

# Course in Heterogeneity: Econ 081

## VIII: Banking Dynamics and Capital Regulation in General Equilibrium

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Based on joint work with Tamon Takamura and Yaz Terajima



- There is a Rep hhold

# A GROWTH MODEL AROUND A BANKING INDUSTRY

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- Mutual Funds
  - Manage Loan firms
  - Own Equity firms
  - Open and own banks with transfer difficulties

# 1 Steady State



- Prices
  - Interest rate  $q$  for bonds: Safe
  - Interest rate  $r^\ell$  for loans: Unsafe
  - Interest rate for deposits  $q^D$  Safe because insured by Gov.
  - Wage function  $w(k, C)$  (I am using a guess and verify based on logs)
- Quantities
  - Employment, and Number of Firms/Plants  $N$
  - Capital per Plant  $K$
  - Output, Cons, Inv,  $C + \delta NK = Y = NAK^\alpha$  – Intermediate Inputs
  - Loans  $L = (1 - \lambda)NK$  **V: (Double check, but similar formula)**
  - Deposits  $D$
  - Bonds  $B$
  - Taxes, Banks Loses  $T$
- Other Elements
  - A Banking Industry with a measure of banks  $x$ , new entrants  $m^E$ , and dividends  $C^b$
  - Mutual funds that manage/own all firms



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- Some banks go bankrupt when they cannot roll over debt. Let the default set be  $M^i(A, L)$
- There is entry of new banks, ( $m^E$  is the measure of entrants), occurs as long as the free-entry condition is satisfied:

$$W^E(a^E, \ell^E) = u^b(\kappa^{Eb})$$

- $a^E, \ell^E$  is the prespecified values of new entrants.
- Function  $u^b(\cdot)$  translates units of the good into units of the objective function of banks
- $\kappa^{E,b}$  is the opening cost of a new bank.

## INDUSTRY EQUILIBRIA

- The definition is exactly like the one in the other paper. But for our purposes we need to link it with the rest of the model.
- We proceed by specifying what are inputs to the banks
- Given safe interest rate,  $1/q$ , deposit rate  $1/q^d$ , loan rate  $r^\ell$  and cost of entry  $\kappa^{Eb}$ , it yields
  - A measure of Banks over their states  $x$ , including entrants  $m^E$ , and fraction of loans in hands of failing banks  $d^B$ .
  - Total Quantity of Bonds  $B$
  - Total Quantity of Deposits  $D$
  - Total Dividends  $C^b$
  - Total Loses  $T$  to be covered by government
  - Total resources needed by new entrants  $m^E \kappa^{Eb}$



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- Output is

$$Y' = (1 - \delta_1)Y + N^n A k^\alpha.$$



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## INVESTMENT AND FIRMS: FINANCING

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- If the bank fails (prob  $d^B$ ), we assume that the loan also terminates **with prob  $\gamma$**  and the Mutual pays the government  $k(1 + r^\ell + \zeta^F)$ . **V: What happens with prob  $(1 - \lambda)$ ?**
- $d^B$  is the endogenous fraction of loans held by defaulting banks:

$$d^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell) \in D_i} \ell \, dm_i(a, \ell)}{\sum_{i=1}^{N_\xi} \int \ell \, dm_i(a, \ell)}$$



- Given capital  $k$ , the maintenance cost  $\delta_2$ , interest rate  $r^\ell$ , wage  $w(k)$ , and the repayment cost  $\zeta^F$  when banks default, the value of a loan firm is

$$\begin{aligned} \Pi^0(k, r^\ell) = & Ak^\alpha - w(k) - (r^\ell + \delta_2)k + (1 - d^B)(1 - \lambda)q(1 - \delta_1)\Pi^0(k, r^\ell) \\ & + q(1 - \delta_1) \left\{ \lambda(1 - d^B) + d^B \right\} (1 - \gamma)\Pi^0(k) \\ & + q(1 - \delta_1) \left[ \lambda(1 - d^B) + d^B \right] \gamma [-k + \Pi^1(k)] - q(1 - \delta_1)d^B\gamma\zeta^F k \end{aligned}$$

