households are forced to hold empirical portfolios (from 2007 SCF). Still make consumption-savings decisions.
- Key plus: more realistic capital losses in Great Recession
- Key minus: Asset price movements do not reflect time-varying appetite for taking on aggregate risk.



- Elasticity of Asset Prices to Output. Key: bond prices fall a lot too (big increase in risk-free rate in recession).

Asset	Endog.	Exog.
Wealth	2.72	2.02
Stock	2.97	2.08
Bond	-0.07	1.31

- Welfare? More significant welfare losses of very young, very old.

Age	Endog.	Exog.
20-29	-1.07%	-2.39%
30-39	-4.78%	-2.91%
40-49	-5.69%	-2.54%
50-59	-7.48%	-7.30%
60-69	-9.61%	-13.73%
70+	-10.00%	-11.37%

WELFARE LOSSES FROM RECESSION BY AGE: SYMMETRIC EARN LOSSES



- Given asset pricing channel, why do the young actually lose?
- Answer: because they are especially hard-hit by the Great Recession in the labor market.

Age	Bench.	Sym. Δ Earn.
20-29	-1.07%	0.32%
30-39	-4.78%	-5.04%
40-49	-5.69%	-5.90%
50-59	-7.48%	-7.64%
60-69	-9.61%	-9.74%
70+	-10.00%	-10.09%

IMPLICATIONS FOR THE DYNAMICS OF THE WEALTH DISTRIBUTION MODEL VS. DATA



Age	Endog. Portfolios			Exog. Portfolios			Data: NW, SCF		
	2007	2010	2013	2007	2010	2010	2007	2010	2013
20-29	0.00	0.00	0.00	0.00	0.00	0.00	2.30	1.27	1.50
30-39	6.29	4.20	7.74	6.25	5.67	5.75	5.95	4.20	6.05
40-49	14.73	11.98	14.61	14.42	14.06	13.35	13.94	13.97	14.25
50-59	25.59	25.20	25.23	25.31	25.28	24.90	24.70	24.52	22.92
60-69	31.76	34.71	31.21	32.03	32.44	31.84	31.45	32.66	30.53
70+	21.62	23.91	21.21	21.99	22.55	24.16	21.67	23.38	24.74

- Wealth share of young cohort (30-39) declines in Great Recession, then rebounds. Both in model and in data.
- Wealth Share of retiring cohort (60-69) increases in Great Recession, then returns to normal. Both in model and in data.



- So far aggregate output z mean reverting, thus in a great recession output and asset prices are expected to recover.
- Robustness to *permanent* shocks to z ? See also Khan (2017). We explored this in a 3-generation OLG model calibrated to the same income losses.
- Three basic results
 - For given risk aversion, asset price decline comparable to model with trend-stationary output if (*and only if*) output growth over ten or twenty years is *negatively* correlated, as in U.S. data ($\text{corr} \approx -0.55$).
 - Absolute welfare losses from the great recession significantly larger in the stochastic growth economy for all (but oldest) generation.
 - *Relative* welfare losses of young vs. middle aged comparable in both economies.



- Assume the wealthy are passive investors.
- Calibrate model to bottom 90% earnings, wealth life cycle profile.
- Requires (on average) less patient individuals.
- Overall: asset price mechanism less relevant to bottom 90%.

Age Group	Economy	
	Baseline	Low Wealth
20-29	-1.07%	-5.12%
30-39	-4.78%	-6.76%
40-49	-5.69%	-7.23%
50-59	-7.48%	-8.20%
60-69	-9.61%	-9.57%
70+	-10.00%	-9.88%

6 Conclusion



- Policy implications?
 - By construction nothing can be done about the recession itself.
 - But: government can of course affect distribution of welfare losses or gains.
 - E.g. by purchasing assets at distressed prices (TARP?) government may have mitigated welfare losses of elderly at expense of welfare gains of young.
 - Same might be true for expansion of outstanding government debt.