## VALUE OF FIRMS: THERE ARE MEASURES $m^0(k, r^\ell)$ AND $m^1(k)$ OF THEM

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- Letting  $R(k) = Ak^\alpha - w(k)$ ,  $\Pi^0 < \Pi^1$  due to loan repayment costs:

$$\begin{aligned}\Pi^1(k) &= \frac{R(k) - \delta_2 k}{1 - q(1 - \delta_1)} \\ \Pi^0(k, r^\ell) &= \frac{R(k) - \delta_2 k}{1 - q(1 - \delta_1)} - \frac{r^\ell + q(1 - \delta_1)\gamma [ \lambda(1 - d^B) + d^B + d^B\zeta^f ]}{1 - q(1 - \delta_1) [ 1 - \gamma \{ \lambda(1 - d^B) + d^B \} ]} k\end{aligned}$$

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$$k^* = \left\{ \frac{(1 - \mu)\alpha A}{\frac{r^\ell + q(1 - \delta_1)\gamma[\lambda(1 - d^B) + d^B + d^B \zeta^f][1 - q(1 - \delta_1)]}{1 - q(1 - \delta_1)[1 - \gamma\{\lambda(1 - d^B) + d^B\}] + \delta_2}} \right\}^{\frac{1}{1 - \alpha}}$$

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- Firms enter until profits are zero:

$$\kappa^{E,f} = q \Pi^0(k^*; r^\ell)$$



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  1. Wage equation

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  4. Feasibility:  $Y = A N k^\alpha = C + I + \text{costs of starting firms and operating banks}$
  5.  $I = (\delta_1 + \delta_2)kN$



## MUTUAL FUNDS

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- Households own Mutual Funds which in turn own firms and banks, but do not trade its shares, just passively receive its dividends.

# MUTUAL FUNDS

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- Mutual Funds manage Loan-firms and own Equity Firms:

$$\begin{aligned} \pi^f &= Y - \mu Y - (1 - \mu) \bar{b} N - r^\ell K^0 \\ &\quad - (1 - d^B) \lambda K^0 - d^B (1 + \zeta^F) K^0 - \kappa^{E,f} N^n \\ &= \int_{k,r^\ell} [R^0(k, r^\ell) - k r^\ell - (1 - d^B) \lambda k - d^B (1 + \zeta^F) k] dm^0(k, r^\ell) \\ &\quad + \int_k R^1(k) dm^1(k) - \kappa^{E,f} N^n \end{aligned}$$

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  3. Cash Flow net of Interest from Loan firms (not zero because of fixed costs)
  4. Loan Repayment
  5. Profits from Equity Firms

# WAGE DETERMINATION

- A bargaining process between the firm and the worker.  $V$ : (We may change this to get more wage rigidity and avoid the Shymer puzzle)
- The bargaining process is repeated every period and if unsuccessful neither firm nor worker can partner with anybody else within a period. We assume that the financial obligations to the bank by the firm do not disappear. Let  $\mu$  be the bargaining weight of the worker and  $\bar{b}$  is workers' outside option. Then, we have

$$w^0(k) = w^1(k) = \mu A k^\alpha + (1 - \mu)\bar{b}$$

- Total (per capita) Labor Income paid in the Economy are

$$W N = N \int [\mu A k^\alpha + (1 - \mu)\bar{b}] di = \mu Y + (1 - \mu)\bar{b}N$$

$$v(a) = \max_{c, b', d'} u(c, d') + \beta v(a') \quad \text{s.t.}$$

$$\begin{aligned} c + q^d d' + q b' &= a + W N + (1 - N) \bar{b} + \pi^f + \pi^B - T \\ a' &= d' + b' \end{aligned}$$

where  $T$  is the taxes needed to pay for bank losses. FOCs:

$$\begin{aligned} u_c &= \frac{\beta}{q} u'_c \\ u_d &= q^d u_c - \beta u'_c \end{aligned}$$

The cost of deposit insurance is the amount of deposits that defaulting banks owe minus liquidated capital.

$$T = \sum_{i=1}^{N_{\xi}} \xi^{i,d} \int_{(a,\ell) \in D} dm^i(a, \ell) - K^0 d^B (1 - \zeta^B)$$

where  $\zeta^B$  is the fraction that the government is unable to recover during the liquidation process.

# OUTPUT OF HOUSEHOLD PROBLEM

---

- Given safe interest rate,  $1/q$ , deposit rate  $1/q^d$ , Taxes  $T$ , wages  $W$ , Profits  $\Pi$ , and Bonds  $B$ , Employment  $N$  we obtain
  1. Consumption  $C$
  2. Deposits  $D$

## Deposits

$$D' = \sum_{i=1}^{N_{\xi}} \xi_i^d \int_{(a,\ell) \notin D_i} dm^{bi}(a, \ell) + \xi^{dE} m^E$$

## Bonds

$$qB' = \sum_{i=1}^{N_{\xi}} \int_{(a,\ell) \notin D_i} q^{ib}(\ell, \ell^{in}(a, \ell), b^{i'}(a, \ell)) b^{i'}(a, \ell) dm^i(a, \ell) + q^{Eb} b'^E m^E$$

# MARKET CLEARING (CONTINUED): V: HOW DOES NIPA TREAT $F$ ? INTERMEDIATE GOODS?

## New loans

$$k^* N^n + (1 - \gamma) \left\{ \lambda(1 - d^B) + d^B \right\} K^0 = \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin D_i} \ell_i^n(a, \ell) dm_i(a, \ell) + \ell_E^n m_E$$

## Goods

$$Y = C + kN^n + \delta_2 kN +$$

$$+ \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin D_i} \xi_i^n \left( \ell_i^n(a, \ell) \right) dm_i(a, \ell) + \xi_E^n \left( \ell_E^n \right) \quad (\text{Bank loan issuance costs})$$

$$+ \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin D_i} \xi_b \left( b'_i(a, \ell) \right) dm_i(a, \ell) + \xi_E^b \left( b'_E \right) \quad (\text{Bank bond issuance costs})$$

$$+ \kappa_E^b m_E + \kappa_E^f N^n \quad (\text{Entry costs})$$

$$+ d^B (\zeta^B + \zeta^F) K^0 \quad (\text{Bank default costs})$$

## STEADY STATE CONDITIONS (1)

Households:  $u(C, D, N) = \log(C) + \eta^D \log(D)$ ,

$$q = \beta \quad (1)$$

$$\frac{\eta^D C}{D} = q^d - \beta \quad (2)$$

Firms:

$$k^* = \left\{ \frac{(1 - \mu)\alpha A}{\frac{[r^\ell + q(1 - \delta_1)\gamma\{\lambda(1 - d^B) + d^B + d^B\zeta^f\}][1 - q(1 - \delta_1)]}{1 - q(1 - \delta_1)[1 - \gamma\{\lambda(1 - d^B) + d^B\}]} + \delta_2} \right\}^{\frac{1}{1 - \alpha}} \quad (3)$$

$$\kappa^{Ef} = q \Pi^0 \quad (4)$$

$$\Pi^0 = \frac{(1 - \mu)(A(k^*)^\alpha - \bar{b}) - \delta_2 k^*}{1 - q(1 - \delta_1)} - \frac{r^\ell + q(1 - \delta_1)\gamma[\lambda(1 - d^B) + d^B + d^B\zeta^f]}{1 - q(1 - \delta_1)[1 - \gamma\{\lambda(1 - d^B) + d^B\}]} k^* \quad (5)$$



## STEADY STATE CONDITIONS (2)

**Wages:**

$$w = \mu A(k^*)^\alpha + (1 - \mu)\bar{b} \quad (6)$$

**Banks:**

$$r^{E,b} = W^E(a^E, \ell^E) \quad (7)$$

$$d^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell) \in D^i} \ell \, dm^i(a, \ell)}{\sum_{i=1}^{N_\xi} \int \ell \, dm^i(a, \ell)} \quad (8)$$