- We have explored **asset price implications** of large recessions. Can rationalize large price drops of (only) risky assets with fear of Great Recession (and IES $1/\sigma < 1$).
- We have explored the **portfolio implications** of the model. It can account for (too much of the) relatively risky portfolios of young and relatively safe portfolios of the old in the data.
- We have explored the **redistributive implications** of such recessions. Old lose a lot, young little. Might have gained if it wasn't for the dismal labor market.
- Heterogeneity within young generation?
 - Winners not the ones that **don't much participate in financial markets**
 - ... but rather those who plan to have **large wealth-to-income ratio** in their 50's.

7 appendix



- Alternative version of the model in which savings is a choice, but in which the portfolio shares are exogenous.
- New parameters: age-varying portfolio shares $\{\lambda_i(z)\}_{i=1}^I$.
- Set equal to age-specific shares of risky assets from SCF:

Age	$\lambda_i(\%)$
20-29	135
30-39	140
40-49	104
50-59	92
60-69	85
70+	79
Aggr.	94

8 Baseline Results



- Asset Prices in a Great Recession
- Portfolio Choices
- Welfare Results
- Quantifying the Asset Price Channel
- Exploring the Sensitivity of Results
 - Exogenous (Data Implied) Portfolios
 - The Importance of Asymmetric Earnings Declines
 - Intracohort Heterogeneity

MOTIVATING FACTS: INCOME AND WEALTH OVER THE LIFE CYCLE

(2007 SCF, \$1,000)



Age	Total Income	Labor Income	Asset Income	Assets	Debts	Net Worth
All	83.43	70.07	13.36	659.00	103.34	555.66
20-29	38.83	39.68	-0.85	130.66	53.30	77.36
30-39	69.83	68.68	1.15	335.87	136.12	199.75
40-49	93.40	84.97	8.43	598.21	132.62	465.59
50-59	117.97	99.56	18.41	959.77	133.24	826.53
60-69	109.06	76.15	32.90	1156.96	104.10	1052.86
70+	57.56	34.46	23.11	756.76	28.48	728.28

▶ [Back to Plot](#)



- The young have lots of future labor income, few financial assets.
- Hurt by lower current wages, might benefit from lower asset prices.
- Welfare consequences of downturn depend on:
 - Size of labor income asset price decline
 - Its persistence
 - Behavioral response of households (consumption-savings and portfolio allocation choices).
- Thus want labor income, asset prices and household choices be *endogenously* determined in quantitative life cycle model.



- Wealth-based welfare measure invariant to remaining lifetime horizon.
- How much must *wealth* be reduced in the no-recession state for households to be indifferent between life with or without the recession in the current period?
- Normalize wealth measure by pc consumption in normal times.

Table

Age	Bench.	Sym. Δ Earn.	Exog.
20-29	-1.98%	0.60%	-3.90%
30-39	-11.20%	-11.87%	-6.30%
40-49	-15.79%	-16.38%	-6.83%
50-59	-22.83%	-23.31%	-20.39%
60-69	-25.90%	-26.24%	-35.77%
70+	-14.95%	-15.08%	-19.11%



Return Stats: Benchmark Model			
Asset	Average	Std. Dev.	Corr. w/ Stock
Stock	4.50%	31.2%	1.00
Bond	4.09%	25.3%	0.79
Return Stats: Model w/o Great Depr.			
Asset	Average	Std. Dev.	Corr. w/ Stock
Stock	4.41%	16.6%	1.00
Bond	3.68%	1.2%	-0.07
Return Stats: Data			
Asset	Average	Std. Dev.	Corr. w/ Stock
Stock	6.62%	36.4%	1.00
Bond	2.29%	30.4%	0.01