# STEADY STATE CONDITIONS (3)

Market clearing conditions:

$$D = \sum_{i=1}^{N_{\xi}} \xi^{i,d} \int_{(a,\ell) \notin D_i} dm^i(a, \ell) + \xi^{E,d} m^E \quad (9)$$

$$qB = \sum_{i=1}^{N_{\xi}} \int_{(a,\ell) \notin D_i} q^{i,b} \left( \ell, \ell^{i,n}(a, \ell), b'^i(a, \ell) \right) b'^i(a, \ell) dm^i(a, \ell) + q^{E,b} b'^E m^E \quad (10)$$

$$k^* N^n + (1 - \gamma) \{ \lambda(1 - d^B) + d^B \} K^0 = \sum_{i=1}^{N_{\xi}} \int_{(a,\ell) \notin D_i} \ell^{i,n}(a, \ell) dm^i(a, \ell) + \ell^{E,n} m^E \quad (11)$$

$$\begin{aligned} Y = C + k^* N^n + \delta_2 k N + \sum_{i=1}^{N_{\xi}} \int_{(a,\ell) \notin D_i} \xi^{i,n} \left( \ell^{i,n}(a, \ell) \right) dm^i(a, \ell) + \xi^{E,n} \left( \ell^{E,n} \right) m^E \\ + \sum_{i=1}^{N_{\xi}} \int_{(a,\ell) \notin D_i} \xi^{i,b} \left( b'^i(a, \ell) \right) dm^i(a, \ell) + \xi^{E,b} \left( b'^E \right) m^E \\ + \kappa^{E,b} m^E + \kappa^{E,f} N^n + d^B (\zeta^B + \zeta^F) K^0 \quad (12) \end{aligned}$$

## STEADY STATE CONDITIONS (4)

Laws of motion:

$$Y = \frac{A(k^*)^\alpha N^n}{\delta_1} \quad (13)$$

$$N = \frac{N^n}{\delta_1} \quad (14)$$

$$K^0 = \frac{k^* N^n}{1 - (1 - \delta_1) [1 - \gamma \{ \lambda(1 - d^B) + d^B \}]} \quad (15)$$

Aggregate endogenous variables:

$$C, D, B, k^*, K^0, N^n, N, Y, d^B, m(a, \ell), m^E, q, q^d, r^\ell, w$$

Parameters:

$$\text{HHs: } \beta, \mu, \bar{b}, \eta^D$$

$$\text{Firms: } \alpha, A, \kappa^{E,f}, \zeta^F$$

$$\text{Banks: } u^b(), \beta^B, \lambda, \xi^{i,d}, \xi^{i,n}, \xi^{i,b}, F(\delta'), \kappa^{E,b}, \zeta^B$$

## ALGORITHM TO GET STEADY STATE

- Set Parameters of Banking:  $u^b(), \beta^B, \lambda, \xi^{i,d}, \xi^{i,n}, \xi^{i,b}, F(\delta')$  and prices  $r^\ell, q^d, q$ . **V: (may come back to this)**
- Compute the banking industry equilibrium. Get loans  $L$ , deposits  $D$  bank dividends  $C^b$ , losses  $T$ , resources for new entrants  $m^E \kappa^{Eb}$ .
- Set HH preference parameters  $\beta, \bar{b}, \eta_D$ , and the bargaining power  $\mu$  so that they are consistent with  $q$ , the observed consumption-to-deposit ratio and the labor share of 2/3.
- Set Technology  $A, \alpha$  as well as  $\delta_2$  and  $\zeta^F$  to solve the firms' problem. Given  $\alpha$  and  $\delta_2$ , adjust  $A$  to make sure that all markets clear.  
**V: (I think that  $\lambda$  doesn't matter much so we should set this to get the equity/debt ratio of the nonfinancial sector and a normalize)**
- Generate key moments of interest.

## SETTING THE BARGAINING PARAMETERS

- We target labor share and the outside option for workers  $\bar{b} = \phi_b w$ :

$$LS = \mu + (1 - \mu) \frac{\bar{b}N}{Y} = \mu + (1 - \mu) \frac{\phi_b w}{A(k^*)^\alpha}$$

$$w = \mu A(k^*)^\alpha + (1 - \mu) \bar{b} = \mu A(k^*)^\alpha + (1 - \mu) \phi_b w$$

- Solving the two conditions simultaneously,

$$\mu = \frac{(1 - \phi_b)LS}{1 - \phi_b LS}$$

$$w(k^*) = \frac{\mu}{1 - (1 - \mu)\phi_b} A(k^*)^\alpha$$

$$\bar{b} = \phi_b w(k^*)$$

- $LS = 2/3$  and  $\phi_b = 0.9$  imply  $\mu = 1/6$ .

- $\beta = q$  by (1)
- $N^n = \delta_1 \bar{N}$  by (14), where  $\bar{N} = 0.9$ .
- The banking industry equilibrium gives  $L^n$ : back out  $k^*$  from (11).
- Set  $A$  so that the loan demand (3) is equal to the loan supply.
- $Y = Ak^* \bar{N}$  and  $I = (\delta_1 + \delta_2)k^* N$ .
- Compute  $K^0$  from (15)
- $C$  is determined as a residual in (12)

- For simplicity, we ignore various intermediate costs for now
- Consumption-deposit ratio:

$$\begin{aligned} \frac{C}{D} &= \frac{C}{L^n} \frac{L^n}{D} = \frac{Y - I}{k^* \delta_1 \bar{N} \left[ 1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]} \right]} \frac{L^n}{D} \\ &= \frac{A(k^*)^{\alpha-1} - \delta_1 - \delta_2}{\delta_1 \left[ 1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]} \right]} \frac{L^n}{D} \\ &= \left[ \frac{1}{\frac{K}{Y} \delta_1 \left[ 1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]} \right]} - \frac{\delta_1 + \delta_2}{\delta_1 \left[ 1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]} \right]} \right] \frac{L^n}{D} \end{aligned}$$

- With  $K/Y = 3$ ,  $L^n/D = 0.9$ ,  $\delta_1 = 0.02$ ,  $\delta_2 = 0.08$ ,  $\gamma = \lambda = 0.5$ ,  $d^B = 0$ , consumption-deposit ratio is about 5.4.

## 2 Equilibrium in Terms of Sequences



## EXISTING BANK'S PROBLEM GIVEN $V_{t+1}, r_t^\ell, q_t^d, \theta_t$

$$V_t^i(a, \ell) = \max \{0, W_t^i(a, \ell)\}$$

$$W_t^i(a, \ell) = \max_{\ell^n \geq 0, c \geq 0, b'} u^b(c^b) + \beta^b \sum_i \Gamma_{i,i'} \sum_{\delta'} \left\{ \pi_t(\delta') V_{t+1}^{i'}[a'(\delta'), \ell'(\delta')] \right\} \text{ s.t.}$$

$$(TL) \quad \ell' = (1 - \lambda)(1 - \delta')\ell + (1 - \bar{\delta})\ell^n$$

$$(TA) \quad a' = (\lambda + r_t^\ell)(1 - \delta')\ell + \lambda(1 - \bar{\delta})\ell^n - \xi^{i,d} - b'$$

$$(BC) \quad c^b + \ell^n + \xi^{i,n}(\ell^n) + \xi^{i,b}(b') \leq a + q_t^{i,b}(\ell, \ell^n, b')b' + q_t^d \xi^{i,d}$$

$$(KR) \quad \frac{\ell^n + \ell - q_t^d \xi^{i,d} - q_t^{i,b}(\ell, \ell^n, b')b'}{\omega_t^r(n + \ell) + \omega_t^s \mathbf{1}_{b' < 0} b' q_t^{i,b}(\ell, \ell^n, b')} \geq \theta_t$$

$\pi_t$  is an exogenous aggregate shock.