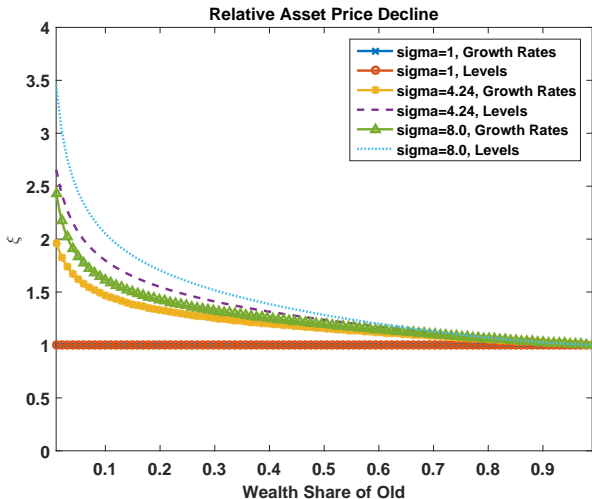
IMPLICATIONS FOR THE DYNAMICS OF THE WEALTH DISTRIBUTION MODEL VS. DATA



Age	Model End. Portf.			Model Exog. Portf.			Data		
	PreR	Rec.	Reco	PreR	Rec.	Reco	2007	2010	2013
20-29	0.0	0.0	0.0	0.0	0.0	0.0	2.3	1.3	1.5
30-39	2.6	-1.4	6.0	4.9	3.9	4.5	6.0	4.2	6.1
40-49	9.9	4.6	12.0	13.6	13.0	12.5	13.9	14.0	14.3
50-59	24.9	24.1	23.4	25.2	25.2	24.8	24.7	24.5	22.9
60-69	36.9	42.6	32.8	33.0	33.7	32.7	31.5	32.7	30.5
70+	25.6	30.1	25.8	23.3	24.2	25.6	21.7	23.4	24.7



- So far aggregate output z mean reverting, thus in a great recession output and asset prices are expected to recover.
- Robustness to permanent shocks to z ? Consider 3-period model but assume that $g' = z'/z$ follows Markov process with $\Gamma_{g,g'}$.
- Calibrate s.t. output falls 9.83% in recession.
- Three basic results
 - For given risk aversion, ξ comparable to model with trend-stationary output if (*and only if*) output. growth over ten or twenty years *negatively* correlated, as in U.S. data ($\text{corr} \approx -0.55$).
 - Absolute welfare losses from the great recession significantly larger in the stochastic growth economy (for all but oldest generation).
 - *Relative* welfare losses by age are comparable in both economies.





Age Group	Economy	
	Shocks to z	Shocks to z'/z
Old (absolute)	-12.3%	-11.4%
Middle (absolute)	-3.7%	-6.0%
Young (absolute)	2.9%	-5.0%
Middle rel.to Old	8.6%	5.4%
Young rel. to Old	15.2%	6.4%



- Are welfare losses of "average household" within an age group representative? Now consider limited intra-cohort heterogeneity.
- Two types of households: a wealthy type and a low-wealth type.
- Assume that wealthy type accounts for a fixed fraction κ_y of aggregate labor earnings, passively holds a fixed fraction κ_a of aggregate debt, equity.
- Thus the wealthy consume a fixed fraction $(1 - \theta)\kappa_y + \kappa_a\theta$ of aggregate output at each date.
- Assets are priced by the low-wealth type, and prices fluctuate such that this type always demands $(1 - \kappa_a)$ shares and $\kappa_a B$ bonds.
- In essence: recalibration of a model with lower income- and wealth households. Key difference: wealth-to-income ratio is lower among asset pricers now.
- Results fairly unchanged relative to baseline model, but asset price channel somewhat less important.



- OLG economies with aggregate risk:
 - Asset pricing: Labadie (1986), Huffman (1987), Constantinides, Donaldson and Mehra (2002), Storesletten, Telmer and Yaron (2007), Kubler and Schmedders (2015)
 - Allocations: a) Business cycles: Rios-Rull (1994, 1996), Gomes, Michaelides and Polkovnichenko (2010), b) Intergenerational risk sharing: Bohn (1998), Shiller (1999), Demange (2002), Smetters (2006), Krueger and Kubler (2006), Ball and Mankiw (2007), Miyazaki, Sato and Yamada (2009), Olovsson (2010).
- Redistributive consequences by age of other aggregate shocks:
 - Inflation: Doepke and Schneider (2006a,b), Meh, Rios-Rull and Terajima (2010).
 - Demographics: Demange and Laroque (1999), Rios-Rull (2001), Abel (2003), Attanasio, Kitao and Violante (2007), Krueger and Ludwig (2007).
- Consumption disasters: Barro (2006, 2009), Nakamura, Steinsson, Barro and Ursua (2013), Gourio (2010).

THANK YOU FOR COMING
AND LISTENING

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