$\theta_t$  is exogenous. A feedback rule to be considered in the next step.

- Entry condition:

$$W_t^E(a^E, \ell^E) = u^b(\kappa^{E,b}) \quad (16)$$

- A fraction of loans destroyed by bank default:

$$d_{t-1}^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell) \in M_t^i} \ell \, dm_{t-1}^i(a, \ell)}{\sum_{i=1}^{N_\xi} \int \ell \, dm_{t-1}^i(a, \ell)} \quad (17)$$

- The value is

$$\Pi_t^1(k) = A_t k^\alpha - w_t(k) - \delta_2 k + q_t(1 - \delta)\Pi_{t+1}^1(k) \quad (18)$$

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- The wage is given by

$$w_t(k) = \mu A_t k^\alpha + (1 - \mu)\bar{b} \quad (19)$$

- The value of bank-financed firm is

$$\begin{aligned} \Pi_t^0(k) = & Ak^\alpha - w(k) - (r_t^\ell + \delta_2)k + (1 - d_t^B)(1 - \lambda)q_t(1 - \delta_1)\Pi_{t+1}^0(k) \\ & + q_t(1 - \delta_1) \{ \lambda(1 - d_t^B) + d_t^B \} (1 - \gamma)\Pi_{t+1}^0(k) \\ & + q_t(1 - \delta_1) [ \lambda(1 - d_t^B) + d_t^B ] \gamma [ -k + \Pi_{t+1}^1(k) ] \\ & - q_t(1 - \delta_1)d_t^B\gamma\zeta^F k \quad (20) \end{aligned}$$

- Given  $q_t$  and  $\Pi_{t+1}^0$ , entrants choose  $k_t^*$ :

$$k_t^* = \arg \max_k \{ q_t \Pi_{t+1}^0(k) - \kappa^{E,f} \} \quad (21)$$

- Entry occurs until firms break even *ex-ante*

$$q_t \Pi_{t+1}^0(k_t^*) = \kappa^{E,f} \quad (22)$$

- Aggregate output:

$$Y_t = A_t(k_t^*)^\alpha N_t^n + (1 - \delta_1)Y_{t-1} \quad (23)$$

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- Aggregate capital:

$$K_t = k_t^* N_t^n + (1 - \delta_1)K_{t-1} \quad (25)$$

- Aggregate capital held by bank-financed firms:

$$\begin{aligned} K_t^0 &= k_t^* N_t^n \\ &+ \left[ (1 - d_{t-1}^B)(1 - \lambda) + (1 - \gamma) \left\{ \lambda(1 - d_{t-1}^B) + d_{t-1}^B \right\} \right] (1 - \delta_1)K_{t-1}^0 \end{aligned} \quad (26)$$

- Consumption Euler equation:

$$u_{c,t} = \beta \frac{u_{c,t+1}}{q_t} \quad (27)$$

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- Consumption-deposit marginal condition:

$$u_{d,t} = q_t^d u_{c,t} - \beta u_{c,t+1} \quad (28)$$

# MARKET CLEARING CONDITIONS

$$D_t = \sum_{i=1}^{N_\xi} \xi^{i,d} \int_{(a,\ell) \notin M_{t-1}^i} dm_{t-1}^i(a, \ell) + \xi^{E,d} m_t^E \quad (29)$$

$$\begin{aligned} k_t^* N_t^n + (1 - \gamma) \{ \lambda(1 - d_{t-1}^B) + d_{t-1}^B \} K_{t-1}^0 \\ = \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin M_t^i} \ell_t^{i,n}(a, \ell) dm_{t-1}^i(a, \ell) + \ell_t^{E,n} m_t^E \end{aligned} \quad (30)$$

$$\begin{aligned} Y_t = C_t + k_t^* N_t^n + \delta_2 K_{t-1} \\ + \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin M_t^i} \xi^{i,n} \left( \ell_t^{i,n}(a, \ell) \right) dm_{t-1}^i(a, \ell) + \xi^{E,n} \left( \ell_t^{E,n} \right) m_t^E \\ + \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin M_t^i} \xi^{i,b} \left( b_t^{i,b}(a, \ell) \right) dm_{t-1}^i(a, \ell) + \xi^{E,b} \left( b_t^{E,b} \right) m_t^E \end{aligned} \quad (31)$$

- Aggregate prices:  $r_t^l$ ,  $q_t$ ,  $q_t^d$

## EQUILIBRIUM OBJECTS TO BE COMPUTED

- Aggregate prices:  $r_t^\ell$ ,  $q_t$ ,  $q_t^d$
- Endogenous aggregate states:  $Y_{t-1}$ ,  $K_{t-1}$ ,  $K_{t-1}^0$ ,  $m_{t-1}^i(a, \ell)$ ,  $d_{t-1}^B$

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- Other endogenous aggregate variables:  $I_t$ ,  $C_t$ ,  $D_t$ ,  $N_t^n$ ,  $k_t^*$ ,  $m_t^E$

- Aggregate prices:  $r_t^\ell, q_t, q_t^d$
- Endogenous aggregate states:  $Y_{t-1}, K_{t-1}, K_{t-1}^0, m_{t-1}^i(a, \ell), d_{t-1}^B$
- Other endogenous aggregate variables:  $I_t, C_t, D_t, N_t^n, k_t^*, m_t^E$
- Banking industry decisions:  
 $\{c_t^{i,b}(a, \ell), \ell_t^{i,n}(a, \ell), b_t^{i'}(a, \ell), M_t^i, c_t^E, \ell_t^{E,n}, b_t^{E'}, q_t^{i,b}(\ell, \ell^n, b')\}$



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- Exogenous aggregate variables:  $\theta_t, A_t, \pi_t$

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- Exogenous aggregate variables:  $\theta_t, A_t, \pi_t$
- $B_t$  can be computed once we know the equilibrium path.

## SOLUTION ALGORITHM: OUTLINE

- The economy is in steady state in  $t = 1$  and  $t \geq T$
- Banks' problem do not depend on endogenous aggregate quantities, but firms' problem depend on  $d_t^B$ . [This isn't the case if a policy rule reacts to, say, aggregate output. But, we can still use what we do here to generate an initial guess.]
- Firm-entry conditions determine  $r_t^\ell$ , given  $q_t$ : This process is inexpensive, as opposed to finding  $q_t^d$  given  $q_t$  and  $r_t^\ell$  from the bank-entry condition
- Thus, our approach is to guess  $\{q_t\}_{t=1}^T$ ,  $\{q_t^d\}_{t=1}^T$ ,  $\{d_t^B\}_{t=1}^T$ ,  $\{m_t^E\}_{t=1}^T$  and  $\{N_t^n\}_{t=1}^T$ , and gradually adjust these objects to meet market-clearing conditions

## SOLUTION ALGORITHM: SOLVING BACKWARDS

Guess  $\{q_t, q_t^d, d_t^B\}_{t=1}^{T-1}$  and start with  $V_T$ ,  $\Pi_T^0$  and  $\Pi_T^1$ . For  $t = T - 1, \dots, 2$ ,

1. Given  $r_t^\ell$ ,  $q_t$ ,  $d_t^B$ ,  $\Pi_{t+1}^0$  and  $\Pi_{t+1}^1$ , compute firms' value functions, (18) and (20), where  $r_t^\ell$  is pinned down by the entry condition (22) given  $q_{t-1}$ :

$$q_{t-1} \Pi_t^0(k_{t-1}^*; r_t^\ell) = \kappa^{E,f}$$
$$k_{t-1}^* = \arg \max_k \Pi_t^0(k; r_t^\ell)$$

2. Solve the bank's problem given  $q_t^d$ ,  $r_t^\ell$ ,  $q_t$  and  $V_{t+1}$
3. Using (21), compute  $k_t^*$  given  $q_t$  and  $\Pi_{t+1}^0$
4. Using (27) and (28), compute  $C_t$  and  $D_t$  given  $q_t$  and  $q_t^d$

## SOLUTION ALGORITHM: INTEGRATING FORWARD

In each  $h$ -th iteration, do the following for  $t = 2, \dots, T - 1$ , given  $Y_1, K_1, K_1^0$ , the decision rules of HHs, banks and firms, and  $\{m_t^{E,(h)}, N_t^{n,(h)}\}_{t=1}^T$ :

1. Aggregate banks' decisions using  $m_{t-1}^i(a, \ell)$
2. Aggregate output:  $Y_t = A_t(k_t^*)^\alpha N_t^{n,(h)} + (1 - \delta_1)Y_{t-1}$
3. Using the goods MCC (31), compute  $N_t^{n,*}$ :

$$Y_t = C_t + k_t^* N_t^{n,*} + \delta_2 K_{t-1} + \text{loan issuance costs given } m_t^{E,(i)} \\ + \text{WSF issuance costs given } m_t^{E,(i)}$$

4. Given  $N_t^{n,*}$ , compute  $m_t^{E,*}$  using the loan MCC (30):

$$k_t^* N_t^{n,*} + (1 - \gamma) \{ \lambda(1 - d_{t-1}^B) + d_{t-1}^B \} K_{t-1}^0 \\ = \sum_{i=1}^{N_\xi} \int_{(a, \ell) \notin M_t^i} \ell_t^{i,n}(a, \ell) dm_{t-1}^i(a, \ell) + \ell_t^{E,n} m_t^{E,*}$$

5. Update the distribution of banks  $m_t^i(a, \ell)$ , based on banks' decisions and  $m_{t-1}^i(a, \ell)$ : we can get  $d_t^{B,*}$  in this process

- The deposit MCC (29) implies excess demand for deposits:

$$X_t^d = \sum_i \xi^{i,d} \int_{(a,\ell) \notin M_t^i} dm_{t-1}^i(a,\ell) + \xi^{E,d} m_t^{E,(h)} - D_t \quad (32)$$

- For  $\lambda^d < 0$ , the updating algorithm for  $q_t^d$  is:

$$q_t^{d,(h+1)} = (1 + \lambda^d X_t^d) q_t^{d,(h)} \quad (33)$$

- An intuition here is to make deposit more expensive when its demand exceeds supply

## SOLUTION ALGORITHM: UPDATING THE DISCOUNT PRICE OF RISK-FREE ASSETS

- From (16), the excess bank-entry condition is:

$$X_t^v = W_t^E(a^E, \ell^E) - \kappa^{E,b} \quad (34)$$

- For  $\lambda^v < 0$ , the updating algorithm for  $q_t$  is:

$$q_t^{(h+1)} = (1 + \lambda^v X_t^d) q_t^{(h)} \quad (35)$$

- An intuition here is to make an entry more costly when the net value of entry is positive

## SOLUTION ALGORITHM: UPDATING OTHER GUESSES

- Updating of  $d_t^B$ :

$$d_t^{B,(h+1)} = \gamma^q d_t^{B,*} + (1 - \gamma^q) d_t^{B,(h)}$$

- Updating of the measure of bank and firm entry :

$$\begin{aligned} m_t^{E,(h+1)} &= \gamma^m m_t^{E,*} + (1 - \gamma^m) m_t^{E,(h)} \\ k_t^* N_t^{n,(h+1)} + (1 - \gamma) \left\{ \lambda (1 - d_{t-1}^{B,(h+1)}) + d_{t-1}^{B,(h+1)} \right\} K_{t-1}^{0,(h+1)} \\ &= \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin M_t^i} \ell_t^{i,n}(a, \ell) dm_{t-1}^i(a, \ell) + \ell_t^{E,n} m_t^{E,(h+1)} \end{aligned}$$